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Hydrodynamic modeling and dimensioning of an oil-water separator with automated control

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Abstract. The treatment of oil-contaminated industrial wastewater presents a significant challenge due to its associated environmental and economic implications. While this research proposes a simple automated system, based on three valve separator controlled by conductivity sensor, it explores the fundamental principles governing the separation and removal of immiscible oils from wastewater, offering a practical approach to separator design.
This article examines the factors influencing oil-water separation efficiency, including key physical properties of the oil (density and viscosity), oil droplet size, and the flow rate of the influent water. Furthermore, the importance of optimized operating conditions, such as maintaining the flow stability within the laminar system, is addressed. The impact of minimum influent water flow rate on oil separator dimensions, specifically length and width, and the relationship between these dimensions and flow regime stability are also investigated. A theoretical analysis of spherical oil droplet trajectory, incorporating relevant influencing parameters, is presented for the design and dimensioning of the oil separator. A two-compartment oil separator model is presented. The first compartment functions as a sludge collector, facilitating the sedimentation of solids such as seeds and mud. The second compartment enables oil-water separation via flotation and decantation, exploiting the lower density of oils relative to the wastewater. The oil separator incorporates an inlet and two outlet ports, each equipped with solenoid valves for automated control. These valves are actuated by a conductivity sensor. A process flow diagram detailing the automated operation of the three solenoid valves is provided.

Keywords: automatic; conductivity sensor; dimensioning; oil-separator; oil-water; solenoid valves; trajectory

1. Introduction

The generation of oily wastewater is an inevitable derivative of global industrial activities, particularly in the petroleum sector [1], [2]. Effective treatment of this wastewater is crucial not only for regulatory conformity and environmental protection but also for the preservation of water resources [3-6]. Central to this treatment challenge is the separation of immiscible oil from water, a process for which a variety of technologies have been developed [7-9].

A wide spectrum of separation technologies exists, broadly categorized by their fundamental

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principles. Physical separation methods, which exploit the density difference between oil and water, are often the first line of treatment. Established technologies like American Petroleum Institute (API) separators, gravity settling tanks, and coalescers are widely employed due to their operational simplicity and cost-effectiveness for free-floating oils [10], [11]. The effectiveness of granular activated carbon for the purification of institutional wastewater is evaluated [12], with removal rates of up to 82% for turbidity and 65% for BOD. However, the primary limitation of these methods is a well-documented inefficacy against stable emulsions and soluble oils, which exhibit a milky appearance and do not separate by gravity alone [13-16]. For these more complex mixtures, advanced filtration or chemical methods are required. Chemical methods use coagulants, adsorbents, and advanced oxidation processes to remove pollutants, specifically, with demulsifiers applied to break oil emulsions promoting the coalescence of oil droplets for easier separation [17-19]. Biological methods, conversely, utilize microorganisms for the degradation of organic contaminants (such as in trickling filter systems, lagoons, and membrane bioreactors) [20-23]. Recent advancements are optimizing wastewater treatment through modeling and artificial intelligence. For instance, artificial neural networks (ANNs) were successfully used to predict performance in a specific treatment process with a near-perfect correlation to experimental data [24]. Concurrently, other research focuses on developing innovative, eco-friendly solutions by integrating combined treatment approaches to enhance efficiency and sustainability [25].

While advanced methods offer solutions for more complex emulsions, they often involve higher capital and operational costs, greater chemical usage, and more complex operational demands [13].

This wide range of methods creates a clear technological divide: sophisticated methods for complex emulsions and fundamental, gravity-driven methods for free oil. For the latter category, which handles a significant volume of industrial wastewater, the design and optimization of separation systems remain heavily reliant on empirical guidelines and standardized designs. Although these established methods are well-documented, there remain discernible scientific gaps in the development of accessible and robust predictive models that can accurately describe the separation dynamics within these systems. Many contemporary studies focus on the development of novel materials or complex processes, leaving the underlying physics of conventional gravity separation somewhat underexplored in modern literature.

Therefore, this work adopts a fundamental approach to address this gap. We focus specifically on the physics governing the separation of immiscible oils in gravity-based systems. The objective is not to introduce a new technology, but to provide a refined mathematical model that enhances the understanding and predictive capability for these fundamental separation units. By doing so, this study offers a practical and cost-effective tool for optimizing the design and operation of gravity separators, ensuring their efficiency in scenarios where advanced treatment is unnecessary, thereby contributing to more sustainable and predictable industrial wastewater management.

2. Diagram of the oil separator with three solenoid control valves

The oil removal operation consists of separating products with a density slightly lower than water, by natural flotation in a liquid enclosure of sufficient volume. The oils that we intend to separate are liquid products that are immiscible with water and as different as vegetable oils, mineral oils, light hydrocarbons and the term oil removal is usually reserved for the elimination of oils present in significant quantities in wastewater, particularly in process water in the oil industry.

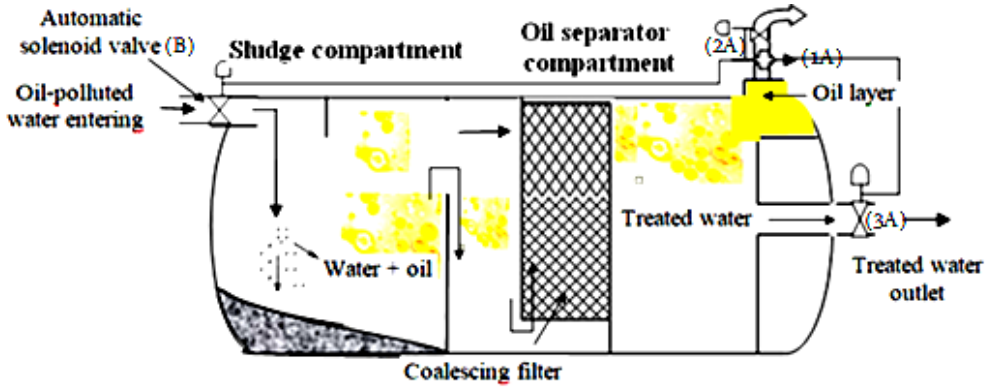

Figure 1. Schematic diagram of an oil separator equipped with a sludge trap and a coalescing filter controlled by automatic solenoid valves

