Membrane Water Treatment, Vol. 2, No. 2 (2011) 75-89
DOI: http://dx.doi.org/10.12989/mwt.2011.2.2.075

Dialysis in double-pass cross-flow rectangular membrane modules with external recycle for improved performance

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(Received April 26, 2010, Accepted April 04, 2011)

Abstract. The predicting equations for mass transfer rate in cross-flow rectangular dialyzers with double flow and recycle, have been derived by mass balances. The recycling operation has two conflicting effects. One is the desirable effect of the increase in fluid velocity, resulting in an increased mass transfer coefficient. The other is the undesirable effect of the reduction in concentration difference due to remixing, resulting in decreased mass-transfer driving force. In contrast a single-pass device without recycling, considerable improvement in mass transfer is achieved if the cross-flow rectangular dialyzer of same size is operated with double pass and external recycling. It is concluded that recycle can enhance mass transfer, especially for larger reflux ratio.

Keywords: dialysis; external recycle; cross-flow; rectangular module; performance improved

1. Introduction

Two applications of dialysis were widely described in the first part of twentieth centry. They are the recovery of NaOH from cellulose steeping liquors and the recovery of sulfuric acid from copper leaching solution (Kessler and Klein 1992). A modern resurgence in applications of dialysis are hemodialysis for removing the metabolic waste from blood such as urea, creatine, phosphate and uric acid (Mulder, 1991), and the recovery of acids from various waste solutions by employing ion exchange membranes (Oh, *et al.* 2000, Palatý and Žakova 2007, Palatý *et al*. 2007, Palatý and Bendova 2010, Tang *et al.* 2006, Xu and Yang 2003, 2004).

Dialysis is a rate-governed membrane process in which a microsolute is driven across semipermeable membrane by means of a concentration gradient. In fact, the driving force for dialysis is the difference in chemical potential between the feed and dialysate. If the feed and dialysate are composed of the same solvent differing only slightly in concentration, the solute concentrations are proportional to their activities. Thus, concentration gradients may be used to satisfactorily describe the driving force of dialysis.

It was reported that the enhancement in separation efficiency of a dialysis process can be substantially achieved by the effect of ultrafiltration for the clearance of the solute (Yeh, *et al.* 1997, 2000), and that the application of reflux indeed has much influence on the performance of dialysis (Yeh 2009, Yeh *et al.* 2010). In these previous works, the effect of external recycle on the performance of dialysis in parallel-flow flat-plate dialyzers was investigated. In contrast to a device without recycle,

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considerable improvement in performance is achievable if dialysis is operated with external recycle, resulting in increase of fluid velocity, as well as reduction of mass-transfer resistance. It is the purpose of present study to investigate the effect of external reflux on dialysis in double-pass rectangular modules operated under cross flow, instead of parallel flow, for improved performance.

2. Theory

The theoretical treatment of mass transfer in parallel-flow dialysis is relatively simple, whereas the computation of mass transfer in cross-flow system is rather complicated because the flow directions of the two fluids cross each other. The assumptions made in this analysis are steady state, no chemical reaction, uniform concentrations, $C_a(x)$ and $C_b(y)$, across the cross sections of flow channels a and b, and uniform velocities over the cross sections of flow, constant rates of flows, constant mass-transfer coefficients and constant distribution coefficients.

2.1 Governing equations

The schematic diagram in Fig. 1 may serve to explain the nomenclature to be employed for dialysis in cross-flow rectangular modules without recycle. This system generally consists of two perpendicular channels for the fluids a and b, respectively, which are separated by a microporous membrane sheet through which solute is extracted and transferred perpendicularly to its exposed surfaces.

By taking the mass balances for membrane dialysis through a differential area dxdy in a rectangular cross-flow dialyzer of length L and width B, two ordinary differential equations for solute concentrations are obtained

$$-(Q_a/B)dC_a dy = K(C_a - C_b)dxdy \tag{1}$$

$$= (Q_b/L)dC_b dx, \tag{2}$$

where K denotes the overall mass-transfer coefficient and Q_b are the volumetric flow rate in channels

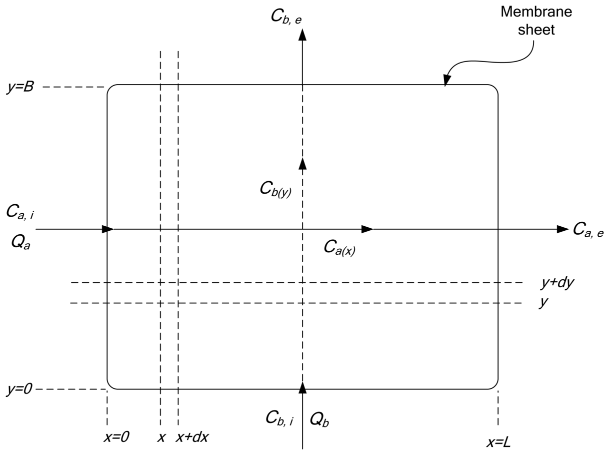

Fig. 1 Flow sheet of cross-flow flat-plate membrane dialysis

a and b, respectively. By solving Eqs. (1) and (2) simultaneously, the expression for the rate of mass transfer, W_0, is easy to be obtained by following the same procedure performed in the pervious work (Yeh and Chen 2000) with the initial conditions:$C_a=C_{a,i}$ at $x=0$ and $C_b=C_{b,i}$ at $y=0$. The result is

