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Thermomechanical interactions in a non local thermoelastic model with two temperature and memory dependent derivatives

Parveen Lata[1] and Sukhveer Singh*[2]

[1] *Department of Basic and applied Sciences, Punjabi University Patiala, India*
[2] *Punjabi University APS Neighbourhood Campus, Dehla Seehan, India*

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Abstract. The present investigation is concerned with two-dimensional deformation in a homogeneous isotropic non local thermoelastic solid with two temperatures due to thermomechanical sources. The theory of memory dependent derivatives has been used for the study. The bounding surface is subjected to concentrated and distributed sources (mechanical and thermal sources). The Laplace and Fourier transforms have been used for obtaining the solution to the problem in the transformed domain. The analytical expressions for displacement components, stress components and conductive temperature are obtained in the transformed domain. For obtaining the results in the physical domain, numerical inversion technique has been applied. Numerical simulated results have been depicted graphically for explaining the effects of nonlocal parameter on the components of displacements, stresses and conductive temperature. Some special cases have also been deduced from the present study. The results obtained in the investigation should be useful for new material designers, researchers and physicists working in the field of nonlocal material sciences.

Keywords: thermoelasticity; nonlocality; nonlocal theory of thermoelasticity; Eringen model of nonlocal theories; two temperature; memory dependent derivative; concentrated and distributed sources

1. Introduction

The concept of nonlocal theory of thermoelasticity considers the dependence of the various physical quantities defined at a point as not just a function of the values of independent constitutive variables at that point only but a function of their values over the whole body. So, the nonlocal theory is just a generalization of the classical field theory. Edelen and Law (1971) discussed a theory of nonlocal interactions. Edelen *et al*. (1971) discussed the consequences of global postulate of energy balance. Artan (1996) proved the superiority of the nonlocal theory by comparing the results of local and nonlocal elasticity theories. Marin (1994) derived the generalized solutions in elasticity. Eringen (2002) developed nonlocal continuum field theories. Marin (2009) and (2010) extended concepts of thermoelasticity to dipolar bodies. Othman and Abbas (2012) developed a solution of thermal-shock problem of generalized thermoelasticity of a non-homogeneous isotropic hollow

*Corresponding author, Assistant Professor, E-mail: sukhveer_17@pbi.ac.in
[a]Associate Professor, E-mail: parveenlata@pbi.ac.in

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cylinder with energy dissipation. Othman *et al*. (2015) studied the effects of rotation on the general model of the equations of generalized thermo-micro stretch. Marin *et al*. (2019) explained partition of energies for thermoelastic materials. Abualnour *et al*. (2019) analyzed composite plates thermomechanically. Mahmoudi *et al* (2019) refined quasi 3d deformation theory for thermomechanical analysis. Belbachir (2019) analyzed bending of cross ply laminated plates. Boussoula *et al*. (2020) gave a shear deformation theory for thermomechanical bending analysis. Zarga *et al*. (2019) studied thermomechanical bending for functionally graded sandwich plates. Tounsi *et al*. (2020) gave a theory for hygro-thermo-mechanical bending analysis.

Belkorissat *et al*. (2015) presented a new nonlocal hyperbolic refined plate model for free vibration properties of functionally graded plates. Ebrahimi and Shafiei (2016) investigated the size dependent vibration behavior of a rotating functionally graded (FG) Timoshenko nanobeam based on Eringen's nonlocal theory. Marin *et al*. (2017a, 2017b) discussed various results and problems for elastic dipolar bodies. Bellifa *et al*. (2017) developed a nonlocal zeroth-order shear deformation theory. Khetir *et al*. (2017) proposed a new nonlocal trigonometric shear deformation theory. Lata (2018a and 2018b) studied the plane waves in a layered medium of two semi-infinite nonlocal solids. Karami and Tounsi (2018) studied nonlocal strain gradient elasticity theory. Mokhtar *et al*. (2018) used the nonlocal differential constitutive relations of Eringen. Hussain *et al*. (2019) studied nonlocal effects on the vibration of armchair and zigzag SWCNTs. Benahmed *et al*. (2019) presented an efficient higher order nonlocal beam theory. Soleimani *et al*. (2019) investigated the effects of inevitable out of plane defects based on nonlocal first order shear deformation theory. Belmahi *et al*. (2019) studied the forced vibration of nanobeam with the application of the non-local continuum or elasticity theory. Balubaid (2019) investigated free vibration using nonlocal refined plate theory. Lata and Singh (2019) focused on the study of effects of nonlocal parameters due to inclined load. Asghar (2020) assessed nonlocal natural frequencies. Bellal *et al*. (2020) used nonlocal model for studying buckling behavior of graphene sheet.

Thermoelasticity with two temperatures was developed by Chen and Gurtin (1968) suggesting that in case of bodies being deformable there is a dependence on two distinct temperatures, namely the thermodynamic temperature and the conductive temperature. Youssef (2005) obtained the uniqueness theorem for equations of two temperature generalized thermoelasticity. Youssef and Al-Lehaibi (2007) gave an indication that the two-temperature generalized thermoelasticity is more realistic in describing the state of an elastic body as compared to one temperature. Abbas *et al*. (2012) explained a nonlinear thermoelasticity model for temperature dependent materials. Abbas and Zenkour (2014) constructed a mathematical model of two-temperature generalized thermoelasticity. Abbas (2014) obtained a general solution to the field equations of two-temperature generalized thermoelastic theory. Atwa and Jahangir (2014) investigated the effect of two temperatures on plane waves propagating through a generalized thermo-microstretch elastic solid. Marin and Florea (2014) studied temporal behavior of porous micropolar bodies. Marin (2014) gave Lagrange method for microstretch thermoelastic materials. Abbas (2014) studied temperature dependent material properties. Sharma *et al*. (2016) studied the thermomechanical interactions in transversely isotropic thermoelastic medium with two temperatures. Said and Othman (2016) applied a general model of the two-temperature theory of generalized thermoelasticity to study the wave propagation in a magneto-thermoelastic medium. Kumar *et al*. (2016a, 2016b) studied the disturbances in a homogeneous transversely isotropic thermoelastic rotating medium with two temperatures.