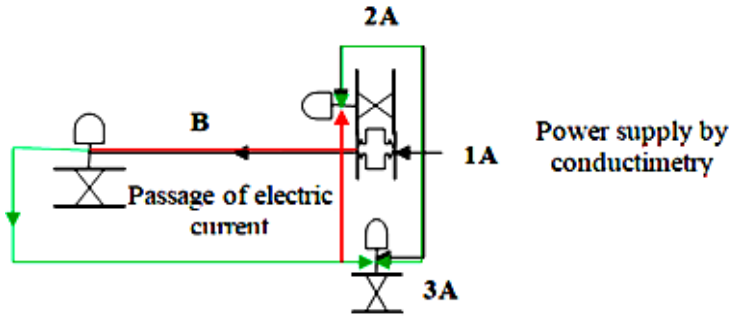

Figure 2. Schematic diagram of the system showing the solenoid valves and their control via a conductivity meter

Conventional oil-water separators are rectangular devices in which wastewater flows lengthwise at a horizontal velocity. Typically, a separator consists of two chambers; a sludge trap and a separator. The compartment considered as a sludge trap retains the sludge and all types of small solids. The separation compartment is the actual chamber that provides the quiescent conditions for oil separation. An oily water separation system consists of a separation unit, a filtering unit and a unit for monitoring and controlling the oil content. This unit consists of three separate stages: coalescence, sediment filtration and emulsion removal.

The core technical specifications of the proposed system are as follows: The separation unit is a horizontal gravity separator constructed from a metal alloy, with a total treatment capacity of 7 m^3. The system operates at inlet and outlet mass flow rates below 0.5 kg/s and a hydraulic pressure of 3-5 bar, under ambient temperature conditions. The feed consists of wastewater contaminated with immiscible oils of varying viscosity, and the separation performance is evaluated based on the reduction of residual oil content (in ppm).

Figs. 1-2 and Table 1 detail the automated oil-water separation system. The system includes a sedimentation compartment and a separation compartment equipped with a coalescing filter. It is important to clarify that this filter, while part of the proposed physical design to improve oil coalescence, is not included in the current hydrodynamic modeling and sizing calculations.

Table 1. Operational sequence of the three-solenoid valve system

Sensor 1A	Electric current reached the conductimeter.
Valves 2A & 3A	The electric current from the conductivity meter controls the solenoid valves 2A and 3A to open when the oil, considered as an insulator of the electric current, passes through the pipe of 2A and the treated water through the pipe 3A.
Valve B	The electric current from the conductivity meter controls solenoid valve B to close when the water to be treated, considered as a carrier of electric current, passes through the piping of 1A and heads towards solenoid valve B to close it

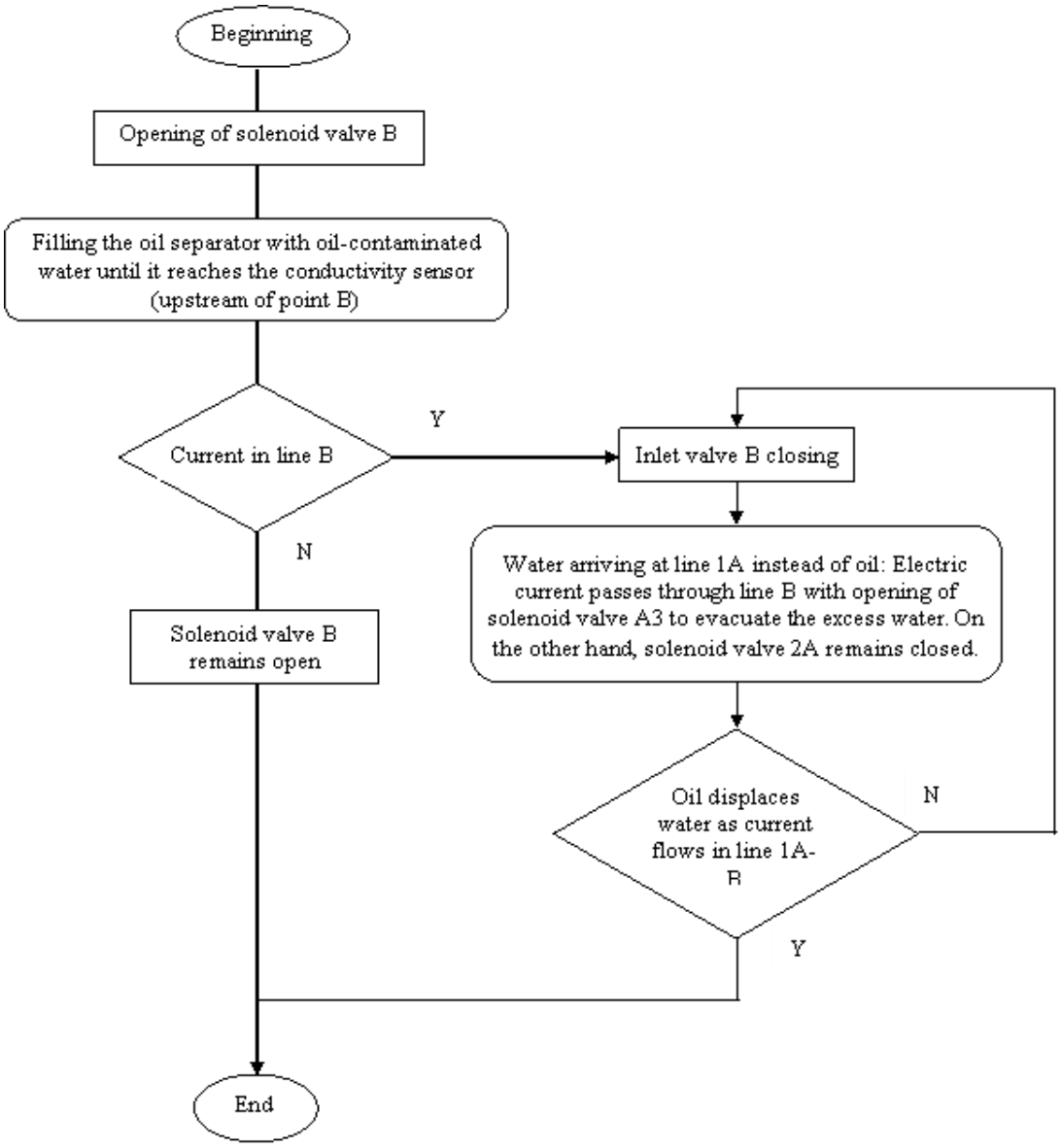

Figure 3. Process flow diagram of the three-solenoid valve system with conductivity control