$$M_0=\frac{Q_a(C_{a,i}-C_{b,i})}{[1/(1-e^{-n})]+[l/(1-e^{-nl})]-(1/n)} \tag{3}$$

where

$$l=\frac{Q_a}{Q_b} \tag{4}$$

$$n=\frac{KS}{Q_a}=\frac{K(BL)}{Q_a} \tag{5}$$

The rate of mass transfer predicted by Eq. (3) confirmed reasonably well with the experimental results (Yeh and Chen 2000), as well as with the exact solution (Yeh and Hsu, 1999), which was derived with the consideration of $C_a(x, y)$ and $C_b(x, y)$.

2.2 Device with recycle

Fig. 2 shows the dialysis system similar to that shown in Fig. 1, except that an impermeable plate with negligible thickness is placed vertical to the upper plate and the membrane sheet at the centerline of channel a (phase a), to divide the channel into two subchannels (subchannels a_1 and a_2) of the same size, and that a pump is installed for recycle. Before entering the subchannels a_1 for double-pass system with a recycle, the feed solution (phase a) at the inlet with volumetric flow rate Q_a and concentration $C_{a,i}$ will mix the fluid exiting from subchannel a_2 with volumetric flow rate RQ_a and outlet concentration $C_{a,e}$, which is controlled by means of a conventional pump situated at the beginning of subchannel a_1. The dialysis phase (phase b) with inlet volumetric flow rate Q_b flows steadily through channel b. The total mass-transfer area of membrane surface, $S=B\times L=2(L\times B/2)$, are

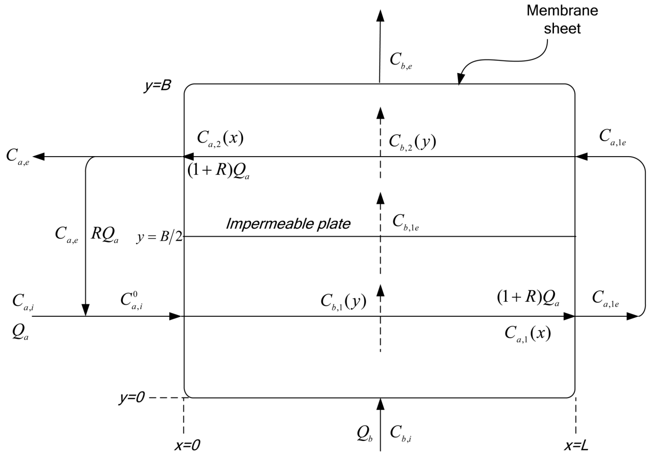

Fig. 2 Flow sheet of cross-flow rectangular membrane dialyzer, with double-pass and external recycle in retentate phase

the same for both devices with and without recycle, as shown in Figs. 1 and 2.

2.2.1 Governing equations

Fig. 2 illustrates the flows in a cross-flow rectangular dialyzer with recycle, while Fig. 3 presents its schematic diagram. The expressions for mass-transfer rate are

$$M=KS(\Delta C)_m \tag{6}$$

$$=Q_a(C_{a,i}-C_{a,e})=(1+R)Q_a(C^0_{a,i}-C_{a,e}) \tag{7}$$

$$=Q_b(C_{b,e}-C_{b,i}) \tag{8}$$

where $(\Delta C)_m$ denotes the mean value of concentration difference. Since the impermeable plate divides the retentate phase into two flow regions, mass balances will also be taken for each flow region. By taking the mass balances through a differential area *dxdy* in flow region 1, one obtains

$$-[(1+R)Q_a/(B/2)]dydC_{a,1}=Kdxdy(C_{a,1}-C_{b,1}) \tag{9}$$

$$=[(Q_b/L)dx]dC_{b,1} \tag{10}$$

Similarly, for flow region 2

$$[(1+R)Q_a/(B/2)]dydC_{a,2}=Kdxdy(C_{a,2}-C_{b,2}) \tag{11}$$

$$=[(Q_b/L)dx]dC_{b,2} \tag{12}$$

The boundary conditions for solving $C_{a,1}$, $C_{a,2}$, $C_{b,1}$ and $C_{b,2}$ from Eqs. (9) to (12) are

