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Fatigue life prediction for radial truck tires using a global-local finite element method

Kyoung Moon Jeong*[1], Hyeon Gyu Beom[2], Kee-Woon Kim[1] and Jin-Rae Cho[3,4]

[1]*R&D Center, Kumho Tires Co. Inc., Gwangju 506-711, Korea*
[2]*Department of Mechanical Engineering, Inha University, Incheon 402-751, Korea*
[3]*School of Mechanical Engineering, Pusan National University, Busan 609-735, Korea*
[4]*Research & Development Institute of Midas IT, Gyeonggi 463-400, Korea*

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Abstract. A global-local finite element modeling technique is employed in this paper to predict the fatigue life of radial truck tires. This paper assumes that a flaw exists inside the tire, in the local model. The local model uses an FEM fracture analysis in conjunction with a global-local technique in ABAQUS. A 3D finite element local model calculates the energy release rate at the belt edge. Using the analysis of the local model, a study of the energy release rate is performed in the crack region and used to determine the crack growth rate analysis. The result considers how different driving conditions contribute to the detrimental effects of belt separation in truck tire failure. The calculation of the total mileage on four sizes of radial truck tires has performed on the belt edge separation. The effect of the change of belt width design on the fatigue lifetime of tire belt separation is discussed.

Keywords: fatigue life; tire; global-local finite element method; crack; energy release rate.

1. Introduction

The tire is a critical safety component for vehicles since it is related to traction, driving, braking, ride, and handling performance. Increasing interests in the field of vehicle safety leads to more and more attention to tire endurance by automotive and tire industry sectors. For the radial truck tires, the design lifetime is generally around 200,000 miles, and the tread wear life 50,000 miles; although by the design intention, a truck tire should exhibit no failures during its useful life or while the tread depth is still adequate (Yin *et al*. 2006). However, due to overload and/or structural materials design problems, some tires do fail unexpectedly. Tire durability due to large local loading, stiffness discontinuities, and production flaws is frequently related to the fracture of tire components such as belt edge separation, ply turn-up separation, or lug cracking in radial truck tire (Ebbott 1996). Influence factors on fracture and durability of rubber material and tires are manifold. Accordingly, understanding the fracture and fatigue behaviors of the tire component is the key to improve durability and assess lifetime of the radial truck tire (Govindjee 2001).

* Corresponding author, Ph. D., E-mail: kmjeong@kumhotire.com

Due to the combination of high stress/strain, high temperature and large thickness, the tire shoulder and bead parts are among the most vulnerable areas in tire components. As for the radial truck tires, the combination of high load and high inflation pressure makes the belt edge or the ply turn-up endurance problem more severe. Several researchers have studied fracture mechanics of tire. Fracture mechanics provides a fundamental approach to analyze the fracture and fatigue crack growth in tires. Ebbott (1996) used a finite element-based method to analyze the severity of internal cracks in cord-rubber structures and to predict tire rolling resistance and temperature distributions (Ebbott *et al.* 1999) in which both the stiffness and the loss properties are updated as a function of strain, temperature, and frequency. Wei *et al.* (1999) demonstrated a virtual crack closure technique for calculation of the energy release rate in the three-dimensional finite element global model, which has an initial flaw inside the tire. Mars (2001) created the concept of the cracking energy density to predict multi-axial fatigue crack initiation in rubber. Yan *et al.* (2002) have studied for the endurance of radial truck tires with finite element modeling by using two approaches: stress analysis parameter approach and tire structure parameter approach. More recently, Han *et al.* (2004) performed a failure analysis of truck tires based on fracture mechanics, using a global-local finite element model. The suggested model has proved effective in a variety of analysis for the prediction of tire lifetime. Zhong (2006) formulated to study fatigue crack growth and durability in tires using a three-dimensional fracture mechanics model. The application of this model in a radial truck tire reveals fracture characteristics of belt edge cracks and helps to explain mechanical and material changes in the tire subject to indoor accelerated durability tests.

The main purpose of this paper is to estimate the fatigue life of the radial truck tires using a three-dimensional finite element modeling fracture analysis based on a steady-state rolling assumption. In order to obtain the fatigue lifetime of a radial truck tire, a steady-state transport scheme is applied for rolling analysis, and different driving conditions such as free rolling, braking, and traction. We assume that a flaw exists inside the tire, in the local model, due to a mechanical inhomogeneity introduced in the manufacturing of the tire. This paper presents a three-dimensional finite element local model to calculate the energy release rate at the belt edge. The local crack model uses a three-dimensional finite element modeling fracture analysis, in conjunction with a global-local technique in ABAQUS. The calculation of the total mileage on four sizes of radial truck tires has performed on the belt edge separation. The effect of the change of belt width design on the fatigue lifetime of tire belt separation is discussed.