Solenoid valves, under the control of a conductivity sensor placed in the oil outlet pipe, automate the separation process, managing the flow of oil and water. The operating flow chart is presented in Fig. 3. The proposed conductivity-sensor and solenoid valves system represents a conceptual automation model. Its practical implementation would require addressing several engineering challenges. These include the potential for sensor fouling from suspended solids or oil coatings, as well as the need for regular calibration to maintain measurement accuracy in complex wastewater matrices.

3. Mathematical modeling of oil droplet dynamics

First of all, and to better study the mathematical modeling of the oil separator, considered as a horizontal type separator, we will present some criteria and hypothesis at the same time. The sizing of this wastewater treatment device requires a set of ideas in the form of tests and to better formulate the physical phenomena in the form of the corresponding mathematical equations.

Criterion 1: The oil droplet is considered as a droplet of variable spherical size r, characterized by the viscosity η and the density ρ_{oil} of its fluid.

Criterion 2: The flow rate of the oily water load to be treated must not disturb the upward trajectory of the oil droplet. For this, a maximum flow rate should respect the nature of the laminar flow regime.

Criterion 3: To theoretically ensure a good separation, the case of the smallest oil droplets must be studied, because this is the type that was most affected by the thrust force caused by the oily water.

Criterion 4: We assume that the size of the oil droplet keeps the spherical shape during its movement in the water.

Criterion 5: We assume the flow rate of the oily water to be constant. Consequently, the horizontal flow velocity is constant v_x.

Criterion 6: We consider a single horizontal velocity at the entrance of the separator ($v_x = v_0$).

Criterion 7: To ensure the upward effect of the oil droplet, its weight must always be less than the effect of Archimedes' thrust P_{ar}.

When an oil droplet is immersed in an aqueous medium and subjected to an external thrust force, it experiences several other forces: its weight P, Archimedes' thrust P_{ar}, and friction forces. The vector resulting from the weight and Archimedes' thrust takes the direction of Archimedes' thrust, because the density of the oil is lower than that of water, which gives Archimedes' thrust a force significantly greater than the weight of the oil droplet, Fig. 4.

The resulting force in the direction of Archimedes' thrust could be added to the external force caused by the wastewater which carries the oil droplet with it. The force resultant obtained is in the opposite direction to the friction forces of the oil droplet when it comes into contact with the water molecules.

The model that we intend to exploit from the physical laws of fluid mechanics will be presented below. Finally the resultant of the forces obtained takes the upward direction inclined towards the right as indicated in Fig. 4.

The forces and weights involved on the spherical oil droplet which moves in the form of an upward trajectory inclined to the right, the forces involved in this phenomenon of movement are as follows:

- The weight of the oil droplet P

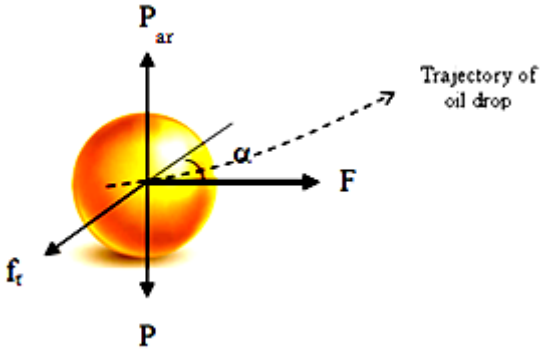

Figure 4. Forces influencing the trajectory of a spherical oil droplet

- The Archimedes' thrust P_{ar}
- The thrust force created by the horizontal displacement of the oily water flow F
- The friction forces created during the movement of the oil droplet f_r

Only one force has the direct projection towards the abscissa axis which is the external force of the waste water. On the other hand, two other forces take the direction of the ordinate axis which is the weight and the Archimedes' thrust. Only the friction forces take the inclined and opposite direction to the movement of the oil droplet.

Theoretically, an oil droplet undergoing four forces during its movement in the form of an upward trajectory to the right should have some forces that followed the two axes of vertical and horizontal motion in a plane of perpendicular axes.

The equations along these two axes are written as follows:

$$-f_{rx} + F = m_{oil} a_x \tag{1}$$

$$-f_{ry} + P_{ar} - P = m_{oil} a_y \tag{2}$$

The horizontal acceleration a_x of the oil droplet is equal to zero (criterion 5).

f_{rx} and f_{ry} are the friction forces on the abscissa and ordinate axes. m_{oil} is the mass of the oil droplet.

The movement of the oil droplet on the abscissa has a constant velocity. Therefore:

$$F = f_{rx} \tag{3}$$

We can replace this force of the wastewater to be treated in mass flow and horizontal velocity v_x:

$$Q_m v_x = 6\pi\eta r v_x \tag{4}$$

With η is the dynamic viscosity, and r is the radius of the oil droplet. Let Q_V be the volume flow rate and μ the kinematic viscosity:

$$Q_V = 6\pi\mu r \tag{5}$$

The upward trajectory of the oil droplet is protected by the ratio (Q_V/r), once the quantity of oil is determined. Similarly the velocity of the wastewater and the entrained oil droplet will have the following expression:

$$Q_V = Sv_x = 6\pi\mu r \tag{6}$$

From which we obtain the following relationship:

$$v_x = 6\frac{\mu}{r} \tag{7}$$

According to this last relation, the horizontal velocity of the water to be treated is indeed that of the oil droplet, because its diameter is small. On the other hand, and according to Eq. (2) given previously for the ordinate axis, we can note that:

$$-6\pi\eta r v_y + \rho_w V_{oil} g - \rho_{oil} V_{oil} g = \rho_{oil} V_{oil} a_y \tag{8}$$