$$\text{at } x=0,\ C_{a,1}=C^0_{a,i} \tag{13}$$

$$C_{a,2}=C_{a,e} \tag{14}$$

$$\text{at } x=\text{L} \tag{15}$$

$$C_{a,1}=C_{a,2}=C_{a,1e} \tag{16}$$

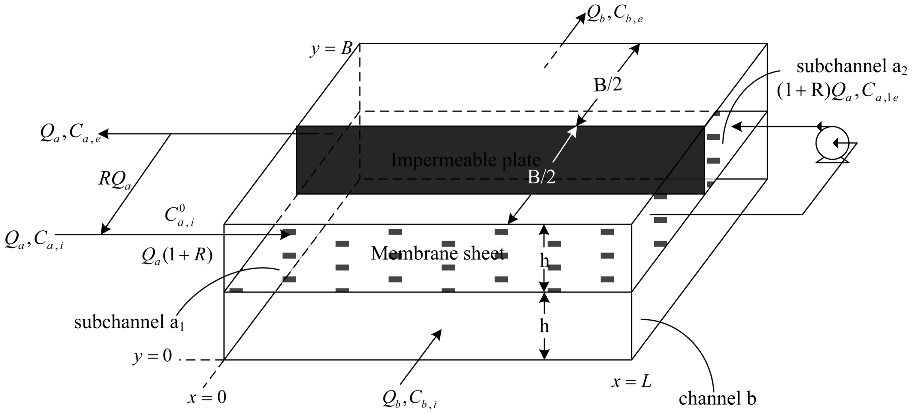

Fig. 3 Schematic diagram of cross-flow rectangular membrane dialyzer with double-pass and external recycle in the retentate phase

$$\text{at } y=0,\ C_{b,1}=C_{b,i} \tag{17}$$

$$\text{at } y=B/2,\ C_{b,1}=C_{b,1e} \tag{18}$$

$$\text{at } y=B,\ C_{b,2}=C_{b,e} \tag{19}$$

Inspection of Eqs. (13)-(19) shows that the outlet concentrations($C_{a,1e}$, $C_{a,e}$, $C_{b,1e}$ and $C_{b,e}$), as well as the mixed inlet concentration, $C^0_{a,i}$, are not specified a priori. Mathematically, more relations for mass-transfer rates, M_1 and M_2, in regions 1 and 2, as well as for the mixing effect at the inlet, are needed for determination of these values. They are

$$M_1=(1+R)Q_a(C^0_{a,i}-C_{a,1e})=Q_b(C_{b,1e}-C_{b,i})=KL(B/2)(\Delta C)_{1,m} \tag{20}$$

$$M_2=(1+R)Q_a(C_{a,1e}-C_{a,e})=Q_b(C_{b,e}-C_{b,1e})=KL(B/2)(\Delta C)_{2,m} \tag{21}$$

$$C_{a,i}+RC_{a,e}=(1+R)C^0_{a,i} \tag{22}$$

where $(\Delta C)_{1,m}$ and $(\Delta C)_{2,m}$ denote the mean concentration differences between phases a and b in flow regions 1 and 2, respectively.

2.2.2 Mean concentrations

Define the following mean concentrations

$$C_{a,1m}=\frac{1}{L}\int_0^L C_{a,1}dx \tag{23}$$

$$C_{a,2m}=\frac{1}{L}\int_0^L C_{a,2}dx \tag{24}$$

$$C_{b,1m}=\frac{2}{B}\int_0^{B/2} C_{b,1}dy \tag{25}$$

$$C_{b,2m}=\frac{2}{B}\int_{B/2}^{B} C_{b,2}dy \tag{26}$$

Integrating Eqs. (9) and (11), respectively, from $y=0$ to $B/2$ and from $y=B/2$ to B, we have

$$-(1+R)Q_a dC_{a,1}=K(B/2)(C_{a,1}-C_{b,1m})dx \tag{27}$$

$$(1+R)Q_a dC_{a,2}=K(B/2)(C_{a,2}-C_{b,2m})dx \tag{28}$$

Integration of Eqs. (10) and (12) from $x=0$ to L yields

$$Q_b dC_{b,1}=KL(C_{a,1m}-C_{b,1})dy \tag{29}$$

$$Q_b dC_{b,2}=KL(C_{a,2m}-C_{b,2})dy \tag{30}$$

Integration of Eqs. (27) from $x=0$ to L with the use of Eqs. (5), (13) and (15) and rearrangement yields the mean concentrations of fluid b in region 1 as

$$C_{b,1m}=\frac{C_{a,1e}-C^0_{a,i}}{1-e^{-n/2(1+R)}}+C^0_{a,i} \tag{31}$$

Similarly, integration of Eq. (28) from $x=0$ to L with the use of Eqs. (5), (14) and (16) yields

$$C_{b,2m} = \frac{C_{a,1e} - C_{a,e}}{1 - e^{n/2(1+R)}} + C_{a,e} \tag{32}$$

Similarly again, integrating Eqs. (29) and (30), respectively, from $y=0$ to $B/2$ and from $y=B/2$ to B, and rearranging, one obtains the mean concentrations of fluid a in regions 1 and 2 as

$$C_{a,1m} = \frac{C_{b,1e} - C_{b,i}}{1 - e^{-nl/2}} + C_{b,i} \tag{33}$$

$$C_{a,2m} = \frac{C_{b,e} - C_{b,1e}}{1 - e^{-nl/2}} + C_{b,1e} \tag{34}$$

2.2.3 Mean concentration differences

Integrating Eq. (27) from $x=0$ to x, we have

$$C_{a,1} - C_{b,1m} = (C^{0}_{a,i} - C_{b,1m})e^{-nx/2L(1+R)} \tag{35}$$

Integrating the above equation again from $x=0$ to L, one obtains

$$C_{a,1m} - C_{b,1m} = \left[\frac{2(1+R)}{n}\right](C^{0}_{a,i} - C_{b,1m})[1 - e^{-n/2(1+R)}] \tag{36}$$

