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# Thermal stability analysis of temperature dependent inhomogeneous size-dependent nano-scale beams

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**Abstract.** Thermal bifurcation buckling behavior of fully clamped Euler-Bernoulli nanobeam built of a through thickness functionally graded material is explored for the first time in the present paper. The variation of material properties of the FG nanobeam are graded along the thickness by a power-law form. Temperature dependency of the material constituents is also taken into consideration. Eringen's nonlocal elasticity model is employed to define the small-scale effects and long-range connections between the particles. The stability equations of the thermally induced FG nanobeam are derived via the principal of the minimum total potential energy and solved analytically for clamped boundary conditions, which lead for more accurate results. Moreover, the obtained buckling loads of FG nanobeam are validated with those existing works. Parametric studies are performed to examine the influences of various parameters such as power-law exponent, small scale effects and beam thickness on the critical thermal buckling load of the temperature-dependent FG nanobeams.

**Keywords:** nonlocal elasticity theory; thermal buckling bifurcation; FG nanobeam; thermal effect

## 1. Introduction

It is well known that the traditional composite materials represent some deficits in drastic thermal environments, which can affect the safety of such structures. To deal with such cases, Japanese scientists in the mid-1980s designed an advanced sort of composite materials (Koizumi 1997), which are named functionally graded materials (FGMs) by changing the fraction of two or more constituents from one surface to another (thickness or axial direction), consequently getting a continuous change of material properties. The advantages of this feature lead to build a new FG structures which can withstand in high mechanical loadings under high temperature environments (Zenkour and Sobhy 2011, Attia *et al*. 2015). More recently, beams and plates made of FGMs have been implemented in the design phases like structural elements in the modern industries, such as thermal barrier in aeronautics/aerospace manufacturing production, mechanical and civil engineering, nuclear reactors and medicine. Displaying such novel properties, functionally graded ceramic/metals have received a great attention of the research community in latest years, which were principally based on the study their stability, static and vibration characteristics of theses functionally graded or sandwich (FG) structures (Li 2008, Aydogdu 2009, Zenkour and Sobhy

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2010, Bouramena *et al*. 2013, Tornabene *et al*. 2014, Bourada *et al*. 2015, Larbi *et al*. 2013, Bensaid *et al*. 2017).

Due to the fast progression of technology, FG miniaturized structures are implemented in various micro/nano-electromechanical systems (MEMS/NEMS) (Zhou and Li 2001, Fleck and Hutchinson 1997, Kuzumaki *et al*. 1998, Schadler *et al*. 1998, Yang *et al*. 2002), such as the components of nanosensors and nanoactuators, thin films and shape memory alloy. The classical continuum theory is limited for mechanical investigation of the macroscopic structures because the classical continuum theory does not consider nanoscale impact by the length scale parameter. Nevertheless, researchers in an effort have established numerous theories to describe attitude of nanoscale structures. The most well-known among them is the nonlocal elasticity theory proposed by Eringen (1978, 1982), this theory suppose that the stress state at any point depends on strains at all points in the body. Based on this model, a number of studies have been carried out to explore the size-dependent behavior of nanosize structures recently based on Eringen's nonlocal elasticity theory (Peddieson *et al*. 2003, Reddy 2007, Murmu and Pradhan 2009a, b, Thai 2012a, b, Berrabah 2013, Benguediab 2014, Tounsi *et al*. 2013a, b, Chakraverty and Behera 2014, Kheroubi *et al*. 2017, Bensaid 2017). For analysis of graphene sheets and nanoplate, Sobhy and his coworkers provided several works taking into account several physical effects such as thermal and hygro-thermal effects (Zenkour and Sobhy 2013, Alzahrani and Sobhy *et al*. 2013, Sobhy 2014a, b, Sobhy 2015, 2016, 2018, Sobhy and Zenkour 2018).

