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Stochastic space vibration analysis of a train-bridge coupling system

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Abstract. The Pseudo-Excitation Method (PEM) is applied to study the stochastic space vibration responses of train-bridge coupling system. Each vehicle is modeled as a four-wheel mass-spring-damper system with two layers of suspension system possessing 15 degrees-of- freedom. The bridge is modeled as a spatial beam element, and the track irregularity is assumed to be a uniform random process. The motion equations of the vehicle system are established based on the d'Alembertian principle, and the motion equations of the bridge system are established based on the Hamilton variational principle. Separate iteration is applied in the solution of equations. Comparisons with the Monte Carlo simulations show the effectiveness and satisfactory accuracy of the proposed method. The PSD of the 3-span simply-supported girder bridge responses, vehicle responses and wheel/rail forces are obtained. Based on the 3σ rule for Gaussian stochastic processes, the maximum responses of the coupling system are suggested.

Keywords: train-bridge system; stochastic vibration; FEM; dynamic response.

1. Introduction

In essence train-bridge coupling vibration is a stochastic process, and the research on this field is very limited owing to the complexity of the coupling system. Previous studies were mainly about the random vibration of beams subjected to moving loads. The random vibration of beams (Fryba 1976), the irregularity of the beam (Yoshimura 1988), the randomness of moving load velocity (Sniady 2001) and the statistical characteristics of random vehicle loads (Ding 1997) were discussed. Up to now, many researches have been done by the moving load model (Di-Paola 1997, Yang 2008, Yau 2009).

As to railway bridges, the weight of a train is much larger than that of a car, thus large errors will be made if the interaction between the train and the bridge is ignored. In most of the researches, track irregularity is taken as a particular time-history sample to compute the responses of the system. That is the deterministic analysis in the strict sense. The Monte Carlo method is commonly used in the statistical characteristics of responses, by which many different track irregularity samples are simulated and the statistical characteristics of responses are obtained. The reliability of the simulation can be ensured only if the number of samples is sufficient. Because of the high

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computational cost, the Monte Carlo method has considerable limitations in practical engineering applications. The covariance analysis method (Jin 2007), the theory of energy random analysis (Zeng 2002) and the statistical analysis technique (Xia 2003) were applied. Fang (2002) and Li (2000) put forward a unified approach to evolutionary random response problem. Yau (2009) This paper is intended to investigate interaction response of a train running over a suspension bridge undergoing support settlements. The suspension bridge is modeled as a single-span suspended beam with hinged ends and the train as successive moving oscillators with identical properties. To conduct this dynamic problem with non-homogeneous boundary conditions, this study first divides the total response of the suspended beam into two parts: the static and dynamic responses. Then, the coupled equations of motion for the suspended beam carrying multiple moving oscillators are transformed into a set of nonlinearly coupled generalized equations by GalerkinThis paper is intended to investigate interaction response of a train running over a suspension bridge undergoing support settlements. The suspension bridge is modeled as a single-span suspended beam with hinged ends and the train as successive moving oscillators with identical properties. To conduct this dynamic problem with non-homogeneous boundary conditions, this study first divides the total response of the suspended beam into two parts: the static and dynamic responses. Then, the coupled equations of motion for the suspended beam carrying multiple moving oscillators are transformed into a set of nonlinearly coupled generalized equations by GalerkinThis paper is intended to investigate interaction response of a train running over a suspension bridge undergoing support settlements. The suspension bridge is modeled as a single-span suspended beam with hinged ends and the train as successive moving oscillators with identical properties. To conduct this dynamic problem with non-homogeneous boundary conditions, this study first divides the total response of the suspended beam into two parts: the static and dynamic responses. Then, the coupled equations of motion for the suspended beam carrying multiple moving oscillators are transformed into a set of nonlinearly coupled generalized equations by GalerkinThis paper is intended to investigate interaction response of a train running over a suspension bridge undergoing support settlements. The suspension bridge is modeled as a single-span suspended beam with hinged ends and the train as successive moving oscillators with identical properties. To conduct this dynamic problem with non-homogeneous boundary conditions, this study first divides the total response of the suspended beam into two parts: the static and dynamic responses. Then, the coupled equations of motion for the suspended beam carrying multiple moving oscillators are transformed into a set of nonlinearly coupled generalized equations by Galerkininvestigated the interaction response of a train running over a suspension bridge undergoing support settlements. Yang (2008) studied the aerostatic instability of cable-supported bridges, with emphasis placed on modeling of the geometric nonlinear effects of various components of cable-supported bridges.

