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Investigation of the accuracy of different finite element model reduction techniques

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Abstract. In this paper, various model reduction methods were assessed using shear frame, plane, and space truss structures. These structures are one-dimensional, two-dimensional and three-dimensional, respectively. Three scenarios namely poor, better, and the best were considered for each of the structures where 25%, 40%, and 60% of the total degrees of freedom (DOFs) were measured in each of them, respectively. Natural frequencies of the full and reduced order structures were compared in each of the numerical examples to assess the performance of model reduction methods. Generally, it was found that the system equivalent reduction expansion process (SEREP) provides full accuracy in the model reduction for all of the numerical examples and scenarios. Iterated improved reduced system (IIRS) was the second-best, providing acceptable results and lower error in higher modes in comparison to the improved reduced system (IRS) method, although Guyan's method has very low levels of accuracy. The structures were classified by the excitation frequency. High-frequency structures compared to low-frequency structures have shown poor performance in the model reduction methods (Guyan, IRS, and IIRS).

Keywords: Model reduction method; Guyan; IRS; IIRS; SEREP

1. Introduction

There is a large number of DOFs in real structures and it's impossible to measure all of them due to extreme experiment costs and lack of laboratory equipment. Therefore, model reduction techniques are performed to reduce the number of DOFs. Model reduction techniques are undertaken in various engineering fields as structural health monitoring (SHM), finite element model(FEM) updating, experimental modal analysis, and experimental-FEM correlation. Guyan reduction method is one of the traditional methods of reducing the size of mass and stiffness matrices. Guyan method is considered as a static method due to negligence of the inertia effects which leads to its lack of accuracy in higher modes. IRS method tends to decrease the errors resulted by neglecting the mass effects through consideration of inertia terms in calculations. On the other hand, IIRS is the improved version of the original IRS method (Humar *et al*. 2006, Boo *et al*. 2017, Naderpour and Fakharian 2016).

SEREP is another model reduction method. Eigenvalue equations are first solved in the method and then, the resulted mode shapes are utilized (Friswell *et al*. 2001) for model reduction. Li (2017)

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utilized a five DOFs mass-spring system to investigate various model reduction techniques as SEREP, IRS, Guyan and Kuhar/Paz by comparing the natural frequencies of a full order to a reduced order system. His investigations revealed that SEREP method was fully accurate. Friswell *et al*. (2001) performed an investigation on a steel plate structure and reduced its DOFs from 80 to 6. They also confirmed the accuracy of SEREP method. Koutsovasilis and Beitelschmidt (2008) compared the performance of different model reduction methods in large mechanical systems. Their study confirmed the desirable performance of the Guyan method in low modes, IRS method and Component Mode Synthesis (CMS) in low and middle modes, and SEREP method in all modes. Jung *et al*. (2004) used an iterative dynamic condensation method for cantilever beam and 2D frame. The outcomes for both numerical examples have been effective and fast. Various articles have been published in recent years on the subject of damage detection using model reduction techniques. Zare Hosseinzadeh *et al*. (2017) utilized Neumann Series Expansion (NSE) to perform model reduction in their published article on damage detection. In another paper, Kourehli (2018) used NSE reduced models for damage detection and then compared the natural frequencies obtained from NSE and Guyan methods. He concluded that while the NSE reduces in errors, yet they would be increased in higher modes. Kourehli (2016) has also used IRS-based model reduction in an investigation on damage detection in structures such as plane truss and beam. Recently, a useful article about the development of a high-sensitivity wireless accelerometer is presented by Zhu *et al*. (2018). The new generation of accelerometers is capable of measuring structural responses in tri-axial and low-noise conditions. One of the other challenges in SHM was found to be optimal sensor placements for measuring modal information. Dinh Cong *et al*. (2018) reduced FEM by IIRS, then they solved optimal sensor placement by one of the novel optimization algorithms called Jaya. These frequencies form the reduced laminated composite beam is acceptable, but the error value in the fifth mode is 16.63%. A summary review on the application of the different model reduction methods was listed in Table 1. In recent articles, the performance of the model reduction is usually assessed on the continuous structures (like plate and beam). That's why this paper investigated discretized (like truss and frame) civil structures. Each structure was studied using different model reduction methods with different measurement scenarios. Sensor placements were selected as sparsely and randomly. The main goals of the present study are: 1) Investigations of error values for different model reduction methods. 2) Investigating the error values by increasing the number of measurements in different scenarios. 3) Evaluation of the performance of model reduction methods in high-frequency and low-frequency structures. 4) Investigating the error of higher modes by different model reduction methods. Optimal sensor placement imposes computational cost. Hence, the use of a model reduction method regardless of the location of the sensor is important. Through this study, it has been shown that SEREP has been successful (error-free) in all structures and scenarios.