Substituting Eq. (31) into Eq. (36) to eliminate the term $C_{b,1m}$, on the right-hand side of Eq. (36), one has

$$C_{a,1m} - C_{b,1m} = \left[\frac{2(1+R)}{n}\right](C^{0}_{a,i} - C_{a,1e}) \tag{37}$$

Similarly, integration of Eq. (28) from $x=0$ to x yields

$$C_{a,2} - C_{b,2m} = (C_{a,e} - C_{b,2m})e^{nx/2L(1+R)} \tag{38}$$

Integrating again from $x=0$ to L, one obtains

$$C_{a,2m} - C_{b,2m} = \left[\frac{2(1+R)}{n}\right](C_{a,e} - C_{b,2m})[e^{n/2(1+R)} - 1] \tag{39}$$

Substitution of Eq. (32) into Eq. (39) to eliminate the term, $C_{b,2m}$, on the right-hand side of Eq. (39) results in

$$C_{a,2m} - C_{b,2m} = \left[\frac{2(1+R)}{n}\right](C_{a,1e} - C_{a,e}) \tag{40}$$

Consequently, suitable definitions of mean concentration differences, $(\Delta C)_m$, $(\Delta C)_{1,m}$, and $(\Delta C)_{2,m}$, can be delineated as follows. From Eqs. (15), (20) and (37)

$$(\Delta C)_{1,m} = C_{a,1m} - C_{b,1m} \tag{41}$$

Similarly, from Eqs. (5), (6), (9), (21), (37) and (40)

$$(\Delta C)_{2,m} = C_{a,2m} - C_{b,2m} \tag{42}$$

$$(\Delta C)_m = \frac{C_{a,1m}+C_{a,2m}-C_{b,1m}-C_{b,2m}}{2} = \frac{(\Delta C)_{1,m}+(\Delta C)_{2,m}}{2} \tag{43}$$

2.2.4 Outlet concentrations

For mathematical convenience, define the following dimensionless groups

$$-\xi^0_{a,e} = \frac{C^0_{a,i}-C_{a,e}}{C_{a,i}-C_{b,i}} \tag{44}$$

$$-\xi^0_{a,1e} = \frac{C^0_{a,i}-C_{a,1e}}{C_{a,i}-C_{b,i}} \tag{45}$$

Accordingly, from Eqs. (8) and (20), respectively

$$\frac{C_{b,e}-C_{b,i}}{C_{a,i}-C_{b,i}} = -l(1+R)\xi^0_{a,e} \tag{46}$$

$$\frac{C_{b,1e}-C_{b,i}}{C_{a,i}-C_{b,i}} = -l(1+R)\xi^0_{a,1e} \tag{47}$$

and also from Eq. (22)

$$\frac{C_{a,i}-C^0_{a,i}}{C_{a,i}-C_{b,i}} = -R\xi^0_{a,e} \tag{48}$$

$$\frac{C_{b,i}-C^0_{a,i}}{C_{a,i}-C_{b,i}} = -1-R\xi^0_{a,e} \tag{49}$$

Substituting Eqs. (31) and (33) into Eq. (37) with the use of Eqs. (44)-(49), we have the superficial dimensionless outlet concentration of subchannel a_1

$$-\xi^0_{a,1e} = \left[\frac{l(1+R)}{1-e^{-nl/2}} + \frac{1}{1-e^{-n/2(1+R)}} - \frac{2(1+R)}{n}\right]^{-1} \tag{50}$$

Substituting Eqs. (32) and (34) into Eq. (40) by using the following relations derived from Eqs. (44)-(49):

$$\frac{C_{b,e}-C_{b,1e}}{C_{a,i}-C_{b,i}} = l(1+R)(-\xi^0_{a,e}+\xi^0_{a,1e}) \tag{51}$$

$$\frac{C_{a,1e}-C_{a,e}}{C_{a,i}-C_{b,i}} = -\xi^0_{a,e}+\xi^0_{a,1e} \tag{52}$$

$$\frac{C_{b,1e}-C_{a,e}}{C_{a,i}-C_{b,i}} = -(1+R)\xi^0_{a,e}-l(1+R)\xi^0_{a,1e}-1 \tag{53}$$

one obtains the superficial dimensionless outlet concentration of subchannel a_2 (*i.e.*, the superficial dimensionless outlet concentration of retentate phase) as

$$-\xi_{a,e}^{0}=\frac{1-\left[\frac{2(1+R)}{n}+l(1+R)+\frac{1}{1-e^{n/2(1+R)}}-\frac{l(1+R)}{1-e^{-nl/2}}\right](-\xi_{a,1e}^{0})}{(1+R)-\frac{2(1+R)}{n}-\frac{1}{1-e^{n/2(1+R)}}+\frac{l(1+R)}{1-e^{-nl/2}}} \tag{54}$$