The memory-dependent derivative is an integral form of a common derivative with a kernel function on a slip in the interval. Yu *et al*. (2014) introduced a generalized model based on memory-dependent derivative (MDD). Ezzat *et al*. (2014) introduced a novel magneto-thermoelasticity theory with memory-dependent derivative while Ezzat *et al*. (2016) extended the results using two temperature with memory-dependent derivative. Sarkar *et al*. (2018) studied a two-dimensional magneto-thermoelastic problem based on a new two-temperature generalized thermoelasticity model with memory-dependent derivative. Marin *et al*. (2013) studied non simple material problems. Alzahrani and Abbas (2016) discussed the effect of magnetic field on a thermoelastic fiber-reinforced material under GN-III theory. Marin *et al*. (2016) and (2017) studied porous bodies. Jahangir *et al*. (2020) studied reflection of photo thermoelastic waves in a semiconductor material with different relaxations. As per study, it has been found that no work has been done yet about the thermomechanical interactions for a nonlocal material with the memory dependent derivatives. So, in the present work, we aimed at investigating Thermomechanical interactions in a non local thermoelastic model with two temperature and memory dependent derivatives.

2. Basic equations

Following Youssef (2005), Eringen (2002) and Sarkar *et al*. (2018), the equations of motion, heat conduction equation with memory dependent derivatives and constitutive relations in a homogeneous non local thermoelastic solid with two temperatures are given by

$$(\lambda + 2\mu)\nabla(\nabla.\boldsymbol{u}) - \mu\,(\nabla \times \nabla \times \boldsymbol{u}) - \beta\nabla\theta \;= (1 - \epsilon^2\nabla^2)\rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2}, \tag{1}$$

$$K^*\nabla^2\varphi = \;\rho\, C^*\frac{\partial\theta}{\partial t} + \;\beta\theta_0\frac{\partial}{\partial t}\,(\nabla.u) + \int_{t-\tau}^{t} K(t-\xi)\,(\rho\, C^*\frac{\partial^2\theta}{\partial\xi^2} + \;\beta\theta_0\frac{\partial^2}{\partial\xi^2}\,(\nabla.u))d\xi, \tag{2}$$

$$where\ \theta = (1 - a\nabla^2)\,\varphi, \tag{3}$$

$$t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\big(u_{i,j} + u_{j,i}\big) - \;\beta\theta\delta_{ij}. \tag{4}$$

where λ, μ are material constants, ϵ is the nonlocal parameter, ρ is the mass density, $\boldsymbol{u}$ = (u, v, w) is the displacement vector, $\boldsymbol{\varphi}$ is the conductive temperature, a is two temperature parameter, θ is absolute temperature and θ_0 is reference temperature, K^* is the coefficient of the thermal conductivity, C^* the specific heat at constant strain, $\beta = (3\lambda + 2\mu)\alpha$ where α is coefficient of liner thermal expansion, e_{ij} are components of strain tensor, e_{kk} is the dilatation, δ_{ij} is the Kronecker delta, t_{ij} are the components of stress tensor.

3. Formulation of the problem

We consider a homogeneous non local isotropic thermoelastic body in an initially undeformed state at temperature θ_0. We take a rectangular Cartesian co-ordinate system (x, y, z) with z axis pointing normally into the half space. The surface of the half-space is subjected to a normal force F_1 or a thermal source F_2 acting at $z = 0$. We restrict our analysis to two-dimensional problem with

$$\boldsymbol{u} = (u, 0, w). \tag{5}$$

Using Eq. (5) in Eqs. (1)-(2), yields

$$(\lambda+\mu)\frac{\partial e}{\partial x}+\mu\,\nabla^2 u-\beta\frac{\partial\theta}{\partial x}=(1-\epsilon^2\nabla^2)\,\rho\frac{\partial^2 u}{\partial t^2}, \tag{6}$$

$$(\lambda+\mu)\frac{\partial e}{\partial z}+\mu\,\nabla^2 w-\beta\frac{\partial\theta}{\partial z}=(1-\epsilon^2\nabla^2)\,\rho\frac{\partial^2 w}{\partial t^2}, \tag{7}$$

$$K^*\nabla^2\varphi=\ \rho\, C^*\frac{\partial\theta}{\partial t}+\ \beta\theta_0\frac{\partial}{\partial t}\ (\nabla.u)+\int_{t-\tau}^{t}K(t-\xi)\left(\rho\, C^*\frac{\partial^2\theta}{\partial\xi^2}+\ \beta\theta_0\frac{\partial^2}{\partial\xi^2}\ (\nabla.u)\right)d\xi. \tag{8}$$

where, $e=\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z},\ \ \nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$.
we define the following dimensionless quantities

$$(x',z',u',w')=\frac{\omega_1}{c_1}(x,z,u,w),\ t'_{ij}=\frac{t_{ij}}{\beta\theta_0},\quad t'=\omega_1 t,\quad a'=\frac{\omega_1^2}{c_1^2}a,\ \varphi'=\frac{\varphi}{\varphi_0}\ \text{and}\ \ {F_1}'=\frac{F_1}{\beta\theta_0} \tag{9}$$

where, $c_1^2=\frac{\mu}{\rho}$ and $\omega_1=\frac{\rho\, C^* c_1^2}{K^*}$.