2. Tire finite element modeling

A tire usually consists of several rubber components, each of which is designed to contribute to some particular factors for tire performance in addition to several cords and rubber composites. These components play a role in maintaining the stiffness and strength required in a tire. It is to be noted that belt edge cracks are common to steel belted radial tires due to the large stress concentration that is present at the belt edge from the material stiffness discontinuity between the steel and rubber. Fig. 1 is an example of a belt edge crack from a passenger and truck tires that developed in a laboratory experiment. Fig. 2 shows the 2D and 3D finite element mesh on the general structure of a radial tire with a size of 12R22.5 found in radial truck and bus tires, where the roles of tire components are well described in a book by Clark (1982). Four radial truck tires with sizes of 11R22.5, 12R22.5, 12.00R24, 315/80R22.5 are used for a 3D finite element modeling

Fig. 1 Belt edge crack example from a steel belted radial tire: (a) a passenger car tire (Govindjee 2001), (b) a radial truck tire

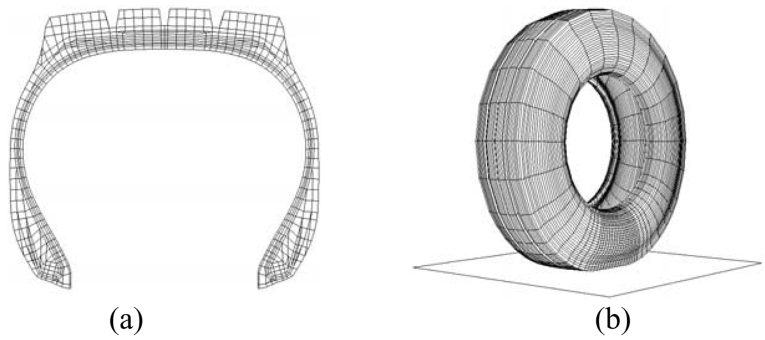


Fig. 2 Global finite element modeling of 12R22.5 tire: (a) 2D model, (b) 3D model

fracture analysis based on a steady-state rolling scheme here. It consists of a radial carcass ply, four belt plies, steel chafer, bead wires and several rubber components. Fig. 2(a) shows a two-dimensional finite element model generated by I-DEAS solid-modeling program, in which 4-node quadrilateral and 3-node triangular elements are used. The global three-dimensional finite element model as shown in Fig. 2(b) is generated by rotating the two-dimensional cross section of a finite element through a 360 deg. and is defined the unequal angular increments. The global model contains very small segments (2.0 deg.) at the contact part as shown in Fig. 2(b).

According to external loading, each of a tire component exhibits considerably distinguished deformation response. Rubbers display a large deformation and almost incompressible response, while steel and polyester cords resisting mostly tension and compression loads produces small strains. Hence, in a tire analysis, it is conventional to employ the hyperelastic model for rubbers and the hypoelastic or hyperelastic model for steel and polyester cords. Rubber compounds or elastomers have non-linear mechanical properties and can undergo very large elastic deformation (Guo and Sluys 2008). Under most circumstances, they act as almost incompressible materials. In this paper, it is assumed that the non-linear mechanical properties of rubbers can be modeled by the strain energy density function in the Mooney-Rivlin form as shown below

$$W(I_1, I_2) = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \tag{1}$$

where I_1 and I_2 represent the first and the second strain invariants, respectively, and C_{10} and C_{01} are the material constants. These constants can be determined from the experimental data in which the rubber specimen is subjected to quasi-static simple elongation.

Referring to Fig. 2, the polyester-cord layer and two steel-cord layers in the belt region are combined respectively into the underlying rubbers using rebar elements (ABAQUS 2008, Helnwein *et al.* 1993), while steel cords and the underlying rubber in the bead region are modeled as homogeneous materials. Rebar is used to define the layer of uniaxial reinforcement in a membrane, a shell and the surface elements. The layer is treated as a smeared layer with a constant thickness equal to the area of each reinforcing bar divided by the reinforcing bar spacing. Since the angle and spacing of fiber reinforcements are changed during manufacturing process, angle and spacing of rebar at each location in a tire should be determined using the following lift equations. Assuming the deformation during manufacturing process in purely due to pantographic action, the angle and spacing of rebar in the cured tire can be obtained by

$$\beta = \cos^{-1}\left[\frac{r\cos\beta'}{r'(1+\varepsilon)}\right], \qquad \alpha = \left[\frac{r'\sin\beta'}{r\sin\beta}\right]\alpha' \tag{2}$$

where β, α, r and ε are angle, spacing, radius and elongation factor of fiber reinforcements after lift. α', β' and r' are spacing, angle and radius of them on a tire building drum.