By reducing the previous equation, we obtain the heterogeneous differential equation of order one:

$$\frac{dv_y}{dt} + \frac{6\pi\eta r}{\rho_{oil} V_{oil}} v_y = \left(\frac{\rho_w}{\rho_{oil}} - 1\right) g \tag{9}$$

In order to simplify the previous differential equation, we change the notation to write the following form:

$$\frac{dv_y}{dt} + a v_y = b \tag{10}$$

Where a and b are two coefficients with:

$$a = \frac{6\pi\eta r}{\rho_{oil} V_{oil}} \quad and \quad b = \left(\frac{\rho_w}{\rho_{oil}} - 1\right) g \tag{11}$$

Eq. (10) is a first-order linear non-homogeneous Ordinary Differential Equation (ODE). It is solved according to the variation of parameters method, by first finding the homogeneous solution $v_y = v_{y,0} e^{-at}$ and then substituting back into Eq. (10) which yields to: $\frac{dv_{y,0}}{dt} e^{-at} = b$. Integrating this equation and applying the initial condition that the vertical velocity is zero when the droplet enters the separator (at t=0) gives the vertical velocity: $v_y = \left(\frac{b}{a} e^{+at} - \frac{b}{a}\right) e^{-at}$.

The vertical position y is then found by integrating the vertical velocity equation with respect to time (Where $y(t = 0) = 0$):

$$y = \frac{b}{a} t + \frac{b}{a^2} (e^{-at} - 1) \tag{12}$$

By substituting the relation for horizontal motion ($x = v_0 t$) into this vertical position solution, we obtain the spatial trajectory of the oil droplet:

$$y = \frac{b}{a} \frac{x}{v_0} + \frac{b}{a^2} \left(e^{-\frac{ax}{v_0}} - 1\right) \tag{13}$$

With the velocity at the inlet of the oil separator, we can have the following relationships:

$$v_0 = \frac{Q_V}{S} = \frac{Q_m}{\rho_{oil} S} \tag{14}$$

$$y = Ax + B(e^{-Cx} - 1) \tag{15}$$

This is the final equation of the oil droplet trajectory in a horizontally circulating waste water bath. The coefficients are:

$$A = \frac{2\pi g(\rho_w - \rho_{oil})r^4}{9\mu Q_m} \tag{16}$$

$$B = \frac{4g(\rho_w - \rho_{oil})r^4}{81\rho_{oil}\mu^2} \tag{17}$$

$$C = \frac{9\pi\rho_{oil}\mu}{2Q_m} \tag{18}$$

4. Parametric analysis of oil droplet trajectory

With a rigorous methodology deployed to explore the trajectory of oil droplets in a horizontal oil-water separator emphasizing mathematical modeling that forms the basis of our analysis, the paper is structured as follows: the next section unveils the results from our mathematical modeling, in particular the effects of some important parameters on the trajectory of the smallest oil droplet that would be entrained by the wastewater flow to be treated.

Each modeling criterion such as droplet size, density, viscosity, and fluid velocity or otherwise the mass flow rate of the water to be treated will be evaluated and interpreted separately to provide meaningful insights into the behavior of oil droplets in the horizontal separator. These results will be accompanied by in-depth analyses, exploring the practical and theoretical implications of each observation. All these mathematical analyses will be used to give separator size limits according to the wastewater flow rate to be treated.

The density of an oil droplet, whether it is light or heavy relative to water, has a significant influence on its trajectory in water. The difference in density between the oil droplet and the water determines whether the droplet will float on the surface of the water or move downward.

If the density of the oil droplet is lower than that of the water, the droplet will be lighter than the water and will tend to float on the surface. In this case, the droplet will be mainly influenced by the drag, turbulence, and resistance forces of the surrounding fluid. These forces can slow down the droplet and affect its horizontal trajectory by causing it to deviate from its original trajectory.

On the other hand, if the density of the oil droplet is higher than that of the water, the droplet will be heavier than the water and will tend to move downward. In this case, the trajectory of the droplet will be mainly determined by the forces of gravity. The droplet will generally move with a more vertical trajectory following the direction opposite to the force of gravity. Fig. 5 expresses two trajectories; each has a sense of direction depending on the density of the oil droplet relative to the water. The black trajectory shows the movement of light oil relative to water. On the other hand, the red trajectory expresses an oil of density greater than water (This trajectory is purely included as a conceptual illustration using a downward buoyancy force).

In our case, we will treat waters infected by oils less dense than water, so their trajectories take the upward direction relative to the horizontal axis.

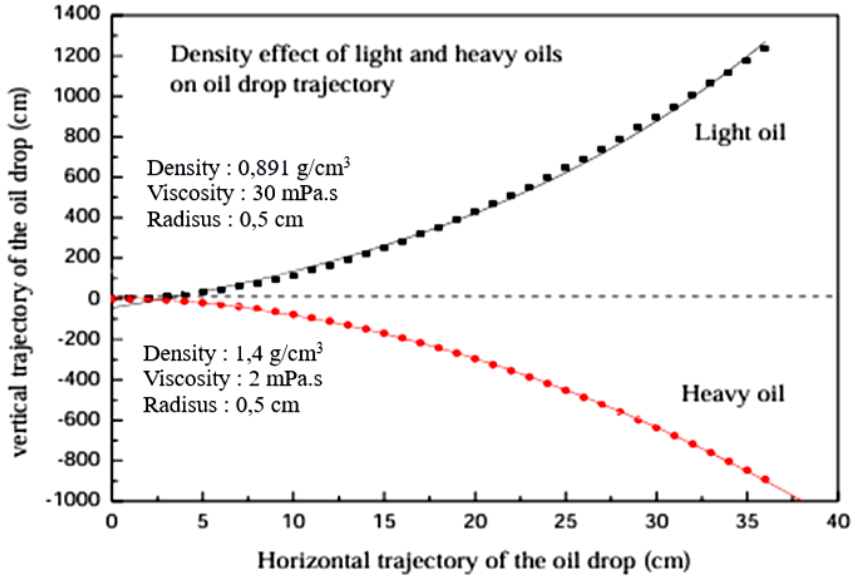

Figure 5. Oil droplet trajectory under external force, considering the droplet density relative to water