The pseudo excitation method (PEM) is an efficient algorithm for structural random vibration analysis and has been applied mainly to linear system (Lin 2004). Zhang (2007) studied the non-sationary random vibration for vehicle-bridge system. Lu (2008 and 2009) adopted a vehicle-bridge interaction (VBI) element proposed by Yang and Wu (2001) and Yang *et al*. (2004) in the stochastic analysis for dynamic interaction of vehicles and structures.

In this paper, PEM is applied to the study on the stochastic space vibration analysis of train-bridge coupling system. Each car body or bogie has 5 DOFs (Degree-Of-Freedom) in directions of Y, Z, R_x, R_y and R_z. The bridge is modeled as a spatial beam element, and the track irregularities are assumed to be Gaussian stochastic processes. The motion equations of the vehicle system are established based on the d'Alembertian principle, and the motion equations of the bridge system are established based on the Hamilton variational principle. Separate iteration (Li 2000) is applied in the solution of equation. A numerical example is given to analyze the stochastic space dynamic responses. Comparisons with the Monte Carlo simulations show the effectiveness and satisfactory accuracy of the proposed method. The PSD responses of bridge mid-point, car body and leading bogie are discussed. The 3σ rule is applied to determine the boundaries for the maximum responses of the coupling system, which can be helpful to the design.

2. Stationary random responses of train-bridge system

The dynamic equation of train-bridge system is as follow

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(\mathbf{t}) + \mathbf{K}\mathbf{u}(\mathbf{t}) = \mathbf{F}_1(t) + \mathbf{F}_2(t) + \mathbf{F}_3(t) + \mathbf{F}_4(t) \tag{1}$$

where $\mathbf{F}_1(t)$ is the deterministic load due to the vehicle weight; $\mathbf{F}_i(t)$ $(i = 2\sim4)$ means the random load related to the vertical profile irregularity, the alignment irregularity, and the elevation difference irregularity between the left and right rail, respectively.

According to the Duhamel integration, solutions of Eq. (1) can be written as

$$\mathbf{u}(\mathbf{t}) = \int_0^t \mathbf{H}(\mathbf{t}-\tau)(\mathbf{F}_1(\tau) + \mathbf{F}_2(\tau) + \mathbf{F}_3(\tau) + \mathbf{F}_4(\tau))d\tau \tag{2}$$

Suppose track irregularities are zero mean valued Gaussian random process, then the mean value vector of the responses is

$$\bar{\mathbf{u}}(\mathbf{t}) = \int_0^t \mathbf{H}(\mathbf{t}-\tau)E[\mathbf{F}_1(\tau)]d\tau \tag{3}$$

From Eq. (3), one can see that the mean values are caused by the vehicle weight.

The track irregularities are assumed to be mutually independent, so that the covariance matrix of the responses can be expressed as

$$R_{\mathbf{uu}}(t_k, t_l) = \int_0^{t_k}\int_0^{t_l} \mathbf{H}_k(t_k-\tau_k)\sum_{i=2}^{4} E(\mathbf{F}_i(\tau_k)\mathbf{F}_i(\tau_l)^T)\mathbf{H}_l(t_l-\tau_l)^T d\tau_k d\tau_l \tag{4}$$

According to the Wiener-Khintchine theorem, Eq. (4) can be written as

$$R_{\mathbf{uu}}(t_k, t_l) = \int_{-\infty}^{+\infty}\int_0^{t_k}\int_0^{t_l} \mathbf{H}(t_k-\tau_k)\sum_{j=2}^{4} S_j(\omega)e^{i\omega(\tau_l-\tau_k)}\mathbf{H}^T(t_l-\tau_l)d\tau_k d\tau_l\omega = \int_{-\infty}^{+\infty} S_{\mathbf{uu}}(\omega, t)d\omega \tag{5}$$

where $S_{\mathbf{uu}}(\omega, t)$ is the auto-PSD matrix of the responses.

