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Reflection of plane harmonic wave in rotating media with fractional order heat transfer

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Abstract. The aim of the present investigation is to examine the propagation of plane harmonic waves in transversely isotropic homogeneous magneto visco thermoelastic rotating medium with fractional order heat transfer and two temperature. It is found that, for two dimensional assumed model, there exist three types of coupled longitudinal waves (quasi-longitudinal, quasi-transverse and quasi-thermal) in frequency domain. phase velocities, specific loss, penetration depth, attenuation coefficients of various reflected waves are computed and depicted graphically. The effects of viscosity and fractional order parameter by varying different values are represented graphically.

Keywords: thermoelastic; transversely isotropic; magneto-visco thermoelastic rotating medium; fractional-order heat transfer; plane harmonic wave propagation

1. Introduction

Magneto-thermoelasticity deals with the interactions of the strain, temperature, and magnetic field. Its applications includes geophysics, for understanding the effects of the earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, etc. Study of the plane wave propagation in a thermoelastic solid gained considerable importance because of its applications in the area of geophysics, nuclear fields, and related topics. In the last few decades, significant attention has been given in the area of magneto-thermoelastic harmonic plane wave propagation in a medium.

Ting (2004) explored a surface wave propagation in an anisotropic rotating medium. Othman and Song (2006, 2008) presented different hypotheses about magneto-thermo-elastic waves in a homogeneous and isotropic medium. Kumar and Chawla (2011) discussed the plane wane propagation in the anisotropic three-phase lag model and two-phase lag model. Deswal and Kalkal (2015) discussed the problem in a surface suffering a time-dependent thermal shock for thermo-viscoelastic interactions in a homogeneous, isotropic three-dimensional medium.

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The effects of reflection and refraction are studied by Kumar and Gupta (2015) at the boundary of elastic and a thermoelastic diffusion media, for plane waves by expanding the Fick law with dual-phase-lag diffusion model with delay times of both mass flow as well as the potential gradient. Besides, Kumar *et al*. (2016a) had depicted the effect of time and thermal and diffusion phase lags for axisymmetric heat supply in a ring by using Laplace and Hankel transform technique for the dual-phase-lag model for the transfer of heat and diffusion for upper and lower surfaces of the ring which were considered as traction free.

Bayones and Abd-Alla (2017) discussed the 2D problem of thermoelasticity regarding thermoelastic wave propagation in a rotating medium under magnetic field and time-dependent heat source effects due to the thermomechanical source. Kumar and Kansal (2017) found reflected and refracted waves occurrence due to longitudinal and transverse wave's incident implicitly at a plane interface between uniform elastic solid half-space and magneto-thermoelastic diffusive solid half-space with voids as a function of the angle of incidence and frequency of the incident wave. Maitya *et al*. (2017) presented plane wave propagation in a rotating elastic fiber-reinforced medium with magnetic and thermal fields under GN –I and II type theories. Said (2017) investigated the effect of hydrostatic initial stress and the gravity field on a thermoelastic medium which is fiber-reinforced with its own heat and constant motion by three-phase-lag model and types II G-N theory. Lata (2018a, b) studied the effect of energy dissipation on plane waves in sandwiched layered thermoelastic medium of uniform thickness, with combined effects of two temperature, rotation, and Hall current in the context of GN Type-II and Type-III theory of thermoelasticity.

Alesemi (2018) demonstrated the efficiency of the thermal relaxation time depending upon LS theory, Coriolis and Centrifugal Forces on the reflection coefficients of plane waves in an anisotropic magneto-thermoelastic rotating with stable angular velocity medium. Othman *et al*. (2019) dealt with the deformation of an infinite micro stretch generalized thermoelastic rotating medium under the effects of initially applied magnetics and gravitational field in GN Theory of thermoelasticity. Despite of this several researchers worked on different theory of thermoelasticity as, Marin (1994, 1995, 2009, 2010), Craciun and Soós (2006), Abbas and Marin (2017), Mohamed *et al*. (2009), Craciun *et al*. (2014), Kumar *et al*. (2016b), Othman *et al*. (2017), Ezzat and El-Bary (2017), Sharma and Marin (2014), Youssef (2013, 2016), Mahmoud *et al*. (2015), Lata *et al*. (2016), Mahmoud (2012), Othman and Marin (2017), Lata and Kaur (2019a, b), Riaz *et al*. (2019), Bhatti *et al*. (2019). Abd-Elaziz *et al*. (2019), Kaur and Lata (2019a, b, 2020), Lata *et al*. (2020a, b) and Lata and Kaur (2019a).

In spite of these, not much work has been carried out in harmonic plane wave propagation due to fractional order heat transfer in transversely isotropic magneto visco thermoelastic rotating medium with two temperature. Keeping these considerations in mind plane harmonic wave propagation problem in transversely isotropic magneto visco thermoelastic rotating medium with viscosity is studied by using reflection techniques.

2. Basic equations

Following Kumar *et al*. (2017), The simplified Maxwell's linear equation of electrodynamics for a slowly moving and perfectly conducting elastic solid are

$$curl\,\vec{h} = \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}, \tag{1}$$

$$curl\, \vec{E} = -\mu_0 \frac{\partial \vec{h}}{\partial t}, \tag{2}$$

$$\vec{E} = -\mu_0 \left(\frac{\partial \vec{u}}{\partial t} + \vec{H}_0 \right), \tag{3}$$

$$div\, \vec{h} = 0. \tag{4}$$

The constitutive relations for anisotropic thermoelastic medium are given by

$$t_{ij} = C_{ijkl} e_{kl} - \beta_{ij} T. \tag{5}$$

Equation of motion as described by Schoenberg and Censor (1973) for a transversely isotropic thermoelastic medium rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega \boldsymbol{n}$, where **n** is a unit vector representing the direction of the axis of rotation and taking into account Lorentz force

$$t_{ij,j} + F_i = \rho\{\ddot{u}_i + (\Omega \times (\Omega \times \mathrm{u})_i + (2\Omega \times \dot{u})_i\}, \tag{6}$$

where $F_i = \mu_0 (\vec{J} \times \vec{H}_0)$ are the components of Lorentz force, $\vec{H}_0$ is the external applied magnetic field intensity vector, $\vec{J}$ is the current density vector, $\vec{u}$ is the displacement vector, μ_0 and ε_0 are the magnetic and electric permeabilities respectively and t_{ij} the component of Maxwell stress tensor. The terms $\Omega \times (\Omega \times \mathrm{u})$ and $2\Omega \times \dot{u}$ are the additional centripetal acceleration due to the time-varying motion and Coriolis acceleration respectively. The heat conduction equation following Youssef (2006, 2010) is

$$K_{ij}\left(1 + \frac{(\tau_t)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\dot{T}_{,ji} + K_{ij}^*\left(1 + \frac{(\tau_v)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)T_{,ji} = \left(1 + \frac{(\tau_q)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{(\tau_q)^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right)\left[\rho C_E \ddot{T} + \beta_{ij} T_0 \ddot{e}_{ij}\right],$$

where

$$\begin{cases} 0 < \alpha < 1 \ \text{for weak conductivity}, \\ \quad \alpha = 1 \ \text{for normal conductivity}, \\ 1 < \alpha \le 2 \ \text{for strong conductivity}, \end{cases} \tag{7}$$

$$\beta_{ij} = C_{ijkl}\alpha_{ij},$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad i,j = 1,2,3.$$

$$\beta_{ij} = \beta_i \delta_{ij}, \qquad K_{ij} = K_i \delta_{ij}, \qquad i \ \text{is not summed}.$$

