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Naval ship's susceptibility assessment by the probabilistic density function

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Abstract

The survivability of the naval ship is the capability of a warship to avoid or withstand a hostile environment. The survivability of the naval ship assessed by three categories (susceptibility, vulnerability and recoverability). The magnitude of susceptibility of a warship encountering with threat is dependent upon the attributes of detection equipment and weapon system. In this paper, as a part of a naval ship's survivability analysis, an assessment process model for the ship's susceptibility analysis technique is developed. Naval ship's survivability emphasizing the susceptibility is assessed by the probability of detection, and the probability of hit. Considering the radar cross section (RCS), the assessment procedure for the susceptibility is described. It's emphasizing the simplified calculation model based on the probability density function for probability of hit. Assuming the probability of hit given a both single-hit and multiple-hit, the susceptibility is accessed for a RCS and the hit probability for a rectangular target is applied for a given threat.

Keywords: Survivability; Vulnerability; RCS (Radar Cross Section); Probability of hit; Probability of detection

1. Introduction

When a naval ship in a modern combat environment is exposed to a threatening weapon and attacked, the combat system and hull structure suffer critical damage. The ability of a naval ship to withstand such threats encountered in a battle environment is defined as the survivability of the ship [1]. The survivability of the ship is evaluated by three types of stochastic indicators—susceptibility, vulnerability, and recoverability [1-3]. The most fundamental method for improving the survivability of the naval ship is to design the ship such that its susceptibility becomes close to zero. However, because it is difficult to attain zero susceptibility in reality, the realistic approach to improving the susceptibility involves considering various situations that may arise on being attacked [4]. The susceptibility of the naval ship refers to the probability of the ship being attacked by threatening weapons after being identified by an enemy's detection technology and equipment [4, 5]. Therefore, the signals generated by a naval ship, the devices for detecting and identifying these signals, and the defense and deception system employed against enemy's threatening weapons following detection need to be analyzed [6]. The susceptibility should be comprehensively evaluated by considering the shape of the naval ship's radar reflection area, infrared signal characteristics, electromagnetic equipment performance, and radiation noises [7]. Many studies have reported on the aforementioned evaluation factors indivi-dually [8], and the analysis field can be divided into Radar Cross Section (RCS), Infrared (IR), Underwater Radiated Noise (URN), and Electro Magnetic Interference (EMI) according to the respective detection characteristics and equipment [6]. In the present study, as a part of a naval ship's survivability analysis, a theoretical process model for the ship's susceptibility analysis technique was developed. The applicability of this model was demonstrated through an example model and arbitrary attack situations.

2. Susceptibility analysis of naval ships

2.1 Susceptibility analysis theory

To analyze the susceptibility of a naval ship, the characteristics and performance of the equipment used for detecting enemy naval ships, and the possibility of being hit by threatening weapons must be analyzed. In this study, the susceptibility analysis process model was constituted as shown in Figure 1 based on the stochastic theory. The attacking probability (P_H) is based on the detection probability (P_D), which is the probability of identifying the friendly naval ship by the enemy detection equipment, and on the hit probability (P_{hit}), which is the probability that the friendly naval ship could be hit by the threatening weapon after being detected. The at-

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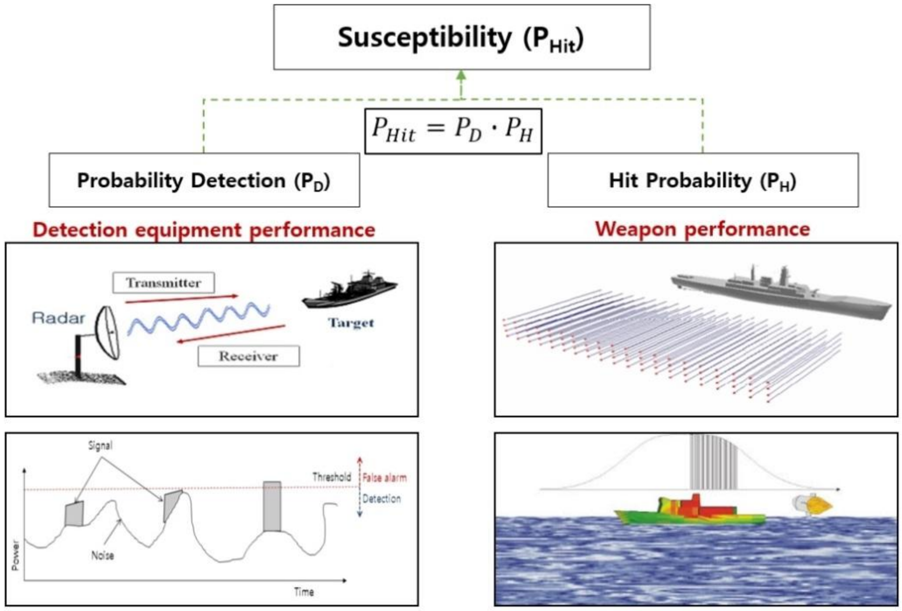

