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Dynamics of electric system for electromechanical integrated toroidal drive under mechanical disturbance

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Abstract. Dynamics of the electric system for the toroidal drive under mechanical disturbance is presented. Using the method of perturbation, free vibrations of the electric system under mechanical disturbance are studied. The forced responses of the electric system to voltage excitation under mechanical disturbance are also presented. We show that as the time grows, the resonance vibration caused by voltage excitation still exists and the vibrations caused by mechanical disturbance are enlarged. The coupled resonance vibration caused by mechanical disturbance and voltage excitation is discussed. The conditions of the occurrence of coupled resonance are studied.

Keywords: toroidal drive; electromechanical integrated; dynamics; free vibration; forced response

Toroidal drive can transmit large torque in a very small size and is suitable for top end technical fields such as aviation and space flight, etc (Kuehnle 1966, Kuehnle *et al*. 1981, Tooten 1983, Peeken *et al*. 1984, Tooten 1985). As more and more electrical and control techniques are utilized in mechanical engineering field, generalized composite drives become advancing edge of the mechanical science. So far, types of the generalized composite drives with integrated structure are still very limited.

The electromagnetic harmonic drive (Delin and Huamin 1993) and piezoelectric harmonic one (Barth 2000) are active drives in which the meshing forces between flexible gear and rigid one are controlled by electromagnetic force or piezoelectric one, and drive and power are integrated.

Based on researching toroidal drive (Xu and Huang 2003), Author presented a kind of active generalized composite drive without contact: electromechanical integrated toroidal drive. In the drive, the toroidal drive, power and control are integrated.

The drive consists of four basic elements, Fig. 1: (a) the central worm; (b) radially positioned planets; (c) a toroidal shaped stator; and (d) a rotor, which forms the central output shaft upon which the planets are mounted. The central worm is fixed and coils are mounted in helical grooves of its surface. The planets have permanent magnets instead of teeth. The N and S polar permanent magnets are mounted alternately on a planet. And the stator has helical permanent magnets instead of helical teeth. In the same manner as planet, The N and S polar helical permanent magnets are mounted alternately on the stator.

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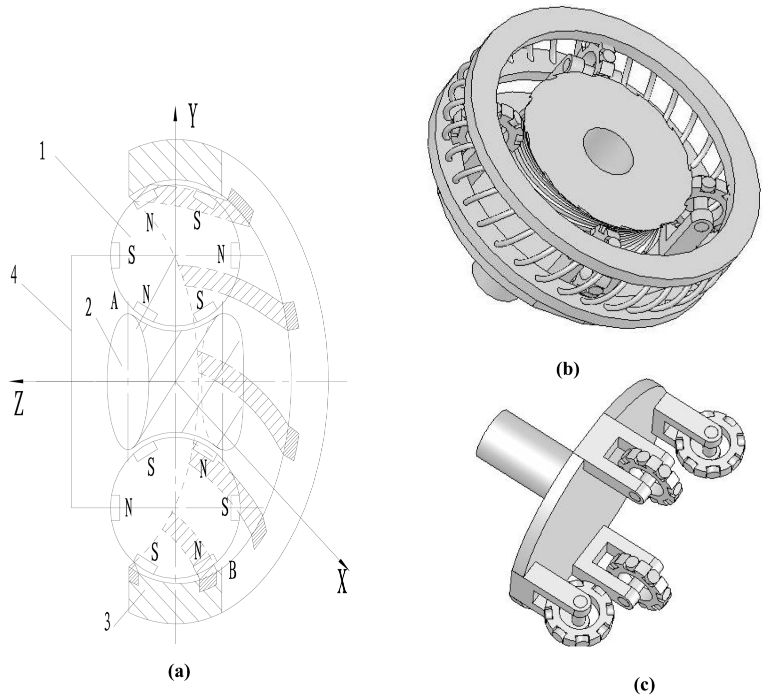


Fig. 1 The electromechanical integrated toroidal drive
(a) Diagram of the drive (1. Planet 2. Worm 3. Stator 4. Rotor) (b) Solid model of the drive (c) Solid model of the rotor parts

If a specific relation between planet pitch, tooth number and lead angle on the stator, and number of pole pairs and lead angle on the worm is realized, N pole of one element will correspond to S pole of the other one all along. The attractive forces between N and S pole of the different elements are driving forces and the meshes without contact are realized. When the alternate current is made in the coils of the worm, a toroidal circular field is formed. It drives several planets rotate about their own axial. And by means of magnetic forces between teeth of the planet and stator, the rotor is driven to rotate about its own axial. Thus, a power of low speed and heavy torque is output.

Compared with toroidal drive, the new drive is easy to produce, without wear, and does not need lubrication. It can be substituted for a servo system to simplify the structure of the existing electromechanical systems. Besides the above-mentioned fields that require compactness, the drive can be used in fields such as robots, etc, that require accurate control.

The electromechanical integrated toroidal drive consists of a mechanical system, an electrical system and a coupled part. The mechanical vibration may occur in the mechanical system, and the electrical current oscillation may occur in the electrical system. With the coupled part, the mechanical vibration and the electrical current oscillation will influence on each other. Hence, the drive system is an electromechanical coupled dynamics system. The electromechanical coupled dynamics was first proposed for the motor (Qiu 1989 and Shatout 1996). Then, the electromechanical coupled dynamics of the electromechanical system consisting of the motor and mechanical system driven by the motor was developed. An electromechanical coupled dynamics model of the electromechanical system consisting of the several motors and mechanical system driven by these motors was proposed. Using the model, the natural frequency of the electromechanical system is analyzed (Kanaan *et al*. 2003). The characteristics of electromechanical coupling self-synchronization of a multi-motor vibration transmission system was investigated (Xiong *et al*. 2001). The electromechanical coupled dynamics of the electromechanical system consisting of the motor and elastic link mechanism was discussed (Li *et al.* 2006). Recently, the electromechanical-fluidic coupled dynamics problem of the piezoceramic plates in fluid was also investigated. However, the electromechanical coupled dynamics of the electromechanical integrated toroidal drive has not been developed yet. To design, evaluate and control dynamics behavior of the drive system effectively, the electromechanical coupled dynamics of the drive system should be investigated.

In this paper, based on electromagnetics and mechanics principle, electromechanical coupled dynamic equations for the drive are developed. And the dynamics of the electric system for the drive under mechanical disturbance is presented. Dynamic equations of the electric system are transmitted into equations independent on each other. And then using method of perturbation, free vibrations of the electric system for the drive under mechanical disturbance are given. The forced responses of the electric system to voltage excitation under mechanical disturbance are also discussed. It is known that for the system with mechanical disturbance, as time grows, small exciting wave still exists when the wave magnitude is tending to zero which is unfavorable to electric system. When mechanical disturbing frequency is near to the natural frequencies of the electric system or their integer multiple, resonance vibrations occur in the electric system. As time grows, vibration quality of the electric system decreases gradually and the resonance vibration will vanish. So the resonance vibration caused by mechanical disturbance is transient. The forced responses of the electric system to voltage excitation under mechanical disturbance are compound vibrations which are decided by voltage excitation, mechanical disturbance and parameters of the electric system in common. As the time grows, the resonance vibration caused by voltage excitation still exists and the vibrations caused by mechanical disturbance are enlarged as well. Last, the coupled resonance vibration caused by mechanical disturbance and voltage excitation is discussed as well. The condition under which above coupled resonance occurs is presented. The coupled resonance is more unfavorable to electric system. These results are useful in design and manufacture of the drive.