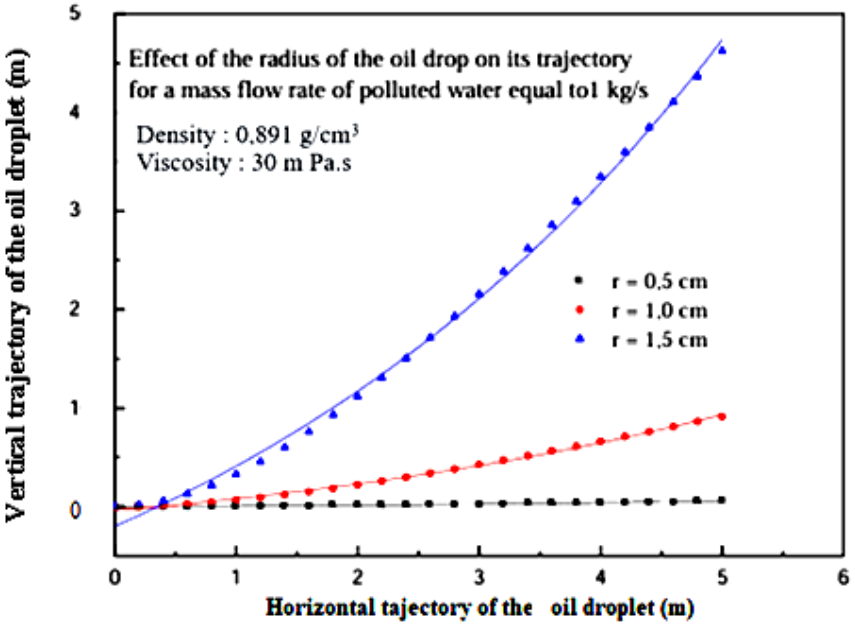

Figure 6. The relationship between diameter and trajectory for spherical oil droplets

4.1 Effect of oil droplet size

For a small water droplet (typically 1 mm), the surface tension is the dominant force. If it is in contact with a highly hydrophobic surface, a droplet adopts a shape close to the sphere.

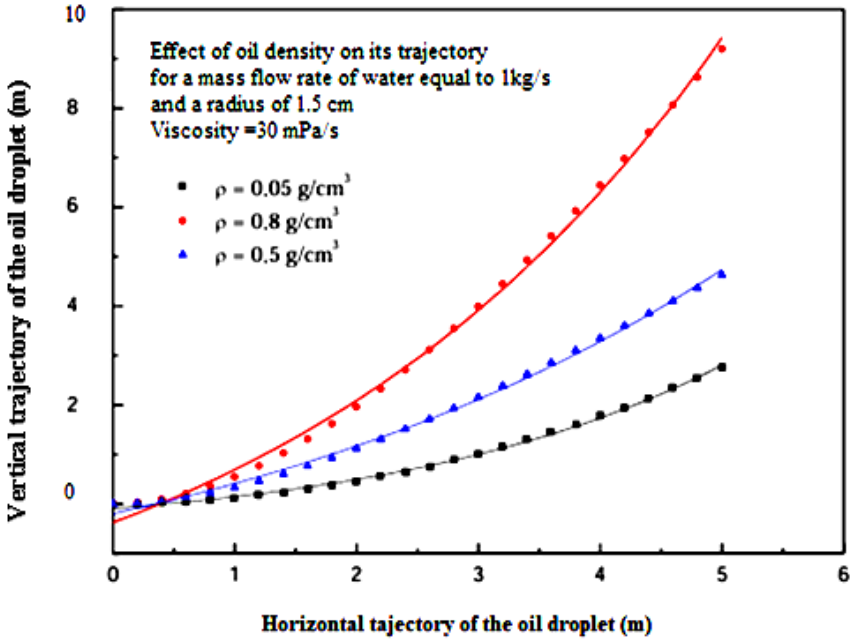

Figure 7. The trajectory of the spherical oil droplet for several densities

The effect of the size of the oil droplet is predominant on its path in water that has a stable horizontal velocity. Using a numerical treatment of the equations determined previously, we can present a graph more or less diverging relative to the x-axis towards the top because the density of the oil droplet is less than that of water. Fig. 6 shows oil droplet trajectories for various diameters.

We can see from Fig. 6 that the smaller the size of the drop, the more its movement is dominated by the horizontal flow of water. On the other hand, the larger the size of the oil droplet, the more it imposes its movement and moves upwards, because Archimedes' thrust gives importance to its radius.

Therefore, the radius of the oil droplet is a key parameter that determines its stability and its interaction with the surrounding environment. An oil droplet with a smaller radius tends to behave more unstable with a greater tendency to disperse because it is more influenced by the drag and turbulence forces present in the water, which makes its trajectory more horizontal. On the other hand, an oil droplet with a larger radius will tend to be less influenced by these forces and to follow a more stable trajectory by rising upwards. According to Fig. 6, the larger the radius of the oil droplet, the more its trajectory takes an elliptical shape and approaches taking a vertical shape.

4.2 Effect of oil density

The density has an influence on the oil droplet shape when it is moving in a flow of water, but we will assume that this influence is less important and assume that its shape is always more or less spherical, especially if its size is considered small.

The density effect is similar to that of its size, the lower the density of the oil droplet, the more its trajectory tends towards the x-axis. Fig. 7 expresses this effect.