Figure 1. Mesh grid of topographic model.

tacking probability is expressed as shown in Eq. (1).

$$P_H = P_D \cdot P_{hit} \tag{1}$$

2.1.1 Detection probability

The detection probability is the probability that the naval ship (target) could be detected by the enemy detection equipment. The RCS detection characteristics and the threshold to noise (T/N) value, which is the minimum detection limit, are calculated on the basis of the signal to noise ratio (S/N) value. The S/N value is the degree of noise in RCS signal according to the cross sectional area (σ) of the radar [9] (Eq. (2)). RCS is the specified area to indicate the reflectivity of the radar's reflector when the electromagnetic waves emitted from the radar are reflected back by the target [7]. Therefore, the reflectivity value of the radar varies depending on the shape of the reflector [7]. Naval systems, including the naval ship's hull and fittings, have their own unique complex shapes, which have to be taken into account when calculating RCS; hence, each shape should be simplified as shown in Figure 2, and then, RCS should be calculated for each simplified shape [9].

$$P_D = \left[1 + \frac{2(T/N)(S/N)}{2 + (S/N)^2}\right] e^{-2(T/N)(2+S/N)} \tag{2}$$

S/N ratio value (dB) represents the extent of unnecessary noise in the signal [10, 12]. In other words, the S/N ratio represents radar signal-to-noise ratio (Figure 3), and it is proportional to the shape of RCS because it is calculated using the radar's cross-sectional area. The S/N value according to the RCS detection characteristics is defined as shown in Eq.

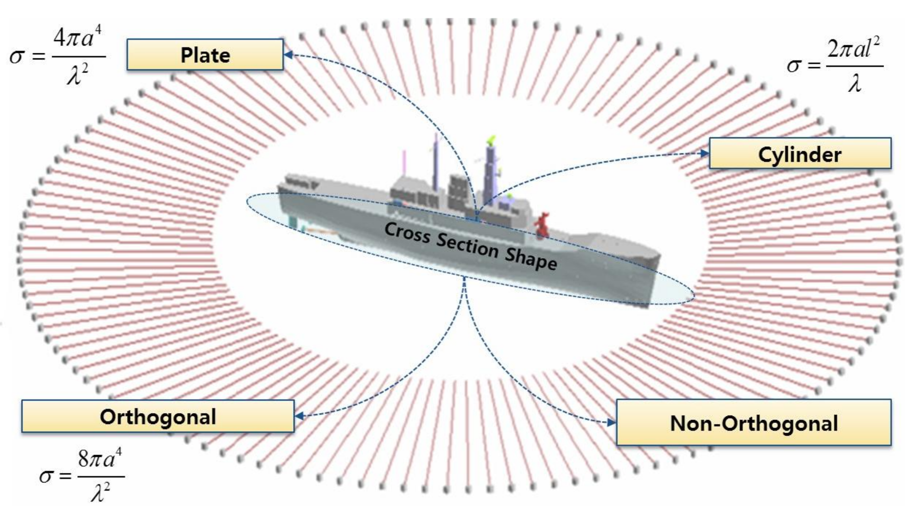

Figure 2. The simplified shape for hull and system.

Table 1. Definition of variable for Eq. (3).

Variable	Definition	Dimension
P	Radar peak power	[Watt]
R	Distance from radar to target	[m]
λ	Wavelength of signal	[m]
G	Antenna gain factor	Const.
K	Boltmann's constant	1.381 × 10-23 [J/deg]
N	Noise factor	Const.
T	Temperature	[℃]
Bn	Radar Receiver bandwidth	[Hz]
L	Signal echo power loss factor	Const.