Furthermore, Eltaher *et al*. (2013) developed a finite element model to investigate the static buckling and vibrational behavior of FG nano beams via the nonlocal classical beam model. Simsek (2012) also examined the static bending and stability of nano-scaled FG beam by using an analytical model. Rahmani and Pedram (2014) explored the size dependency effects on dynamic characteristic of FG nanobeams according to the nonlocal TBT. Ebrahimi and Salari (2015) applied for the first time the differential transformation method to examine the applicability of on vibrational characteristics of FG size-dependent nanobeams. Based on a nonlocal third-order shear deformation theory, Rahmani and Jandaghian (2015) researched the buckling behavior of functionally graded nanobeams. Ebrahimi and Barati (2015) provided a nonlocal higher-order beam model for dynamic analysis of nano-sized FG beams. Chaht *et al*. (2015) explored size-dependent bending and buckling examination of higher-order FG nanobeams based on a new simple shear and normal deformations theory. Zemri *et al*. (2015) researched the mechanical response of functionally graded nanosize beam by analyzing the bending, vibration and buckling of FG nanoscale beams applying higher order refined beam model. Ahouel *et al*. (2015) inspected the size effect behavior of FG higher order shear deformable nanobeams considering neutral axis location. Nguyen *et al*. (2014) developed closed analytical solutions for static bending of transversely or axially nonlocal FG beams. In another work, Ebrahimi and Barati (2017) examined the buckling behavior of nonlocal strain gradient axially functionally graded nanobeams lying on variable elastic medium, in which two parameters were introduced taking into account of both nonlocal stress field and strain gradient effects to capture size effect more accurately. A novel quasi 3-D nonlocal hyperbolic plate theory for free vibration and buckling of FGM nanoplates was developed by Sobhy and Radwan (2017). Recently, Bensaid (2017) investigated for the first time the static bending and buckling of nonhomogenous nanobeams having porosities and geometrical neutral surface position.

However, as can be seen from the studies cited previously, most of them have been elaborated based on the negligence of the thermal environment impacts. Only few works have dealt with the thermal effect of temperature changes. Ebrahimi and Salari (2015) carried out an analytical

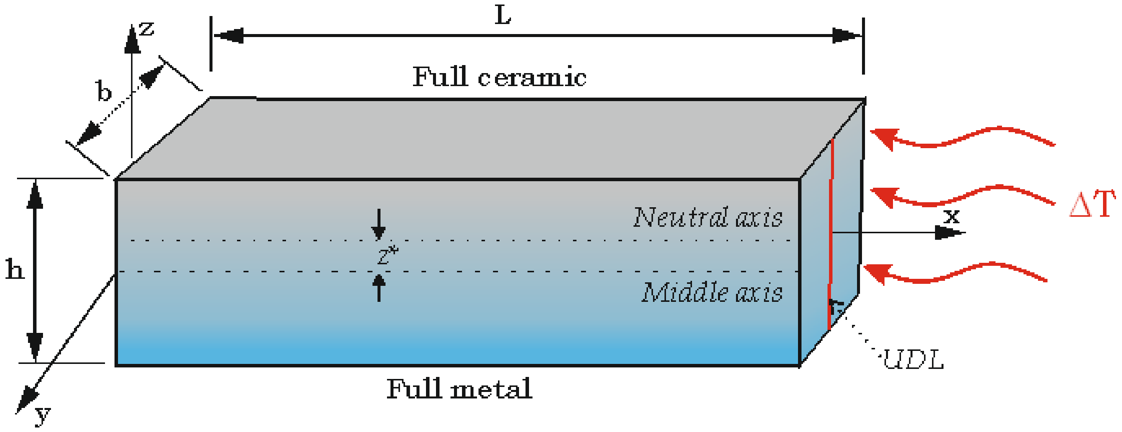


Fig. 1 Schematic arrangement of thermally induced a functionally graded nanobeam

solution to explore the thermal effects on buckling and free vibration characteristics of FG size-dependent Timoshenko nanobeams exposed to an in-plane thermal loading. The impact of various types of thermal loadings on buckling and vibrational characteristics of nonlocal temperature-dependent functionally graded nanobeams has been investigated by Ebrahimi and Salari (2016). Mouffoki *et al*. (2017) developed a novel two-unknown trigonometric shear deformation beam mode to analyze dynamic behavior of nonlocal advanced nanobeams in hygro-thermal environment. Mashat *et al*. (2016) carried out an investigation on the vibration and thermal buckling of nanobeams embedded in an elastic medium considering various boundary conditions. More recently, Dehrouyeh-Semnani (2017) presented a comparison study between the use of simplified and original boundary conditions of functionally graded beams subjected to an in plane thermal loading. He discussed the advantage and the disadvantage of the two approaches on the thermo mechanical vibrations characteristics of FG beams.