2. Model reduction methods procedure

In a dynamic system, a certain number of DOFs are measured using sensors. These values are called master, while the rest of unmeasured DOFs are called slave.

In model reduction techniques, the transformation matrix is used to reduce the size of FEM by being multiplied by the mass and stiffness matrices. The general form of reduced mass and stiffness matrices are represented as follow

Table 1 A summary review on the application of different model reduction methods

Author(s)	Model Reduction method(s)	Type of Structure(s)	Descriptions
Friswell *et al.* (2001)	Static reduction, IRS, SEREP	Plate	Reduced number of DOFs 80 to 6, SEREP method was fully accurate
Koutsovasilis and Beitelschmidt (2008)	Guyan, IRS, SEREP, CMS, Dynamic	Large Mechanical Systems	Guyan and Dynamic: low modes. CMS and IRS: low and middle modes. SEREP: low, middle and high modes.
Jung *et al*. (2004)	Iterative dynamic condensation method	Cantilever beam and plane frame	Proposed Accurate and fast dynamic model reduction method
Li (2017)	Guyan, Kuhar/Paz, IRS, SEREP	Mass-Spring system	SEREP has a complete accuracy than other methods.
Kourehli (2016)	IRS	Plane truss and Beam	Damage detection with reduced models and least squares support vector machine
Zare Hosseinzadeh *et al.* (2017)	NSE	Plane frame, shear frame, plane truss	Damage detection with reduced models and Particle Swarm Optimization
Kourehli (2018)	NSE	Plane frame, shear frame, plane truss	Damage detection with reduced models and extreme learning machine
Dinh Cong *et al.* (2018)	IIRS	Laminated composite	Damage detection with optimal sensor measurements and Jaya optimization algorithm

Fig. 1 Schematic representation of the model reduction process (Avitabile, 2005)

$$[K]_{reduced} = [T]^T [K][T] \tag{1}$$

$$[M]_{reduced} = [T]^T [M][T] \tag{2}$$

Where, K and M are stiffness and mass matrices of the full model, respectively. T^T is the transpose of the transformation matrix. $[K]_{reduced}$ and $[M]_{reduced}$ are reduced stiffness and mass matrices, respectively. A schematic representation of the model reduction process is presented in Fig. 1.

3. Different model reduction methods

Guyan is a traditional model reduction method, introduced in 1965, for reducing the size of stiffness and mass matrices (Guyan 1965). The equation of motion of an undamped dynamic system with the applied external force (F) is represented in Eq. (3).

$$[M]\ddot{X} + [K]X = \{F\} \tag{3}$$

Where, X and $\ddot{X}$ are the displacement and acceleration vectors.
Eq. (3) can be written by master and slave DOFs. The inertia effects are neglected.

$$\begin{bmatrix} K_{mm} & K_{ms} \\ K_{sm} & K_{ss} \end{bmatrix} \begin{Bmatrix} \{X_m\} \\ \{X_s\} \end{Bmatrix} = \begin{Bmatrix} \{F_m\} \\ \{F_s\} \end{Bmatrix} \tag{4}$$

Subscripts *m* and *s* in the following equations are representative of master and slave DOFs, respectively.

$$\{X_s\} = -[K_{ss}]^{-1}[K_{sm}]\{X_m\} + [K_{ss}]^{-1}\{F_s\} \tag{5}$$

It is assumed in the Guyan method that no external force is applied to slave DOFs ($F_S = 0$).
With this account, the transformation matrix for the Guyan reduction method is as follows

$$T_g = \begin{bmatrix} [I] \\ -[K]_{ss}^{-1}.[K]_{sm} \end{bmatrix} \tag{6}$$

Where, $[I]$ is the identity matrix. The transformation matrix of the Guyan reduction methods was assembled with stiffness matrices, only. Guyan is the static model reduction method and does not have sufficient accuracy.