Following Sarkar *et al*. (2018), the kernel function form can be chosen freely as

$$\mathrm{K}(t-\xi)=1-\frac{2b}{\omega}(t-\xi)+\frac{a^2}{\omega^2}(t-\xi)^2$$
$$=\begin{cases}1 & if\ a=0,b=0\\ 1-\frac{(t-\xi)}{\omega} & if\ a=0,b=\frac{1}{2}\\ 1-(t-\xi) & if\ a=0,b=\frac{\omega}{2}\\ \left(1-\frac{t-\xi}{\omega}\right)^2 & if\ a=b=1\end{cases} \tag{10}$$

Upon introducing the quantities defined by Eq. (9) in equations Eqs. (6)-(8), and suppressing the primes, yields

$$(1+a_1)\frac{\partial^2 u}{\partial x^2}+a_1\frac{\partial^2 w}{\partial x\partial z}+\frac{\partial^2 u}{\partial z^2}-a_2(1-a\nabla^2)\frac{\partial\varphi}{\partial x}=(1-\epsilon^2\nabla^2)\frac{\partial^2 u}{\partial t^2}, \tag{11}$$

$$\frac{\partial^2 w}{\partial x^2}+a_1\frac{\partial^2 u}{\partial x\partial z}+(1+a_1)\frac{\partial^2 w}{\partial z^2}-a_2(1-a\nabla^2)\frac{\partial\varphi}{\partial z}=(1-\epsilon^2\nabla^2)\frac{\partial^2 w}{\partial t^2}. \tag{12}$$

$$\text{and, }\nabla^2\varphi=(1+\omega D_\omega)[a_3(1-a\nabla^2)\frac{\partial\varphi}{\partial t}C^*\frac{\partial\theta}{\partial t}+\ a_4\frac{\partial e}{\partial t}], \tag{13}$$

where, $\ \ a_1=\frac{\lambda+\mu}{\mu},\ a_2=\frac{\beta\theta_0}{\mu},\ a_3=\frac{\rho C^* c_1^2}{K^*\omega_1}$ and $\ a_4=\frac{\beta c_1^2}{K^*\omega_1}$

The initial and regularity conditions are given by

$$u(x,z,0)=0=\dot{u}(x,z,0),$$
$$w(x,z,0)=0=\dot{w}(x,z,0),$$
$$\varphi(x,z,0)=0=\dot{\varphi}(x,z,0)\ \text{ for }\ z\geq 0,-\infty<x<\infty,$$
$$u(x,z,t)=w(x,z,t)=\varphi(x,z,t)=0\ for\ t>0\ when\ z\to\infty.$$

Applying Laplace and Fourier Transform defined by

$$\bar{f}(\mathrm{x},\mathrm{z},\mathrm{s})=\int_0^\infty f(x,z,t)\,e^{-st}dt, \tag{14}$$

$$\hat{f}(\xi,z,s)=\int_{-\infty}^{\infty}\bar{f}(x,z,s)\,e^{i\xi x}dx. \tag{15}$$

on Eqs. (11)-(13), we obtain a system of equations

$$[(1+a_1)(-\boldsymbol{\xi}^2)+D^2(1+\epsilon^2\mathrm{s}^2)-(1+\epsilon^2\xi^2)s^2]\tilde{u}+\iota a_1\xi D\tilde{w}-[\iota\xi a_2(1+a\xi^2-aD^2)]\tilde{\varphi}=0, \tag{16}$$

$$\iota a_1 \xi D\tilde{u} + [(1 + a_1 + \epsilon^2 s^2)D^2 - \xi^2 - (1 + \epsilon^2 \xi^2)s^2]\tilde{w} - [a_2(1 + a\xi^2 - aD^2)D]\tilde{\varphi} = 0, \quad (17)$$

$$(1 + G)[\iota a_4 s\xi \tilde{u} + a_4 sD\tilde{w}] + [a_3 s(1 + a\xi^2 - aD^2)(1 + G) - D^2 + \xi^2]\tilde{\varphi} = 0. \quad (18)$$

where,

$$G(s) = (1 - e^{-s\omega})\left(1 - \frac{2b}{s\omega} + \frac{2a^2}{s^2\omega^2}\right) - e^{-s\omega}\left(a^2 - 2b + \frac{2a^2}{s\omega}\right) \quad (19)$$

and a, b are constants such that

$$\mathrm{L}(\omega D_\omega f(t) = \begin{cases} 1 - e^{-s\omega} & if\ a = 0, b = 0 \\ 1 - \frac{(1 - e^{-s\omega})}{s\omega} & if\ a = 0, b = \frac{1}{2} \\ (1 - e^{-s\omega}) - \frac{1}{s}(1 - e^{-s\omega}) + \omega e^{-s\omega} & if\ a = 0, b = \frac{\omega}{2} \\ \left(1 - \frac{2}{s\omega}\right) + \frac{2(1 - e^{-s\omega})}{s^2\omega^2} & if\ a = b = 1 \end{cases} \quad (20)$$