Here C_{ijkl} are elastic parameters and having symmetry $(C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk})$. The basis of these symmetries of C_{ijkl} are

i. The stress tensor is symmetric, which is only possible if $(C_{ijkl} = C_{jikl})$
ii. If a strain energy density exists for the material, the elastic stiffness tensor must satisfy $C_{ijkl} = C_{klij}$

iii. From stress tensor and elastic stiffness tensor symmetries infer ($C_{ijkl} = C_{ijlk}$) and $C_{ijkl} = C_{klij} = C_{jikl} = C_{ijlk}$

β_{ij} is the thermal elastic coupling tensor, T is the absolute temperature, T_0 is the reference temperature, φ is the conductive temperature, t_{ij} are the components of the stress tensor, e_{ij} are the components of strain tensor, u_i are the displacement components, ρ is the density, C_E is the specific heat, K_{ij} is the materialistic constant, α_{ij} is the coefficient of linear thermal expansion, τ_0 is the relaxation time, which is the time required to maintain steady-state heat conduction in an element of volume of an elastic body when sudden temperature gradient is imposed on that volume element, δ_{ij} is the Kronecker delta, Ω is the angular velocity of the solid, τ_t is the phase lag of heat flux, τ_v is the phase lag of temperature gradient, τ_q is the phase lag of thermal displacement, α is the fractional parameter.

3. Formulation and solution of the problem

We consider a homogeneous transversely isotropic magneto-visco-thermoelastic medium initially at a uniform temperature T_0. Following Kaliski (1963) the viscoelastic nature of the material is described by the Voigt model of linear viscoelasticity as

$$\bar{C}_{ijkl} = C_{ijkl}\left(1 + \tau\frac{\partial}{\partial t}\right), \tag{8}$$

where, τ is viscosity.

Now using (8) in (5) we have

$$t_{ij} = \bar{C}_{ijkl}e_{kl} - \beta_{ij}T. \tag{9}$$

Let the medium be permeated by an initial magnetic field $\vec{H}_0 = (0, H_0, 0)$ acting along $the\ y$-axis. The rectangular Cartesian coordinate system (x, y, z) having origin on the surface $(z = 0)$ with z-axis pointing vertically into the medium is introduced. In addition, we consider that

$$\boldsymbol{\Omega} = (0, \Omega, 0).$$

From the generalized Ohm's law (Lata and Kaur 2018)

$$\boldsymbol{E} = -\mu_0 H_0(-\ddot{w}, 0, \ddot{u}) \tag{10}$$

$$\boldsymbol{j} = \left(-\frac{\partial Hy}{\partial z} - \varepsilon_0\dot{E}_x, 0, -\frac{\partial Hy}{\partial x} - \varepsilon_0\dot{E}_z\right). \tag{11}$$

$$\boldsymbol{F} = \left(\mu_0 H_0^2\left(\frac{\partial e}{\partial x} - \varepsilon_0\mu_0\ddot{u}\right), 0, \mu_0 H_0^2\left(\frac{\partial e}{\partial z} - \varepsilon_0\mu_0\ddot{w}\right)\right) \tag{12}$$

In addition, the equations of displacement vector (u, v, w) and conductive temperature φ for transversely isotropic magneto visco thermoelastic solid

$$u = u(x,z,t), v = 0, w = w(x,z,t) \ and \ \varphi = \varphi(x,z,t). \tag{13}$$

following Slaughter (2002) and using appropriate transformations on (7) and (9) with the aid of (12)-(13), yield

$$\begin{gathered}\bar{C}_{11}\frac{\partial^2 u}{\partial x^2} + \bar{C}_{13}\frac{\partial^2 w}{\partial x \partial z} + \bar{C}_{44}\left(\frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 w}{\partial x \partial z}\right) - \beta_1 \frac{\partial}{\partial x}\left\{\varphi - \left(a_1\frac{\partial^2 \varphi}{\partial x^2} + a_3\frac{\partial^2 \varphi}{\partial z^2}\right)\right\} \\ +\mu_0 H_0^2\left(\frac{\partial e}{\partial x} - \varepsilon_0\mu_0\ddot{u}\right) = \rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right),\end{gathered} \tag{14}$$

$$\begin{gathered}(\bar{C}_{13} + \bar{C}_{44})\frac{\partial^2 u}{\partial x \partial z} + \bar{C}_{44}\frac{\partial^2 w}{\partial x^2} + \bar{C}_{33}\frac{\partial^2 w}{\partial z^2} - \beta_3\frac{\partial}{\partial z}\left\{\varphi - \left(a_1\frac{\partial^2 \varphi}{\partial x^2} + a_3\frac{\partial^2 \varphi}{\partial z^2}\right)\right\} \\ +\mu_0 H_0^2\left(\frac{\partial e}{\partial z} - \varepsilon_0\mu_0\ddot{w}\right) = \rho\left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right),\end{gathered} \tag{15}$$

$$\begin{aligned}&K_1\left(1 + \frac{(\tau_t)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\frac{\partial^2 \varphi}{\partial x^2} + K_3\left(1 + \frac{(\tau_t)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\frac{\partial^2 \dot{\varphi}}{\partial z^2} \\ &+K_1^*\left(1 + \frac{(\tau_v)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\frac{\partial^2 \varphi}{\partial x^2} + K_3^*\left(1 + \frac{(\tau_v)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right)\frac{\partial^2 \varphi}{\partial z^2} \\ &= \left(1 + \frac{(\tau_q)^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha} + \frac{(\tau_q)^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right) \\ &\quad\left[\rho C_E\frac{\partial^2}{\partial t^2}\left\{\varphi - \left(a_1\frac{\partial^2 \varphi}{\partial x^2} + a_3\frac{\partial^2 \varphi}{\partial z^2}\right)\right\} + T_0\left\{\beta_1\frac{\partial \ddot{u}}{\partial x} + \beta_1\frac{\partial \ddot{w}}{\partial z}\right\} + bT_0\ddot{\phi}\right],\end{aligned} \tag{16}$$

and

$$t_{11} = \bar{C}_{11}e_{11} + \bar{C}_{13}e_{13} - \beta_1 T, \tag{17}$$

$$t_{33} = \bar{C}_{13}e_{11} + \bar{C}_{33}e_{33} - \beta_3 T, \tag{18}$$

$$t_{13} = 2\bar{C}_{44}e_{13}, \tag{19}$$

$$\begin{aligned}\beta_1 &= (\bar{C}_{11} + \bar{C}_{12})\alpha_1 + \bar{C}_{13}\alpha_{3,} \\ \beta_3 &= 2\bar{C}_{13}\alpha_1 + \bar{C}_{33}\alpha_{3,} \\ e &= \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\end{aligned}$$