If a pseudo excitation $\tilde{F}_j(t) = \sqrt{S_j(\omega)}e^{i\omega t}$, $(j = 2\sim4)$ is constituted, the corresponding response will be

$$\tilde{\mathbf{u}}(\mathbf{t}) = \int_0^t \mathbf{H}(\mathbf{t}-\tau)\sum_{j=2}^{4}\sqrt{S_j(\omega)}e^{i\omega\tau}d\tau \tag{6}$$

The auto-PSD matrix of the responses can be expressed as

$$\tilde{\mathbf{u}}^*(t)\tilde{\mathbf{u}}^{\mathrm{T}}(t) = \int_0^{t_k}\int_0^{t_l}\mathbf{H}(t_k-\tau_k)\sum_{i=2}^{4}S_{jj}(\omega)e^{i\omega(\tau_l-\tau_k)}\mathbf{H}^T(t_l-\tau_l)d\tau_k d\tau_l = S_{\mathbf{uu}}(\omega,t) \tag{7}$$

The agreement between Eq. (5) and Eq. (7) validates the pseudo excitation method.

3. Constitution of pseudo excitation of the train-bridge coupling system

Suppose track irregularities are zero mean valued random processes. Corresponding to the location of the i-th wheel-set, the track irregularity can be written as (Li 2010, Zhu 2010)

$$\mathbf{F}_i(t) = \alpha(\mathbf{t})\mathbf{F}_i(t-\tau_m) = \alpha(\mathbf{t})[F_i(t-\tau_1)\ F_i(t-\tau_2)\ F_i(t-\tau_3)\ F_i(t-\tau_4)]^{\mathrm{T}}$$

$$(i = 2\sim4,\ m = 1\sim4) \tag{8}$$

The pseudo-excitations are constituted

$$\alpha(\mathbf{t})\sum_{i=2}^{4}\mathbf{Q}e^{i\omega t}\sqrt{S_{ii}(\omega)} \tag{9}$$

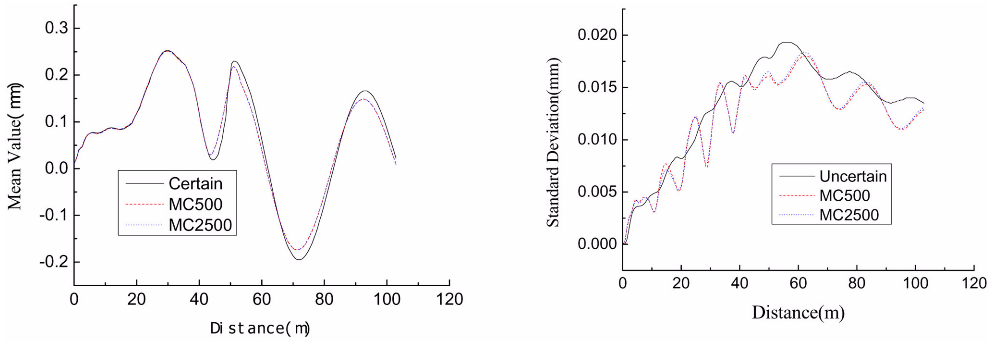
where: $\alpha(\mathbf{t})$ is a transformation matrix, which represents the location of the wheel; $\mathbf{F}(t-\tau)$ is a vector consisting of track irregularity at the wheel-rail contacts; $Q = (e^{-i\omega\tau_1}\ e^{-i\omega\tau_2}\ e^{-i\omega\tau_3}\ e^{-i\omega\tau_4})^T$