From Eq. (16), Eq. (17) and Eq. (18), we obtain a set of homogeneous equations which will have a nontrivial solution if determinant of coefficient $[\tilde{u}, \tilde{w}, \tilde{\varphi}]^T$ vanishes so as to give a characteristic equation as

$$[D^6 + QD^4 + RD^2 + S](\tilde{u}, \tilde{w}, \tilde{\varphi}) = 0. \quad (21)$$

where,

$$Q = \frac{1}{P}\{[\zeta_1\zeta_4(a_3 s\zeta_3\zeta_5 + \xi^2] + \zeta_6[\xi^2\zeta_7 + s^2(\zeta_2 - \zeta_1)] + aa_2a_4 s\xi^2\zeta_3(\zeta_1 + \zeta_7)$$
$$+a_2a_4 s\zeta_3(\zeta_1\zeta_5 + as^2\zeta_2) - a_1\xi^2[as\zeta_3(a_1a_3 + a_2a_4) + a_1]\},$$
$$R = \frac{1}{P}\{-s^2\zeta_1\zeta_2(sa_3\zeta_3\zeta_5 + \xi^2) + \xi^2\zeta_7[s^2 - \xi^2 + sa_3\zeta_7(as^2 - \zeta_5\zeta_4)]$$
$$-\zeta_3\zeta_5 s(a_2a_4\zeta_7 + a_3 s^2\zeta_2\zeta_4) + s^4{\zeta_2}^2(s^2 - \xi^2 + aa_3 s\zeta_3\},$$
$$S = \frac{1}{P}\{a_3 s^3\zeta_3\zeta_5\zeta_2(s^2\zeta_2 - \xi^2\zeta_7) - s^2\xi^2(\xi^2\zeta_7 + s^2\zeta_2)$$
$$+sa_2a_4\xi^2\zeta_3\zeta_5(s^2\zeta_2 + \xi^2)\},$$
$$P = -\zeta_1[\zeta_4\zeta_6 + \zeta_3 aa_2a_4 s].$$

where, $D = \frac{d}{dz}$, $\zeta_1 = 1 + \epsilon^2 s^2$, $\zeta_2 = 1 + \epsilon^2\xi^2$, $\zeta_3 = 1 + G$, $\zeta_4 = 1 + a_1 + \epsilon^2 s^2$, $\zeta_5 = 1 + a\xi^2$, $\zeta_6 = 1 + aa_3 s(1 + G)$, $\zeta_7 = 1 + a_1$.

The roots of the Eq. (21) are $\pm\lambda_i (i = 1, 2, 3)$ satisfying the radiation condition that $\tilde{u}, \tilde{w}, \tilde{\varphi} \rightarrow 0$ as $z \rightarrow \infty$, the solutions of equation can be written as

$$\tilde{u} = A_1 e^{-\lambda_1 z} + A_2 e^{-\lambda_2 z} + A_3 e^{-\lambda_3 z}, \quad (22)$$

$$\tilde{w} = d_1 A_1 e^{-\lambda_1 z} + d_2 A_2 e^{-\lambda_2 z} + d_3 A_3 e^{-\lambda_3 z}, \quad (23)$$

$$\tilde{\varphi} = l_1 A_1 e^{-\lambda_1 z} + l_2 A_2 e^{-\lambda_2 z} + l_3 A_3 e^{-\lambda_3 z}. \quad (24)$$

where

$$d_i = \frac{P^*{\lambda_i}^3 + Q^*\lambda_i}{S^*{\lambda_i}^4 + T^*{\lambda_i}^2 + U^*} \quad i = 1,2,3. \quad (25)$$

$$l_i = \frac{P^{**}{\lambda_i}^2 + Q^{**}}{S^*{\lambda_i}^4 + T^*{\lambda_i}^2 + U^*} \quad i = 1,2,3. \quad (26)$$

where,

$P^{*} = \iota\xi[a_1 + as\zeta_3(a_1a_3 - a_2a_4)]$,
$Q^{*} = \iota\xi[\zeta_3\zeta_5 s(a_2a_4 - a_1a_3) - a_1\xi^2]$,
$S^{*} = -\zeta_4\zeta_6$,
$T^{*} = \zeta_4(sa_3\zeta_3\zeta_5 + \xi^2) + (s^2\zeta_2 + \xi^2)(asa_3\zeta_3 - 1)$,
$U^{*} = -(sa_3\zeta_3\zeta_5 + \xi^2)(s^2\zeta_2 + \xi^2)$,
$P^{**} = -\iota\xi a_4 s\zeta_3\zeta_1$,
$Q^{**} = \iota\xi a_4 s\zeta_3(\xi^2 + s^2\zeta_2)$.

4. Applications

On the half-space ($z = 0$) normal point force and thermal point source are applied. The boundary conditions are

$$\begin{aligned} &(1)\ t_{zz}(x,z,t) = -F_1\psi_1(x)\delta(t),\\ &(2)\ t_{xz}(x,z,t) = 0,\\ &(3)\ \frac{\partial}{\partial z}\varphi(x,z,t) = F_2\psi_2(x)\delta(t)\ \text{ at }\ z = 0. \end{aligned} \tag{27}$$

where, F_1 is the magnitude of the force applied, F_2 is constant force applied on the boundary, $\psi_1(x)$ specify the source distribution function along x axis.