To simplify the solution, mention below dimensionless quantities are used

$$\begin{gathered}x' = \frac{x}{L}, \quad z' = \frac{z}{L}, \quad u' = \frac{\rho c_1^2}{L\beta_1 T_0}u, \quad w' = \frac{\rho c_1^2}{L\beta_1 T_0}w, \quad t' = \frac{c_1}{L}t, \\ T' = \frac{T}{T_0}, \quad t'_{11} = \frac{t_{11}}{\beta_1 T_0}, \quad t'_{33} = \frac{t_{33}}{\beta_1 T_0}, \quad t'_{31} = \frac{t_{31}}{\beta_1 T_0}, \quad h' = \frac{h}{H_0}, \\ \Omega' = \frac{\mathrm{L}}{C_1}\Omega, \quad (\tau', \tau'_T, \tau'_v, \tau'_q, t') = \frac{c_1}{L}(\tau, \tau_T, \tau_v, \tau_q, t).\end{gathered} \tag{20}$$

Making use of (20) in Eqs. (14)-(16), after suppressing the primes, yield

$$\left[\left(1+\tau\frac{\partial}{\partial t}\right)+\delta_5\right]\frac{\partial^2 u}{\partial x^2}+\left[\delta_4\left(1+\tau\frac{\partial}{\partial t}\right)+\delta_5\right]\frac{\partial^2 w}{\partial x\partial z}+\delta_2\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial x\partial z}\right)$$
$$-\frac{\partial}{\partial x}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\}=\left(\frac{\varepsilon_0\mu_0^2H_0^2}{\rho}+1\right)\frac{\partial^2 u}{\partial t^2}-\Omega^2 u+2\Omega\frac{\partial w}{\partial t}, \tag{21}$$

$$\left[\delta_1\left(1+\tau\frac{\partial}{\partial t}\right)+\delta_5\right]\frac{\partial^2 u}{\partial x\partial z}+\delta_2\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial^2 w}{\partial x^2}+\left[\delta_3\left(1+\tau\frac{\partial}{\partial t}\right)+\delta_5\right]\frac{\partial^2 w}{\partial z^2}$$
$$-\frac{\beta_3}{\beta_1}\frac{\partial}{\partial z}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\}=\left(\frac{\varepsilon_0\mu_0^2H_0^2}{\rho}+1\right)\frac{\partial^2 w}{\partial t^2}-\Omega^2 w-2\Omega\frac{\partial u}{\partial t}, \tag{22}$$

$$\left[1+\frac{C_1}{L}\frac{\tau_T^\alpha}{\alpha!}\frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right]\left[K_1\frac{\partial^2}{\partial x^2}+K_3\frac{\partial^2}{\partial z^2}\right]\varphi+\left[1+\frac{\tau_v^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}\right]\left[K_1^*\frac{\partial^2}{\partial x^2}+K_3^*\frac{\partial^2}{\partial z^2}\right]\varphi$$
$$=\left[1+\frac{\tau_q^\alpha}{\alpha!}\frac{\partial^\alpha}{\partial t^\alpha}+\frac{\tau_q^{2\alpha}}{2\alpha!}\frac{\partial^{2\alpha}}{\partial t^{2\alpha}}\right]$$
$$\left[C_1^2\rho C_E\frac{\partial^2}{\partial t^2}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\}+\beta_1 T_0\left(\beta_1\frac{\partial\ddot{u}}{\partial x}+\beta_3\frac{\partial\ddot{w}}{\partial z}\right)\right], \tag{23}$$

where

$$\delta_1=\frac{C_{13}+C_{44}}{C_{11}},\qquad \delta_2=\frac{C_{44}}{C_{11}},\qquad \delta_3=\frac{C_{33}}{C_{11}},\qquad \delta_4=\frac{C_{13}}{C_{11}},\qquad \delta_5=\frac{\beta_1 T_0\mu_0 H_0^2}{L\rho^2 C_1^4}.$$

Making use of dimensionless quantities defined by (20) in Eqs. (17)-(19) and after suppressing the primes, yields

$$t_{11}(\mathrm{x,z,t})=\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x}+\delta_4\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z}-\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\} \tag{24}$$

$$t_{33}(\mathrm{x,z,t})=\delta_4\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial x}+\delta_3\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial z}-\frac{\beta_3}{\beta_1}\left\{\varphi-\left(a_1\frac{\partial^2\varphi}{\partial x^2}+a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\}, \tag{25}$$

$$t_{13}(\mathrm{x,z,t})=\delta_2\left(1+\tau\frac{\partial}{\partial t}\right)\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right), \tag{26}$$

4. Plane-wave propagation

We pursue plane wave to be a time-harmonic solution of the equations of the form

$$\begin{pmatrix}u\\ w\\ \varphi\end{pmatrix}=\begin{pmatrix}U\\ W\\ \varphi^*\end{pmatrix}e^{i(\xi(xn_1+z\,n_3)-\omega t)} \tag{27}$$

where $n_1 = \sin\theta\,, n_3 = \cos\theta$ denotes the projection of wave normal on to the x-z plane, ξ and ω are the wavenumbers and angular frequency of plane waves propagating in x-z plane respectively.

Upon using Eq. (27) in Eqs. (21)-(23) we get

$$\begin{aligned} &U[\zeta_1\xi^2 + \zeta_2] + W[\zeta_3\xi^2 + \zeta_4] + \varphi^*(\zeta_5\xi + \zeta_6\xi^3) = 0,\\ &U[\zeta_7\xi^2 - \zeta_4] + W[\zeta_8\xi^2 + \zeta_2] + \varphi^*(\zeta_9\xi + \zeta_{10}\xi^3) = 0,\\ &-\zeta_{11}U - \zeta_{12}W + \varphi^*[\zeta_{13}\xi^2 + \zeta_{14}\xi^4] = 0. \end{aligned}$$

and then eliminating $U, W$and φ^* from the resulting equations yields the following characteristic equation