O'Callahan has developed the IRS method in 1989 by adding the inertia effects to the Guyan method (O'Callahan 1989). The transformation matrix for the IRS reduction method is as follows

$$T_i = [T_g] + [S][M][T_g][M_g]^{-1}[K_g] \tag{7}$$

where the S matrix is

$$S = \begin{bmatrix} 0 & 0 \\ 0 & K_{SS}{}^{-1} \end{bmatrix} \tag{8}$$

An objection of the IRS method is that the necessity to establich Guyan method. This can increase the cost of computing.

The SEREP method was developed by O'Callahan and Avitabile (1989). In this method, mode shapes (master and slave) are used to generate the transformation matrix. Mode shapes are obtained through solving the eigenvalue equation (Chopra 2012) as shown in Eq. (10)

$$\left([K]-\omega_j^2[M]\right)\left\{\phi_j\right\}=\{0\} \tag{9}$$

where, ω_j and ϕ_j are the j[th] natural frequency and j[th] mode shape, respectively.

The transformation matrix of the SEREP reduction method is as follows

$$T_s=\begin{bmatrix}\phi_m\\ \phi_s\end{bmatrix}\phi_m^+ \tag{10}$$

$$\phi_m^+=(\phi_m^T.\phi_m)^{-1}\phi_m^T \tag{11}$$

where the superscript "+" represent pseudo-inverse. Transformation matrix of the SEREP method is assembled with mode shapes. So, the inertia terms has not been neglected.

The IIRS method was developed by Friswell *et al*. (1997) improving the transformation matrix of the IRS method. The transformation is as follows

$$T_{i+1}=[T_g]+[S][M][T_i][M_i]^{-1}[K_i] \tag{12}$$

The IIRS transformation matrix encompassed Guyan and IRS terms. So this method has more computational costs than IRS methods. On the other hand, the accuracy of this method is greater than that of Guyan and IRS methods.

4. Numerical examples

In this section, each of the presented model reduction methods was performed for a plane truss with 25-elements, a space truss with 25-elements, and a 16-story shear frame. DOFs were measured with the three scenarios namely poor, better, and the best in which 25%, 40%, and 60% of the major. For example, if the total number of DOFs is 25, then the number of poor scenarios would be [6.25]=6. The FEM in this paper was performed by MATLAB (2018) software.

4.1 Plane truss with twenty- five elements

The plane truss shown in Fig. 2 consists of 25 elements and 14 nodes (Wikiversity 2018). Each node has two translational DOFs (X and Y direction). Boundary conditions were applied to the support locations.

The properties of the materials in use for this example are presented in Table 2. The length of horizontal and vertical members was considered to be 0.3 meters.

Also, the natural frequencies were compared to reduced and full measurements of different scenarios in Table 3. Measured nodes are put down between brackets.

Table 2 Material properties for the plan truss

cross-section (m^2)	Mass Density (kg/m^3)	Young's modulus (GPa)
A= 0.0001	ρ = 5000	E = 100

Table 3 The Natural frequencies (Rad /sec) for different scenarios and reduced measurements in the plane truss