1. Electromechanical coupled dynamic equations

The electromechanical coupled dynamic model of an electromechanical integrated toroidal drive is shown in Fig. 2. Space phase relation of the coils on the worm is shown in Fig. 3. Symbol ϕ_v denotes face width angle of the worm. Then, in the transverse plane of the planet, the phase angle of the phase-i is

$$\phi_i = \frac{i-1}{n}\frac{\phi_v}{p} \qquad (i = 1 \text{ to n})$$

Where n-phase number of the worm coils ($n = 3$)
p-the number of the pole-pairs

Let position angle $\theta = 0$ of the planet when tooth of the planet is aligned completely with phase-1

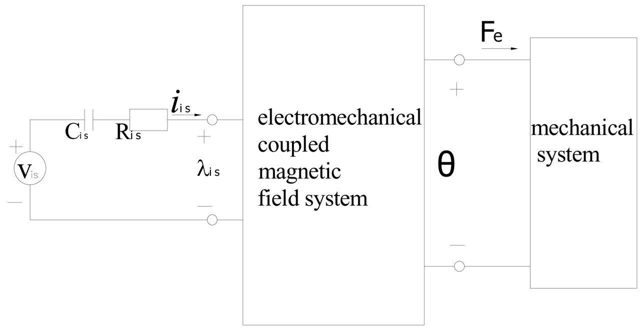


Fig. 2 Dynamic model of an electromechanical integrated toroidal drive

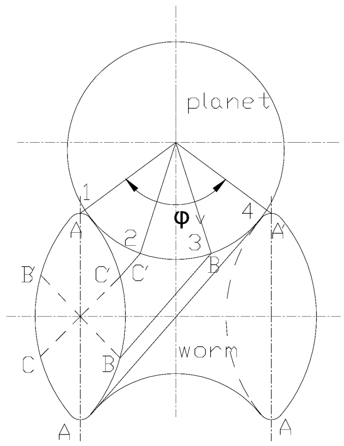


Fig. 3 Space phase relation of the coils on the worm

coil. Let the tooth number of the planet equal z_1. Inductances of the every phase can be calculated as shown in Appendix.

The differential equations of the electrical system of the drive are as follows

$$v_{is} = R_{is}i_{is} + \frac{1}{C_i}\int i_{is}dt + \frac{d\lambda_{is}}{dt} \qquad (i = 1 \text{ to n}) \tag{1}$$

Where v_{is}, R_i and C_i are voltage, resistance and capacitance of the i th phase worm coil, respectively. If n phase alternate currents are made in the coils, n differential equations are obtained.

The magnetic linkage λ_{is} of the coils on the fixed central worm can be calculated as follows

$$\lambda_{is} = \sum_{j=1}^{n} L_{ij}i_{is} \tag{2}$$

Let q_i denote quantity, then $i_{is} = \frac{dq_i}{dt}$ Substituting Eq. (2) and $i_{is}\ \frac{dq_i}{dt}$ into Eq. (1), yields

$$v_{is} = \sum_{j=1}^{n} L_{ij}\frac{d^2q_j}{dt^2} + R_{is}\frac{dq_i}{dt} + \sum_{j=1}^{n}\frac{\partial L_{ij}}{\partial\theta}\frac{\partial\theta}{dt}\frac{dq_j}{dt} + \frac{q_i}{C_i} \tag{3}$$

Hence, the differential equations of the electric system for the drive can be given in matrix form as

$$\mathbf{L}_v \ddot{\mathbf{q}} + \mathbf{R}_v \dot{\mathbf{q}} + \mathbf{C}_v \mathbf{q} = \mathbf{v}_s \tag{4}$$

The quality and voltage vectors, $\mathbf{q}$ and $\mathbf{v}_s$, and equivalent inductance, resistance and capacitance matrices, $\mathbf{L}_v$, $\mathbf{R}_v$ and $\mathbf{C}_v$, are given in Appendix.

In Eq. (4), $\frac{\partial \theta}{dt}$ is rotational speed of the relative vibration between planet and worm, and it shows influence of the mechanical system to electric system.

The equations of motion of the mechanical system for the drive are given in matrix form as

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{C}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F} \tag{5}$$

Where $\mathbf{X}$ and $\mathbf{F}$ are displacement and load vectors, respectively; $\mathbf{M}$, $\mathbf{K}$ and $\mathbf{C}$ are mass, stiffness and damping matrixes, respectively.

In Eq. (5), stiffness matrix $\mathbf{K}$ includes electromagnetic meshing stiffness and shows influence of the electric system to mechanical system.

Therefore, Combining Eq. (4) with Eq. (5), the electromechanical coupled dynamic equations of the drive system are obtained. In this paper, dynamics of the electric system for the drive under mechanical disturbance is developed.

2. Free vibration of the electric system under mechanical disturbance

The Eq. (4) decides a team of equations which are coupled to each other. For simplicity purposes, the formulation (4) should be transmitted into equations independent on each other. Then, Eq. (4) is changed into the following form

$$\mathbf{L}_{vN} \ddot{\mathbf{q}}_N + \mathbf{R}_{vN} \dot{\mathbf{q}}_N + \mathbf{C}_{vN} \mathbf{q}_N = \mathbf{v}_{sN} \tag{6}$$

Where $\mathbf{L}_{vN}$, $\mathbf{R}_{vN}$ and $\mathbf{C}_{vN}$ are regular diagonal equivalent inductance, resistance and capacitance matrices, respectively. $\mathbf{v}_{sN}$ and $\mathbf{q}_N$ are transmitted equivalent exciting voltage and quality vectors, respectively (see Appendix).

Thus, Eq. (6) includes n equations independent on each other. The i th equation can be expressed as below

$$a_i \ddot{q}_{Ni} + b_i \dot{q}_{Ni} + c_i q_{Ni} = v_{Ni} \qquad (i = 1 \text{ to } n) \tag{7}$$

Where $a_i = 1$, $b_i = (R_i + \varepsilon L_i \,\omega_m \cos \omega_m t)$, $c_i = \omega_i^2$.