$$(A\xi^6 + B\xi^4 + C\xi^2 + D)\xi^2 = 0, \tag{28}$$

Where

$$\begin{aligned}
A &= \zeta_1\zeta_{14}\zeta_8 - \zeta_3\zeta_7\zeta_{14},\\
B &= \zeta_1\zeta_8\zeta_{13} + \zeta_8\zeta_2\zeta_{14} + \zeta_2\zeta_{14}\zeta_1 + \zeta_{12}\zeta_1\zeta_{10} - \zeta_3\zeta_7\zeta_{13}\\
&\quad -\zeta_4\zeta_7\zeta_{14} - \zeta_3\zeta_4\zeta_{14} - \zeta_3\zeta_{10}\zeta_{11} - \zeta_6\zeta_7\zeta_{12} + \zeta_{11}\zeta_8\zeta_6,\\
C &= \zeta_2\zeta_8\zeta_{13} + \zeta_1\zeta_2\zeta_{13} + \zeta_2\zeta_2\zeta_{14} + \zeta_1\zeta_{12}\zeta_9 + \zeta_{12}\zeta_{10}\zeta_2 - \zeta_4\zeta_7\zeta_{13}\\
&\quad -\zeta_4\zeta_4\zeta_{14} - \zeta_3\zeta_{11}\zeta_9 - \zeta_7\zeta_{12}\zeta_5 - \zeta_6\zeta_4\zeta_{12} + \zeta_5\zeta_8\zeta_{11} - \zeta_3\zeta_{11}\zeta_9 + \zeta_6\zeta_{11}\zeta_2,\\
D &= -\zeta_2\zeta_2\zeta_{13} + \zeta_1\zeta_{12}\zeta_9 - \zeta_4\zeta_4\zeta_{13} - \zeta_{11}\zeta_4\zeta_9 - \zeta_4\zeta_{12}\zeta_5 + \zeta_{11}\zeta_2\zeta_5,\\
\zeta_1 &= -(1+\delta_5)(1-i\omega\tau)n_1^2 - \delta_2(1-i\omega\tau)n_3^2,\\
\zeta_2 &= \left(\frac{\varepsilon_0\mu_0^2H_0^2}{\rho} + 1\right)\omega^2 + \Omega^2,\\
\zeta_3 &= (\delta_4 + \delta_5 + \delta_2)(1-i\omega\tau)n_1n_3,\\
\zeta_4 &= -2\omega\Omega i,\\
\zeta_5 &= in_1,\\
\zeta_6 &= n_1 i(a_1n_1^2 + a_3n_3^2),\\
\zeta_7 &= (\delta_1 + \delta_5)(1-i\omega\tau)n_1n_3,\\
\zeta_8 &= -\delta_2(1-i\omega\tau)n_1^2 - (\delta_3+\delta_5)(1-i\omega\tau)n_3^2,\\
\zeta_9 &= -i\frac{\beta_3}{\beta_1}n_3,\\
\zeta_{10} &= -\frac{\beta_3}{\beta_1}n_3 i(a_1n_1^2 + a_3n_3^2)\\
\zeta_{11} &= -\frac{\beta_1^2T_0}{\rho}\omega^2 in_1\,,\\
\zeta_{12} &= \frac{\beta_1\beta_3}{\rho}T_0\omega^2 in_3,\\
\zeta_{13} &= \left[1 + \frac{C_1}{L}\frac{\tau_T^\alpha}{\alpha!}(i\omega)^{\alpha+1}\right][-K_1n_1^2 - K_3n_3^2] + \left[1 + \frac{\tau_v^\alpha}{\alpha!}(i\omega)^\alpha\right][-K_1^*n_1^2 + K_3^*n_3^2]\\
&\quad -C_1^2\rho C_E\omega^2\left[1 + \frac{\tau_q^\alpha}{\alpha!}(i\omega)^\alpha + \frac{\tau_q^{2\alpha}}{2\alpha!}(i\omega)^{2\alpha}\right],\\
\zeta_{14} &= (a_1n_1^2 + a_3n_3^2)\zeta_{13},
\end{aligned}$$

The six non zero roots of Eq. (28) give six roots of ξ that is, $\pm\xi_1$,$\pm\xi_2$ and $\pm\xi_3$, in which we are interested in those roots whose imaginary parts are positive. Corresponding to these roots, there exist three waves corresponding to descending order of their velocities namely a quasi-longitudinal (QL), quasi-transverse (QTS) and quasi-thermal waves (QT). The phase velocities, attenuation coefficients, specific loss and penetration depth of these waves are obtained by the following expressions.

4.1 Phase velocity

The phase velocities are given by

$$V_i = \frac{\omega}{Re(\xi_i)}, \qquad \mathrm{i} = 1,2,3$$

where V_1, V_2, V_3 are the velocities of QL, QTS, and QT waves respectively.

4.2 Attenuation coefficient

The attenuation coefficient is defined as

$$Q_i = Img(\xi_i), \qquad \mathrm{i} = 1,2,3.$$

where Q_1, Q_2, Q_3 are the attenuation coefficients of QL, QTS, and QT waves respectively.

4.3 Specific loss

The specific loss is the ratio of energy (ΔW) dissipated in taking a specimen through the cycle, to elastic energy (W) stored in a specimen when the strain is maximum. The specific loss is the most direct method of defining internal friction for a material. For a sinusoidal plane wave of small amplitude, it was shown by Kaliski (1963) that specific loss $\frac{\Delta W}{W}$ equals 4π times the absolute value of the imaginary part of ξ to the real part of ξ i.e.

$$W_i = \left(\frac{\Delta W}{W}\right) i = 4\pi \left|\frac{Img(\xi_i)}{Re(\xi_i)}\right|, \qquad \mathrm{i} = 1,2,3.$$

where W_1, W_2, W_3 are specific loss of QL, QTS, and QT waves respectively.

4.4 Penetration depth

The penetration depth is defined by

$$S_i = \frac{1}{Img(\xi_i)}, \qquad \mathrm{i} = 1,2,3.$$

where S_1, S_2, S_3 are penetration depth of QL, QTS, and QT waves respectively.

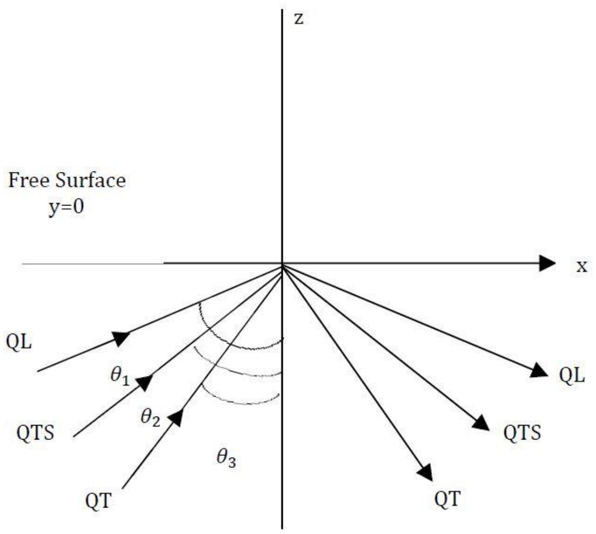

Fig. 1 Geometry of the problem

5. Reflection and transmission at the boundary surfaces

We consider a homogeneous transversely isotropic magneto visco thermoelastic half-space occupying the region z ≥ 0. Incident quasi-longitudinal or quasi-transverse or quasithermal waves at the stress free, thermally insulated surface (z = 0) will generate reflected QL, reflected QTS and reflected QT waves in the half-space z > 0. The total displacements, conductive temperature are given by

$$u = \sum_{j=1}^{6} A_j e^{iM_j}, \tag{29}$$

$$w = \sum_{j=1}^{6} d_j A_j e^{iM_j}, \tag{30}$$

$$\varphi = \sum_{j=1}^{6} l_j A_j e^{iM_j}, \qquad j = 1,2,3,\dots,6 \tag{31}$$

Where

$$M_j = \omega t - \xi_j(xn_{1j} - zn_{3j}), \qquad j = 1, 2, 3,$$
$$M_j = \omega t - \xi_j(xn_{1j} + zn_{3j}), \qquad j = 4, 5, 6.$$