It can be seen from Fig. 7 that the higher the density, the more the oil droplet dominates its

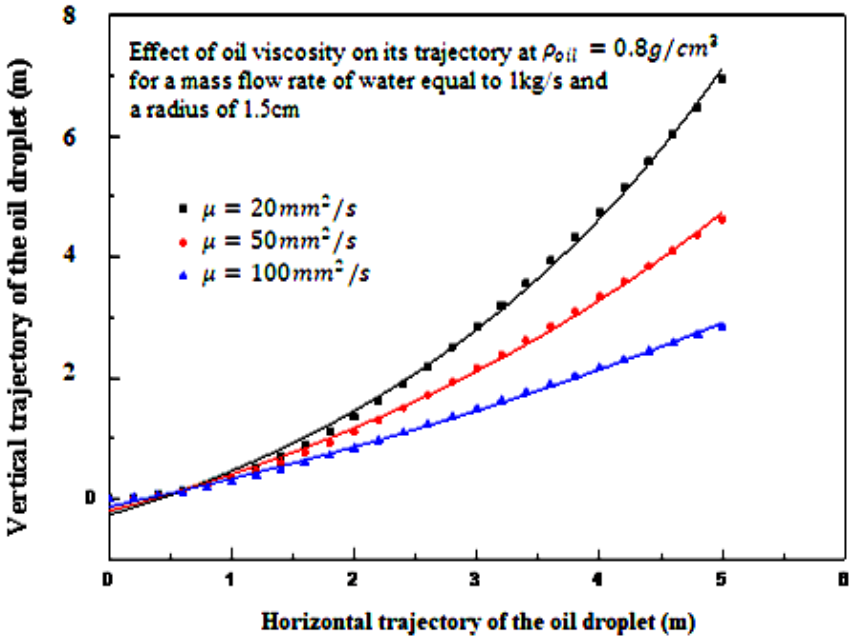

Figure 8. influence of kinematic viscosity on oil droplet trajectory within a horizontal water flow

movement and the Archimedes' thrust would become important compared to the other forces. Since the density of the oil droplet is generally lower than that of water, we note that this difference in density will cause a buoyancy force that acts on the oil droplet. The density of the oil droplet determines the position in the water and significantly affects its trajectory of movement as shown in the curves, the higher the density of the oil, the more vertical its path to floating is.

The density of the oil droplet determines the position in the water and thus affects its horizontal trajectory. A droplet lighter than water will tend to float near the surface and will be more likely to be influenced by the movements of the surrounding fluid. A droplet denser than water will be more sunk in the water and may have a more direct trajectory downwards.

4.3 Effect of oil viscosity

If the oil droplet viscosity is low, it can be considered that the external forces exerted on the liquid droplet can change its shape. It is considered that this effect does not exist because the size of the oil droplet is small. The viscosity parameter has a different role compared to the size and density of the oil droplet. Fig. 8 expresses this effect.

According to Fig. 8, the lower the kinematic viscosity, the more its trajectory is directed upwards, because Archimedes' thrust dominates its movement.

The kinematic viscosity and dynamic viscosity of the oil determine its resistance to movement and flow in water. More viscous oil will tend to be less mobile, to encounter greater resistance when moving in water and to spread less quickly, which influences its trajectory and makes its movement heavier. Viscosity can also affect the interactions between the oil droplet and the surrounding water, particularly with regard to the coalescence or fragmentation of the droplet.

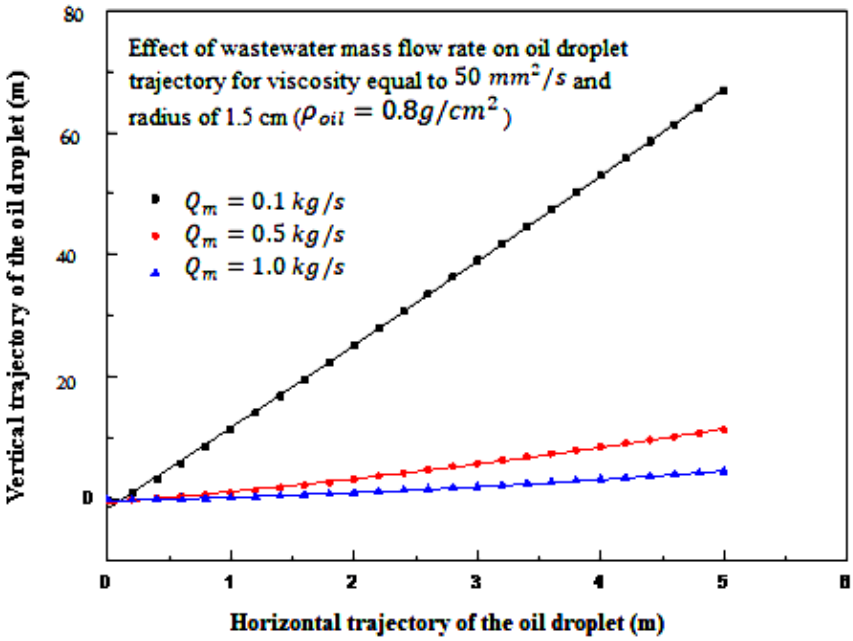

Figure 9. The influence of water mass flow rate on oil droplet trajectory in a horizontal flow regime

4.4 Impact of influent water flow rate

It seems interesting to us to study the evolution of the oil droplet trajectory as a function of the water flow rate. For the specific design parameters considered in this study, model results indicate that flow rates exceeding approximately 0.5 $kg.s^{-1}$ would significantly reduce separation efficiency by impeding the buoyant rise of oil droplets. Flow rates that are too extreme considerably reduce the upward property of our oil droplet and Archimedes' thrust would no longer be the dominant force of the oil droplet movement. Below 0.5 $kg.s^{-1}$, the movement is acceptable, but above this flow rate it becomes dependent on the flow rate of water which moves horizontally. Fig. 9 expresses the shape of the oil droplet trajectory under the effect of several mass flow rates of the water to be treated. We have therefore presented the trajectory of the oil droplet for different mass flow rates between 0.1 $kg.s^{-1}$ and 1.0 $kg.s^{-1}$.

We notice that the movement of the oil droplet rises sharply upwards when the water flow rate weakens. This phenomenon is explained by the mobility of the oil molecules in the water: the decrease in weight is accompanied by an increase in the density of the oil droplet, which has the effect that the cohesive forces between the oil molecules are sufficient for them to rise quickly upwards.