There are 15 DOFs in the spatial vehicle analytical model of the train-bridge coupled system, that is, $\{y_c, z_c, \theta_c, \phi_c, \psi_c, y_{t1}, z_{t1}, \theta_{t1}, \phi_{t1}, \psi_{t1}, y_{t2}, z_{t2}, \theta_{t2}, \phi_{t2}, \psi_{t2}\}^T$, in which the subscript c indicates the car body, and the subscript t_1 and t_2 indicate the leading bogie and the trailing bogie. Spatial beam finite element method is applied in the bridge model. Every element node has 6 DOFs: 3 translation DOFs along the three axes x, y, z and 3 rotation DOFs around the three axes. The vertical profile irregularity, the alignment irregularity, and the elevation difference irregularity between left and right rail are regarded as the excitations. PEM is applied to constitute the pseudo excitation of the analytical model. The separate iteration method is adopted to solve the system equations, and then the study of non-stationary random vibrations of train-bridge system is performed.

4. Numerical example

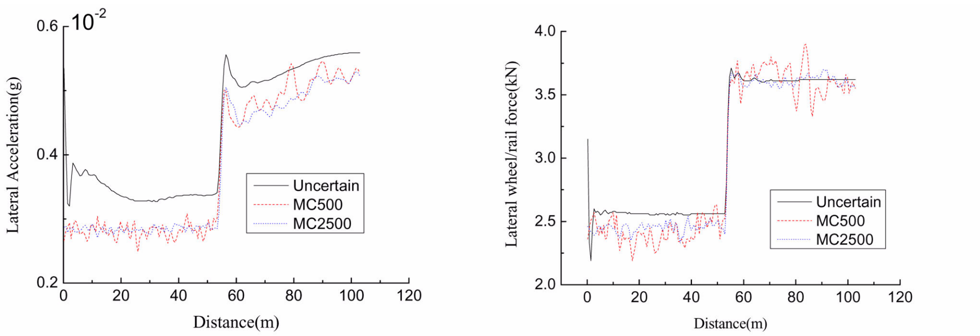
4.1 Basic information

The three-span simply-supported beam bridge (pier height of 15 m) on the Harbin-Dalian passenger special line is selected to analyze the stochastic space dynamic responses of the coupling system. The German Low-interference Track Spectrum is adopted. The German ICE high-speed trains move over the bridge with the speed of 220 km/h, the train is composed of one motor car plus two trailers plus one motor car. The line deviates 2.5 m from the center of the bridge deck. The train will travel 100 meters on the same line as on the bridge before it moves onto the bridge, and will travel 50 meters more when it moves out of the bridge. Two hundred points were taken for the frequency, f, ranging from 0.3 to 60 Hz.



(a) Mean value (b) Standard deviation

Fig. 1 Verification of lateral displacement at bridge mid-span



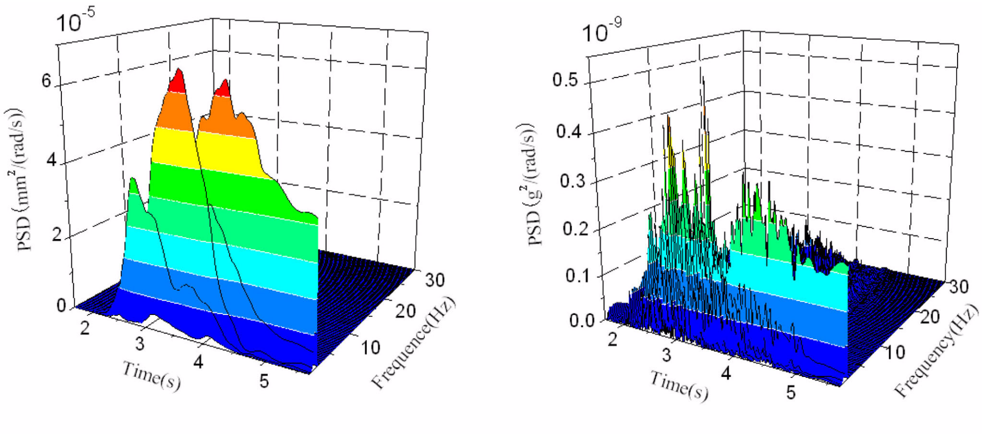
(a) Lateral acceleration of the car body (b) Lateral wheel/rail force acting on the first wheel set of the motor car