In Eq. (7), b_i changes along with time t. So this is a variable coefficient differential equation. For Eq. (7), following transformation is done

$$q_i = x e^{-\frac{1}{2}\int \frac{b_i(t)}{a_i} dt} \tag{8}$$

Hence, Eq. (7) is changed as below

$$\ddot{x} + \alpha(t) x = v' \tag{9}$$

Where $\alpha(t) = \frac{c_i}{a_i} - \frac{1}{4}\frac{b_i^2(t)}{a_i^2} - \frac{1}{2}\frac{d}{dt}\left[\frac{b_i(t)}{a_i}\right]$, $v' = v_{Ni}e^{\int \frac{b_i(t)}{2a_i}dt}$

Changes of the angle θ between planet and worm for the mechanical system are as below

$$\theta = \theta_0 + \Delta\theta \sin(\omega_m t) \quad (10)$$

Where θ_0 is static relative rotating angle between planet and worm, $\Delta\theta$ is magnitude of the dynamic rotating displacement, ω_m is frequency of the angular vibration between planet and worm.
Eq. (10) are substituted into above equations Eq. (9)and then Eq. (9) can be transformed into following form

$$\ddot{x} + \delta(1 + \varepsilon_1 \cos 2\omega_m t + \varepsilon_2 \cos\omega_m t + \varepsilon_3 \sin\omega_m t)x = v' \quad (11)$$

From analysis, it is known that ε_1 and ε_2 are much smaller than ε_3. So ε_1 and ε_2 can be neglected. And Eq. (11) is simplified as (here $\varepsilon = \varepsilon_3$)

$$\ddot{x} + \delta(1 + \varepsilon \sin\omega_m t)x = v' \quad (12)$$

As only free vibration of the electric system under mechanical disturbance is discussed, let $v' = 0$ in Eq. (12), then

$$\ddot{x} + \delta(1 + \varepsilon \sin\omega_m t)x = 0 \quad (13)$$

Let

$$x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \cdots\cdots \quad (14)$$

and

$$\omega^2 = \delta(1 + \varepsilon\sigma_1 + \varepsilon^2\sigma_2 + \cdots\cdots) \quad (15)$$

Let $\tau = \omega t$, and substituting Eqs. (14) and (15) into Eq. (13), and then let sum of the coefficients with the same order power of the parameter ε equal zero, following equations can be given

$$\ddot{x}_0 + x_0 = 0 \quad (16a)$$

$$\ddot{x}_1 + x_1 = x_0 \sin e\tau - \sigma_1\ddot{x}_0 \quad (16b)$$

$$\ddot{x}_2 + x_2 = x_1 \sin e\tau - \sigma_1\ddot{x}_1 - \sigma_2\ddot{x}_0 \quad (16c)$$

Where $e = \frac{\omega_m}{\omega}$, and $\ddot{x}_1$ is the second order derivative of the variable x_i to new variable $\tau = \omega t$.

Here, initial conditions are

$$\begin{cases} x_0(0) = A \ \dot{x}_0(0) = 0 \\ x_1(0) = 0 \ \dot{x}_1(0) = 0 \\ x_2(0) = 0 \ \dot{x}_2(0) = 0 \\ \cdots\cdots \end{cases}$$

Then, solution of zero order equation under above initial conditions is

$$x_0 = A\cos\tau = A\cos\omega t$$

Substituting Eq. (17) into Eq. (16b), yields

$$\ddot{x}_1 + x_1 = \frac{A}{2}[\sin(e+1)\tau + \sin(e-1)\tau] + A\sigma_1\cos\tau \tag{18}$$

In order to remove secular item, let $\sigma_1 = 0$, then

$$x_1 = -\frac{A}{2[(e+1)^2-1]}[\sin(e+1)\tau - \sin\tau] - \frac{A}{2[(e+1)^2-1]}[\sin(e-1)\tau - \sin\tau] \tag{19}$$

Substituting Eqs. (17) and (19) into Eq. (16c), yields

$$\ddot{x}_2 + x_2 = A\left[\sigma_2 - \frac{1}{4[(e+1)^2-1]} - \frac{1}{4[(e-1)^2-1]}\right]\cos\tau + \frac{1}{4[(e+1)^2-1]}\cos(2e+1)\tau$$
$$+\frac{1}{4[(e-1)^2-1]}\cos(2e-1)\tau + \left\{\frac{1}{4[(e+1)^2-1]} + \frac{1}{4[(e-1)^2-1]}\right\}[\cos(e-1)\tau - \cos(2e+1)\tau] \tag{20}$$

In order to remove secular item, let $\sigma_2 = \dfrac{A}{4[(e+1)^2-1]} + \dfrac{A}{4[(e-1)^2-1]}$, then

$$x_2 = -\frac{A}{4[(e+1)^2-1][(2e+1)^2-1]}[\cos(2e+1)\tau - \cos\tau] -$$
$$\frac{A}{4[(e-1)^2-1][(2e-1)^2-1]}[\cos(2e-1)\tau - \cos\tau]$$
$$-\left[\frac{A}{4[(e+1)^2-1]} + \frac{A}{4[(e-1)^2-1]}\right]\left[\frac{\cos(e-1)\tau - \cos\tau}{(e-1)^2-1} - \frac{\cos(e+1)\tau - \cos\tau}{(e+1)^2-1}\right] \tag{21}$$

In a same manner, the solution of n th order equation under above initial conditions can be obtained as well, so the solution of the Eq. (13) is

$$x = A\cos\omega t - \varepsilon\left\{\frac{A}{2[(e+1)^2-1]}[\sin(e+1)\omega t - \sin\omega t] - \frac{A}{2[(e-1)^2-1]}[\sin(e-1)\omega t - \sin\omega t]\right\}$$
$$-\varepsilon^2\left\{\frac{A}{4[(e+1)^2-1][(2e+1)^2-1]}[\cos(2e+1)\omega t - \cos\omega t] + \frac{A}{4[(e-1)^2-1][(2e-1)^2-1]}\right.$$
$$\cdot[\cos(2e-1)\omega t - \cos\omega t] +$$
$$\left.\left\{\frac{A}{4[(e+1)^2-1]} + \frac{A}{4[(e-1)^2-1]}\left[\frac{\cos(e-1)\omega t - \cos\omega t}{(e-1)^2-1} - \frac{\cos(e+1)\omega t - \cos\omega t}{(e+1)^2-1}\right]\right\}\right.$$
$$+\cdots\cdots \tag{22}$$

From Eq. (15), natural frequency of the electric system can be given as well

$$\omega^2 = \delta\left\{1 + \frac{\varepsilon^2 A}{4[(e+1)^2 - 1]} + \frac{\varepsilon^2 A}{4[(e-1)^2 - 1]} + \cdots\cdots\right\} \tag{23}$$

Substituting Eq. (22) into Eq. (8), and then using equation $\mathbf{q} = \mathbf{A}_N\,\mathbf{q}_N$, the free quality vibration q_i can be given.

3. Forced response of the electric system to voltage excitation under mechanical disturbance

In order to analyze forced responses of the electric system to electric excitation under mechanical disturbance, expressing Eq. (7) as following form

$$\ddot{q}_{Ni} + (R_i + \varepsilon L_i \omega_m \cos\omega_m t)\dot{q}_{Ni} + \omega_i^2 q_{Ni} = v_{Ni} \tag{24}$$

Where voltage excitation is given as $v_{Ni} = \varepsilon\, v_0 \sin\left(\omega_e t + \frac{i-1}{3}\right)$, ω_e is exciting frequency of the voltage,

Let v_0 is magnitude of the exciting voltage.

$$q_i = q_0 + \varepsilon q_i + \varepsilon^2 q_2 + \cdots\cdots \tag{25}$$

Substituting Eq. (24) into Eq. (23), and then let sum of the coefficients with the same order power of the parameter equal zero, following equations can be given

$$\begin{cases} \ddot{q}_0 + R_i\dot{q}_0 + \omega_i^2 q_0 = 0 \\ \ddot{q}_1 + R_i\dot{q}_1 + \omega_i^2 q_1 = L_i\omega_m\cos\omega_m t\dot{q}_0 + v_0\sin\left(\omega_e t + \frac{i-1}{3}\pi\right) \\ \ddot{q}_2 + R_i\dot{q}_2 + \omega_i^2 q_2 = L_i\omega_m\cos\omega_m t\dot{q}_1 \\ \cdots\cdots \end{cases} \tag{26}$$

In a same manner as solving Eq. (16), the solutions of the Eq. (26) can be obtained as well, and then substituting the solutions into Eq. (25), and then using equation $\mathbf{q} = \mathbf{A}_N\,\mathbf{q}_N$, forced vibration quality q_i can be obtained.