Using the dimensionless quantities defined by Eq. (9) and using Eqs. (3), (4), (13), 17) in Eq. (27) and substituting values of $\tilde{u}$, $\tilde{w}$ and $\tilde{\varphi}$ from Eqs. (22)-(24), and solving, we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\tilde{u} = \frac{F_1\widetilde{\psi_1}(\xi)}{\Delta}\left\{\Delta_{11}e^{-\lambda_1 z} + \Delta_{21}e^{-\lambda_2 z} + \Delta_{31}e^{-\lambda_3 z}\right\} + \frac{F_2\widetilde{\psi_2}(\xi)}{\Delta}\left\{\Delta_{12}e^{-\lambda_1 z} + \Delta_{22}e^{-\lambda_2 z} + \Delta_{32}e^{-\lambda_3 z}\right\}, \tag{28}$$

$$\begin{aligned} \tilde{w} = &\frac{F_1\widetilde{\psi_1}(\xi)}{\Delta}\left\{d_1\Delta_{11}e^{-\lambda_1 z} + d_2\Delta_{21}e^{-\lambda_2 z} + d_3\Delta_{31}e^{-\lambda_3 z}\right\}\\ &+ \frac{F_2\widetilde{\psi_2}(\xi)}{\Delta}\left\{d_1\Delta_{12}e^{-\lambda_1 z} + d_2\Delta_{22}e^{-\lambda_2 z} + d_3\Delta_{32}e^{-\lambda_3 z}\right\}, \end{aligned} \tag{29}$$

$$\begin{aligned} \tilde{\varphi} = &\frac{F_1\widetilde{\psi_1}(\xi)}{\Delta}\left\{l_1\Delta_{11}e^{-\lambda_1 z} + l_2\Delta_{21}e^{-\lambda_2 z} + l_3\Delta_{31}e^{-\lambda_3 z}\right\}\\ &+ \frac{F_2\widetilde{\psi_2}(\xi)}{\Delta}\left\{l_1\Delta_{12}e^{-\lambda_1 z} + l_2\Delta_{22}e^{-\lambda_2 z} + l_3\Delta_{32}e^{-\lambda_3 z}\right\}, \end{aligned} \tag{30}$$

$$\begin{aligned} \widetilde{t_{zz}} = &\frac{-F_1\widetilde{\psi_1}(\xi)}{\Delta}\left\{N_{21}\Delta_{11}e^{-\lambda_1 z} + N_{22}\Delta_{21}e^{-\lambda_2 z} + N_{23}\Delta_{31}e^{-\lambda_3 z}\right\}\\ &- \frac{F_2\widetilde{\psi_2}(\xi)}{\Delta}\left\{N_{21}\Delta_{12}e^{-\lambda_1 z} + N_{22}\Delta_{22}e^{-\lambda_2 z} + N_{23}\Delta_{32}e^{-\lambda_3 z}\right\}, \end{aligned} \tag{31}$$

$$\begin{aligned} \widetilde{t_{zx}} = &\frac{F_1\widetilde{\psi_1}(\xi)}{\Delta}\left\{\nabla_{11}\Delta_{11}e^{-\lambda_1 z} + \nabla_{12}\Delta_{21}e^{-\lambda_2 z} + \nabla_{13}\Delta_{31}e^{-\lambda_3 z}\right\}\\ &+ \frac{F_2\widetilde{\psi_2}(\xi)}{\Delta}\left\{\nabla_{11}\Delta_{12}e^{-\lambda_1 z} + \nabla_{12}\Delta_{22}e^{-\lambda_2 z} + \nabla_{13}\Delta_{32}e^{-\lambda_3 z}\right\} \end{aligned} \tag{32}$$

$$\begin{aligned} \widetilde{t_{xx}} = &\frac{F_1\widetilde{\psi_1}(\xi)}{\Delta}\left\{N_{31}\Delta_{11}e^{-\lambda_1 z} + N_{32}\Delta_{21}e^{-\lambda_2 z} + N_{33}\Delta_{31}e^{-\lambda_3 z}\right\}\\ &+ \frac{F_2\widetilde{\psi_2}(\xi)}{\Delta}\left\{N_{31}\Delta_{12}e^{-\lambda_1 z} + N_{32}\Delta_{22}e^{-\lambda_2 z} + N_{33}\Delta_{32}e^{-\lambda_3 z}\right\}. \end{aligned} \tag{33}$$

$$\Delta = -M_{11}\Delta_{12} + M_{22}\Delta_{22} + M_{33}\Delta_{32} \tag{34}$$

where,

$$\Delta_{11} = \mathrm{M}_{33}\nabla_{12} - \mathrm{M}_{22}\nabla_{13},\ \Delta_{21} = \mathrm{M}_{33}\nabla_{11} - \mathrm{M}_{11}\nabla_{13},\ \Delta_{31} = \mathrm{M}_{22}\nabla_{11} - \mathrm{M}_{11}\nabla_{12}.$$
$$\Delta_{12} = \mathrm{N}_{13}\nabla_{12} - \mathrm{N}_{12}\nabla_{13},\ \Delta_{22} = \mathrm{N}_{13}\nabla_{11} - \mathrm{N}_{11}\nabla_{13},\ \Delta_{32} = \mathrm{N}_{12}\nabla_{11} - \mathrm{N}_{11}\nabla_{12}.$$
$$\mathrm{N}_{1j} = \lambda_j d_j(\lambda + 2\mu) - \beta l_j(1 + a\xi^2 - a\lambda_j^2),\ \mathrm{N}_{2j} = \lambda_j d_j(\lambda + 2\mu) + \beta l_j(1 + a\xi^2 - a\lambda_j^2),$$
$$\mathrm{N}_{3j} = \iota\xi(\lambda + 2\mu) - \beta l_j\left(1 + a\xi^2 - a\lambda_j^2\right),\ \nabla_{1j} = \iota\xi d_j - \lambda_j,\ \mathrm{M}_{jj} = l_j\lambda_j\ ;\quad j = 1,2,3.$$

5a. Mechanical force on the surface of half-space:

Taking $F_2 = 0$ in Eqs. (28)-(33), we obtain the components of displacement, normal stress, tangential stress and conductive temperature due to mechanical force.