Fig. 2 Verification of the vehicle responses

4.2 Verifications of the pseudo excitation method (PEM)

The Monte Carlo simulation has been applied to verify the PEM. Using the Trigonometric Series Method which has been adopted to simulate the track irregularities in (Xia 2005), 500 samples and 2500 samples are calculated by the Monte Carlo simulation. In order to decrease the calculation time, an example of only one motor car running across a single-span simply-supported beam is applied. Fig. 1 shows the mean value and the standard deviation of lateral displacement in the mid-span of bridge. Fig. 2 shows the standard deviation of the motor car responses. In the figures, “Certain” means the responses due to the deterministic load and “Uncertain” means the responses obtained with the pseudo excitation method; “MC500” and “MC2500” mean the sample number is 500 and 2500 in the Monte Carlo simulation, respectively.

Fig. 1 shows good agreement between the two methods. It makes no great difference to the mean value and the standard deviation of the bridge displacement responses with 500 samples simulation and 2500 samples simulation.



(a) Lateral displacement (b) Lateral acceleration

Fig. 3 PSD distribution of bridge midpoint responses

Fig. 2 shows very obvious randomness in the lateral acceleration of the car body and the lateral wheel/rail force, so the simulation with 2500 samples has better close to the result of the PEM than that with 500 samples.

4.3 Power spectral analysis of the train-bridge coupling system

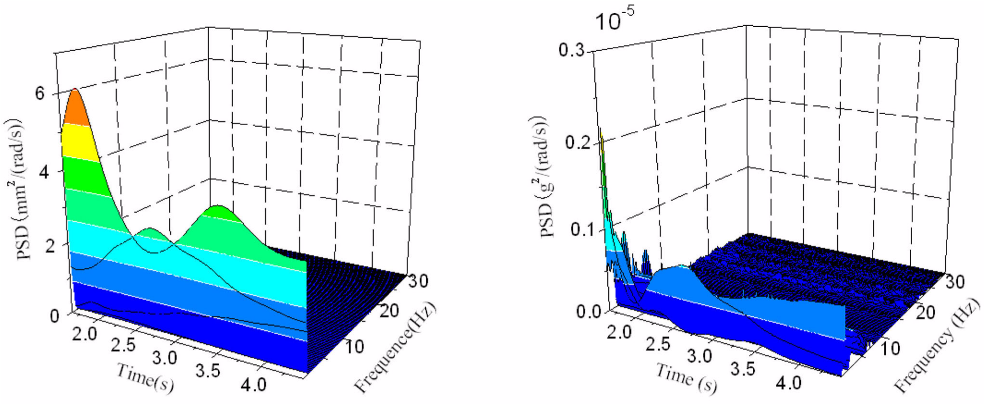
Fig. 3 gives the PSD distributions of the bridge mid-span responses versus time and frequency.

From Fig. 3(a) one can see that the frequency distribution range is from 0.3 Hz to 2.5 Hz as to the PSD of bridge displacement, and the frequency value corresponding to the peak value is 0.9 Hz. However, the first lateral natural frequency of the bridge is 3.33 Hz. As to the PSD of the bridge acceleration, Fig. 3(b) shows that the frequency distribution range is from 0.3 Hz to 30 Hz, and the frequency value corresponding to the peak value is 3.3 Hz.

Shown in Fig. 4 and Fig. 5 are, respectively, the PSD distributions of the car-body responses and the leading bogie responses of the first motor car.