Scenarios	Mode	Guyan	IRS	IIRS	SEREP
Poor	1	411.54 (0.139)	410.97 (0.000)	410.97 (0.000)	410.97 (0.000)
Measured Nodes:	2	1669.00 (2.186)	1633.55 (0.016)	1633.40 (0.007)	1633.29 (0.000)
	3	3498.50 (32.024)	2950.82 (11.356)	2802.58 (5.762)	2649.89 (0.000)
	4	5838.70 (71.858)	3404.37 (0.205)	3399.11(0.050)	3397.40 (0.000)
[1,3,5,9,12,14]	5	11372.01 (129.642)	5099.98 (2.987)	5014.34 (1.257)	4952.07 (0.000)
Y direction.	6	11728.29 (86.317)	6796.80 (7.975)	6519.25 (3.566)	6294.80 (0.000)
Better	1	411.10 (0.032)	410.97 (0.000)	410.97 (0.000)	410.97 (0.000)
	2	1645.53 (0.749)	1633.44 (0.009)	1633.37 (0.005)	1633.29 (0.000)
Measured Nodes:	3	3447.83 (30.112)	2847.92 (7.473)	2753.51 (3.910)	2649.89 (0.000)
	4	5115.98 (50.585)	3399.14 (0.051)	3398.08 (0.020)	3397.40 (0.000)
	5	6523.38 (31.730)	4961.35 (0.187)	4956.05 (0.080)	4952.07 (0.000)
[1,2,3,4,5,6,7, 9,10,	6	7751.49 (23.141)	6330.11 (0.561)	6308.97 (0.225)	6294.80 (0.000)
13]	7	11417.54 (60.501)	7292.35 (2.511)	7235.97 (1.719)	7113.69 (0.000)
Y direction.	8	12603.01 (68.690)	8747.30 (17.082)	8119.70 (8.682)	7471.09 (0.000)
	9	13536.83 (60.950)	12328.79 (46.587)	11294.18 (34.285)	8410.59 (0.000)
	10	14405.39 (45.163)	13509.09 (36.131)	13283.49 (33.858)	9923.59 (0.000)
Best	1	410.98 (0.002)	410.97 (0.000)	410.97 (0.000)	410.97 (0.000)
Measured Nodes:	2	1638.17 (0.299)	1633.29 (0.000)	1633.29 (0.000)	1633.29 (0.000)
	3	2995.98 (13.061)	2650.57 (0.026)	2650.01 (0.005)	2649.89 (0.000)
	4	3419.74 (0.658)	3397.50 (0.003)	3397.42 (0.001)	3397.40 (0.000)
[1,2,3,4,5,6,7, 10,	5	4993.73 (0.841)	4953.58 (0.030)	4952.47 (0.008)	4952.07 (0.000)
11,12,13]	6	6375.85 (1.288)	6307.18 (0.197)	6299.18 (0.070)	6294.80 (0.000)
Y direction,	7	7325.94 (2.984)	7208.73 (1.336)	7159.68 (0.646)	7159.69 (0.000)
	8	9851.32 (31.859)	7720.60 (3.340)	7556.22 (1.139)	7471.09 (0.000)
And	9	11590.54 (37.809)	8969.70 (6.648)	8634.82 (2.666)	8410.59 (0.000)
[9, 14]	10	13007.54 (31.077)	11513.26 (16.019)	10554.57 (6.358)	9923.59 (0.000)
Both X, Y	11	14111.99 (15.989)	12845.19 (5.577)	12720.44 (4.552)	12166.66 (0.000)
direction.	12	14477.26 (12.281)	14286.26 (10.800)	13911.55 (7.894)	12893.77 (0.000)
	13	14671.54 (11.662)	14373.82 (9.396)	14353.62 (9.242)	13139.26 (0.000)
	14	16154.78 (12.612)	15083.86 (5.147)	14634.97 (2.018)	14345.54 (0.000)
	15	18519.64 (27.567)	17427.89 (20.047)	16608.83 (14.405)	14517.59 (0.000)

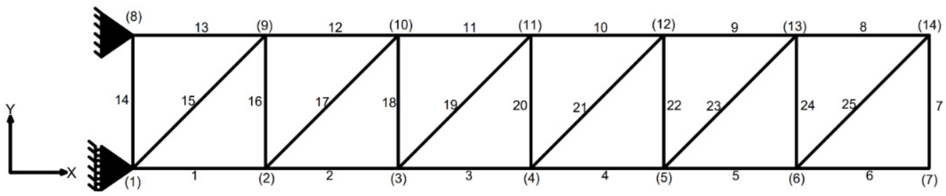

Fig. 2 The plane truss with twenty-five elements

The error was calculated by Eq. (14) and these values are written between parentheses.

$$Error = \frac{(\omega_{reduced} - \omega_{full})}{\omega_{full}} \times 100 \tag{13}$$

where, '$\omega_{reduced}$' and 'ω_{full}' are natural frequencies of reduced and full measurements. These values of error will be taken with three decimal places. These calculations have been carried out by the Microsoft excel software.