For rolling analysis, a steady-state transport scheme is applied with different driving conditions: free-rolling, braking, traction loading. The expression for a tire rigid body rotation is described as Eulerian and deformation as Lagrangian description. With the utilization of this kinematical relation, the steady contact problem is converted into a purely spatially dependent problem. The steady-state rolling contact analysis permits a local fine mesh near the contact zone. It leads to large reduction of computation time in the steady-state rolling analysis. During a steady-state straight rolling on a flat surface case, the tire undergoes combined load effects. In this paper, the coefficient of friction is applied as 0.8 between the tire and the road surfaces.

The steady-state transport analysis needs the traveling straight-line velocity of the tire axle v_0 and tire spinning angular velocity ω, in order to specify the rolling condition. In a straight-line steady-state rolling, both the velocity and the acceleration at an arbitrary point P of a tire are expressed by v_0 and ω, since a tire is rolling in the state of planar motion. Classification of the driving conditions depends on the sense of torque and a given vehicle speed. The range of an angular velocity might be estimated by Eq. (3), with the dimension of the free radius of a tire r_0, and distance from the rim center to the contact point of a rigid surface r_f when the tire is deflected.

$$\frac{v_0}{r_0} < \omega < \frac{v_0}{r_f} \tag{3}$$

It is found from this relation that the range of the angular velocity for a free rolling tire is around 40.04 ~ 43.03 rad/sec. The free rolling condition can be determined from the torque-angular velocity curve, or from the deformed circumferential configuration. We found that it becomes 41.9 rad/sec in case of the traveling velocity of 80km/h for a 12R22.5 truck tire.

3. Local crack model

There are many locations in a tire where a crack might exist. However, cracks at limited locations play dominant roles in the tire failure process. To perform an efficient and effective tire fracture

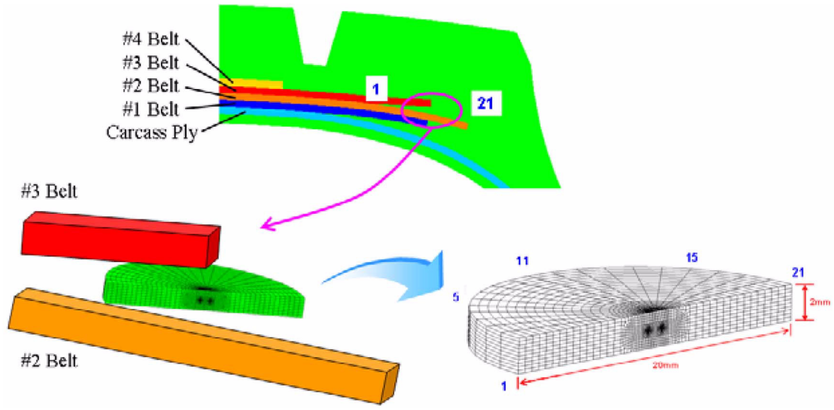


Fig. 3 Crack location (1: center direction, 11: circumferential direction, 21: side direction)

mechanics analysis, one has to identify critical locations first. The critical locations, such as belt edge and ply turn-up, can be determined either from experience, such as observations of tire failures in the field or in lab tests, or from finite element analysis of tires. Regions with highly localized deformation or stress concentrations or abrupt changes in material stiffness, such as the interfaces between cord and cord, are likely locations for cracks. In tire FEA modeling, the critical locations can be selected based on maximum cyclic strain energy density or maximum shear strain depending on material properties and local geometry. It is found that maximum cyclic strain energy density or maximum shear strain generates at the belt edge region between #2 and #3 belts using the global analysis results. Therefore, our attention is restricted to the case in which the local crack exists at the belt edge region between #2 and #3 belts in radial truck tire. The local crack model is made of only belt edge rubber and has very small dimension to be embedded into the belt edge region as shown in Fig. 3. A symmetry model that has a rectangular type of crack surface can cause a singular problem at the sharp corner crack fronts in numerical calculation. Therefore, we select the crack surface with the half-circular shape. This model, however, overcomes the difficulties such as singularity.