4. Results and discussions

4.1 Free vibrations under mechanical disturbance

Eqs. (8) and (21) are used for free vibration analysis of the electric system for the drive under mechanical disturbance. Parameters of the numerical example are shown in Table 1. Free vibrations

Table 1 Parameters of the example system

ϕ_v (^{0})	$\Delta\theta$ (*rae*)	ω_m (*rad/s*) *rad*	*p*	*n*	L_0 (*H*)	L_1 (*H*)	C_i (F)	R_i (Ω)	ε
80	0.007	10^7	**1**	**3**	10^{-3}	10^{-3}	1.6×10^{-10}	10	**0.01**

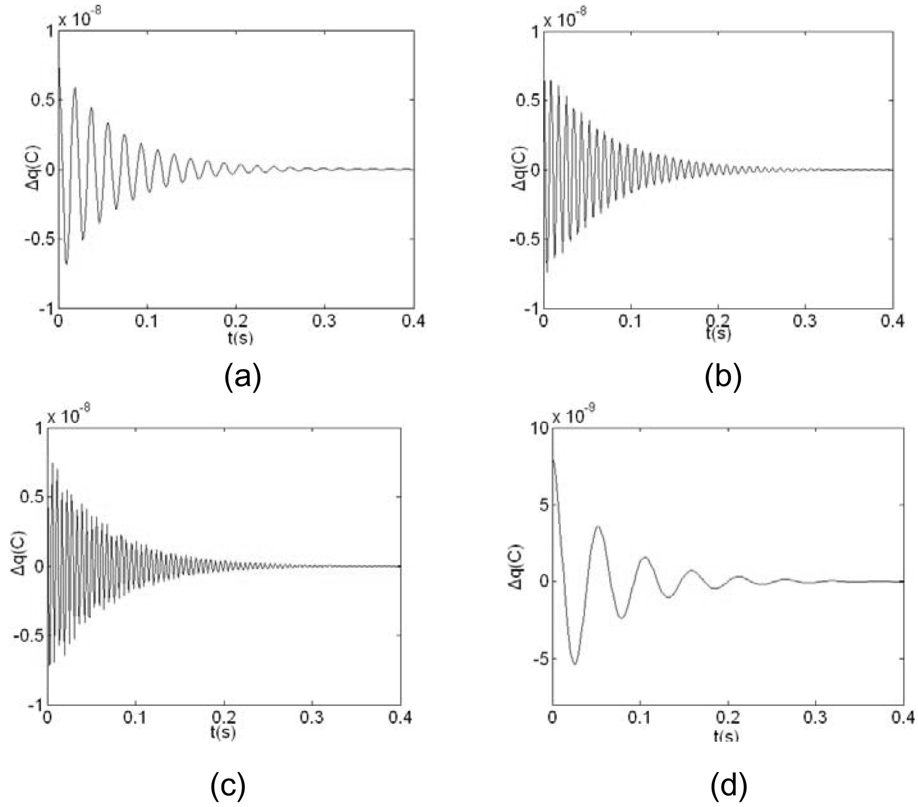
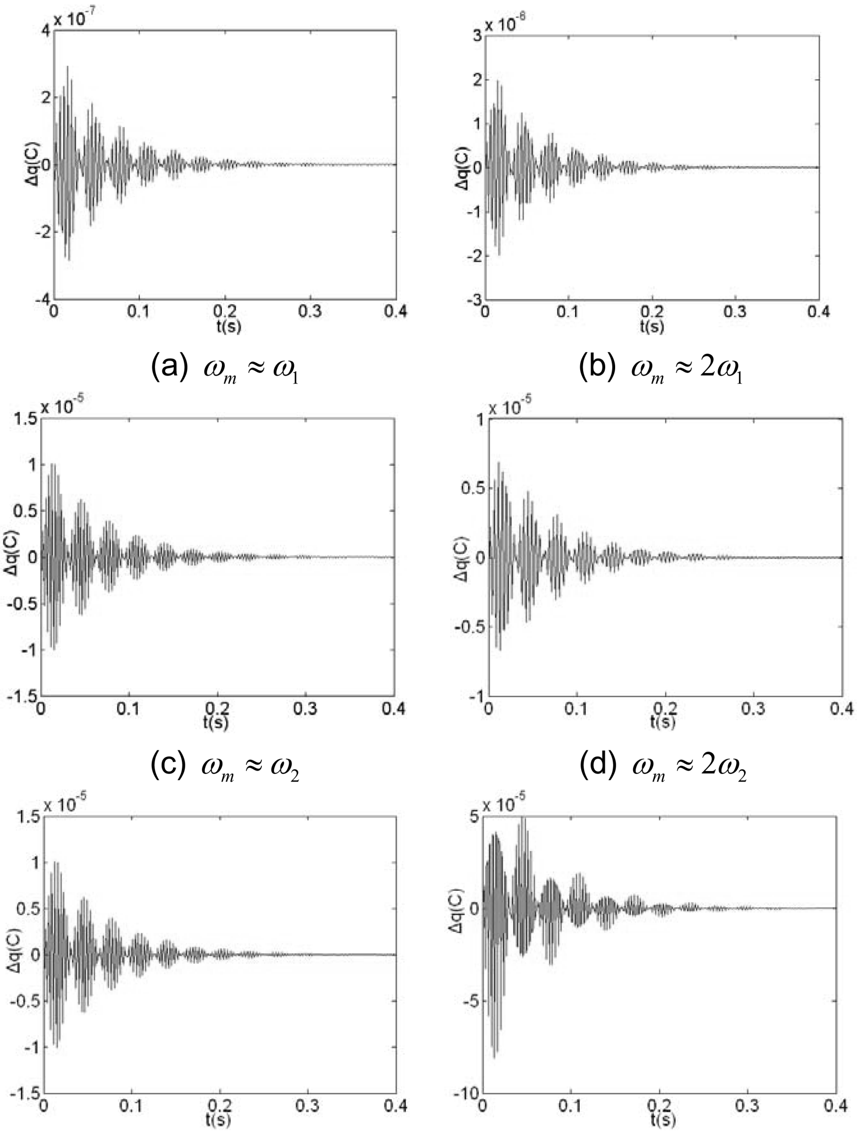