4.2 Space truss with twenty-five element

The space truss shown in Fig. 3 consists of 25 elements and 10 nodes (Bureerat and Pholdee 2018). Each node has three translational DOFs (X, Y and Z direction). Boundary conditions were applied to the support locations.

The properties of the materials used for this example are presented in Table 4.

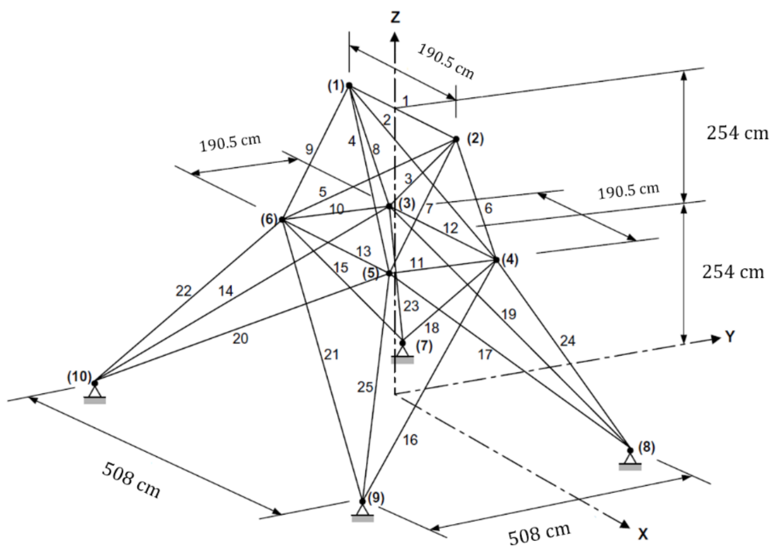

Fig. 3 The space truss with twenty-five elements (Bureerat and Pholdee 2018)

Table 4 Material properties for the space truss

cross-section (m^2)	Mass Density (kg/m^3)	Young's modulus (GPa)
A= 6.4165×10^{-6}	ρ = 7850	E = 200

Table 5 The Natural frequencies (Rad /sec) for different scenarios and reduced measurements in the space truss

Scenarios	Mode	Guyan	IRS	IIRS	SEREP
Poor	1	441.24 (0.627)	438.50 (0.002)	438.49 (0.000)	438.49 (0.000)
Measured Nodes:	2	732.35 (60.045)	610.93 (33.510)	603.06 (31.790)	457.59 (0.000)
[1, 2, 3, 6]	3	833.04 (38.285)	764.93 (26.978)	763.51 (26.743)	602.41 (0.000)
X direction.	4	2510.48 (232.554)	1184.15 (56.859)	834.25 (10.510)	754.91 (0.000)
Better	1	440.38 (0.431)	438.50 (0.002)	438.49 (0.000)	438.49 (0.000)
Measured Nodes:	2	666.20 (45.589)	481.49 (5.223)	468.40 (2.362)	457.59 (0.000)
[1, 2, 3, 4, 5]	3	757.10 (25.679)	610.86 (1.403)	604.23 (0.302)	602.41 (0.000)
X direction,	4	807.11 (6.915)	763.97 (1.200)	763.45 (1.131)	754.91 (0.000)
And	5	1980.95 (159.494)	887.76 (16.292)	819.54 (7.356)	763.39 (0.000)
[6]	6	2065.78(163.003)	1186.19 (51.019)	847.93 (7.953)	785.46 (0.000)
Both X and Z direction	7	2563.65 (205.291)	2200.26 (162.017)	2115.26 (151.895)	839.74 (0.000)
Best	1	444.80 (1.439)	438.50 (0.002)	438.49 (0.000)	438.49 (0.000)
	2	496.65 (8.536)	458.34 (0.164)	457.83 (0.052)	457.59 (0.000)
Measured Nodes:	3	684.29 (13.592)	604.60 (0.364)	603.15(0.123)	602.41 (0.000)
[1, 4, 6]	4	773.57 (2.472)	755.38 (0.062)	755.02 (0.015)	754.91 (0.000)
Both X, Z direction,	5	811.78 (6.339)	764.00 (0.080)	763.43 (0.005)	763.39 (0.000)
[2, 3]	6	883.11 (12.432)	793.26 (0.993)	787.96 (0.318)	785.46 (0.000)
Z direction,	7	1081.41 (28.779)	849.51 (1.163)	842.01 (0.270)	839.74 (0.000)
And	8	1654.72 (45.544)	1254.41 (10.334)	1190.85 (4.744)	1136.92 (0.000)
[1, 5]	9	2192.52 (24.923)	1885.54 (7.432)	1826.72 (4.081)	1755.10 (0.000)
Y direction.	10	2571.57 (37.904)	2500.75 (34.106)	2416.97 (29.613)	1864.76 (0.000)