As one can observe above, few papers are reported on thermal buckling analysis of FG nanobeams that take into consideration the thermal environment effects. Moreover, most of earlier works on thermal stability of FG nanobeams have been examined only simply supported boundary conditions and neglected the clamped boundaries and geometries neutral surface positions, which can lead to inaccurate results as pointed by Kiani and Eslami 2013, Shen and Wang 2014. So, it is important to understand the thermal buckling behavior of FG nanobeams in considering the effects of temperature changes and clamped boundary conditions.

According to this deficiency, in this work, thermal buckling bifurcation of temperature-dependent FG nanosize beam having neutral surface position and considering the effect of thermal loading is investigated. In this study, incorporation of small scale parameter is achieved via nonlocal elasticity theory of Eringen. The material properties of the FG nanobeam are supposed changing continuously across the beam thickness via a power-law model and are temperature dependency. The governing equations of motion have derived via the minimum total potential energy based on classical beam theory. These are solved analytically by Galerkin procedure for clamped boundary conditions. Comparison survey is also carried out to check the present model with those of preceding works. Finally, a parametric study is presented to show the impacts of thickness of the nanobeam, power law index, nonlocal parameter and temperature change on the critical buckling temperature of FG nanobeam.

Table 1 Temperature dependency coefficients material properties for Si3N4 and SUS 304 FG Nanobeams

| Material | Properties | $P_0$ | $P_{-1}$ | $P_1$ | $P_2$ | $P_3$ |
|---|---|---|---|---|---|---|
| $Si_3N_4$ | $E$ (Pa) | 348.43e+9 | 0 | -3.070e-4 | 2.160e-7 | -8.946e-11 |
| | $\alpha$ ($K^{-1}$) | 5.8723e-6 | 0 | 9.095e-4 | 0 | 0 |
| | $\rho$ ($Kg/m^3$) | 2370 | 0 | 0 | 0 | 0 |
| | $\kappa$ (W/mk) | 13.723 | 0 | -1.032e-3 | 5.466e-7 | -7.876e-11 |
| | $\nu$ | 0.24 | 0 | 0 | 0 | 0 |
| SUS304 | $E$ (Pa) | 201.04e+9 | 0 | 3.079e-4 | -6.534e-7 | 0 |
| | $\alpha$ ($K^{-1}$) | 12.330e-6 | 0 | 8.086e-4 | 0 | 0 |
| | $\rho$ ($Kg/m^3$) | 8166 | 0 | 0 | 0 | 0 |
| | $\kappa$ (W/mk) | 15.379 | 0 | -1.264e-3 | 2.092e-6 | -7.223e-10 |
| | $\nu$ | 0.3262 | 0 | -2.002e-4 | 3.797e-7 | 0 |

## 2. Theory and formulation

### 2.1 The material properties of FG nanobeams

Let assume a ceramic-metal nanobeam having length L, width b and uniform thickness h in the reference configuration. The FG nanobeam is subjected to uniform distributed thermal loadings which can be seen in Fig. 1. Material properties are supposed to be graded continuously through in the thickness direction (***z***-axis direction) of nanobeam by a power law form of the volume fraction of the constituents as (Natarajan *et al*. 2012, Larbi *et al*. 2013, Wattanasakulpong and Chaikittiratana 2015, Zemri *et al*. 2015, Ahouel *et al*. 2015, Chaht *et al*. 2015, Bourada *et al*. 2015, Ebrahimi and Salari 2016, Bensaid and Kerboua 2017, Zidi *et al*. 2017). Thus, the material inhomogeneous properties of FG nanobeam $P$, such as Young's modulus ($E$), Poisson's ratio ($v$), the shear modulus ($G$) and the mass density ($\rho$), can be described by

$$P(z) = (P_t - P_b)\left(\frac{z}{h} + \frac{1}{2}\right)^p + P_b \tag{1a}$$

Here $P_t$ and $P_b$ are the corresponding material property at the top and bottom surfaces of the FG nanobeam. And ***p*** is the power law index which takes the value greater or equal to zero and determines the material distribution through the thickness of the beam.

To explore the behavior of FGM materials in high temperature environment more exactly, it is required to consider the temperature dependency on material properties. The nonlinear equation of thermo-elastic material properties in relation of temperature *T(K)* can be expressed as (Touloukian 1967)

$$P = P_0\left(P_{-1}T^{-1} + 1 + P_1 T + P_2 T^2 + P_3 T^3\right) \tag{1b}$$

in which $P_0$, $P_{-1}$, $P_1$, $P_2$ and $P_3$ are the temperature-dependent coefficients, in which their values can be taken from the table of materials properties (Table 1) for $Si_3N_4$ and SUS 304. The base surface ($z = -h/2$) of FG nanobeam is full metal (SUS 304), while the top surface ($z = h/2$) is pure ceramics ($Si_3N_4$).