A global-local technique can be applied quite generally in finite element analysis. The material response defined for the local model may be different from that defined for the global model (Cho *et al*. 2008). This numerical technique is used to obtain the detailed local response of a local part of interest in the model with a refined mesh using the interpolation of the solution obtained with an initial, relatively coarse, global model. It is well know that this technique is most useful when an accurate, detailed solution in a local region is necessary and the detailed modeling of that local part region has negligible effect on the overall solution. The model of which solution is mapped onto the relevant parts of the boundary of the local model is referred to as the global model. Driven variables are defined as those variables in the local model that are constrained to match with the results of the global model. In this paper, the driven variables are the degrees of freedom at nodes in the node-based numerical approximation techniques (ABAQUS 2008).

The question of crack orientation can be easily resolved if there are observations of typical failures in the radial truck tire under consideration. Otherwise, one has to make an assumption. Since shear deformation is dominant in tire components, one can assume that a macroscopic crack orients in the direction of maximum shear at the critical location. This assumption typically leads to

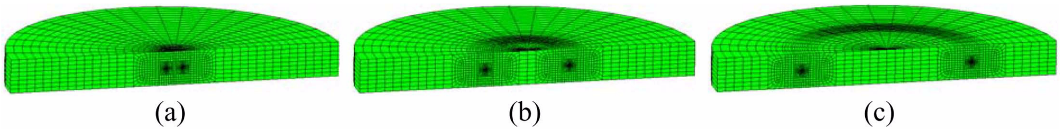


Fig. 4 Local crack model: (a) 1 mm, (b) 5 mm, (c) 10 mm

a crack parallel to tire component interfaces in a tire cross section. Depending on tire types and loading conditions, some tire components can be subject to tensile deformation. In this case, the crack can be perpendicular to component interfaces, or perpendicular to the maximum principal tension at the location (Zhong 2006). As shown in Fig. 3, the local crack model is centrally located at the #3 belt edge region, and the crack surface parallels #3 belt. The local model has 21 crack tips, 2mm in height, and 20mm in diameter. Fig. 4 shows the local crack model with the different crack size, respectively. In order to analyze the crack propagation, ten different crack sizes are chosen (Diameter: 1~10 mm). The maximum size limitation of the crack is restricted to the dimension of the local crack model, since the local crack model was generated to be located inside of the belt edge rubber.

4. Fatigue crack growth analysis

In tire fracture mechanics, the general energy release rate concept is used to describe the crack growth driving force since tires are complex non-homogeneous composite structures. Energy release rate G by definition is the rate of change in potential energy with respect to the newly created crack area. It is a measure of the energy available for an increment of crack extension. The energy release rate is defined as

$$G = -\frac{d\Pi}{dA} \tag{4}$$

where Π is the potential energy of a material system, and A is the fracture surface area. Investigations of energy release for non-linear material, especially for rubber, have been performed by Rivlin and Thomas (1953). In computational mechanics, energy release rate is used as same meaning of the J integral, by conservation of the contour integral theorem. The same definition holds for non-linear elastic materials. The value of G can be determined by the J integral. In the absence of body forces and thermal strains, the J integral is generally defined as (Rice 1968)

$$G = J = \int_{\Gamma}\left(W dy - T_i \frac{\partial u_i}{\partial x} ds\right) \tag{5}$$

where W is the strain energy density, T_i are components of the traction vector, u_i are the displacement vector components, and ds is a length increment along the contour Γ.

Fatigue cracks typically initiate from regions of high stress concentrations. Therefore, radial truck tire has many possible failure regions such as groove, belt and bead area. But most failures, which result in severe accidents, come from the belt edge separation due to large stress concentrations induced by material discontinuity in a macrosense. Under a monotonic load, crack growth is determined by the critical energy release rate. Under cyclic loading conditions, crack growth is also called fatigue crack growth. Fatigue crack growth is commonly classified into three main stages: crack initiation, stable crack growth and propagation, and unstable crack growth leading to complete fracture. These three stages are often studied using fracture mechanics. If fracture toughness range is less than some threshold fracture toughness which is often assumed a material constant, a crack cannot initiate. As the fracture toughness is large than the threshold value, a crack initiate, allowing for steady crack growth to follow in the second stage. The second stage of fatigue crack growth is perhaps the most studied stage of all fatigue research. This is when fatigue crack growth is steady and the growth rate increases in a linear fashion. Fatigue crack growth is typically determined by a fatigue crack growth law. Commonly used fatigue crack growth law has a power law form

$$\frac{da}{dN} = m(\Delta G)^n \qquad (6)$$

where a is the crack length in the unstrained state, N is the number of load cycles. m and n are material constants calculated from the test data (Govindjee 2001). This equation is often referred as Paris' law, where ΔG is the range of energy release rate in a cycle. From the finite element based fracture mechanics analysis, ΔG may be defined as $\Delta G = G_{max} - G_{min}$ for one revolution. Assuming that G_{max} and G_{min} are the largest and smallest energy release rates among the results obtained for different positions in circumferential direction during rotation. In general, G_{max} is the value of rolling analysis results and G_{min} is the value of inflation analysis results. Using the Eq. (6), it is possible to predict the total lifetime of a radial truck tire in number of cycles. The third stage occurs when maximum fracture toughness approaches the material critical fracture toughness. During this stage, crack growth is unstable and occurs at a rapid rate leading to ultimate fracture.