The natural frequencies were compared to reduced and full measurements of different scenarios in Table 5.

4.3 Shear frame with sixteen stories

The shear frame shown in Fig. 4 consists of 16 stories and 16 DOFs (Roy 2017). Each story has one DOFS (X direction). The lateral stiffness and mass of each story are 250 kN/m and 200 Kg, respectively. The simplified model of the building is obtained by assuming that all of the mass is lumped at the floor levels. The natural frequencies were compared to reduced and full measurements for different scenarios in Table 6.

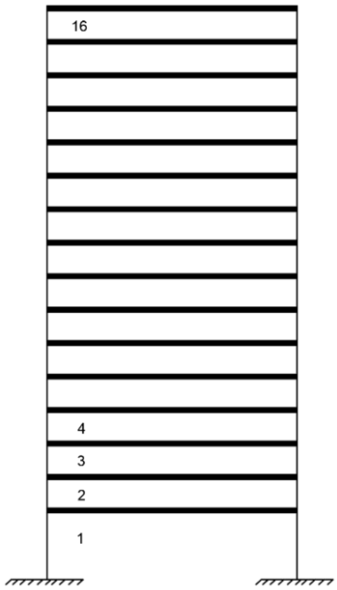

Fig. 4 The shear frame with sixteen story

Table 6 The Natural frequencies (Rad /sec) for different scenarios and reduced measurements in the shear frame

Scenarios	Mode	Guyan	IRS	IIRS	SEREP
Poor	1	3.41 (1.488)	3.36 (0.000)	3.36 (0.000)	3.36 (0.000)
Measured Nodes:	2	11.07 (10.040)	10.08 (0.199)	10.06 (0.000)	10.06 (0.000)
[1,3,9,16]	3	18.17 (8.998)	16.77 (0.600)	16.70 (0.180)	16.67 (0.000)
X direction.	4	41.96 (81.488)	31.83 (37.673)	27.54 (19.118)	23.12 (0.000)
Better	1	3.38 (0.595)	3.36 (0.000)	3.36 (0.000)	3.36 (0.000)
Measured Nodes:	2	10.43 (3.678)	10.06 (0.000)	10.06 (0.000)	10.06 (0.000)
[1,3,6,8,11,14]	3	18.32 (9.898)	16.71 (0.240)	16.67 (0.000)	16.67 (0.000)
X direction.	4	26.01 (12.500)	23.53 (1.773)	23.26 (0.606)	23.12 (0.000)
	5	36.47 (24.174)	31.53 (7.354)	30.45 (3.677)	29.37 (0.000)
	6	45.47 (28.628)	42.55 (20.368)	40.62 (14.908)	35.35 (0.000)
Best	1	3.36 (0.000)	3.36 (0.000)	3.36 (0.000)	3.36 (0.000)
	2	10.14 (0.795)	10.06 (0.000)	10.06 (0.000)	10.06 (0.000)
	3	17.04 (2.220)	16.67 (0.000)	16.67 (0.000)	16.67 (0.000)
Measured Nodes:	4	24.11 (4.282)	23.14 (0.087)	23.12 (0.000)	23.12 (0.000)
[1,3,5,7,9,11,13,15,16]	5	31.31 (6.605)	29.46 (0.306)	29.38 (0.034)	29.37 (0.000)
X direction.	6	38.44 (8.741)	35.72 (1.047)	35.42 (0.198)	35.35 (0.000)
	7	44.88 (9.437)	42.12 (2.707)	41.36 (0.853)	41.01 (0.000)
	8	49.30 (6.479)	48.36 (4.449)	47.56 (2.721)	46.30 (0.000)
	9	53.79 (5.120)	52.49 (2.580)	51.94 (1.505)	51.17 (0.000)