Fig. 4 Free vibrations of the electric system under mechanical disturbance

are shown in Fig. 4. Here, initial quality *A* is 8.0×10^{-9} *C*. Free vibration analysis without mechanical disturbance shows that natural frequencies of above electric system are $\omega_1 = 0.10869 \times 10^7$ *rad/s*, $\omega_2 = 0.4135 \times 10^7$ *rad/s* and $\omega_3 = 4.1430 \times 10^7$ *rad/s.* Fig. 4 shows results under condition that mechanical disturbing frequency is far from natural frequencies of the electric system (here $\omega_m = 1 \times 10^7$ *rad/s*). Fig. 4(a) shows results of the first mode ($\omega_1 = 0.10869 \times 10^7$ *rad/s*); Fig. 4(b) shows results of the second mode ($\omega_2 = 0.4135 \times 10^7$ *rad/s*); and Fig. 4(c) shows results of the third mode ($\omega_3 = 4.1430 \times 10^7$ *rad/s*). Fig. 4(d) shows results without mechanical disturbance. Fig. 5 shows results under condition that the mechanical disturbing frequency is near to the natural frequencies of the electric system. Fig. 5(a) shows results under condition that the mechanical disturbing frequency is near to the first natural frequency ($\omega_m \approx \omega_1$) of the electric system; Fig. 5(b) shows results under condition that the mechanical disturbing frequency is near to the double of the first natural frequency ($\omega_m \approx 2\omega_1$) of the electric system. Fig. 5(c) shows results under condition that the mechanical disturbing frequency is near to the second natural frequency ($\omega_m \approx \omega_2$) of the electric system; Fig. 5(d) shows results under condition that the mechanical



(a) $\omega_m \approx \omega_1$ (b) $\omega_m \approx 2\omega_1$

(c) $\omega_m \approx \omega_2$ (d) $\omega_m \approx 2\omega_2$

(e) $\omega_m \approx \omega_3$ (f) $\omega_m \approx 2\omega_3$

Fig. 5 Resonance vibrations of the electric system under mechanical disturbance

disturbing frequency is near to the double of the second natural frequency $(\omega_m \approx 2\omega_2)$ of the electric system. Fig. 5(e) shows resulst under condition that the mechanical disturbing frequency is near to the third natural frequency $(\omega_m \approx \omega_3)$ of the electric system; Fig. 5(f) shows results under condition that the mechanical disturbing frequency is near to the double of the third natural frequency $(\omega_m \approx 2\omega_3)$ of the electric system. From Figs.4 and 5, ones know:

(1) As time grows, vibration quality of the electric system decreases gradually. The vibration of the quality is periodical damped wave. Magnitude of the wave is decided by initial excitation and time. The decaying speed is decided by damping term R_i.

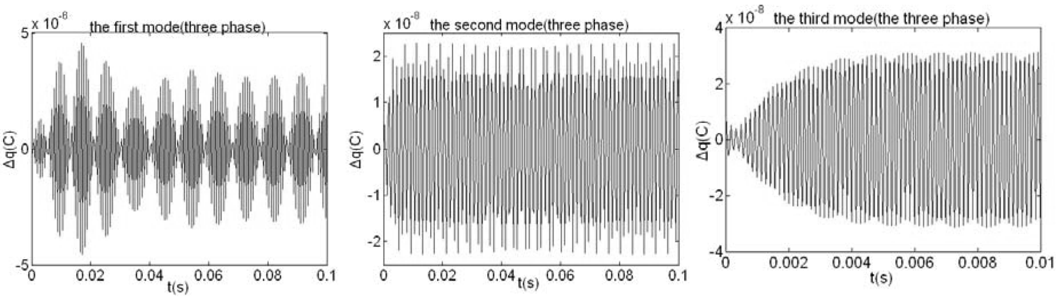
(2) For higher natural frequency of the system, periodic time of the quality vibration is shorter. So for the natural frequency 4.1430×10^7 *rad/s*, the shortest periodic time of the quality vibration occurs. It is because the periodic time of the quality vibration for the electric system is decided by natural frequency of the electric system and disturbing frequency of the mechanical system.

(3) For the system without mechanical disturbance, the periodic time of the quality vibration is longest. The periodic time of the quality vibration is only decided by natural frequency of the electric system.

(4) For the system with mechanical disturbance, as time grows, small exciting wave still exists when the wave magnitude is tending to zero. This is unfavorable to electric system.

(5) When mechanical disturbing frequency is near to the natural frequencies of the electric system or their integer multiple, resonance vibrations occur in the electric system.

(6) The resonance vibrations are more obvious under condition that disturbing frequency is near to the natural frequencies of the electric system than those under condition that disturbing frequency is near to the integer multiple of the natural frequencies of the electric system.



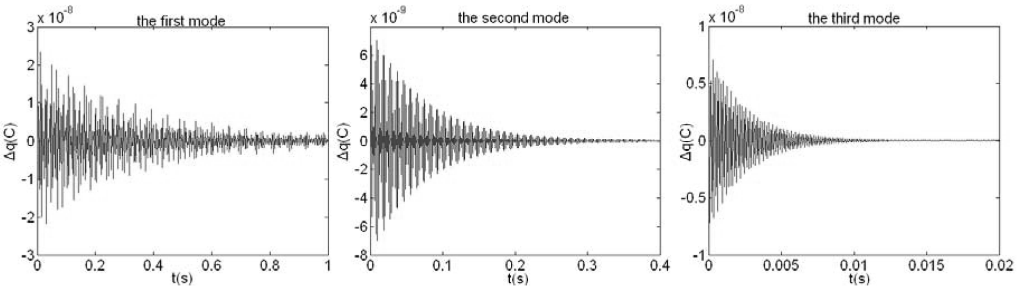
(a) $\omega_e \approx \omega_1$ (b) $\omega_e \approx \omega_2$ (c) $\omega_e \approx \omega_3$

Fig. 6 Forced responses of the electric system to voltage excitation under mechanical disturbance

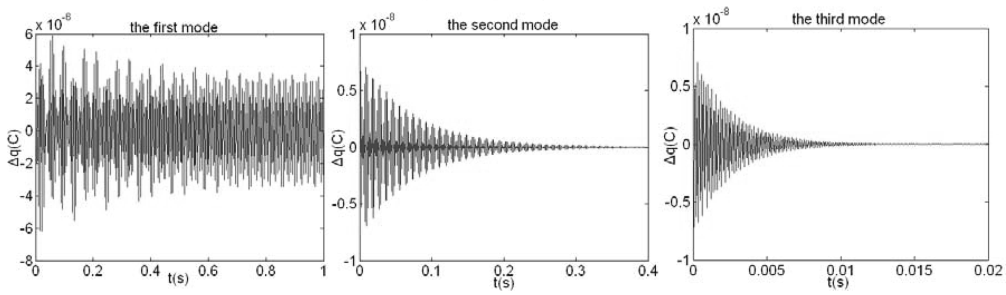
Table 2 Changes of the resonance vibrations along with disturbance and excitation frequency

ω_m	ω_e	resonance vibration from disturbance	coupled resonance vibration from disturbance and excitation
$\omega_m = 2\omega_1$	far from ω_i (i = 1, 2 and 3)	occur for the first mode	do not occur
	$\omega_e = \omega_1$	do not occur	occur for the first mode
	$\omega_e = \omega_2$	occur for the first mode	occur for the second mode
	$\omega_e = \omega_3$	occur for the first mode	occur for the third mode
$\omega_m = 2\omega_2$	far from ω_1 (i = 1, 2 and 3)	occur for the first and second modes	do not occur
	$\omega_e = \omega_1$	occur for the first mode	occur for the second mode
	$\omega_e = \omega_2$	do not occur	occur for the second mode
	$\omega_e = \omega_3$	occur for the second mode	occur for the third mode
$\omega_m = 2\omega_3$	far from ω_i (i = 1, 2 and 3)	occur for the first and third modes	do not occur
	$\omega_e = \omega_1$	occur for the third mode	occur for the first mode
	$\omega_e = \omega_2$	occur for the third mode	occur for the second mode
	$\omega_e = \omega_3$	do not occur	occur for the third mode