5. Results and discussions

Using the formulation of a tire fracture mechanics model outlined in the previous section, a comprehensive study of a radial truck tire with a belt edge crack is conducted. The local crack is formed using a displacement boundary condition in a global finite element model analysis of radial truck tire in the presence of an initial flaw of half-circular shape. In order to obtain the results of global finite element analysis, the conditions of inflation pressure and vertical loading for four radial truck tires are as shown in Table 1. When a truck tire is loaded and rolling, no variation in the energy release rate was found from the top to just ahead of contact region, 140 deg., and energy release rate increases linearly from a possible contact zone to the contact center point (Han *et al.* 2004). Therefore, we are only concerned with the case in which the local crack model lies at the contact center point. Stress distributions with respect to the crack size at the crack tip for a size of 12R22.5 truck tire are as shown in Fig. 5. It is found that the maximum stress at the crack tip increases as crack size increases. As mentioned in this previous study (Han *et al.* 2004), a mode shape of the crack surface shows an opening mode at the top location and a mixed mode at the

Table 1 Inflation pressure and vertical loading for four truck tires

	12R22.5	11R22.5	12.00R24	315/80R22.5
Inflation pressure	120 psi	120 psi	120 psi	120 psi
Vertical loading	3,350 kgf	3,000 kgf	4,190 kgf	3,750 kgf

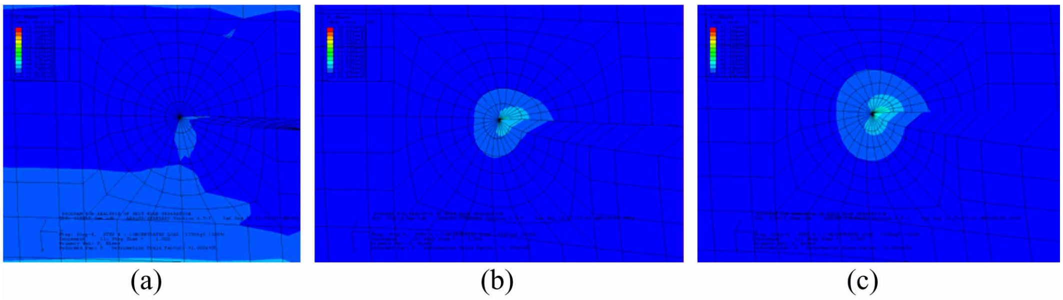


Fig. 5 Stress distribution at the crack tips for a size of 12R22.5: (a) 1 mm, (b) 5 mm, (c) 10 mm