5. Criterion of a unidirectional horizontal flow of water

To have a unidirectional horizontal movement of water, the flow must often be laminar. It creates fewer pressure losses. More stable, it allows us to anticipate the equations. That is to say that the water to be treated is stationary, the observed medium is stable and presents very few

spatial and temporal variations. To have a laminar flow, we must calculate the Reynolds number, which is a fundamental data to determine the flow of a fluid. Theoretically, the type of flow is characterized by four fluid variables: Tube diameter (d); velocity (v), density (ρ) and dynamic viscosity (η). Taking into account all the variables makes it possible to provide what is called the Reynolds number R_e. This parameter is crucial to deduce whether the flow is laminar or turbulent. It is said that a number less than 2300 designates a laminar flow. Its relationship is written as follows:

$$Re = \frac{vd}{\mu} = \frac{4Q_m}{\pi\rho_w\,\mu_w\,d} \tag{19}$$

This Reynolds number for a laminar regime must not exceed the value 2300:

$$Re\ < 2300 \tag{20}$$

Expressed in terms of mass flow rate, this becomes:

$$Qm<575\ \pi\rho w\ \mu w\ d \tag{21}$$

Or for a volume flow rate:

$$QV<575\ \pi\ \mu w\ d \tag{22}$$

6. Estimation of the maximum flow rate of wastewater

The choice of the oil separator depends on the flow rate of wastewater to be treated. According to the previous inequality, this maximum flow rate should be estimated as shown in Fig. 10. The mass flow rate of the water to be treated depends on its viscosity and on the diameter of the pipe through which it flows. The maximum flow rate used should not exceed the line drawn in Fig. 10; more than the calculated value of Q_m, the regime is no longer laminar.

7. Dimensions of the oil-water separation compartment

When sizing a water treatment system, two main factors must be taken into account: The treatment flow rate and the discharge capacity. We will base our sizing on the flow rate of water to be treated. Most oil-water separators also involve a filtration process that can be in several stages and the principle of adsorption, which means that the oil is attracted and absorbed by a filter medium. The first stage of filtration, generally made of fibers, absorbs and separates the oil from the water. This part is integrated into the coalescing filter of our separator.

7.1 Estimation of the oil separator diameter

We have already presented in Eq. (21) the relationship between the mass flow rate of water to be treated and the diameter of the inlet manifold of the separator (Q_m, d). If we use the equation of the continuity of fluids between two tubes of different diameters such as the inlet manifold of the separator and the internal diameter of the latter, we can write the following relationship:

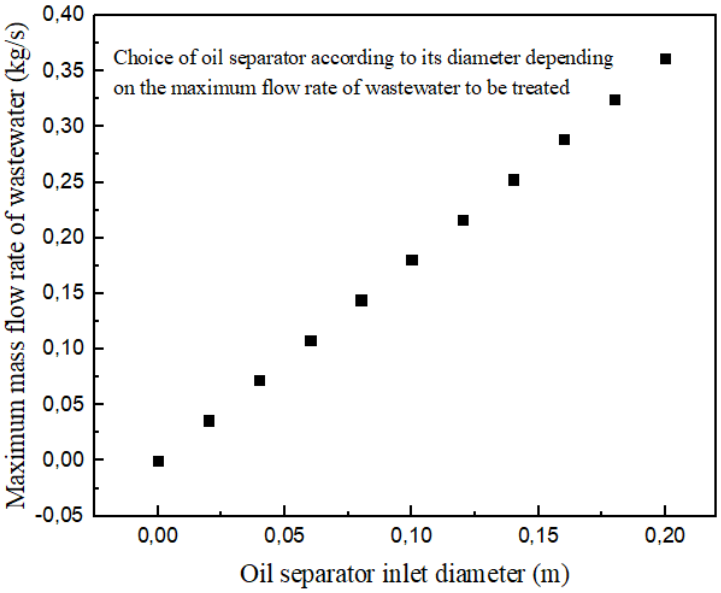

Figure 10. Maximum flow rate vs. inlet diameter (oil separator)

$$Q_{m,inlet} = Q_{m,inside} = \rho_w S_{inside} v_{inside} \tag{23}$$

The diameter of the oil separator inlet must have the same flow rate as that of the oil separator in order to have a laminar regime. In other words and according to the equation of the continuity of fluid flow:

$$Q_{m,inside} = \rho_w \frac{\pi}{4} {d_{inside}}^2 v_{inside} \tag{24}$$

Applying the laminar regime condition, to have a stable and unidirectional flow, we write:

$$2300 > \frac{4Q_m}{\rho_w \mu_w \pi d_{inside}} \tag{25}$$

Thus, we obtain:

$$Q_{m,inside} < 1{,}8055\, d_{inside} \tag{26}$$

The plot of this inequality is shown in Fig. 11, the mass flow rate of water to be treated as a function of the internal diameter of the separator. According to this figure and for a given diameter d_{inside}, the mass flow rate of water to be treated must be less than $1.8055\, d_{inside}$ to be laminar. For example, if the separator has an internal diameter of 15 cm, the allowable mass flow rate must not exceed approximately 0.27 kg/s to maintain a laminar regime.

7.2 Estimation of the oil separator length

To estimate the length of the separator compartment of our oil separator, Eq. (15) is needed. The value of **y** expresses the radius of the oil separator, so we can take the value of $d_{inside}/2$

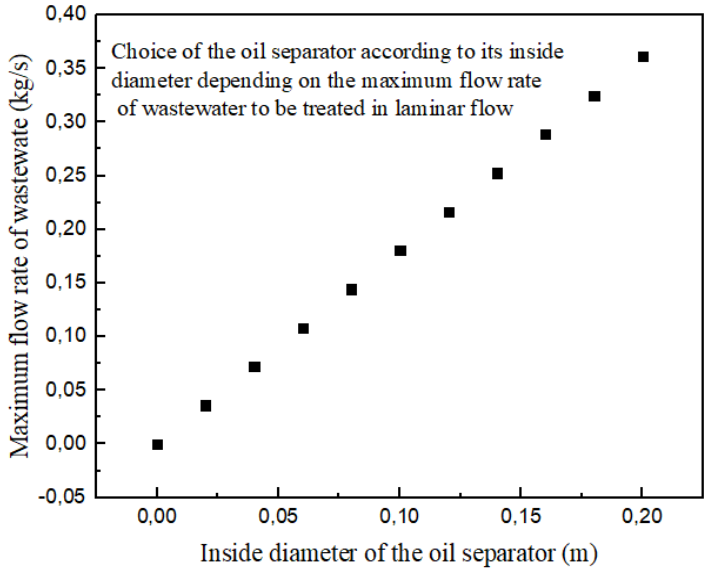

Figure 11. The maximum mass flow rate of water to be treated as a function of the internal diameter of the oil-water separator