There is still an unknown value of $C_{a,i}^{0}$ (the mixed inlet concentration) in $\xi_{a,e}^{0}$, as shown in Eq. (44). Practically, we define the actual dimensionless outlet concentration of retentate phase as

$$-\xi_{a,e}=\frac{C_{a,i}-C_{a,e}}{C_{a,i}-C_{b,i}} \tag{55}$$

With the use of Eqs. (44) and (48), Eq. (55) becomes

$$-\xi_{a,e}=-(1+R)\xi_{a,e}^{0} \tag{56}$$

Eq. (56) is also readily obtained from Eq. (22) by using Eqs. (44) and (55). Accordingly, when l, n and R are specified, $-\xi_{a,1e}^{0}$, $-\xi_{a,e}^{0}$ and $-\xi_{a,e}$ are readily calculated from Eqs. (50), (54) and (56), respectively. Finally, the outlet concentration, $C_{a,e}$, of retentate phase is obtained by Eq. (55).

2.2.5 Mass-transfer rate

Once the outlet concentration of retentate phase is obtained, the mass-transfer rate is calculated from Eq. (7) which may rewritten by using Eqs. (55) and (56) as

$$M=-\xi_{a,e}\,Q_a(C_{a,i}-C_{b,i})=-(1+R)\,\xi_{a,e}^{0}\,Q_a(C_{a,i}-C_{b,i}) \tag{57}$$

2.3 Mass transfer coefficients

The overall mass-transfer coefficient K may be defined as (Yeh and Hsu, 1999)

$$\frac{1}{K}=\frac{H_{am}}{k_a}+\frac{1}{k_b}+\frac{H_{bm}}{k_m} \tag{58}$$

Where H_{am} and H_{bm} are the partition coefficients between two phases at the interface, while k_a, k_b and k_m denote the mass-transfer coefficients in subchannel a, in subchennel b and within membrane, respectively, *i.e.* (Porter, 1990)

$$k_a=0.816\left[\frac{6Q_a(1+R)D_a^2}{L(B/N)h^2}\right]^{\frac{1}{3}} \tag{59}$$

$$k_b=0.816\left[\frac{6Q_bD_b^2}{LBh^2}\right]^{\frac{1}{3}} \tag{60}$$

$$k_m=\frac{D_a\varepsilon}{\tau t} \tag{61}$$

In the above equation, D_a and D_b are the diffusivities of solute in phase a (retentate), and phase b (dialysate), respectively; ε, τ, and t are, respectively, the porosity of the membrane, the pore tortuosity of membrane and the thickness of membrane sheet; while R, N, and h denote, respectively, the reflux ratio, the pass number, and height of flow channels.

3. Numerical calculation

3.1 Numerical example

For the purpose of illustration, let us employed some numerical values for dialyzing urea from aqueous solution with the use of membrane sheet (L=B=0.6m) made of microporous polypropylene (Gelman Science, ε=70%, t=1.78×10^{-4}m, τ=2.6) as a permeable barrier. Further, h=0.02m, D_a=D_b=1.378×10^{-9} m^2/s(Geankoplis 1983) and $C_{b,i}$=0; R=0 and N=1 in Eq. (59) for single-pass device, while N=2 for double-pass one with reflux ratio R. With the use of above numerical values, the dialysis rates (M_0 and M) were calculated from Eqs. (3) and (57) for single-pass operation and double-pass operation with recycle, respectively. The results are presented in Tables 1 and 2 for $C_{a,i}$=1 *kg mol/cm*^{3} and 5 kg *mol/m*^{3}, respectively.

3.2. Effects of operating parameters on mass-transfer rate

As expected, the mass-transfer rates for the operation with and without recycle (M and M_0),

Table 1 Dialysis rates in cross flow device with external reflux for $C_{a,i}$=1 *kg mol/m*^{3} and $C_{b,i}$=0

			R=1		R=3		R=5	
Q_a×10^6 (m^3/s)	Q_b×10^6 (m^3/s)	M_0×10^8 (*kg mol/s*)	M×10^8 (*kg mol/s*)	I (%)	M×10^8 (*kg mol/s*)	I (%)	M×10^8 (*kg mol/s*)	I (%)
0.1	0.1	2.1754	2.4957	14.73	2.6431	21.50	2.7206	25.06
0.25	0.1	2.6100	2.9514	13.08	3.0992	18.74	3.1763	21.70
0.5	0.1	2.8809	3.2097	11.41	3.3470	16.18	3.4179	18.64
1.0	0.1	3.1090	3.4104	9.69	3.5324	13.62	3.5949	15.63
0.1	0.25	2.6100	3.1105	19.18	3.3477	28.26	3.4759	33.18
0.25	0.25	3.2671	3.8402	17.54	4.0982	25.44	4.2363	29.67
0.5	0.25	3.7064	4.2846	15.60	4.5363	22.39	4.6695	25.98
1.0	0.25	4.0960	4.6479	13.47	4.8807	19.16	5.0025	22.13
0.1	0.5	2.8809	3.5151	22.01	3.8238	32.73	3.9935	38.62
0.25	0.5	3.7064	4.4712	20.63	4.8266	30.22	5.0203	35.45
0.5	0.5	4.2836	5.0823	18.65	5.4417	27.04	5.6353	31.56
1.0	0.5	4.8136	5.5999	16.33	5.9425	23.45	6.1246	27.24
0.1	1.0	3.1090	3.8684	24.43	4.2481	36.64	4.4600	43.45
0.25	1.0	4.0960	5.0571	23.46	5.5179	34.71	5.7733	40.95
0.5	1.0	4.8136	5.8511	21.55	6.3336	31.58	6.5980	37.07
1.0	1.0	5.4941	6.5463	19.15	7.0201	27.78	7.2761	32.43