Here subscripts j = 1, 2, 3 respectively denote the quantities corresponding to incident QL, QTS and QT-mode, whereas the subscripts j = 4, 5, 6 denote the corresponding reflected waves, ξ_j are the roots obtained from Eq. (25), $n_{1j} = \sin\theta_j$; $n_{3j} = \cos\theta_j$.

$$d_j = \frac{(\zeta_2\zeta_{13j} + \zeta_{12j}\zeta_{9j}) + (\zeta_{8j}\zeta_{13j} + \zeta_2\zeta_{14j} + \zeta_{12j}\zeta_{10j})\xi_j^2 + \zeta_8 j\zeta_{14j}\xi_j^4}{\zeta_{7j}\zeta_{14j}\xi_j^4 + (\zeta_{1j}\zeta_{13j} + \zeta_2\zeta_{14j} + \zeta_{6j}\zeta_{11j})\xi_j^2 - (\zeta_2\zeta_{13j} + \zeta_{11j}\zeta_{5j})}, \qquad j = 1, 2, 3.$$

$$l_j = \frac{(\zeta_2^2+\zeta_4^2) + \left(\zeta_2\zeta_{1j} + \zeta_2\zeta_{8j} + \zeta_4\zeta_{3j} - \zeta_4\zeta_{7j}\right)\xi_j^2 + \left(\zeta_4 j\zeta_{8j} - \zeta_7 j\zeta_{3j}\right)\xi_j^4}{\zeta_{7j}\zeta_{14j}\xi_j^4 + \left(\zeta_{1j}\zeta_{13j} + \zeta_2\zeta_{14j} + \zeta_{6j}\zeta_{11j}\right)\xi_j^2 - \left(\zeta_2\zeta_{13j} + \zeta_{11j}\zeta_{5j}\right)}, \qquad \mathrm{j} = 1, 2, 3.$$

$$d_j = \frac{\left(\zeta_2\zeta_{13j} - \zeta_{12j}\zeta_{9j}\right) + \left(\zeta_{8j}\zeta_{13j} + \zeta_2\zeta_{14j} + \zeta_{12j}\zeta_{10j}\right)\xi_j^2 + \zeta_8 j\zeta_{14j}\xi_j^4}{-\zeta_{7j}\zeta_{14j}\xi_j^4 + \left(\zeta_{1j}\zeta_{13j} + \zeta_2\zeta_{14j} + \zeta_{6j}\zeta_{11j}\right)\xi_j^2 - \left(\zeta_2\zeta_{13j} + \zeta_{11j}\zeta_{5j}\right)}, \qquad \mathrm{j} = 4, 5, 6.$$

$$l_j = \frac{(\zeta_2^2+\zeta_4^2) + \left(\zeta_2\zeta_{1j} + \zeta_2\zeta_{8j} - \zeta_4\zeta_{3j} + \zeta_4\zeta_{7j}\right)\xi_j^2 + \left(\zeta_4 j\zeta_{8j} + \zeta_7 j\zeta_{3j}\right)\xi_j^4}{-\zeta_{7j}\zeta_{14j}\xi_j^4 + \left(-\zeta_{1j}\zeta_{13j} + \zeta_2\zeta_{14j} + \zeta_{6j}\zeta_{11j}\right)\xi_j^2 - \left(\zeta_2\zeta_{13j} + \zeta_{11j}\zeta_{5j}\right)}, \qquad \mathrm{j} = 4, 5, 6.$$

6. Boundary conditions

The dimensionless boundary conditions at the free surface z = 0, are given by

$$t_{33} = 0, \tag{32}$$

$$t_{31} = 0, \tag{33}$$

$$\frac{\partial \varphi}{\partial z} = 0. \tag{34}$$

Making use of Eq. (26) into the boundary conditions Eqs. (32)-(34), and using Eqs. (29)-(31) we obtain

$$\begin{aligned} &\sum_{j=1}^{3} A_j e^{i\left(\omega t - \xi_j(x sin\theta_j)\right)} \Big[-\delta_4(1-i\omega\tau)i\xi_j sin\theta_j + \delta_3(1-i\omega\tau)id_j\xi_j cos\theta_j \\ &\qquad - \frac{\beta_3}{\beta_1} l_j\left(1 + a_1\xi_j^2\sin^2\theta_j + a_3\xi_j^2\cos^2\theta_j\right)\Big] \\ &+ \sum_{j=4}^{6} A_j e^{i(\omega t - \xi_j(x sin\theta_j))} \Big[-\delta_4(1-i\omega\tau)i\xi_j sin\theta_j - \delta_3(1-i\omega\tau)id_j\xi_j cos\theta_j \\ &\qquad - \frac{\beta_3}{\beta_1} l_j\left(1 + a_1\xi_j^2\sin^2\theta_j + a_3\xi_j^2\cos^2\theta_j\right)\Big] = 0, \end{aligned} \tag{35}$$

$$\begin{aligned} &\sum_{j=1}^{3} A_j e^{i\left(\omega t - \xi_j(x sin\theta_j)\right)} \left[\xi_j cos\theta_j - d_j\xi_j sin\theta_j\right] \\ &- \sum_{j=4}^{6} A_j e^{i(\omega t - \xi_j(x sin\theta_j))} \left[\xi_j cos\theta_j + d_j\xi_j sin\theta_j\right] = 0, \end{aligned} \tag{36}$$

$$\sum_{j=1}^{3} A_j e^{i(\omega t - \xi_j(x sin\theta_j))} \left[il_j\xi_j cos\theta_j\right] - \sum_{j=4}^{6} A_j e^{i(\omega t - \xi_j(x sin\theta_j))} \left[il_j\xi_j cos\theta_j\right] = 0, \tag{37}$$

The Eqs. (35)-(37) are satisfied for all values of x, therefore we have

$$M_1(x,0) = M_2(x,0) = M_3(x,0) = M_4(x,0) = M_5(x,0) = M_6(x,0) \tag{38}$$

From Eqs. (26) and (33), we obtain

$$\xi_1 sin\theta_1 = \xi_2 sin\theta_2 = \xi_3 sin\theta_3 = \xi_4 sin\theta_4 = \xi_5 sin\theta_5 = \xi_6 sin\theta_6 \tag{39}$$

which is the form of Snell's law for the stress-free, the thermally insulated surface of transversely isotropic magneto-visco-thermoelastic medium with rotation. Eqs. (35)-(37) and (39) yield

$$\sum_{j=1}^{3} X_{ij}A_j + \sum_{j=4}^{6} X_{ij}A_j = 0, \qquad (i = 1,2,3) \tag{40}$$