5. Discussion on errors

In this section the minimum, average and maximum natural frequencies errors have been listed for different scenarios and model reduction methods. These error values have been evaluated in section four. Tables 7, 8 and 9 report these values for the plane truss, space truss, and shear frame, respectively. Also, the desired tables include minimum, average and maximum errors collected from the previous section. It can be seen that the error values for space truss is more than other structures. In the next step, the shear frame errors are less than those of other structures. Results show that model reduction methods (Guyan, IRS, IIRS) had better performance in the one-dimensional structure. SEREP model reduction method has been completely accurate in all structures. In this study, the space truss with first frequency of 438.49 (Rad/sec), and the shear frame with first frequency of 3.36 (Rad/sec) were classified as high-frequency and low-frequency, respectively. High-frequency structures have had poor performance in the model reduction methods (Guyan, IRS, and IIRS) compared to low-frequency structures.

Table 7 The comparison of natural frequencies errors for the plane truss

Scenarios	Error (%)	Guyan	IRS	IIRS	SEREP
Poor	Minimum	0.139	0.000	0.000	0.000
	Average	53.694	3.757	1.774	0.000
	Maximum	129.642	11.356	5.762	0.000
Better	Minimum	0.032	0.000	0.000	0.000
	Average	37.165	11.059	8.278	0.000
	Maximum	68.690	46.587	34.285	0.000
Best	Minimum	0.002	0.000	0.000	0.000
	Average	13.333	5.238	3.267	0.000
	Maximum	37.809	20.047	14.405	0.000

Table 8 The comparison of natural frequencies errors for the space truss

Scenarios	Error (%)	Guyan	IRS	IIRS	SEREP
Poor	Minimum	0.627	0.002	0.000	0.000
	Average	82.878	29.338	17.261	0.000
	Maximum	232.554	56.860	31.790	0.000
Better	Minimum	0.431	0.002	0.000	0.000
	Average	86.629	33.879	24.428	0.000
	Maximum	205.291	162.017	151.895	0.000
Best	Minimum	1.439	0.002	0.000	0.000
	Average	18.196	5.470	3.922	0.000
	Maximum	45.544	34.106	29.613	0.000

Table 9 The comparison of natural frequencies errors for the shear frame

Scenarios	Error (%)	Guyan	IRS	IIRS	SEREP
Poor	Minimum	1.488	0.000	0.000	0.000
	Average	25.503	9.618	4.824	0.000
	Maximum	81.488	37.673	19.118	0.000
Better	Minimum	0.595	0.000	0.000	0.000
	Average	13.246	4.956	3.198	0.000
	Maximum	28.628	20.368	14.908	0.000
Best	Minimum	0.000	0.000	0.000	0.000
	Average	4.853	1.242	0.590	0.000
	Maximum	9.437	4.449	2.721	0.000

6. Conclusions

In this paper, the accuracy of different finite element model reduction techniques consisting of Guyan, improved reduced system (IRS), Iterated improved reduced system (IIRS) and system equivalent reduction expansion process (SEREP) was investigated. Results show that:

- The errors in Guyan are higher compared to the rest of the examples and scenarios.
- The errors of the IRS method are considerably lower in comparison to those of Guyan method.
- In IIRS, the value of errors is reduced compared to the IRS.
- The error values in the 'better scenario' and 'best scenario' are reduced.
- The errors increase in all of the three model reduction methods of Guyan, IRS, and IIRS in higher modes. It should be noted that the errors of the IIRS method are so much lower in higher modes in comparison with Guyan and IRS methods.
- Structures were categorized by excitation frequency, high-frequency structures compared to low-frequency structures have had poor performance in the model reduction methods (Guyan, IRS, and IIRS). But SEREP method presented accurate results for all ranges of frequencies.
- In each of the numerical examples, different scenarios were designed for measured DOFs. This shows that the SEREP method did not require optimal sensor locations for complete accuracy.