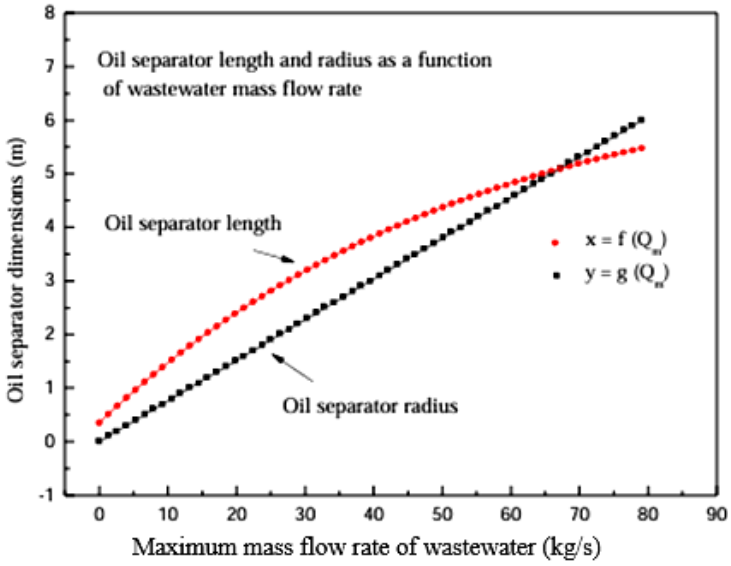

Figure 12. Length and radius of the separator compartment as a function of the mass flow rate of the wastewater to be treated

noted in Eq. (26) with the maximum mass flow rate of water to be treated. By these two Eqs. (15) and (26), we estimate the value of x which is the length of the separator compartment. It is sufficient to know the maximum quantity Q_m per unit of time or the diameter of the oil separator $r = d_{inside}/2$. Fig. 12 expresses the length and radius of the separator compartment as a function of the maximum mass flow rate of wastewater (red and black lines respectively).

As a logical design principle, a higher flow rate generally requires a larger separator. The model defines a continuous relationship where any chosen flow rate corresponds to an optimal pair of dimensions (radius and length) to ensure effective separation. For a credible industrial design, the key is to select a flow rate that results in a balanced proportion between length and radius, typically within a length-to-diameter ratio of 3:1 to 5:1, to ensure both hydraulic efficiency and practical geometry. To ensure robust performance in a real application, the theoretical length derived from this model is often multiplied by a safety factor (typically 1.5 to 3) to achieve the effective length-to-radius ratio (3:1-5:1). This practice balances the dimensions, preventing hydraulically inefficient units and ensuring effective oil-water separation.

8. Conclusion

In response to the environmental challenge of oil-polluted water, this study has developed a theoretical hydrodynamic model to size a horizontal oil separator, automated by a conductivity-sensor and solenoid valve system. The key outcome of this work is the establishment of specific design recommendations derived from the model.

The separator is designed with a length-to-width ratio between 3:1 and 5:1. This is achieved in practice by multiplying the theoretical length from this model by a safety factor (typically 1.5 to 3) to transform the minimum theoretical dimension into a robust and practical size. The horizontal velocity should be maintained below $0.015\ m/s$ to preserve a laminar flow regime which is crucial for the buoyant rise of oil droplets. The design should incorporate a dedicated sludge compartment (one-third the volume of the separation compartment) and an automated control system for efficient phase separation and discharge.

As a design tool, this model allows for the direct sizing of a separator based on a set of critical input variables: the mass flow rate of the wastewater and the key properties of the oil (droplet size, density, and viscosity).

For this model to be adopted in industrial design, first future efforts must concentrate on physical prototype validation which provides the crucial experimental data to confirm the predictive accuracy of the under real operating conditions. The automated control system must be tested for long-term reliability. This involves evaluating sensor durability, determining optimal calibration protocol, and verifying its response to fluctuations in wastewater quality. For further refining the design criteria beyond the current simplified assumptions, advanced computational techniques are prioritized in future studies. This includes employing Computational Fluid Dynamics (CFD) to model complex flow patterns and turbulence effects, coupled with Discrete Phase Modeling (DPM) and Population Balance Models (PBM) to accurately simulate droplet trajectories and coalescence behavior, respectively.

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Nomenclature

Roman Symbols

symbol	***Definition***	***unit***
A, B, C	*Coefficients in the final droplet trajectory equation*	m^{-1}, m, m^{-1}
a_x, a_y	*Accelerations*	$m.s^{-2}$
a, b	*Coefficients in the simplified differential equation*	$s^{-1}, m.s^{-2}$
d	*Diameter (typically of inlet pipe or separator)*	m
F	*Thrust force created by the water flow*	N
f_r	*Friction force*	N
g	*Gravitational acceleration*	$m.s^{-2}$
m_{oil}	*Mass of the oil droplet*	kg
P	*Weight of the oil droplet*	N
P_{ar}	*Archimedes' thrust (buoyancy force)*	N
Q_m	*Mass flow rate*	$kg.s^{-1}$

Q_V	*Volume flow rate*	$m^3.s^{-1}$
R_e	*Reynolds number*	-
r	*Radius of the oil droplet*	m
S	*Cross-sectional area*	m^2
t	*Time*	s
v	*Velocity*	$m.s^{-1}$
v_0	*Inlet velocity*	$m.s^{-1}$
v_x	*Horizontal velocity component*	$m.s^{-1}$
v_y	*Vertical velocity component*	$m.s^{-1}$
V_{oil}	*Volume of the oil droplet*	m^3

Greek Symbols

η	*Dynamic viscosity*	$Pa.s, kg.m^{-1}.s^{-1}, N.s.m^{-2}$
μ	*Kinematic viscosity*	$m.s^{-2}$
ρ	*Density*	$Kg.m^{-3}$

Subscripts

Inside	*Pertaining to the inside of the separator*
Inlet	*Pertaining to the inlet of the separator*
w	*water*