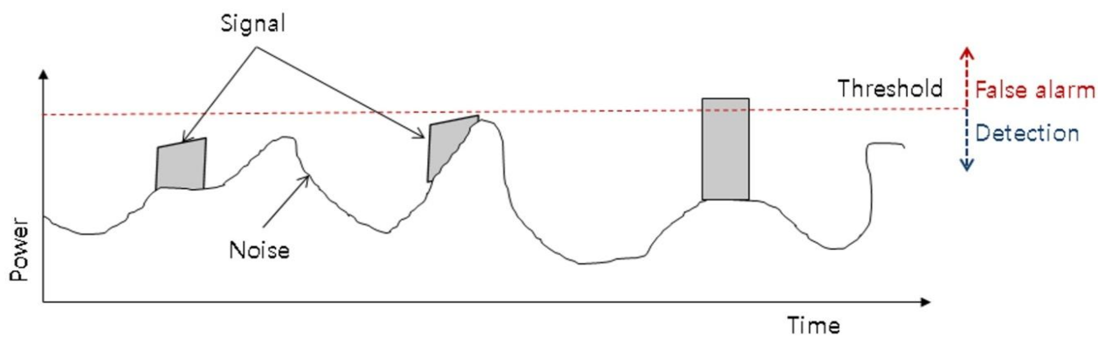

Figure 3. Conditional detection by threshold setting.

(3); the variables are defined in Table 1.

$$S/N = \left[\frac{PG^2\lambda^2\sigma}{(4\pi)^3R^4FkTB_nL}\right] \quad (3)$$

Here, radar peak power indicates the antenna reception power, and kTB_n value indicates noise power (Eq. (4)) [10].

$$N(Noise\ power) = kTB_n \quad (4)$$

As shown in Figure 2, the threshold to noise ratio (*T/N*) indicates the minimum signal threshold for detection [11], and it is calculated using Eq. (5). In Eq. (5), P_{false} indicates the probability of error (false alarm) when the enemy detection equipment identifies a target. In fact, it is difficult to calculate the error probability (P_{false}) accurately because these data are obtained through Live Fire Testing and Evaluation (LFT&E). In the pre-sent study, the detection probability is calculated by specifying arbitrary error probability.

$$N(Noise\ power) = kTB_n \quad (5)$$

2.1.2 Hit probability

Hit probability is the probability that the enemy threat weapon, which is targeted at the friendly ship, hits the target area, and it is calculated according to the characteristics and effectiveness of the threat weapon [5].

In the present study, the hit probability was calculated for two-dimensional target area using the probability density function under the assumption the threat weapon characteristics are classified as single hit and multiple hit according to the number of hits by the war head, as mentioned in Figure 1. The procedure to calculate the hit probability for a single hit is as follows:

(1) First, the target area (T_a) for the probability analysis is defined; this target area is an arbitrary area (T_a) from the total area of the naval ship.

(2) The probability distribution of the hit locations in the length and depth direction of the naval ship is defined. The data on probability distribution of the hit locations are obtained through the actual LFT&E. In the case of the naval ship survivability analysis program, MOTISS™, the probability distributions of the hit location in the length and depth direction of the naval ship are defined by the normal distribution and Weibull distribution, respectively [6]. In the present study, the hit probability was calculated by assuming that the probability distributions of each hit location in the length and depth directions of the naval ship as normal distributions (Figure 4).

(3) Thereafter, the area of the probability density function (PDF) with regard to the target area of analysis is calculated by the hit probability (Figure 5). The hit probability of a single hit can be defined by Eqs. (6)-(9). In these equations, x and y represent the variables in the length and depth direction, respectively, x_1~ x_2 and y_1~ y_2 indicate the hit area section in each plane. The linear error probability (LEP) of identifying the threat weapon's target in the normal-distribution-based hit shape of the naval ship is defined by Eq. (11). In the cal-

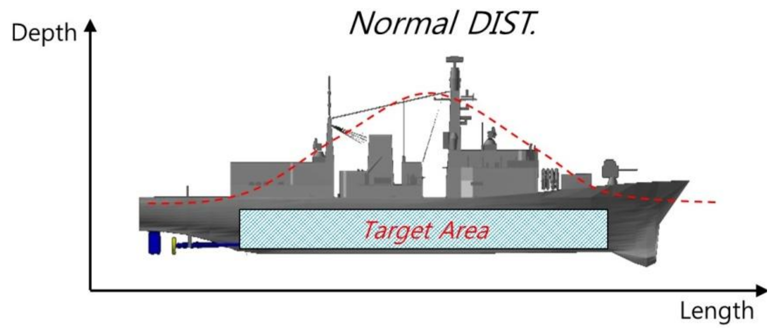

Figure 4. The example of probability distribution for hit on target area.