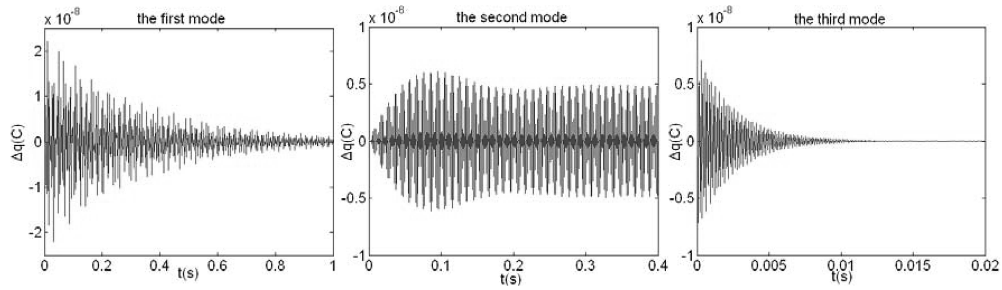
(7) As time grows, vibration quality of the electric system decreases gradually and the resonance vibration will vanish. So the resonance vibration caused by mechanical disturbance is transient, but their hazard is assignable.



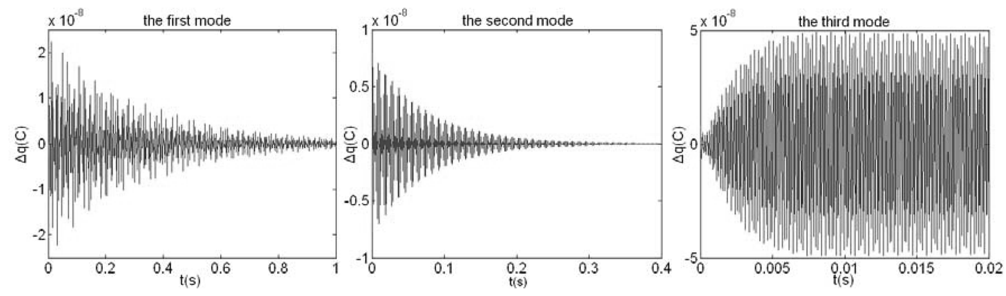
(a) $\omega_m = 2\omega_1$, $\omega_e = 0.001\times10^7$



(b) $\omega_m = 2\omega_1$, $\omega_e = \omega_1$



(c) $\omega_m = 2\omega_1$, $\omega_e = \omega_2$

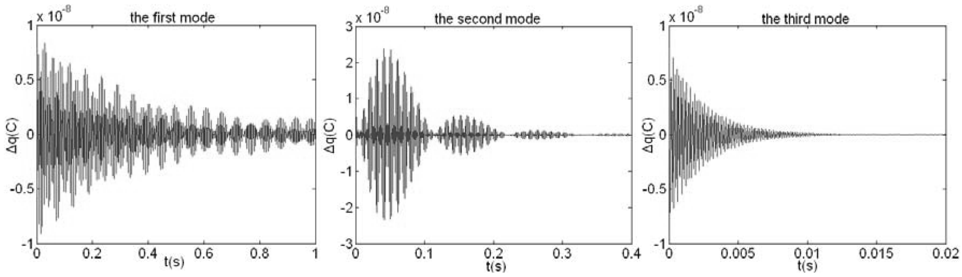


(d) $\omega_m = 2\omega_1$, $\omega_e = \omega_3$

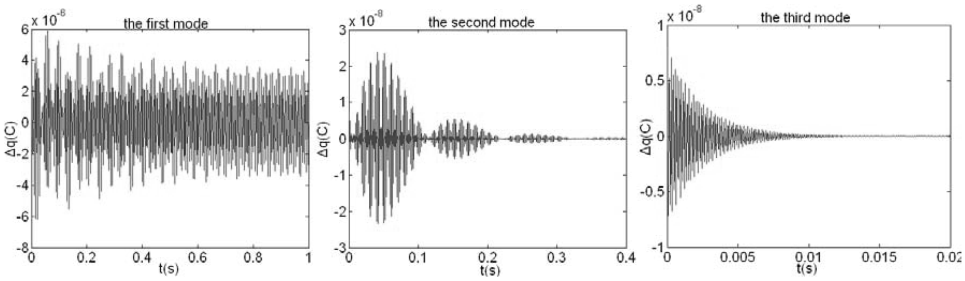
Fig. 7 Coupled vibrations under mechanical disturbance and voltage excitation ($\omega_m = 2\omega_1$)

4.2 Forced responses to electric excitation under mechanical disturbance

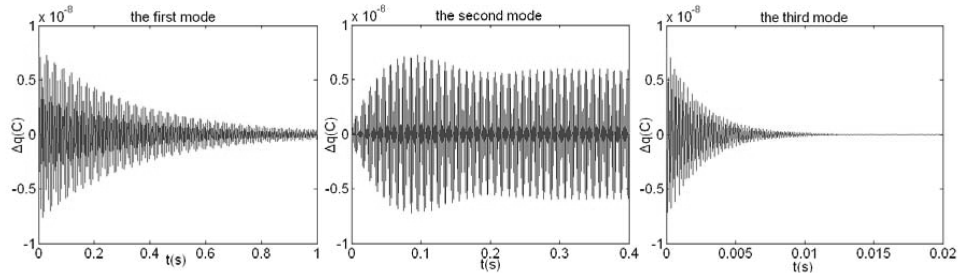
Eqs. (26), (27) and (24) are used for forced response analysis of the electric system to voltage excitation under mechanical disturbance. The forced responses are shown in Fig. 6. Fig. 6(a) shows



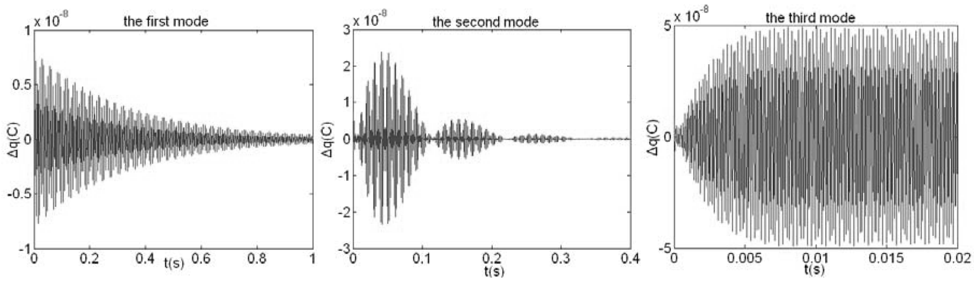
(a) $\omega_m = 2\omega_2$, $\omega_e = 0.001\times 10^7$