### *2.2 Kinematics and strains*

Based on the classical beam theory, the displacement field at any point of the beam taking into accounts the exact position of the neutral surface written as (Zhang and Zhou 2008, Zhang 2013, Barati and Zenkour 2018, She *et al*. 2017)

$$u_1(x,z) = u(x) - (z - z^*)\frac{\partial w}{\partial x} \tag{2a}$$

$$u_3(x,z) = w(x) \tag{2b}$$

And

$$z^* = \frac{\int_{-h/2}^{h/2} E(z)\,z\,dz}{\int_{-h/2}^{h/2} E(z)\,dz} \tag{3}$$

In which $u$ is the displacement of the mid-surface and $w$ is the bending displacement. The associated nonzero strains of the current beam model are expressed as

$$\varepsilon_{xx} = \varepsilon_{xx}^0 - zk_x^0, \quad \varepsilon_{xx}^0 = \frac{\partial u(x)}{\partial x}, \quad k_x^0 = \frac{\partial^2 w(x)}{\partial x^2} \tag{4}$$

In order to obtain the governing equations of motion, the principal of the minimum total potential energy in static stability form can be use as follow (Simsek 2012, Chaht *et al*. 2015)

$$\delta \Pi = \delta\left(U_{\text{int}} - W_{ext}\right) = 0 \tag{5}$$

in which $\Pi$ is the total potential energy. $\delta U_{\text{int}}$ is the virtual variation of the strain energy; and $\delta W_{\text{ext}}$ is the variation of work induced by external forces. The first variation of the strain energy is given as

$$\delta U_{\text{int}} = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x \delta\varepsilon_x)\,dz\,dx \tag{6}$$

$$= \int_0^L \left( N\frac{d\delta u_0}{dx} + M\frac{d^2\delta w}{dx^2} \right) dx \tag{7}$$

where $N$ and $M$ are the stress resultants defined as

$$\left(N, M\right) = \int_{-\frac{h}{2}}^{\frac{h}{2}} (1, z)\sigma_x\,dx \tag{8}$$

The first-order variation of the work to temperature change can be obtained by

$$\delta W = \int_0^L N^T \frac{\partial w}{\partial w} \frac{\partial \delta w}{\partial x} dx \tag{9}$$

where $N^T$ is thermal axial force can be written as

$$N^T = E(z,T)\alpha(z,T)(T - T_0)dz \tag{10}$$

where $T_0$ in Eq. (10) is the reference temperature.

Substituting the expressions for $\delta U_{\text{int}}$ and $\delta W_{\text{ext}}$ from Eqs. (7) and (9) into Eq. (5) and integrating by parts and collecting the coefficients of $\delta u_0$ and $\delta w_0$, the following governing equations of thermally induced FGM nanobeam are obtained

$$\delta u_0 = \frac{dN}{dx} = 0 \tag{11a}$$

$$\delta M : \frac{\partial^2 M}{\partial x^2} - N^T \frac{\partial^2 w}{\partial x^2} = 0 \tag{11b}$$

### 2.3 The nonlocal elasticity model

Contrary to the classical (local) theory, in the nonlocal elasticity theory of Eringen (1972, 1983), the stress at a reference point $x$ is considered to be a functional of the strain field at every point in the body. For example, in the non-local elasticity, the uniaxial constitutive law is expressed as elasticity Eringen (1972, 1983).

$$\sigma_{xx} - \mu \frac{d^2 \sigma_{xx}}{dx^2} = E(z)\varepsilon_{xx} \tag{12}$$

$\mu = (e_0 a)^2$ is a nonlocal parameter revealing the nanoscale effect on the response of nanobeams, $e_0$ is a constant appropriate to each material and $a$ is an internal characteristic length. In general, a conservative estimate of the nonlocal parameter is $e_0 a < 2.0$ nm for a single wall carbon nanotube (Wang 2005, Heireche *et al*. 2008a, b, Tounsi *et al*. 2013b, c)