Where for p = 1, 2, 3 we have

$$\begin{aligned} X_{1p} &= -\delta_4(1-i\omega\tau)i\xi_p sin\theta_p + \delta_3(1-i\omega\tau)id_p\xi_p cos\theta_p \\ &\quad -\frac{\beta_3}{\beta_1}l_p\left(1 + a_1\xi_p^2\sin^2\theta_p + a_3\xi_p^2\cos^2\theta_p\right) \\ X_{2p} &= \xi_p cos\theta_p - d_p\xi_p sin\theta_p, \\ X_{3p} &= il_p\xi_p cos\theta_p \end{aligned}$$

And for j = 4, 5, 6 we have

$$\begin{aligned} X_{1j} &= -\delta_4(1-i\omega\tau)i\xi_j sin\theta_j - \delta_3(1-i\omega\tau)id_j\xi_j cos\theta_j \\ &\quad -\frac{\beta_3}{\beta_1}l_j\left(1 + a_1\xi_j^2\sin^2\theta_j + a_3\xi_j^2\cos^2\theta_j\right), \\ X_{2j} &= -\xi_j cos\theta_j - d_j\xi_j sin\theta_j, \\ X_{3j} &= -il_j\xi_j cos\theta_j \end{aligned}$$

Incident QL-wave

In the case of a quasi-longitudinal wave, the subscript p takes only one value, that is p = 1, which means $A_2 = A_3 = 0$. Dividing the set of Eq. (40) throughout by A_1, we obtain a system of three homogeneous equations in three unknowns which can be solved by Cramer's rule and we have

$$A_{1i} = \frac{A_{i+3}}{A_1} = \frac{\Delta_i^1}{\Delta} \tag{41}$$

Incident QTS-wave

In the case of a quasi-transverse wave, the subscript q takes only one value, that is q = 2, which means. Dividing the set of Eq. (40) throughout by we obtain a system of three homogeneous equations in three unknowns which can be solved by Cramer's rule and we have

$$A_{2i} = \frac{A_{i+3}}{A_2} = \frac{\Delta_i^2}{\Delta} \tag{42}$$

Incident QT-wave

In the case of a quasi-thermal wave, the subscript q takes only one value, that is q = 3, which means. Dividing the set of Eq. (40) throughout we obtain a system of three homogeneous equations in three unknowns which can be solved by Cramer's rule and we have

$$A_{3i} = \frac{A_{i+3}}{A_3} = \frac{\Delta_i^3}{\Delta} \tag{43}$$

Where Z_i (i = 1, 2, 3) is the amplitude ratios of the reflected QL, reflected QTS, reflected QT - waves to that of the incident QL-(QTS or QT) waves respectively.

Here

$$\Delta = \left|A_{i(i+3)}\right|_{3X3}$$
$$\Delta_i^p, \qquad (i = 1,2,3)$$

can be obtained by replacing, respectively, the 1st, 2nd and 3rd columns of Δ by $\left[-X_{1p}, -X_{2p}, -X_{3p}\right]'$.

Following Achenbach (1973), the energy flux across the surface element, which is the rate at which the energy is communicated per unit area of the surface is represented as

$$P^* = t_{lm} n_m \dot{u}_l \tag{44}$$

Where t_{lm} is the stress tensor, n_m are the direction cosines of the unit normal and $\dot{u}_l$ are the components of the particle velocity.

The time average of P^* over a period, denoted by $< P^* >$ represents the average energy transmission per unit surface area per unit time and is given at the interface $z = 0$ as

$$< P^* > = < Re(t_{13}).Re(\dot{u}) + Re(t_{33}).Re(\dot{w}) > \tag{45}$$

Following Achenbach (1973), for any two complex functions f and g, we have

$$< Re(f) >< Re(g) > = \frac{1}{2} Re(f\bar{g}) \tag{46}$$

Hence

$$< P^* \geq \frac{1}{2} Re(t_{13}\bar{\dot{u}}) + \frac{1}{2} Re(t_{33}\bar{\dot{w}})$$
$$= \frac{1}{2} Re\left(\delta_2(1 - i\omega\tau)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\bar{\dot{u}}\right)$$
$$+ \frac{1}{2} Re\left(\delta_4(1 - i\omega\tau)\frac{\partial u}{\partial x} + \delta_3(1 - i\omega\tau)\frac{\partial w}{\partial z} - \frac{\beta_3}{\beta_1}\left\{\varphi - \left(a_1\frac{\partial^2\varphi}{\partial x^2} + a_3\frac{\partial^2\varphi}{\partial z^2}\right)\right\}\bar{\dot{w}}\right)$$

The expressions for energy ratios E_i (I = 1, 2, 3) for reflected QL, QTS, QT wave are given as

(i) In case of incident QL- wave

$$E_{1i} = \frac{< P_{i+3}^* >}{< P_1^* >}, \qquad i = 1, 2, 3. \tag{47}$$

(ii) In case of incident QTS- wave

$$E_{2i} = \frac{< P_{i+3}^* >}{< P_2^* >}, \qquad i = 1, 2, 3. \tag{48}$$

(iii) In case of incident QT- wave

$$E_{3i} = \frac{< P_{i+3}^* >}{< P_3^* >}, \qquad i = 1, 2, 3. \tag{49}$$

Where $< P_i^* >$ i = 1, 2, 3 are the average energies transmission per unit surface area per unit time corresponding to incident QL, QTS, QT waves respectively and $< P_{i+3}^* >$ i = 1, 2, 3 are the average energies transmission per unit surface area per unit time corresponding to reflected QL, QTS, QT waves respectively.

7. Particular cases

(1) If $\tau_T \neq 0, \tau_v \neq 0, \tau_q \neq 0$ we obtain results for plane harmonic wave propagation in transversely isotropic magneto-visco-thermoelastic solid with rotation, and with and without energy dissipation and TPL (three-phase lag) effects and fractional order heat transfer with two temperature.

(2) If $\tau_T = 0, \tau_v = 0, \tau_q = 0$ and $K^* \neq 0$ we obtain results for plane harmonic wave propagation in transversely isotropic magneto-visco-thermoelastic solid with rotation and GN III theory (thermoelasticity with energy dissipation) and fractional order heat transfer with two temperature.

(3) If $\tau_T = 0, \tau_v = 0, \tau_q = 0$, and $K^* = 0$ we obtain results for plane harmonic wave propagation in transversely isotropic magneto-visco-thermoelastic solid with rotation and GN II theory (generalized thermoelasticity without energy dissipation) and fractional order heat transfer with two temperature.