(b) $\omega_m = 2\omega_2$, $\omega_e = \omega_1$



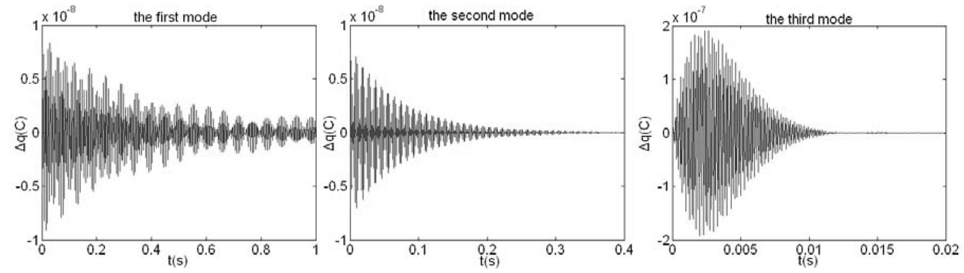
(c) $\omega_m = 2\omega_2$, $\omega_e = \omega_2$



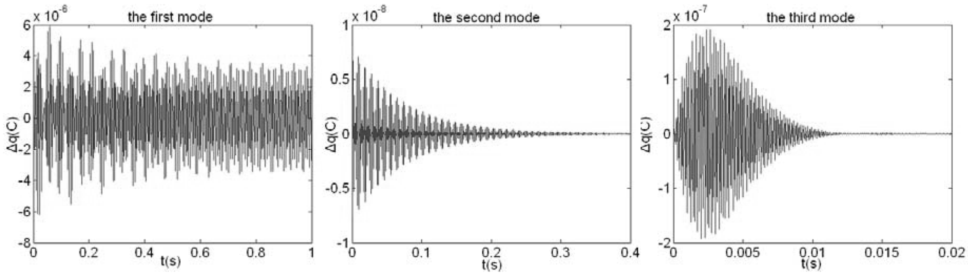
(d) $\omega_m = 2\omega_2$, $\omega_e = \omega_3$

Fig. 8 Coupled vibrations under mechanical disturbance and voltage excitation ($\omega_m = 2\omega_2$)

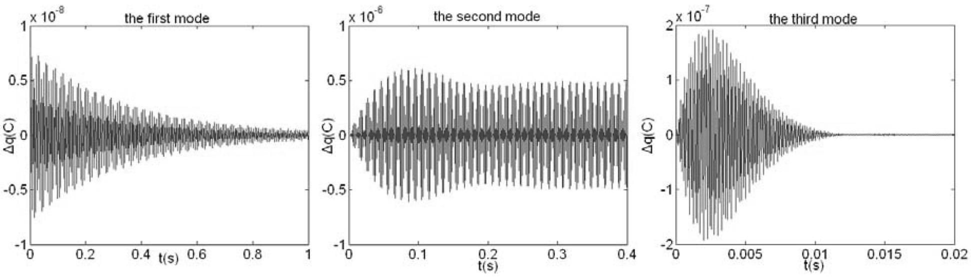
results under condition that exciting frequency is near to the first natural frequency ($\omega_e \approx \omega_1$); Fig. 6(b) shows results under condition that exciting frequency is near to the second natural frequency ($\omega_e \approx \omega_2$); Fig. 6(c) shows results under condition that exciting frequency is near to the third natural frequency ($\omega_e \approx \omega_3$). Here, only results of the first mode (ω_1 = 0.10869×10^7 *rad/s*) are presented. From Fig. 6, ones know:



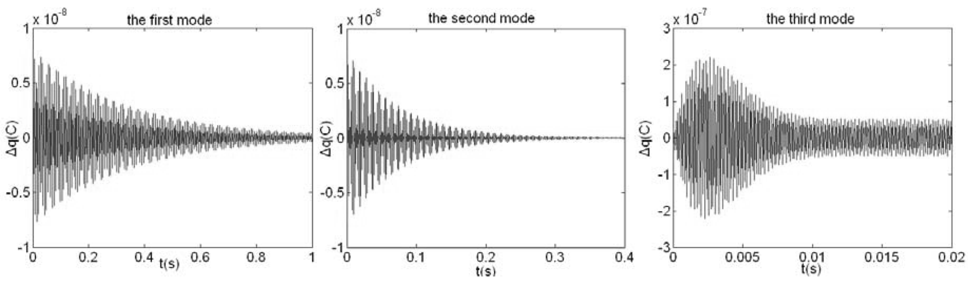
(a) $\omega_m = 2\omega_3$, $\omega_e = 0.001\times10^7$



(b) $\omega_m = 2\omega_3$, $\omega_e = \omega_1$



(c) $\omega_m = 2\omega_3$, $\omega_e = \omega_2$



(d) $\omega_m = 2\omega_3$, $\omega_e = \omega_3$

Fig. 9 Coupled vibrations under mechanical disturbance and voltage excitation ($\omega_m = 2\omega_3$)

(1) When voltage exciting frequency ω_e is near to one of the natural frequencies for the electric system, the resonance occurs in the electric system.

(2) As the time grows, the resonance vibration caused by voltage excitation still exist which is different from ones caused by mechanical disturbance.

(3) The forced responses of the electric system to voltage excitation under mechanical disturbance are compound vibrations which are decided by voltage excitation, mechanical disturbance and natural frequency of the electric system.

(4) As vibrations caused by voltage excitation exist, the vibrations caused by mechanical disturbance are enlarged.

Under condition that voltage exciting frequency is near to the natural frequencies and the mechanical disturbing frequency ω_m is near to one-half of them for the electric system, the coupled resonance occurs in the electric system as shown in Figs. 7-9. From Figs. 7-9, ones know: Under the mechanical disturbance and the voltage excitation, the resonance vibration from mechanical disturbance and the coupled resonance vibration from mechanical disturbance and voltage excitation may occur. Changes of the two types of the resonance vibration along with disturbance frequency and the excitation frequency are summarized in Table 2. The coupled resonance vibrations are much larger than those caused only by mechanical disturbance or voltage excitation and they are more

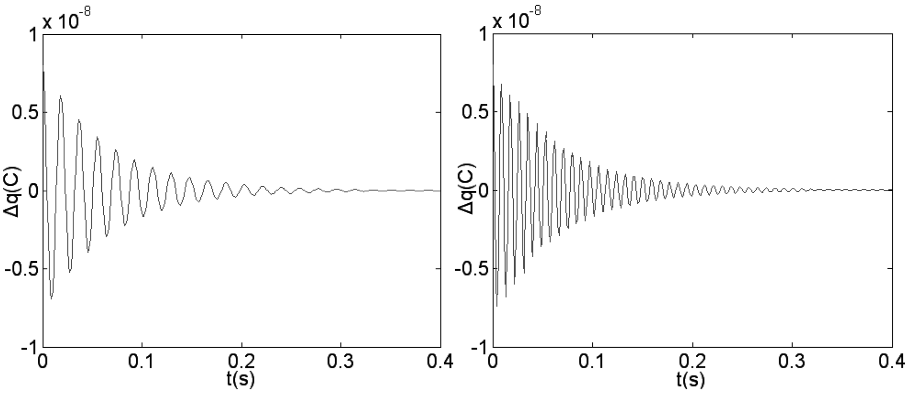
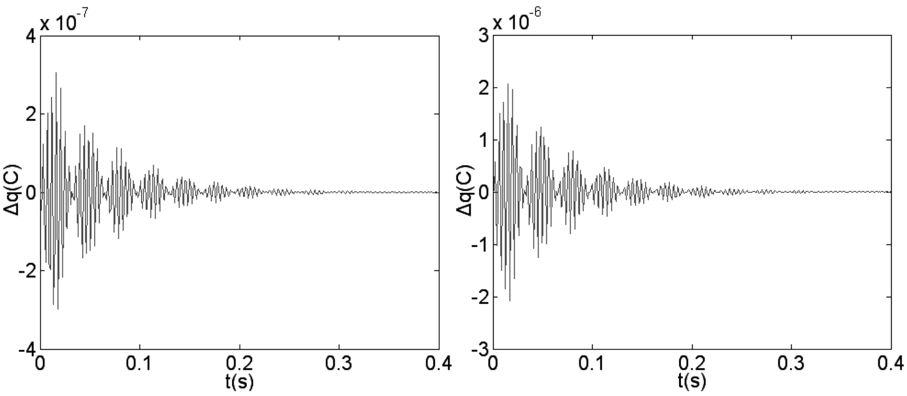