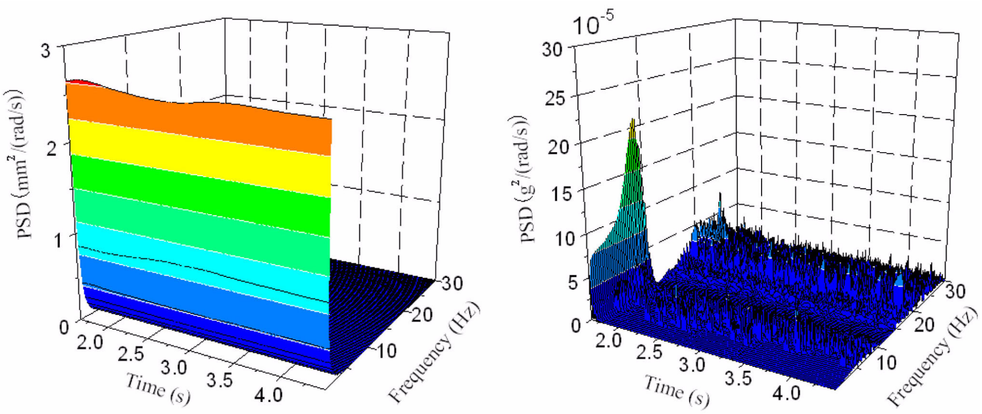
Fig. 4 shows the peak value corresponding frequency is 0.3 Hz for the lateral car-body displacement and acceleration of the motor car. The frequency distribution range is from 0.3 Hz to 2.5 Hz as to the lateral displacement and 0.3 Hz to 30 Hz as to the lateral acceleration. The PSD of lateral displacement varies strongly with time when the frequency is smaller than 1.5 Hz, but varies slightly when the frequency is larger than 1.5 Hz. For the lateral acceleration, the critical frequency is 5 Hz.

Fig. 5 shows the peak value corresponding frequency is 0.3 Hz for the lateral displacement of the leading bogie. The frequency distribution range is from 0.3 Hz to 1.5 Hz as to the PSD of displacement, and the PSD of displacement varies slightly with time. For the PSD of lateral acceleration, the distribution range is from 0.3 Hz to 40 Hz and the peak value corresponding frequency is 7.8 Hz. When the frequency is between 0.3 Hz and 7.8 Hz, the PSD of lateral acceleration reaches its maximum when the motor car is on the bridge, and the PSD varies slightly with time. When the frequency is between 10 Hz and 40 Hz, the PSD of lateral acceleration varies little with time.



(a) Lateral displacement (b) Lateral acceleration

Fig. 4 PSD distribution of the motor car body responses



(a) Lateral displacement (b) Lateral acceleration

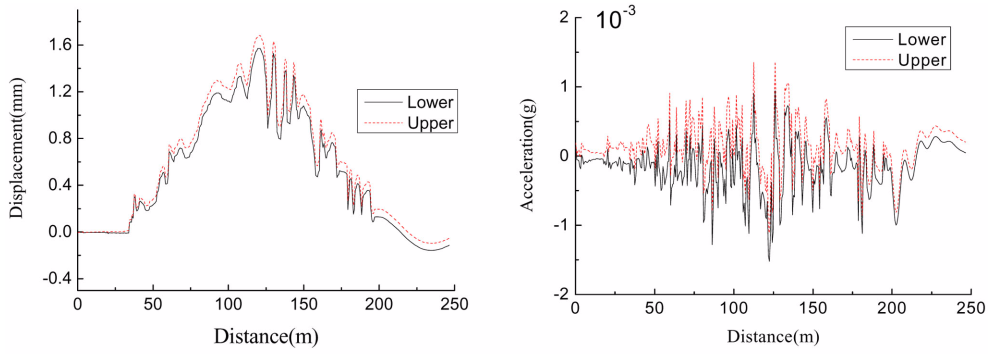
Fig. 5 PSD of the leading bogie responses

4.4 Estimated boundaries of the response in the train-bridge coupled system

Since the track irregularities are assumed to be Gaussian stochastic processes, the 3σ rule is applied to determine the boundaries for the maximum responses, that is, the mean value ± 3 times the standard deviations. Fig. 6 shows the response curves at the mid-span of the bridge.