(4) If $\tau_T \neq 0, \tau_v \neq 0, \tau_q \neq 0$ and $K^* = 0$ we obtain results for plane harmonic wave propagation in transversely isotropic magneto-visco-thermoelastic solid with rotation and GN II theory with TPL effect and fractional order heat transfer with two temperature.

(5) If $\tau_T = 0, \tau_v = 0, \tau_q = \tau_0 > 0$ and $K^* = 0$, and ignoring τ_q^2 we obtain results for Rayleigh wave propagation in transversely isotropic magneto-visco-thermoelastic solid with diffusion and Lord–Shulman model and fractional order heat transfer with two temperature.

(6) If $\tau_T = 0, \tau_v = 0, \tau_q = 0$ and if the medium is not permeated with the magnetic field i.e., $\mu_0 = H_0 = 0$ then we obtain results for plane harmonic wave propagation in transversely isotropic visco-thermoelastic solid with rotation and without TPL effect and fractional order heat transfer with two temperature.

(7) If $C_{11} = C_{33} = \lambda + 2\mu, C_{12} = C_{13} = \lambda, C_{44} = \mu,$ $\alpha_1 = \alpha_3 = \alpha',$ $a_1 = a_3 = a$, $K_1 = K_3 = K,$ $K_1^* = K_3^* = K^*$ we obtain expressions for plane harmonic wave propagation in magneto-visco-thermoelastic isotropic materials with rotation and with and without energy dissipation with TPL effect and fractional order heat transfer with two temperature.

(8) If $\alpha = 1$ and $\tau_T \neq 0, \tau_v \neq 0, \tau_q \neq 0$ we obtain results for plane harmonic wave propagation in transversely isotropic magneto-visco-thermoelastic solid with rotation, and

with and without energy dissipation and TPL (three-phase lag) effects with two temperature.

(9) If $\tau = 0,$ we obtain results for plane harmonic wave propagation in transversely isotropic magneto- thermoelastic solid with two temperature for all the above 8 cases.

(10) If $a_1 = a_3 = o,$ we obtain results for plane harmonic wave propagation in transversely isotropic magneto-visco-thermoelastic solid without two temperature for all the above 8 cases.

8. Numerical results and discussion

To demonstrate the theoretical results and effect of fractional order parameter and two temperature, the physical data for cobalt material, which is transversely isotropic, is taken from Dhaliwal and Singh (1980) is given as

$C_{11} = 3.07 \times 10^{11} Nm^{-2},$ $C_{33} = 3.581 \times 10^{11} Nm^{-2},$ $C_{13} = 1.027 \times 10^{10} Nm^{-2},$
$C_{44} = 1.510 \times 10^{11} Nm^{-2},$ $\beta_1 = 7.04 \times 10^6 Nm^{-2} deg^{-1},$ $\beta_3 = 6.90 \times 10^6 Nm^{-2} deg^{-1},$
$\rho = 8.836 \times 10^3 Kgm^{-3},$ $C_E = 4.27 \times 10^2 jKg^{-1} deg^{-1},$
$K_1 = 0.690 \times 10^2 Wm^{-1} Kdeg^{-1},$ $K_3 = 0.690 \times 10^2 Wm^{-1} K^{-1},$
$K_1^* = 1.313 \times 10^2 Wsec,$ $K_3^* = 1.54 \times 10^2 Wsec,$
$\mathrm{T_0} = 298\ \mathrm{K},$ $\mathrm{H_0} = 1\ \mathrm{Jm^{-1} nb^{-1}},$ $\varepsilon_0 = 8.838 \times 10^{-12} \mathrm{Fm^{-1}},$ $\mathrm{L} = 1.$

The values of frequency, rotation Ω, magnetic effect H_0, are taken as 0.03,0.5, 10, respectively. The software MATLAB 8.0.4 has been used to determine the amplitude ratios of reflected QL, QTS and QT waves with respect to incident QL, QTS, and QT waves respectively. The variation of the magnitude of amplitude ratios has been plotted in the Figs. 2-10 with respect to the angle of incidence. A comparison has been made to show the effect of fractional order parameter and two temperature on the various quantities.

(1) The black line represents $\alpha = 0.5, \tau = 0.0$,
(2) The red line represents to $\alpha = 1.5, \tau = 0.0,$
(3) The blue line represents to $\alpha = 0.5, \tau = 1.0,$
(4) The green line represents to $\alpha = 1.5, \tau = 1.0,$

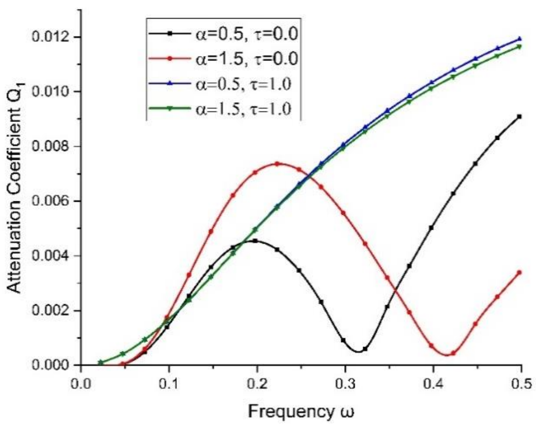

Fig. 2 Variations of Attenuation Coefficient Q_1 with frequency ω

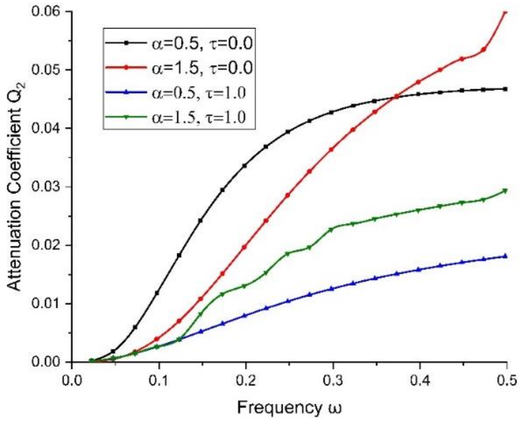

Fig. 3 Variations of attenuation coefficient Q_2 with frequency ω

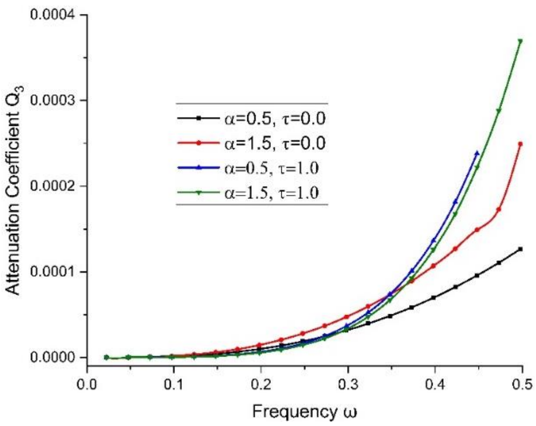

Fig. 4 Variations of attenuation coefficient Q_3 with frequency ω

8.1 Attenuation coefficients

Figs. 2-4 show the variations of attenuation coefficients Q_1, Q_2, Q_3 of QL, QTS and QT waves with respect to frequency ω. From the graphs, we observe that the attenuation coefficients Q_1, Q_2, Q_3 of QL, QTS and QT waves increase sharply with the increase in frequency ω with variations in magnitude for different values of α and viscosity.