Fig. 10 Numerical simulation of the free vibration



(a) $\omega_m \approx \omega_1$ (b) $\omega_m \approx 2\omega_1$

Fig. 11 Numerical simulation of the resonance vibration under mechanical disturbance unfavorable to electric system.

To illustrate validity of the theoretical model, the calculating results are compared with numerical simulation. Fig. 10 gives free vibrations done by numerical simulation. Fig. 11 gives forced vibrations done by numerical simulation. They are done from four order Runge-Kutta method. Fig. 10(a) and (b) corresponds to Fig. 4(a) and (b), respectively. Fig. 11(a) and (b) corresponds to Fig. 5(a) and (b), respectively. These figures shows that the analytical solutions are in good agreement with the numerical simulation.

These figures also show that the accuracy of the 3rd solution is high enough. For the free vibration analysis, the relative error of the amplitude is smaller than 3.5%. For the forced vibration analysis, the relative error of the amplitude is smaller than 5%.

5. Conclusions

In this paper, the electromechanical coupled dynamic equations for the drive are developed. And the dynamics of the electric system for the drive under mechanical disturbance is discussed. Using method of perturbation, free vibrations and forced responses of the electric system to voltage excitation under mechanical disturbance are researched. It is known that for the system with mechanical disturbance, as time grows, small exciting wave still exists when the wave magnitude is tending to zero which is unfavorable to electric system. When mechanical disturbing frequency is near to the natural frequencies of the electric system or their integer multiple, resonance vibrations occur in the electric system. The forced responses of the electric system to voltage excitation under mechanical disturbance are compound vibrations caused by voltage excitation, mechanical disturbance and parameters of the system. As the time grows, the resonance vibration caused by voltage excitation still exists and the vibrations caused by mechanical disturbance are enlarged as well. The coupled resonance vibration caused by mechanical disturbance and voltage excitation is discussed as well. The conditions under which above coupled resonance vibrations occur are presented. The coupled resonance is more unfavorable to electric system and must be avoided in design.

Acknowledgments

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Appendix

Self-inductances:

$$L_{ii} = L_0 + L_1 \cos\left(z_1\theta - (i-1)\frac{\phi_v}{pn}\right) \qquad (i = 1 \text{ to n})$$

mutual-inductances between adjacent phases:

$$L_{i-1,i} = L_{01} + L_1 \cos\left(z_1\theta - (2i-3)\frac{\phi_v}{2pn}\right) \qquad (i = 2 \text{ to n})$$

mutual-inductances between two spacing phases:

$$L_{i-2,i} = L_{02} + L_1 \cos\left(z_1\theta - (2i-4)\frac{\phi_v}{2pn}\right) \qquad (i = 3 \text{ to n})$$

Where $L_{0i} = L_0 \cos\frac{i\phi_v}{pn} \qquad (i = 1 \text{ to n})$

$\mathbf{q} = \{q_1, \cdots\cdots, q_i, \cdots\cdots, q_n)^T$, $\mathbf{v}_s = \{v_{1s}, \cdots\cdots, v_{is}, \cdots\cdots, v_{ns})^T$

$$\mathbf{L}_v = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ L_{21} & L_{22} & \cdots & L_{2n} \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ L_{n1} & L_{n2} & \cdots & L_{nn} \end{bmatrix}, \mathbf{R}_v = Diag[R_1\ R_2\ \cdots\ R_n] + \frac{d\theta}{dt}\begin{bmatrix} l_{11} & l_{12} & \cdots & l_{1n} \\ l_{21} & l_{22} & \cdots & l_{2n} \\ \vdots & \vdots & \vdots\vdots\vdots & \vdots \\ l_{n1} & l_{n2} & \cdots & l_{nn} \end{bmatrix}$$

$$\mathbf{C}_v = Diag\left[\frac{1}{C_1}\ \frac{1}{C_2}\ \cdots\ \frac{1}{C_n}\right]$$

$$\mathbf{L}_{vN} = \mathbf{A}_N^T\mathbf{L}_v\mathbf{A}_N = \begin{bmatrix} \ddots & & \\ & 1 & \\ & & \ddots \end{bmatrix} = \mathbf{I},\ \mathbf{R}_{vN} = \mathbf{A}_N^T\mathbf{R}_v\mathbf{A}_N = \begin{bmatrix} \ddots & & \\ & \mathbf{R}_N & \\ & & \ddots \end{bmatrix}$$

$$\mathbf{C}_{vN} = \mathbf{A}_N^T\mathbf{C}_v\mathbf{A}_N = \begin{bmatrix} \ddots & & \\ & C_N & \\ & & \ddots \end{bmatrix} = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} = \begin{bmatrix} \omega_1^2 & & \\ & \ddots & \\ & & \omega_n^2 \end{bmatrix}$$

$$\mathbf{v}_{sN} = \mathbf{A}_N^{-1}\mathbf{v}_s = \begin{bmatrix} A_{N1}^{(1)} & A_{N2}^{(1)} & \cdots & A_{Nn}^{(1)} \\ A_{N1}^{(2)} & A_{N2}^{(2)} & \cdots & A_{Nn}^{(2)} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N1}^{(n)} & A_{N2}^{(n)} & \cdots & A_{Nn}^{(n)} \end{bmatrix}^{-1} \begin{Bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{(n-1)} \\ v_{ns} \end{Bmatrix}$$

$$\mathbf{q}_N = \mathbf{A}_N^{-1}\mathbf{q} = \begin{bmatrix} A_{N1}^{(1)} & A_{N2}^{(1)} & \cdots & A_{Nn}^{(1)} \\ A_{N1}^{(2)} & A_{N2}^{(2)} & \cdots & A_{Nn}^{(2)} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N1}^{(n)} & A_{N2}^{(n)} & \cdots & A_{Nn}^{(n)} \end{bmatrix}^{-1} \begin{Bmatrix} q_{1s} \\ q_{2s} \\ \vdots \\ q_{(n-1)s} \\ q_n \end{Bmatrix}$$

Here $\mathbf{A}_N$ is regular mode matrix of Eq. (4)