Using Eqs. (12), (8) and (4), the force-strain and the moment-strain relations of the nonlocal FG Euler-Bernouli beam hypothesis can be obtained as

$$N - \mu \frac{\partial^2 N}{\partial x^2} = A_{xx} \frac{\partial u_0}{\partial x} - B_{xx} \frac{\partial^2 w_0}{\partial x^2} \tag{13}$$

$$M - \mu \frac{\partial^2 M}{\partial x^2} = B_{xx} \frac{\partial u_0}{\partial x} - D_{xx} \frac{\partial^2 w_0}{\partial x^2} \tag{14}$$

In which the constants ($A_{xx}$, $B_{xx}$, $D_{xx}$) are defined as

$$\left(A_{xx}, B_{xx}, D_{xx}\right) = \int_L E(z,T)\left(1, z, z^2\right) dA \tag{15}$$

By substituting Eqs. (13), (14), (15) into Eqs. (11a), (11b), the nonlocal equations of motion can be expressed in terms of displacements ($u_0$, $w_0$) as follows

$$A_{xx}\frac{\partial^2 u_0}{\partial x^2}-B_{xx}\frac{\partial^3 w_0}{\partial x^3}=0 \tag{16}$$

$$B_{xx}\frac{\partial^3 u_0}{\partial x^3}+D_{xx}\frac{\partial^4 w_0}{\partial x^4}-N^T\left(\frac{\partial^2 w_0}{\partial x^2}-\mu\frac{\partial^4 w_0}{\partial x^4}\right)=0 \tag{17}$$

### *2.4 Uniform temperature rise (UTR)*

In this study we assume that the FG nanobeam has an initial temperature ($T_0$ = 300K), which is a stress-free state, uniformly changed to final temperature with $\Delta T$. The temperature rise is stated by

$$\Delta T = T - T_0 \tag{18}$$

## 3. Solution procedure

In this part, the governing equations of graded nanobeam with simply-supported (S) and clamped (C) boundaries are resolved, which are presented as follows

– *Simply-supported* (*S*)

$$w = N_{xx} = M_{xx} = 0 \quad at\ \ x = 0, L \tag{19}$$

– *Clamped* (*C*)

$$u = w = \frac{\partial w}{\partial x} = 0 \quad at\ \ x = 0, L \tag{20}$$

To assure previous-mentioned boundary conditions, the new displacement functions are introduced (Ebrahimi and Barati 2017b) as follows

$$u = \sum_{n=1}^{\infty} U_m \frac{\partial X_m}{\partial x} e^{i\omega_n t} \tag{21}$$

$$w = \sum_{m=1}^{\infty} W_m X_m e^{i\omega_n t} \tag{22}$$

in which ($U_m$, $W_m$) are the unknown coefficients. Placing equations (22) and (21) into equations (16) and (17), respectively, leads to

$$\left\{\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}\right\}\begin{Bmatrix} U_m \\ W_m \end{Bmatrix} = 0 \tag{23}$$

where

$$k_{11} = A_{xx}\kappa_{12}\,, \qquad k_{12} = B_{xx}\kappa_{13},$$
$$k_{21} = -B_{xx}\kappa_{12}\,,$$
$$k_{22} = -(B_{xx} + D_{xx})\kappa_{13} + N^T\left(-\kappa_{11} + \mu\kappa_{13}\right)$$

In which

$$\kappa_{11} = \int_0^L \left(X''_m X_m\right)dx$$
$$\kappa_{12} = \int_0^L \left(X'''_m X'_m\right)dx \tag{24}$$
$$\kappa_{13} = \int_0^L \left(X''''_m X_m\right)dx$$

The above system of equations is resolved by calculating determinant of the coefficients of exceeding matrix and making it equal to zero, with the help of the Eq. (10), we can get the thermal buckling loads. The shape functions cited previously for different boundary conditions are specified by

$$S-S \quad X\left(x\right) = \sin(\lambda_m x)$$
$$\lambda_m = \frac{m\pi}{L} \tag{25}$$

$$C-C \quad X_m\left(x\right) = \sin\left(\lambda_m x\right) - \sinh\left(\lambda_m x\right) - \xi_m\left(\cos\left(\lambda_m x\right) - \cosh\left(\lambda_m x\right)\right)$$
$$\xi_m = \frac{\sin\left(\lambda_m x\right) - \sinh\left(\lambda_m x\right)}{\cos\left(\lambda_m x\right) - \cosh\left(\lambda_m x\right)} \tag{26}$$
$$\lambda_1 = 4.730,\ \lambda_2 = 7.853,\ \lambda_3 = 10.996,\ \lambda_4 = 14.137, \lambda_{m\geq 5} = \frac{\left(m+0.5\right)\pi}{L}$$