Table 2 Dialysis rates in cross flow device with external reflux for and $C_{a,i}=5$ *kg mol/m*^{3} and $C_{b,i}=0$

			R=1		R=3		R=5	
$Q_a \times 10^6$ (m^3/s)	$Q_b \times 10^6$ (m^3/s)	$M_0 \times 10^8$ (*kg mol/s*)	$M \times 10^8$ (*kg mol/s*)	I (%)	$M \times 10^8$ (*kg mol/s*)	I (%)	$M \times 10^8$ (*kg mol/s*)	I (%)
0.1	0.1	10.8770	12.4787	14.73	13.2154	21.50	13.6030	25.06
0.25	0.1	13.0498	14.7571	13.08	15.4960	18.75	15.8815	21.70
0.5	0.1	14.4045	16.0485	11.41	16.7349	16.18	17.0897	18.64
1.0	0.1	15.5451	17.0521	9.69	17.6622	13.62	17.9745	15.63
0.1	0.25	13.0498	15.5526	19.18	16.7387	28.27	17.3793	33.18
0.25	0.25	16.3357	19.2012	17.54	20.4909	25.44	21.1813	29.66
0.5	0.25	18.5321	21.4231	15.60	21.4231	15.60	22.6813	22.39
1.0	0.25	20.4798	23.2397	13.48	24.4036	19.16	25.0125	22.13
0.1	0.5	14.4045	17.5754	22.01	19.1192	32.73	19.9675	38.62
0.25	0.5	18.5321	22.3561	20.63	24.1329	30.22	25.1014	35.49
0.5	0.5	21.4178	25.4117	18.65	27.2085	27.04	28.1767	31.56
1.0	0.5	24.0681	27.9995	16.33	29.7124	23.45	30.6231	27.24
0.1	1.0	15.5451	19.3420	24.43	21.2407	36.64	22.3001	43.45
0.25	1.0	20.4798	25.2856	23.47	27.5895	34.72	28.8666	40.95
0.5	1.0	24.0681	29.2557	21.55	31.6678	31.58	32.9898	37.07
1.0	1.0	27.4704	32.7316	19.15	35.1003	27.78	36.3804	32.43

increase with inlet concentration difference ($C_{a,i}-C_{b,i}$), as well as merely with $C_{a,i}$ for the numerical example of present interest where $C_{b,i}=0$. They also increase with the volumetric flow rates in both phases, Q_a and Q_b. For the operation with recycling, increasing the reflux ratio R will further increase the mass-transfer rate M.

3.3 Improvement in performance

The improvement in performance by operating a double-pass dialyzer with recycle is beat illustrated by calculating the percentage increase in mass-transfer rate based on that M_0 obtained in a single-pass device of same size without recycling

$$I=\frac{M-M_0}{M_0} \tag{62}$$

3.3.1 Effects of reflux ratio and concentration difference upon improvement

The results of improvement in performance I are also presented in Tables 1 and 2, and some of them are expressed graphically in Fig. 4. From these results we see that operation with a higher reflux ratio substantially enhances the mass-transfer rate. The enhancement I increases with increasing reflux ratio R. However, I is independent of the inlet concentration difference, $C_{a,i}-C_{b,i}$, as well as $C_{a,i}$ for the numerical example of present interest, where $C_{b,i}=0$. This is easy to check from Eqs. (3) and

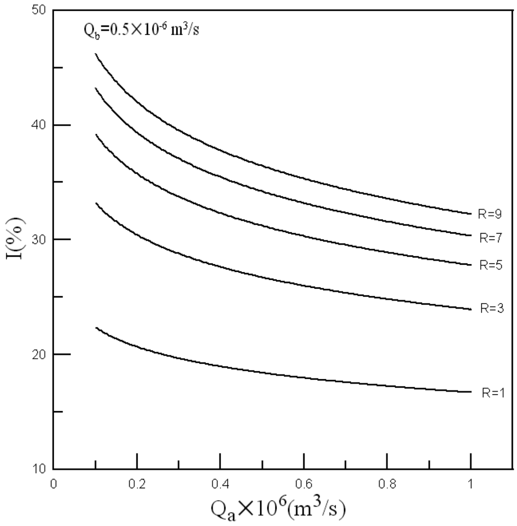

Fig. 4 Improvement of performance vs. Q_a with Q_b=0.5×10^{-6} m^3/s and R as parameter

(57) for M_0 and M, respectively, in which the term, $C_{a,i}$–$C_{b,i}$, will be cancelled in Eq. (62).