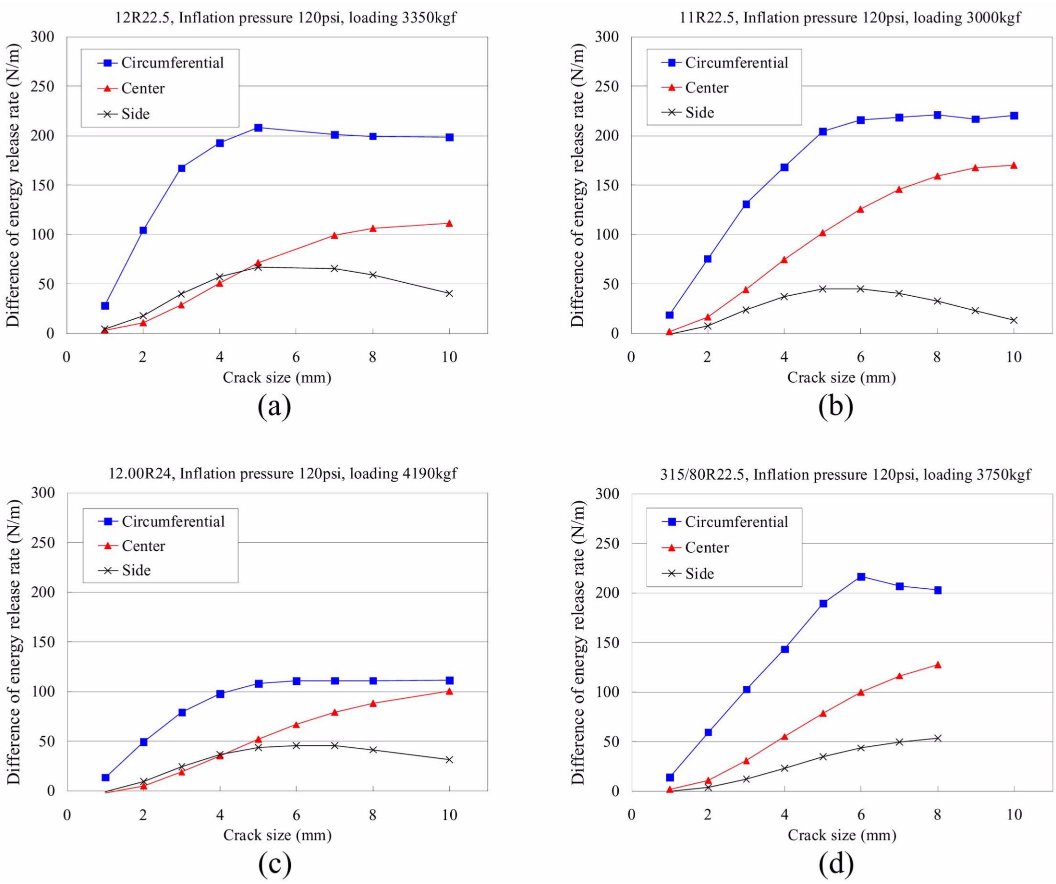


Fig. 6 Difference of energy release rate with respect to the direction of crack propagation: (a) 12R22.5, (b) 11R22.5, (c) 12.00R24, (d) 315/80R22.5 radial truck tires

contact region as shown in Fig. 5. The same mode shape is obtained in all other driving conditions. Fig. 6 shows the difference of energy release rate with respect to the direction of crack propagation under static contact condition for four radial truck tires. It is shown that the energy release rate increases as crack size increases. In almost all cases, the energy release rate in the circumferential direction is higher than that of the center direction. The results predicted here agree with the previous results in Han *et al*. (2004). It is found that crack growth in the side direction can not be