5b. Thermal source on the surface of half-space

Taking $F_1 = 0$ in Eqs. (28)-(33), we obtain the components of displacement, normal stress, tangential stress and conductive temperature due to thermal force.

5.1 Influence function

The method to obtain the half-space influence function, i.e. the solution due to distributed load applied on the half space is obtained by setting

$$\psi_1(x) = \begin{cases} 1\ if\ \ |x| \le m \\ 0\ if\ \ |x| > m \end{cases}. \tag{35}$$

in Eq. (27). The Laplace and Fourier transform of $\psi_1(x)$ with respect to the pair (x, ξ) for the case of a uniform strip load of non-dimensional width 2 m applied at origin of co-ordinate system $x = z = 0$ in the dimensionless form after suppressing the primes is given by

$$\widehat{\psi_1}(\xi) = \left[{}^{2\sin(\xi m)}/_{\xi}\right] \xi \neq 0. \tag{36}$$

The expressions for displacement, stresses and conductive temperature can be obtained for uniformly distributed normal force and thermal source by replacing $\widehat{\psi_1}(\xi)$ from Eq. (36) in Eqs. (28)-(33) respectively.

6. Particular cases

- If $a = 0$, then from Eqs. (28)-(33), the corresponding expressions for displacements, stresses and conductive temperature for nonlocal isotropic solid without two temperature are obtained .
- If $\epsilon = 0$, then from Eqs. (28)-(33), the corresponding expressions for displacements, stresses and conductive temperature for isotropic solid without nonlocal effects and with two temperature are obtained.
- If $\epsilon = a = 0$, then from Eqs. (28)-(33), the corresponding expressions for displacements, stresses and conductive temperature for isotropic solid without nonlocal effects and two temperature are obtained.

7. Inversion of the transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (28)-(33). Here the displacement components, normal and tangential stresses and conductive temperature are functions of z and the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f(\xi,z,s)$. To obtain the function $f(x,z,t)$ in the physical domain, we first invert the Fourier transform as used by Sharma *et al*. (2008), using

$$\tilde{f}(x,z,s)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-i\xi x}\hat{f}(\xi,z,s)d\xi=\frac{1}{2\pi}\int_{-\infty}^{\infty}|\cos(\xi x)\,f_e-i\sin(\xi x)\,f_0|\,d\xi. \tag{37}$$

where, f_e and f_0 are respectively the even and odd parts of $\hat{f}(\xi,z,s)$. Thus the expression (37) gives the Laplace transform $\tilde{f}(x,z,s)$ of the function $f(x,z,t)$. Following Honig and Hirdes, the Laplace transform function $\tilde{f}(x,z,s)$ can be inverted to $f(x,z,t)$.

The Last step is to calculate the integral in Eq. (37). The method for evaluating this integral is decscribed in Press at al. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion:

Magnesium material is chosen for the purpose of numerical calculation which is homogeneous isotropic and according to Dhaliwal and Singh (1980), physical data for which is given as

$$\lambda=9.4\times10^{10}Nm^{-2},\mu=3.278\times10^{10}Nm^{-2},\ K^*=1.7\times10^{2}Wm^{-1}K^{-1},$$
$$\rho=1.74\times\ 10^{3}Kgm^{-3},\theta_0=298\ K,C^*=10.4\times10^{2}JKg^{-1}deg^{-1},\ \omega_1=3.58\ \ a=0.05$$

A comparison of values of displacement components u and w, stress components $t_{zz},\ t_{xx},\ t_{zx}$ and conductive temperature φ for a transversely isotropic thermoelasic solid with distance x has been made for the local parameter $(\epsilon=0)$ and nonlocal parameter $(\epsilon=2)$.

1) The solid black colored line with center symbol square corresponds to local parameter $(\epsilon=0)$.

2) The dashed reddish colored line with center symbol circle represents local parameter $(\epsilon=2)$.

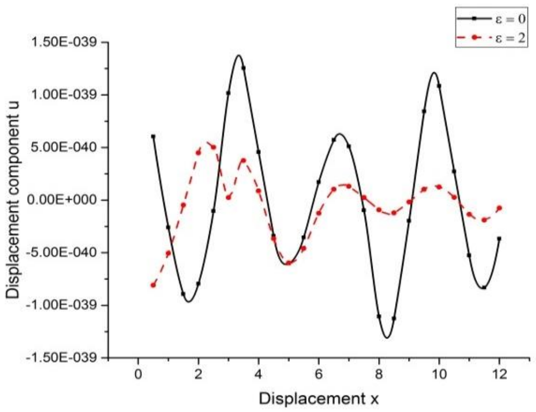

Fig. 1 Variation of displacement component u with displacement *x* (mechanical force)