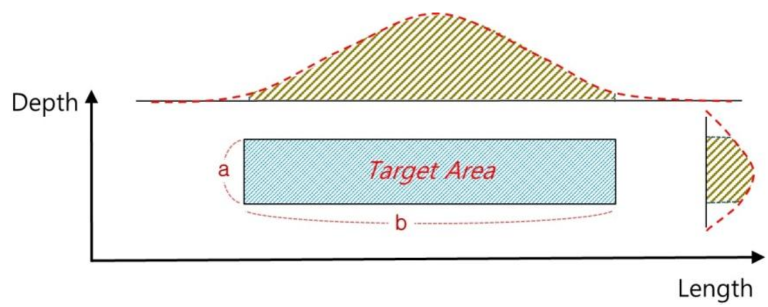

Figure 5. The probability distribution for target area on 2-dimensional surface.

culation of the hit probability with LEP, μ and σ represent the mean and standard deviations, respectively, of the corresponding probability distributions.

$$P_H(x,y) = P(Langth) \cdot P(Depth) = PDF(x) \cdot PDF(y) \quad (6)$$

$$PDF(x) \cdot PDF(y) = \int_{-\frac{a}{2}}^{\frac{a}{2}} P(x)dx \cdot \int_{-\frac{b}{2}}^{\frac{b}{2}} P(y)dy \quad (7)$$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(x-\mu)^2}{\sigma^2}\right] dx \quad (8)$$

$$P(y_1 < y < y_2) = \int_{y_1}^{y_2} \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(y-\mu)^2}{\sigma^2}\right] dy \quad (9)$$

$$LEP = 0.6745\sigma \quad (10)$$

The hit probability of multiple hits is determined by the total number of hits as calculated by using the hit probability calculation equation for a single hit, which was defined earlier [5]. The procedure for calculating the hit probability of multiple hits is as follows. (1) First, the total number of hits on the entire naval ship is determined (Eq. (11)). (2) The expected value ($E(x)$) of the hits on the pertinent target area is calculated (Eq. (12)). (3) Finally, the hit probability is calculated by Eq. (13) according to the expected hit values.

$$M = \frac{A_p}{A_c} \cdot K \quad (A_c = range^2\theta) \quad (11)$$

$$E(x) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] \quad (12)$$

$$P_H(H \geq 1) = \sum_{n=1}^{n} \left(1 - P_{single}\right)^{n-1} \cdot P_{single} = P_{single} \frac{1-\left(1-P_{single}\right)^n}{1-\left(1-P_{single}\right)} = 1 - \left(1-P_{single}\right)^n \quad (13)$$

2.2 Example of susceptibility analysis

In this section, an example model of the naval ship susceptibility analysis is described according to arbitrary multiple penetration threats and RCS detection characteristics. For this example model, the susceptibility analysis of arbitrary target

Table 2. Detection properties for radar.

Property	Value	Dimension
Receiver sensitivity	1E-14	W(m)
Operation frequency	0.25	[GHz]
Distance from radar	25,000	[m]
Angle	90	[θ]
Noise factor	2.5E-16	[w]
Temperature	195	[F]