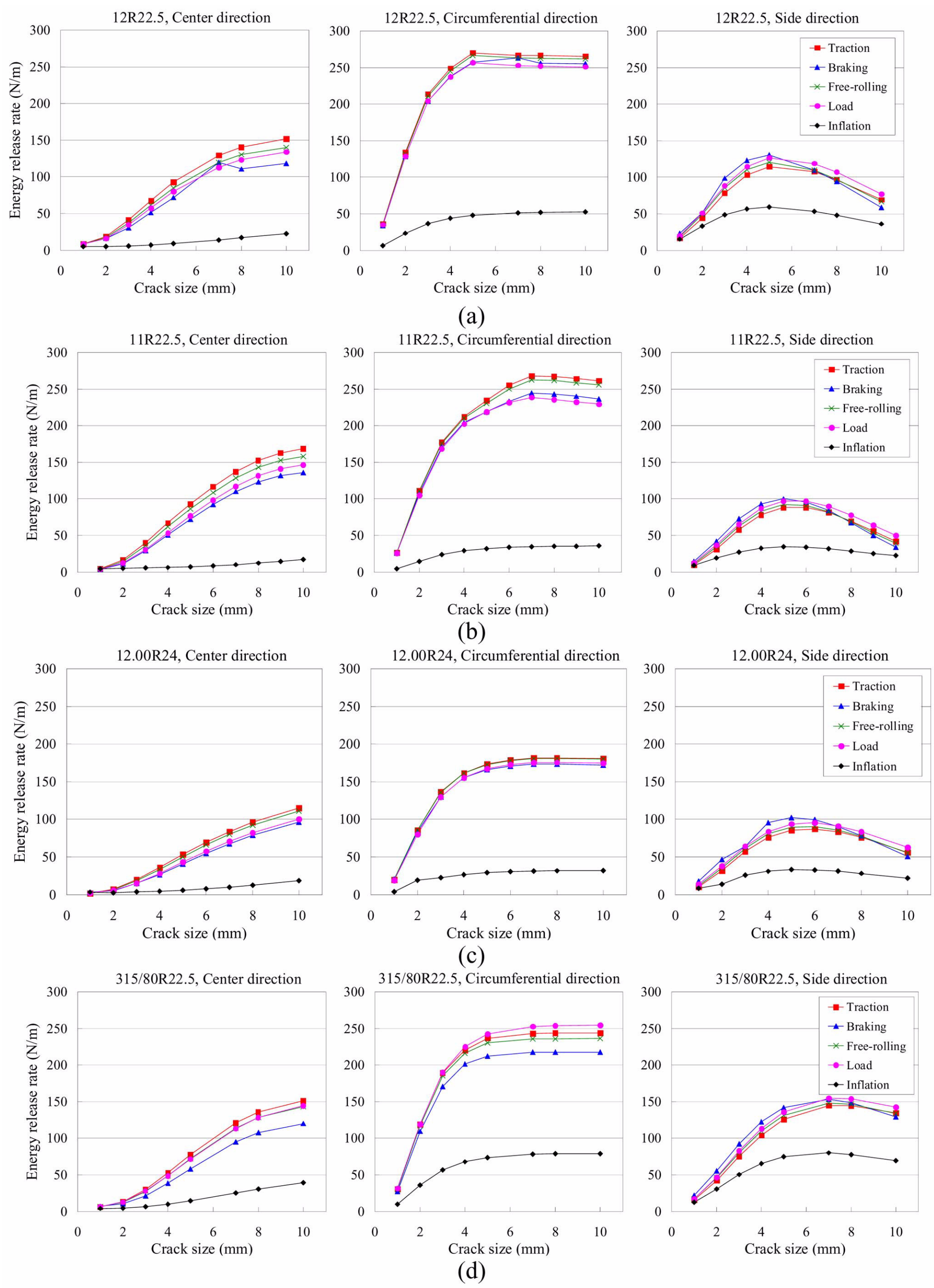


Fig. 7 Energy release rate with respect to the direction of crack propagation for the driving condition: (a) 12R22.5, (b) 11R22.5, (c) 12.00R24, (d) 315/80R22.5 radial truck tires

caused since the difference of the energy release rate has a reduced value together with an increased crack length. This result agrees with the experimental observation in Fig. 1(b).

The steady-state rolling contact analysis permits a local fine mesh near the contact zone while the transient rolling contact analysis requires a fine mesh along the entire tire surface. It leads to large reduction of computation time in the steady-state rolling analysis. It is assumed that vehicle travels at 80km/h for classification of an angular velocity of a tire. Driving conditions considered in this paper are free-rolling, traction and braking in order to understand how different driving contribute to the detrimental effects of belt separation in truck tire failure.

A small variation of energy release rate is found to each different driving condition for center, circumferential, and side directions. Fig. 7 shows the energy release rate with respect to the direction of crack propagation for the driving condition. It is shown in Fig. 7 that the propagation of the crack size increases gradually under the crack size, 5mm. The crack growth above about 5mm of crack size approaches the constant value.

The lifetime of a tire that is limited by belt edge separation can be approximately predicted by the energy release rates for four radial truck tires. From Eq. (6), the lifetime of a tire can be expressed by the number of cycle, N, which for the number l of crack size intervals takes the form of

$$N = \int_{a_1}^{a_2} \frac{1}{m(\Delta G_1(a))^n} ds + \int_{a_2}^{a_3} \frac{1}{m(\Delta G_2(a))^n} ds + \ldots + \int_{a_{l-1}}^{a_l} \frac{1}{m(\Delta G_l(a))^n} ds \tag{7}$$

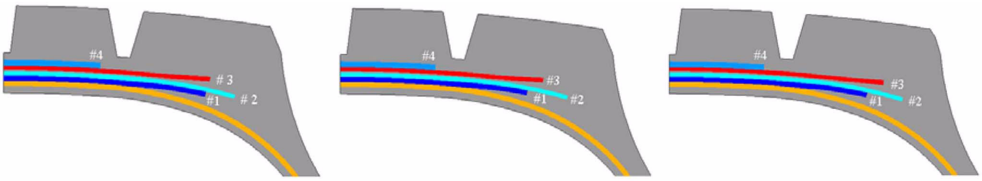
The fatigue lifetime of radial truck tire depends on the crack size, material parameters and energy release rate. The material parameters used in the present paper are introduced in Govindjee (2001): $m = 8.2 \times 10^{-8}$ and $n = 2.5$. The values of energy release rate are shown in Fig. 7, with the different driving conditions and different crack sizes for the four truck tires. The total mileage worn-out truck tire without belt separation is around 319,394~417,125 km for a size of 12R22.5 tire. Table 2 represents the total mileage with respect to different driving conditions for four truck tires when the crack propagates up to 10 mm at the belt edge region.

Because several factors such as temperature rise, chemical aging, cornering and severe inflation/loading conditions under driving is not considered in the study, the predicted total mileage of truck tire with belt edge separation can be calculated higher than the mileage of worn-out truck tire.