3.3.2 Effects of volumetric flow rates of phases upon improvement

It is seen in the tables and Fig. 4 that I increases with the volumetric flow rate of dialysate phase, Q_b, but decreases with that of feed (retentate) stream, Q_a, where the recycle is applied. Actually, applying the recycling operation to heat or mass exchanger produces two conflicting effects. One is the desirable effect of increasing fluid velocity, resulting in increase in the hest-or mass-transfer coefficient. The other is undesirable effect of decreasing temperature or concentration difference due to remixing, resulting in decrease of driving force. As shown in the tables and Fig. 4, the introduction of reflux really has a substantial effect on dialysis (mass-transfer rate M). However, the improvement in performance I decreases with the volumetric flow rate Q_a of the recycle phase (phase a), as indicated in Fig. 4. This is because that at higher volumetric flow rate, the fluid velocity is already large enough, and the desirable effect of further increasing fluid velocity by applying recycle though still compensates for the undesirable effect of decreasing mass-transfer driving force, the strength of desirable effect increases slower that the strength of undesirable effect does, during the increase of Q_a. Accordingly, application of recycling is rather beneficial to the operation with lower inlet volumetric flow rate of recycling phase.

3.3 Hydraulic-dissipated energy

As shown in previous section, considerable improvement of mass transfer in a dialyzer really can be obtained by double-pass operation coupled with recycle, however, the hydraulic dissipated energy due to the friction loss of fluid flow should be also discussed. The Hydraulic-dissipated loss in a rectangular channel may be estimated by

$$H = (\text{volumetric flow rate})\Delta P \tag{63}$$

If laminar flow in the flow channel is assumed, the pressure drop ΔP through the flow channel a

Table 3 Hydraulic-dissipated powers in cross flow device with external reflux

			R=1		R=3		R=5	
$Q_a\times10^6$ (m^3/s)	$Q_b\times10^6$ (m^3/s)	$H_0\times10^{13}$ (W)	$H\times10^{10}$ (W)	E	$H\times10^{10}$ (W)	E	$H\times10^{10}$ (W)	E
0.1	0.1	0.30	1.65	4.50	7.65	24.50	18.70	60.50
0.5	0.1	3.90	37.65	8.65	187.65	47.12	457.65	116.35
1.0	0.1	15.15	150.15	8.91	750.15	48.51	1830.15	119.80
0.1	0.5	3.90	5.25	0.35	11.25	1.88	22.05	4.65
0.5	0.5	7.50	41.25	4.50	191.25	24.50	461.25	60.50
1.0	0.5	18.75	153.75	7.20	753.75	39.20	1833.75	96.80
0.1	1.0	15.15	16.50	0.09	22.50	0.49	33.30	1.20
0.5	1.0	18.75	52.50	1.80	202.50	9.80	472.50	24.20
1.0	1.0	30.00	165.00	4.50	776.59	24.50	1845.00	60.50

of length L is

$$\Delta P=\frac{12\mu L\times(\text{volumetric flow rate})}{h^2\times(\text{cross}-\text{section area of channel})} \tag{64}$$

For the flow channel b of length B, L in above equation should be replaced by B. Accordingly, one has the hydraulic-dissipated losses for single-pass device and double-pass device with recycle, respectively, as

$$H_0=\frac{12\mu_a L Q_a^2}{h^3 B}+\frac{12\mu_b B Q_b^2}{h^3 L}=\frac{1.2\times10^{-2}}{h^3}(Q_a^2+Q_b^2) \tag{65}$$

$$H=2\times\frac{12\mu_a L[Q_a(1+R)]^2}{h^3(B/2)}+\frac{12\mu_b B Q_b^2}{h^3 L}=\frac{1.2\times10^{-2}}{h^3}\{[2Q_a(1+R)]^2+Q_b^2\} \tag{66}$$

where we let $\mu_a=\mu_b=\mu=1\times10^{-6}$ kPas, to neglect the effect of concentration on fluid viscosity, as well as L=B=0.6m for present interest. The enhancement of hydraulic-dissipated energy in the double-pass device with recycle, based on the single-pass one without recycle, may be calculated by

$$E=\frac{H-H_0}{H_0}=\frac{4(1+R)^2-1}{1+(Q_b/Q_a)^2} \tag{67}$$

Some of the results are given in Table 3. It is seen in Eq. (67), as well as in Table 3 that E increases when R or (Q_a/Q_b) increases. Nevertheless, the hydraulic-dissipated losses, H_0 and H, are too small to be concerned about.

3.4 Examination for application limit

The Reynolds number in the flow channels a1 and a2 for double-pass operation with recycle may be defined by

$$\mathrm{Re}=\frac{2h\left[\dfrac{Q_a(1+R)}{(B/2)h}\right]\rho_a}{\mu_a}=\frac{4Q_a(1+R)B\rho_a}{\mu_a}$$

For the typical case that $Q_a=1\times10^{-6} m^3/s$, $B=0.6\ m$, $R=9$, and $\rho_a=1\times10^3\ kg/m^3$ and $\mu_a=1\times10^{-6}$ kPas, one has Re=24. Therefore, the assumption of laminar flow is confirmed.