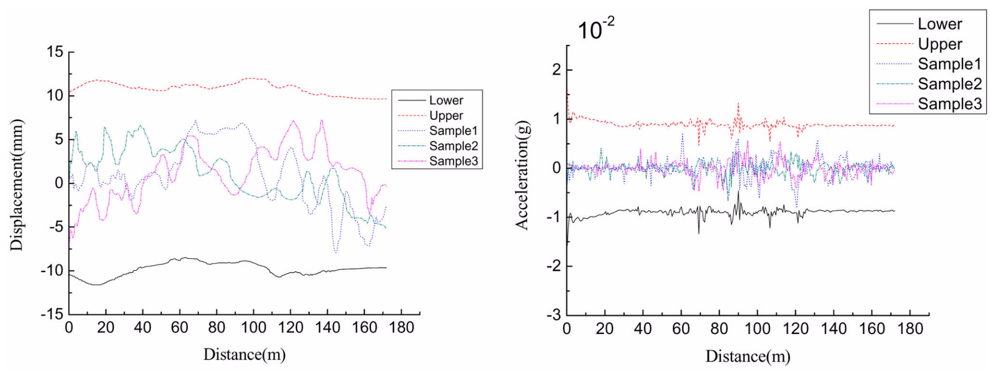
From Fig. 6 one can see that for the bridge displacement, the upper boundary curve and the lower boundary curve are very close, while for the bridge acceleration, most of the upper boundary curve and the lower boundary curve are respectively above and below the x axis when the vehicles is moving across the bridge. It can be concluded that track irregularities have very little influence on the bridge displacement but have a major effect on the acceleration.

Shown in Fig. 7 and Fig. 8 are, respectively, the history curves of the car-body responses and the



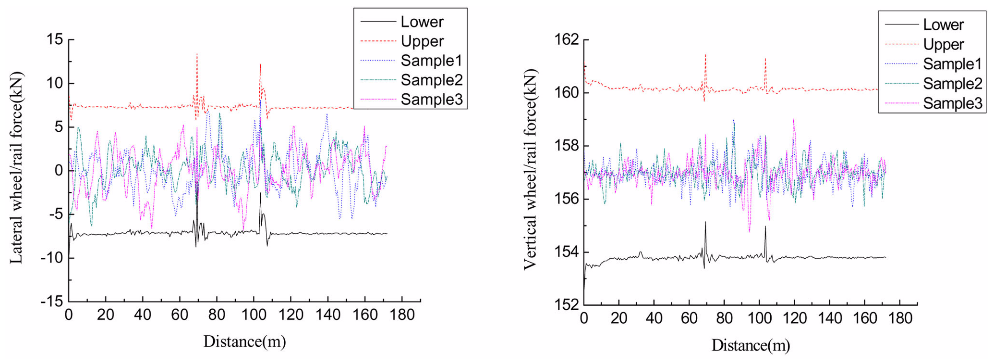
(a) Lateral displacement (b) Lateral acceleration

Fig. 6 Estimated boundaries for bridge mid-span responses



(a) Lateral displacement (b) Lateral acceleration

Fig. 7 Estimated boundaries for motor car body



(a) Lateral wheel/rail force (b) Vertical wheel/rail force

Fig. 8 Estimated boundaries for the wheel/rail force acting on the first wheel-set of motor car wheel/rail forces acting on the first wheel-set of the first motor car.

In Fig. 7, the upper boundary curve and the lower boundary curve as to the vehicle responses are all above and below the horizontal ordinate, respectively, from which one can see that track irregularities are the primary factor leading to the vehicle vibration.

Fig. 8 shows that for the lateral wheel/rail force, the upper boundary curve and the lower boundary curve are respectively above and below the horizontal ordinate, which indicates that track irregularities are the major factor leading to the lateral wheel/rail force. While for the vertical wheel/rail force, the upper boundary curve and the lower boundary curve are very close, which indicates that track irregularities have very little influence on the vertical wheel/rail force. In fact, the self-weight of the vehicle is the primary factor of the vertical wheel/rail force.

5. Conclusions

In this paper, the random vibration characteristic of the train-bridge coupling system has been discussed with the PEM. Comparisons with the Monte Carlo simulations show the effectiveness and satisfactory accuracy of the proposed method. Numerical examples are given and lead to the following conclusions.

(1) The PSD peak value of the bridge mid-span lateral acceleration corresponding frequency is close to its first natural frequency of lateral vibration.

(2) Estimated boundaries for the maximum responses of the train-bridge coupling system can be obtained using the 3σ rule.

(3) Track irregularities have great effect on the bridge lateral acceleration, vehicle vibration and the lateral wheel/rail force.

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