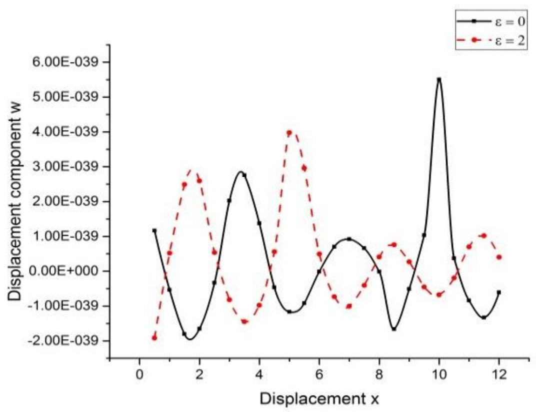

Fig. 2 Variation of displacement component w with displacement *x* (mechanical force)

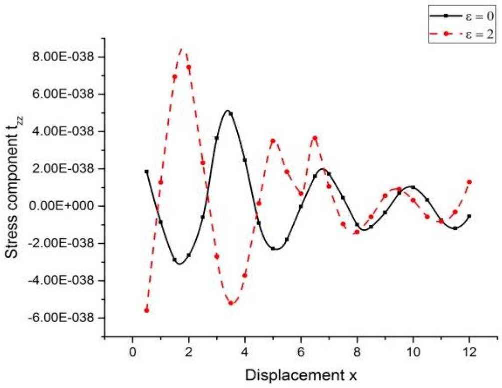

Fig. 3 Variation of stress component t_{zz} with displacement x (mechanical force)

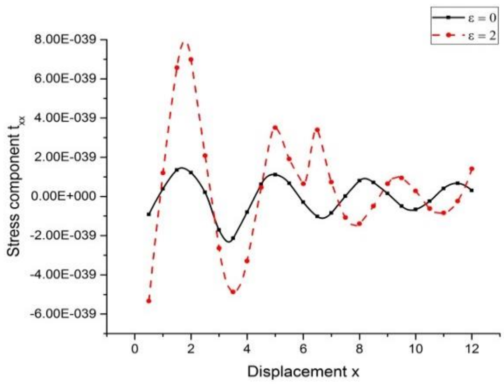

Fig. 4 Variation of stress component t_{xx} with displacement x (mechanical force)

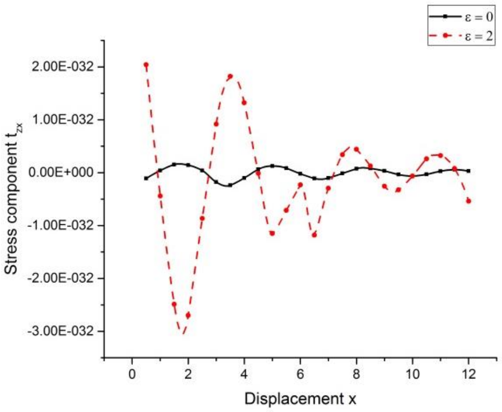

Fig. 5 Variation of stress component t_{zx} with displacement x (mechanical force)

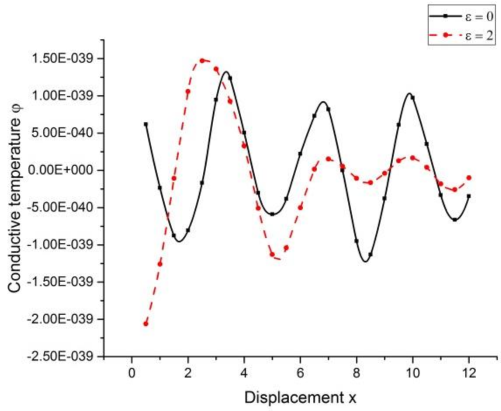

Fig. 6 Variation of conductive temperature φ with displacement x (mechanical force)

a) Mechanical force on the surface of half-space

Uniformly distributed normal force:

Fig. 1, shows the variations in values of displacement component u. It is clear that the values of u follow oscillatory pattern. For $\epsilon = 0$, the variations are more oscillatory as compared to $\epsilon = 2$. Fig. 2 depicts the variation of values of displacement component w. The pattern is oscillatory with a clear difference between values for local and non-local parameters. Fig. 3 and Fig. 4 describe the variations of stress components t_{zz} *and* t_{xx} with respect to displacement. For both local and non-local parameters, the behavior is oscillatory but the nonlocality effects can be visibly seen. Fig. 5 shows the variation of stress component t_{zx}. Here too the behavior followed is oscillatory with more variations for $\epsilon = 2$ as compared to $\epsilon = 0$. Fig. 6 illustrates the variation of conductive temperature φ. The behavior followed is oscillatory with non-local parameter causing the effects.

b) Thermal source on the surface of half-space

Uniformly distributed normal force:

Fig. 7 and Fig. 8, shows the variations in values of displacement components u and w

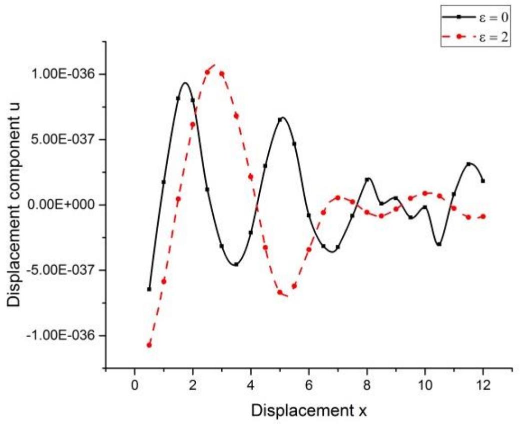

Fig. 7 Variation of displacement component u with displacement x (thermal source)