For parametric study using a three-dimensional finite element modeling fracture analysis, a radial truck tire, 12R22.5, is selected as the design change. It is decided to vary the belt widths, which have a significant impact on the belt edge separation. All other construction features, including the carcass profile, and compounds are held constants. The design change in the belt edge region in the radial truck tires with the different belt widths are shown in Fig. 8. Fig. 8(a) shows that the width of #1 belt is similar to the one of #3 belt. In Fig. 8(b), the width of #1 belt is reduced to 10mm. And, the widths of #2 and #3 belts increase to 6 and 10 mm as shown in Fig. 8(c). Difference of energy

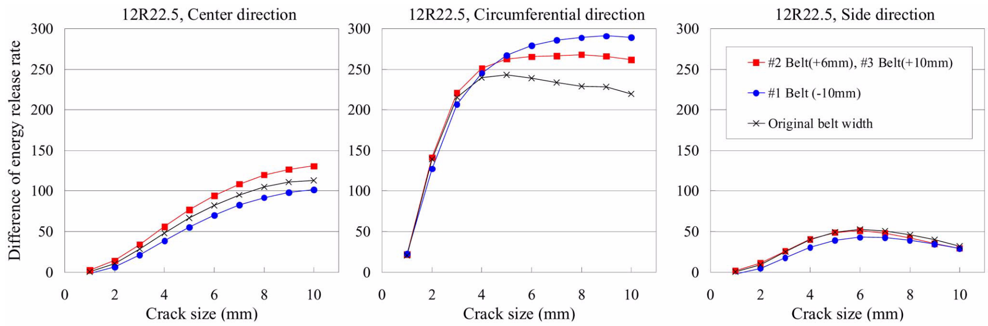
Table 2 Total mileages with respect to different driving conditions

Size	Braking	Free-rolling	Traction
12R22.5	247,242 km	202,841 km	189,358 km
11R22.5	214,055 km	231,096 km	226,852 km
12.00R24	461,866 km	442,119 km	441,424 km
315/80R22.5	408,103 km	342,043 km	324,463 km



(a) (b) (c)

Fig. 8 Design with the different belt width: (a) original belt width, (b) #1 Belt (-10 mm), (c) #2 Belt (+6 mm), #3 belt (+10 mm)



(a) (b) (c)

Fig. 9 Difference of energy release rate with respect to the direction of crack propagation: (a) center direction, (b) circumferential direction, (c) side direction

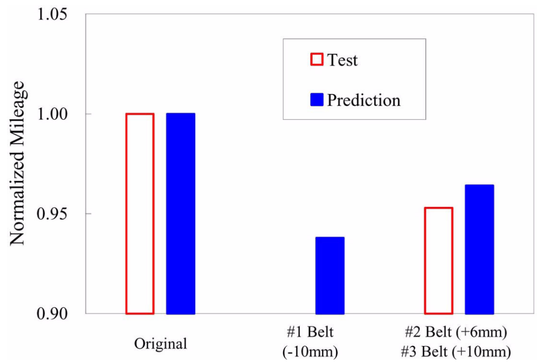


Fig. 10 Comparison of predicted mileage to measurement from tests

release rate with respect to the direction of crack propagation is shown in Fig. 9. In the circumferential direction, the difference of energy release rate is higher than the center or side direction. The fracture mechanics based tire durability approach, along with the proprietary fatigue crack growth law, has been applied to two different truck tires under different accelerated tire testing conditions at Kumho. Here tire durability test is presented to demonstrate the success of the fracture mechanics based tire durability analysis approach. Two of the experimental tires have the different tire construction with a change in belt width in the tire. Tire fatigue life is designated as the mileage in the simulated test, at which the assumed initial crack grows to a size, e.g. 10 mm. Fig. 10 shows the comparison of predicted tire life to measurement from tests. The failure mileage of each tire is normalized with the mileage of the original tire for either prediction or indoor test. It is found that the best design for the belt edge endurance of a radial truck tire, 12R22.5, is original belt width. The fracture mechanics predicts the correct ranking and reasonable relative percentage change for this test case.

6. Conclusions

A global-local finite element modeling technique is employed for the calculation of the energy release rate at the belt edge region to predict the fatigue life of radial truck tires, based on the steady-state rolling analysis. The local model including the crack surface uses an FEM fracture analysis in conjunction with a global-local technique in ABAQUS. In order to obtain the fatigue lifetime of a radial truck tire, a steady-state transport scheme is applied for rolling analysis, and different driving conditions such as free rolling, braking, and traction. The calculation of the total mileage on four sizes of radial truck tires has performed on the belt edge separation. The effect of the change of belt width design on the fatigue lifetime of tire belt separation is carried out. It is found that the prediction of fatigue lifetime using global-local finite element method will be useful for the reliability design of radial truck tire.

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