8.2 Specific loss

Figs. 5-7 show the variations of specific loss W_1, W_2, W_3 of QL, QTS and QT waves with respect to frequency ω. From the graphs, we observe that specific loss W_1, W_2, W_3 of QL, QTS and QT waves decrease with increase in frequency ω with variations in magnitude for different values of fractional order parameter α and viscosity.

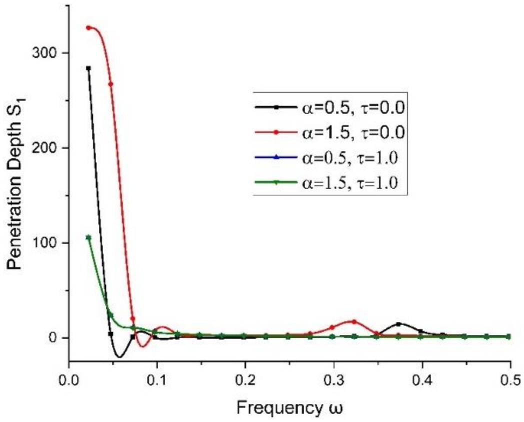

Fig. 5 Variations of Specific Loss W_1 with frequency ω

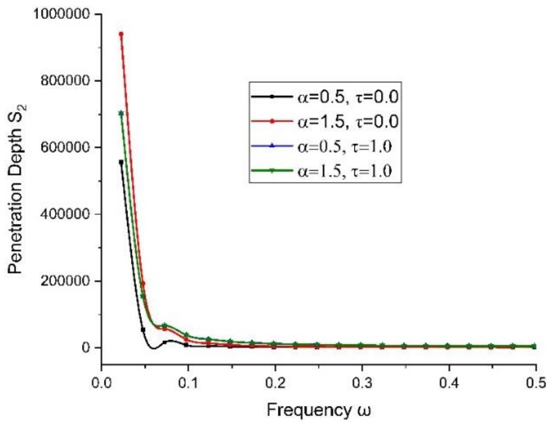

Fig. 6 Variations of specific loss W_2 with frequency ω

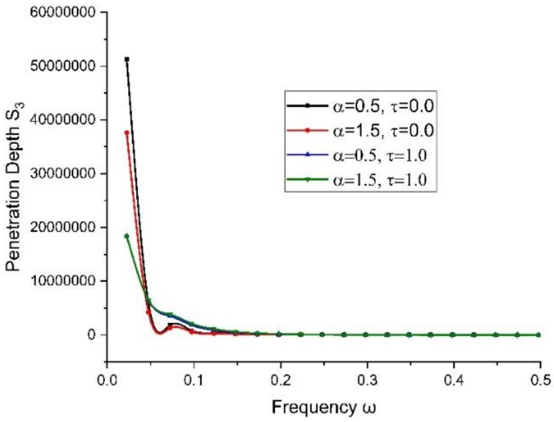

Fig. 7 Variations of specific loss W_3 with frequency ω

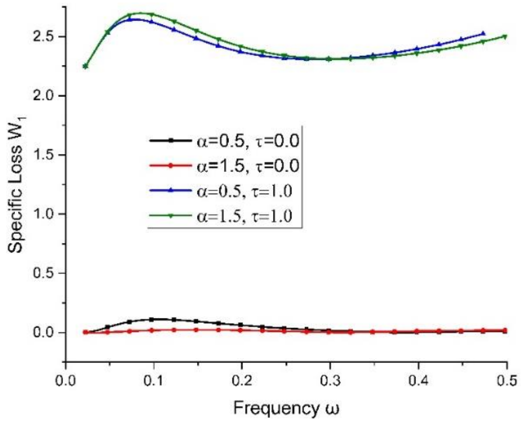

Fig. 8 Variations of penetration depth S_1 with frequency ω

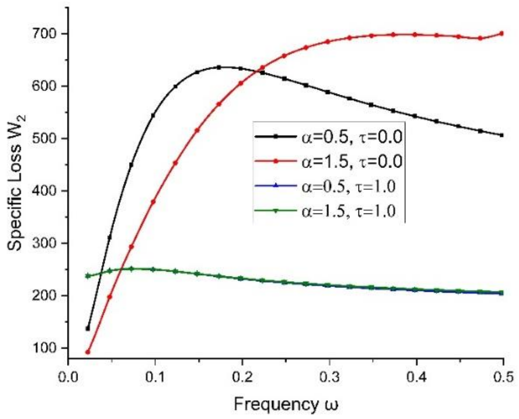

Fig. 9 Variations of penetration depth S_2 with frequency ω

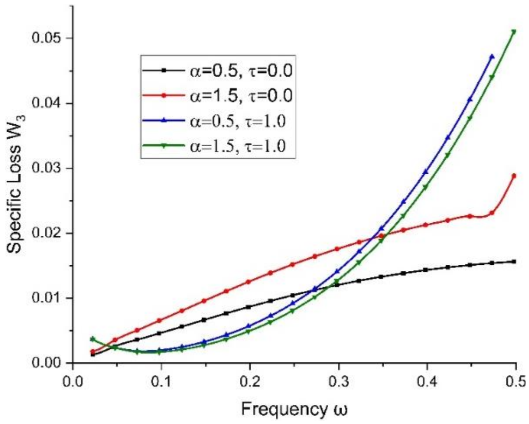

Fig. 10 Variations of Penetration Depth S_3 with frequency ω

8.3 Penetration depth

Figs. 8-10 show the variations of penetration depth S_1, S_2, S_3 of QL, QTS and QT waves with respect to frequency ω. From the graphs, we observe that the penetration depth S_1, S_2, S_3 of QL, QTS and QT waves show an increase in values with a small difference in magnitude for the four different cases showing the effect of fractional order parameter α and viscosity.

9. Conclusions

The propagation of plane harmonic wave in homogeneous transversely isotropic magneto-visco-thermoelastic rotating medium with fractional order heat transfer and viscosity have been studied. From the graphs, we observe the following concluding remarks:

(1) viscosity has a significant effect on the specific loss, penetration depth, attenuation coefficients of the plane harmonic wave.
(2) The specific loss, penetration depth, attenuation coefficients also show variations with weak and strong conductivity which shows a significant effect of fractional order parameter on the plane harmonic wave.
(3) Study of these waves are not only helpful in providing information about the internal structures of the earth but also helpful in geophysics, for understanding the effects of the earth's magnetic field on seismic waves, damping of acoustic waves in a magnetic field, emission of electromagnetic radiations from nuclear devices, etc.

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