## 4. Results and discussion

In this part, the size-dependent critical thermal bifurcation responses of FG nanobeams will be tabulated and illustrated based nonlocal Euler-Bernoulli beam model. The functionally graded nanosize beam is made of Steel (SUS 304) and Silicon nitride (Si3N4), in which properties are provided in Table 1. In the first step of this analysis, different types of boundary conditions are considered (*S-S* and *C-C*). It is supposed that the FG beam has the following dimensions: $L$ (length) = 10 nm, $b$ (width) = 1 nm and $h$ (thickness) changeable. It is presumed that the temperature increases in metal surface to reference temperature $T_0$ of the FG nanobeam is $T_m$ - $T_0$ = 5 $K$ (Kiani and Eslami 2013b).

Table 2 Nondimensional buckling loads of FG nanobeam for various gradient indices and nonlocal parameters ($L/h$ = 20)

| | $p$=0 | | $p$=0.2 | | $p$=1 | | $p$=5 | |
|---|---|---|---|---|---|---|---|---|
| $\mu$ (nm)$^2$ | REF[(a)] | Present | REF[(a)] | Present | REF[(a)] | Present | REF[(a)] | Present |
| 1 | 8.98302 | 8.9830 | 10.4585 | 10.4585 | 12.4479 | 12.4478 | 14.3191 | 14.3191 |
| 2 | 8.24258 | 8.2425 | 9.59648 | 9.5964 | 11.4219 | 11.4218 | 13.1389 | 13.1388 |
| 3 | 7.61492 | 7.6149 | 8.86572 | 8.8657 | 10.5521 | 10.5521 | 12.1384 | 12.1383 |
| 4 | 7.07608 | 7.0760 | 8.23837 | 8.2383 | 9.80543 | 9.8054 | 11.2794 | 11.2794 |
| 5 | 6.60846 | 6.6084 | 7.69394 | 7.6939 | 9.15744 | 9.1574 | 10.534 | 10.5340 |

[(a)] Ebrahimi and Barati (2017a)

Table 3 Thermal buckling loads of FG nanobeam for various gradient indices, nanobeam thickness and nonlocal parameters ($L$ = 10nm)

| | | Power law index | | | | |
|---|---|---|---|---|---|---|
| $\mu$ (nm)$^2$ | $h$ (nm) | 0 | 0.2 | 1 | 2 | 5 |
| 0 | 0.15 | 472.4957 | 408.2533 | 331.7339 | 310.3996 | 292.5423 |
| | 0.25 | 1312.4883 | 1134.0371 | 921.4832 | 862.221 | 812.6177 |
| | 0.35 | 2572.4770 | 2222.7127 | 1806.1071 | 1689.9534 | 1592.7301 |
| 1 | 0.15 | 336.5917 | 290.8273 | 236.3172 | 221.1193 | 208.3943 |
| | 0.25 | 934.9771 | 807.8538 | 656.4362 | 614.2203 | 578.8843 |
| | 0.35 | 1832.5551 | 1583.3935 | 1286.6162 | 1203.8719 | 1134.6133 |
| 2 | 0.15 | 261.4040 | 225.8625 | 183.5288 | 171.7258 | 161.8464 |
| | 0.25 | 726.1224 | 627.3958 | 509.8023 | 477.0161 | 449.5734 |
| | 0.35 | 1423.1999 | 1229.6959 | 999.2125 | 934.9516 | 881.1640 |
| 3 | 0.15 | 213.6737 | 184.6218 | 150.0179 | 140.3700 | 132.2945 |
| | 0.25 | 593.5381 | 512.8383 | 416.7164 | 389.9167 | 367.4848 |
| | 0.35 | 1163.3348 | 1005.1631 | 816.7164 | 764.2368 | 720.2703 |
| 4 | 0.15 | 180.6825 | 156.1162 | 126.8551 | 118.6969 | 111.8682 |
| | 0.25 | 501.8959 | 433.6561 | 352.3754 | 329.7136 | 310.7452 |
| | 0.35 | 983.7160 | 849.9660 | 690.6558 | 646.2387 | 609.0607 |

To check the correctness of the developed model, the obtained thermal buckling results are compared with those predicted by Ebrahimi and Barati 2017a based on classical beam theory and analytical solution. Table 2 presents the comparison results for a simply supported FG nanobeam taking a different nonlocal parameters and power law indices and a good correlation is noticed between two results.