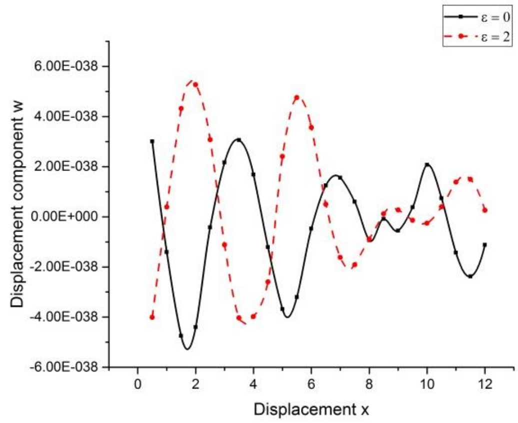

Fig. 8 Variation of displacement component w with displacement x (thermal source)

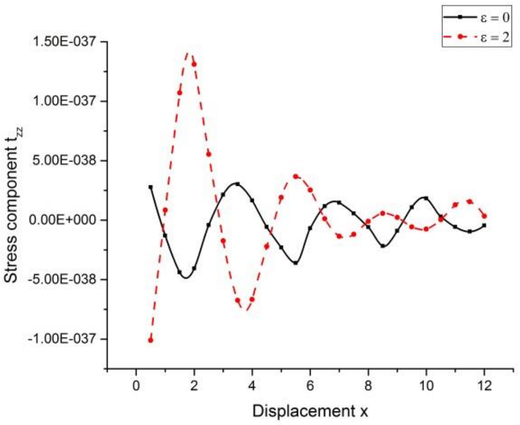

Fig. 9 Variation of stress component t_{zz} with displacement x (thermal source)

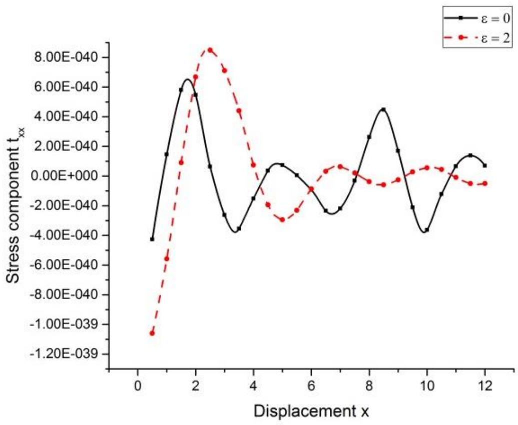

Fig. 10 Variation of stress component t_{xx} with displacement x (thermal source)

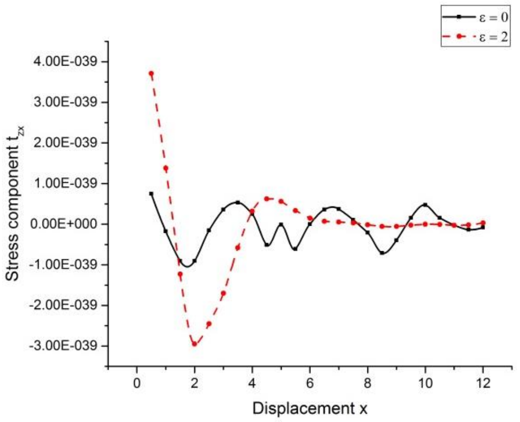

Fig. 11 Variation of stress component t_{zx} with displacement x (thermal source)

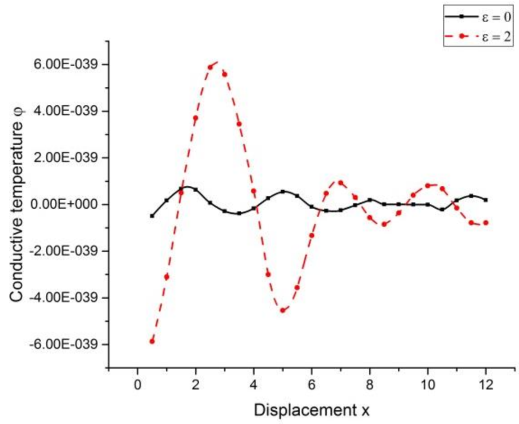

Fig. 12 Variation of conductive temperature φ with displacement x (thermal source)

respectively. The behavior followed is oscillatory in both figures. Non-locality is clearly playing its part. Fig. 9 depicts the variations of values of stress component t_{zz}. The behavior followed is oscillatory and the effects of non-local parameter can be clearly noticed. Fig. 10 describes the variations of stress component t_{xx}. Nonlocality is visibly causing the differences for all frequencies. Fig. 11 illustrates the variation of stress component t_{zx}. Nonlocality is clearly causing differences in variations. Fig. 12 shows the variation of conductive temperature φ. As the trend goes the variations are oscillatory with difference for local and non-local parameter values.

8. Conclusions

In the present discussion the numerical results showing the effects of nonlocal parameter on the components of displacements, stresses and conductive temperature have been shown graphically. From above discussion it is observed that nonlocality is playing a significant effect on all the components viz. displacements, stresses and conductive temperature. It is observed from the figures (1-12) that the trends in the variations of the characteristics mentioned are similar with difference in their magnitude when the mechanical forces or thermal sources are applied. The results of this paper can be helpful for the researchers working in the field of material engineering, geophysics, marine engineering, acoustics etc., for analysis of deformation field around mining tremors and for the theoretical considerations of volcanic and seismic sources. Further, the results can play a role for those scientists and researchers who are working on the development of the theory of nonlocal thermoelasticity.

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