4. Conclusions

The effect of external recycle on mass transfer in a double-pass cross-flow rectangular dialyzer has been investigated. The theory of mass transfer was developed from mass balances with the assumption that the solute concentrations are uniform over the cross-sections of flow channels. The predicting equation for mass-transfer rate was derived, and the improvement in mass transfer by recycle operation was estimated.

It is found that by applying the recycle operation in a dialyzer, the desirable effect of increasing fluid velocity, as well as the increase in mass-transfer coefficient, compensates for the undesirable effect of decreasing mass-transfer driving force (concentration difference) due to remixing, resulting in increasing mass-transfer rate. Accordingly, considerable improvement in performance is achievable if the recycle effect is applied, and the improvement in performance increases with reflux ratio and the volumetric flow rate of dialysate phase, but decreases as the volumetric flow rate of feed stream (retentate) phase increases. This may be because that when the volumetric flow rate of retentate phase increases, the strength of undesirable effect increases more fast than the strength of undesirable effect does. Therefore, recycling operation is rather beneficial to the operation with lower inlet volumetric flow rate of recycling phase. It is also found that the mass transfer rates increase with the solute concentration difference, while the improvement in performance does not depend on it.

It is mentioned that the increases in fixed charge (such as installing a pump for recycling) and operating cost (due to the transportation of recycling streams) should be also taken into considerations with the economic sense when a recycled device will be employed.

In present study of interest, though the enlargement of hydraulic-dissipated loss increases rapidly with reflux ratio based on that of the device without recycle, it is extremely small and thus, the operating costs in all devices may be ignored.

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CC

Nomenclature

B	membrane width (cm)
C_a, C_b	solute concentration distribution in phase a, phase b (kg mol/m^3)
$C_{a,1}$, $C_{a,2}$	solute concentration distribution in channels a_1 and a_2, of retentate phase (kg mol/m^3)
$C_{a,1m}$, $C_{a,2m}$	average concentration in subchannels a_1 and a_2, defined by Eqs. (23) and (24) (mol/m^3)
$C_{b,1}$, $C_{b,2}$	solute concentration distribution within $0 \le y \le \frac{B}{2}$, within $\frac{B}{2} \le y \le B$, of diaysate phase (kg mol/m^3)
$C_{b,1m}$, $C_{b,2m}$	average concentration in phase b of regions 1 and 2, defined by Eqs. (25) and (26) (kg mol/m^3)
$C_{a,i}$, $C_{b,i}$	inlet solute concentration in phase a, in phase b (kg mol/m^3)
$C_{a,e}$, $C_{b,e}$	outlet solute concentration in phase a, in phase b (kg mol/m^3)
$C_{a,1e}$, $C_{b,1e}$	solute concentration of phase a at $x=L$ and phase b at $y=\frac{B}{2}$ (kg mol/m^3)
$C^0_{a,i}$	mixed inlet concentration in retentate phase (kg mol/m^3)
$(\Delta C)_m$	total average value of concentration difference defined by Eqs. (6) and (43) (kg mol/m^3)
$(\Delta C)_{1,m}$	average concentration difference between channel a_1 and the contact part of phase b defined by eq. (41) (kg mol/m^3)
$(\Delta C)_{2,m}$	average concentration difference between channel a_2 and the contact part of phase b defined by eq. (42) (kg mol/m^3)
D_a, D_b	solute diffusivity in phase a, in phase b (m^2/s)
E	enhancement of hydraulic-dissipated loss
h	height of flow channel (m)
H_0, H	hydraulic-dissipated loss in a single-pass device, in a double-pass device with recycle (W)
I	improvement of mass-transfer rate defined by Eq. (62)

K	overall mass-transfer coefficient (m/s)
L	length of membrane sheet, effective length of membrane extractor (m)
k_a, k_b, k_m	mass-transfer coefficient in phase a, in phase b, and within membrane, respectively (m/s)
M	total mass-transfer rate in a dialyzer (kg mol/s)
M_0	M obtained in a device of same size without recycle (kg mol/s)
M_1, M_2	mass transfer rate in region 1, in region 2 (kg mol/s)
l, n	dimensionless group defined by Eqs. (4) and (5)
Q_a, Q_b	volume flow rate in phase a, in phase b (m^3/s)
R	reflux ratio
ΔP	pressure drop (kPa/m^2)
Re	Reynolds number
S	total mass-transfer area in a membrane sheet, BL (cm^2)
t	thickness of membrane sheet (cm)
x, y	rectangular coordinates (cm)

Greek letters

ε	porosity of membrane sheet
ρ	fluid density (kg/m^3)
τ	tortuosity of membrane
μ	fluid viscosity ($kPas$)
$\xi_{a,e}$	outlet value of ξ_a from channel a_2, defined by Eq. (55)
$\xi^0_{a,e}$, $\xi^0_{a,1e}$	value of $\xi_{a,e}$ and $\xi_{a,1e}$, based on mixed inlet concentration, defined by Eqs. (44) and (45)