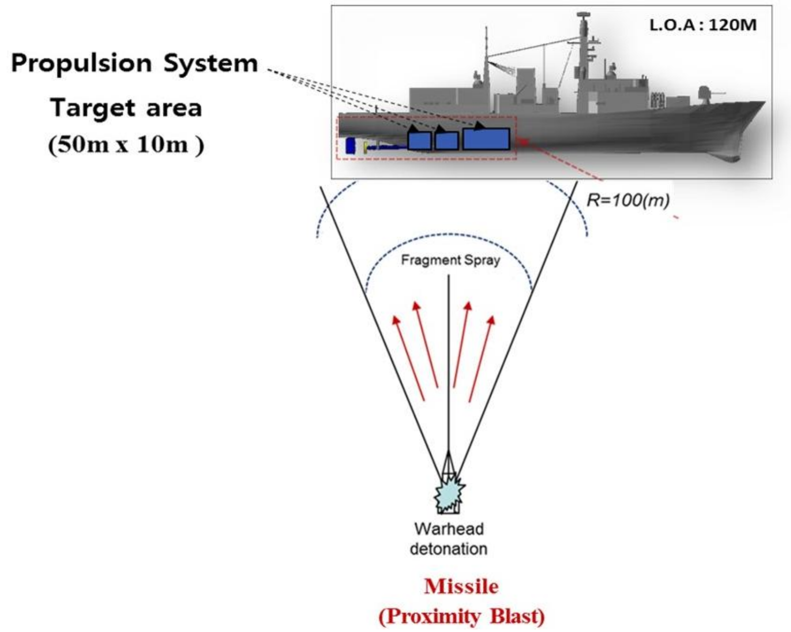

Figure 6. The example of threat scenario.

areas, including the propulsion system, was performed by assuming that the radar detection is expected to have the attributes listed in Table 2 for a naval ship whose overall length and the height of the hull outer plate exposed above the water surface are 120 m and 2 m, respectively. Furthermore, to calculate the detection probability in the susceptibility analysis, the naval ships' outer plate was assumed to be a flat plate, and the susceptibility analysis was performed for the hull outer plate excluding the super structure on the main deck.

Eq. (14) shows the calculation of the RCS value (σ) of the flat plate shape that was assumed as an example in this study. Using this value, the S/N value depending on the RCS detection is calculated, as shown in Eq. (15). Next, the detection probability is calculated based on Eq. (3) by considering the T/N and S/N values (Eq. (16)), where the T/N value is calculated using Eq. (17) by assuming an error probability (T_{false}) of 10^{-3}.

$$\sigma = \frac{4\pi(240)^2}{1.2^2} = 502{,}400 \quad (With\ wave\ length = 1.2) \tag{14}$$

$$S/N = \frac{PG^2\lambda^2\sigma}{(4\pi)^3R^4FkTB_nL} = 12.81 \tag{15}$$

$$P_D = \left[1 + \frac{2(6.91)(12.81)}{2+(12.81)^2}\right] e^{-2(6.91)(2+12.81)} = 0.813 \tag{16}$$

$$\frac{T}{N} = -\ln(0.001) = 6.91 \tag{17}$$

In the present example model, the threat weapon whose attributes are as shown in Figure 6 was assumed to be the fragments from a near-explosion hitting the naval ship. Eq. (18) is the calculation example for the number of hits (M) on the pertinent target areas, including the propulsion system. The distance range, incidence angle (θ), and total number of hits (K) were as-sumed to be 80 m, 90°, and 70, respectively.

$$M = \frac{500}{10048} \times 70 = 3.5 \tag{18}$$

With regard to the number of hits calculated by Eq. (18), the hit probability, which is based on the PDF, is calculated by Eq. (19). Therefore, the final hit probability of the present example model is calculated as shown in Eq. (20) by using the detection probability (T_D) and hit probability (T_{hit}) calculated earlier. In other words, the final hit probability is identified as approximately 70% during multiple hits on a 120 m long naval ship, i.e., the present example model.

$$P_H(H \geq 3.48) = 1 - (1-0.446)^{3.48} = 0.872 \tag{19}$$

$$P_H = 0.813 \times 0.872 = 0.709 \tag{20}$$

3. Conclusions

In the present study, a theoretical procedure was proposed to analyze quantitatively the susceptibility, which is one of the analysis components of the naval ship's survivability. The susceptibility analysis procedure was divided into two steps determining the detection probability that is based on the characteristics of the detection equipment, and determining the hit probability that is based on the characteristics of the threat weapon. In particular, the hit probability calculation procedure for single hit and multiple hits by the threat weapon was proposed by using the two-dimensional PDF. Furthermore, the susceptibility analysis procedures were organized for RCS detection and dealing with multiple penetration using examples based on the proposed theory. However, the detection characteristics, which are identified upon exposure of the naval ship in a battle environment, vary depending on the detection equipment; hence, the detection probability calculation must be formulated based on the S/N value for various detection characteristics such as infra-red and underwater radiated noise. In addition, the probability distribution, which is the core of the present susceptibility analysis procedure, is assumed as a normal distribution; in future, a probability distribution model with a quantitative base should be established.

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