References

Avitabile, P. (2005), "Model reduction and model expansion and their applications–part 1 theory", *Proceedings of the 23rd International Modal Analysis Conference*, Orlando, Jan.

Boo, S.H. and Lee, P.S. (2017), "A dynamic condensation method using algebraic substructuring", *Int. J. Numer. Meth. Eng.*, **109**(2), 1701-1720.

Bureerat, S. and Pholdee, N. (2018), "Inverse problem based differential evolution for efficient structural health monitoring of trusses", *Appl. Soft Comput.*, **66**, 462-472.

Chopra, A.K. (2012), Dynamics of Structures: Theory and Applications to Earthquake Engineering. Prentice

Hall, Boston, Massachusetts, USA.
Dinh-Cong, D., Dang-Trung, H. and Nguyen-Thoi, T. (2018), "An efficient approach for optimal sensor placement and damage identification in laminated composite structures", *Adv. Eng. Softw.*., **119**, 48-59.
Friswell, M., Penny, J. and Garvey, S. (2001), "Model reduction for structures with damping and gyroscopic effects", *Proceedings of the international seminar on modal analysis*, Belgium.
Friswell, M., Garvey, S. and Penny, J. (1997), "Using iterated IRS model reduction techniques to calculate eigensolutions", Proceedings- *SPIE* the International *Society for Optical Engineering*, February.
Guyan, R.J. (1965), "Reduction of stiffness and mass matrices", *AIAA J.*, **3**(2), 380-380.
Humar, J., Bagchi, A., and Xu, H. (2006), "Performance of vibration-based techniques for the identification of structural damage", *Struct. Health Monit.*, **5**(3), 215-241.
Jung, Y.K., Qu, Z.Q. and Jung, D.S. (2004), "Structural dynamic condensation method with an iterative scheme", *KSCE J. Civil Eng.*, **8**(2), 205-211.
Kourehli, S.S. (2018), "Damage identification of structures using second-order approximation of Neumann series expansion", *J. Rehab. Civil Eng.*, doi: http://dx.doi.org/10.22075/jrce.2018.13348.1242 .
Kourehli, S.S. (2016), "LS-SVM regression for structural damage diagnosis using the iterated improved reduction system", *Int. J. Struct. Stab. Dyn.*, **16**(6), 1-16.
Koutsovasilis, P. and Beitelschmidt M. (2008), "Comparison of model reduction techniques for large mechanical systems", *Multibody Syst. Dyn.*, **20**(2), 111-128.
Li, D. (2017), "Discussion of model reduction and reservation", *Procedia Eng.*, **188**, 354-361.
MATLAB (2018), MATLAB Documentation.; MathWorks, Massachusetts,USA.
Naderpour, H. and Fakharian, P. (2016), "A synthesis of peak picking method and wavelet packet transform for structural modal identification", *KSCE J. Civil Eng.*, **20**(7), 2859-2867.
O'Callahan, J., Avitabile, P. and Riemer, R. (1989), "System equivalent reduction expansion process (SEREP)", *Proceedings of the 7th Inter. Modal Analysis Conf.*
O'Callahan, J.C. (1989), "A procedure for an improved reduced system (IRS) model", *Proceedings of the 7th IMAC*, Las Vegas, November.
Roy, K. (2017), "Structural damage identification using mode shape slope and curvature", *J. Eng. Mech.*, **143**(9), 1-12.
Wikiversity (2018), Team Negative Damping (3): Report 5, Wikimedia community, San Francisco, California, USA, https://en.wikiversity.org/wiki/User:Eml4507.s13.team3.steiner/Team_Negative_Damping_(3):_Report_5
Zare Hosseinzadeh, A., Ghodrati Amiri, G. and Seyed Razzaghi, S.A. (2017), "Model-based identification of damage from sparse sensor measurements using Neumann series expansion", *Inverse Probl. Sci. Eng.*, **25**(2), 239-259.
Zhu, L., Fu, Y., Chow, R., Spencer, B.F., Park, J.W. and Mechitov, K. (2018), "Development of a High-Sensitivity Wireless Accelerometer for Structural Health Monitoring", *Sensors*, **18**(1), 1-16.

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