Next, after the validation step of the present model, the effects of different parameters, such as, nonlocal parameter, nanobeam thickness, magnitude of thermal loading and power law index on the thermal buckling of FG nanobeam will be explored.

Table 3 presents the effect of the thickness of FG nanobeam ($h$), power law index ($p$) and nonlocal parameter ($\mu$) on the pick value of critical buckling temperature of the C-C graded nanobeam based on analytical solution method. According to these tabulated values, increasing in nonlocal scale parameter yields the decrease in the critical buckling temperature. Additionally, it is seen that the $\Delta T_{cr}$ reduce by rising power law index ($p$) due to the augmentation in percentage of metal phase. On the other hand, increasing beam thickness ($h$) will lead to a considerable increase of $\Delta T_{cr}$.

Fig. 2 displays the variations of the critical buckling temperature $\Delta T_{cr}$ of clamped FG nanosize beam under uniform temperature change for numerous values of nonlocal parameters and power law indexes. One can see from this figure that thermal buckling loads increase with increasing the nanobeam thickness. Also, it is concluded that the thermal buckling temperature reduces by rising nonlocality parameters due to softening effect.

Critical buckling temperature load of FG nanobeam as function of thickness ($h$) for various power-law indices and fixed values of nonlocal parameter ($\mu$= 0, 2, 4) is illustrated in Fig. 3. As the previous discussion, it can be observed that an increment in the thickness ($h$) causes an increase in the thermal buckling load. It is also noticed that $\Delta T_{cr}$ becomes less important at bigger values of gradient index. Moreover, as the variation parameter increases, the thermal buckling loads of the FG nanobeam tend to be diminished.

Variation of thermal bifurcation load of clamped-clamped FG nanobeam with respect to change of the power law index parameter ($p$) for various values of nanosize parameters and beam thickness is displayed in Figs. 4 and 5, respectively. One can see from these figures that an increase in the beam thickness generates a rising in critical buckling temperature of FG nanobeam. Moreover in Figs. 4 and 5, it is shown that the thermal buckling diminishes with remarkable rate where the power indices between values of 0 to 2 than that where the power law exponent is comprised between 2 and 10. Also, increasing the nonlocal parameter generates a considerable decrease in the critical buckling temperature at fixed material distribution.

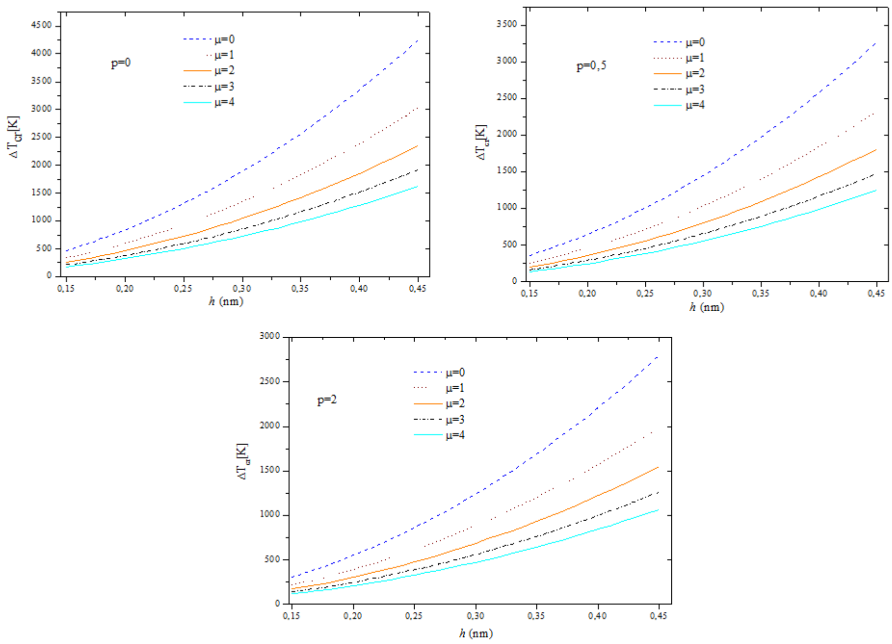


Fig. 2 Variation of thermal buckling load of the C-C FGP nanobeam versus nanobeam thickness under uniform temperature rise for different values of nonlocal parameters and power index

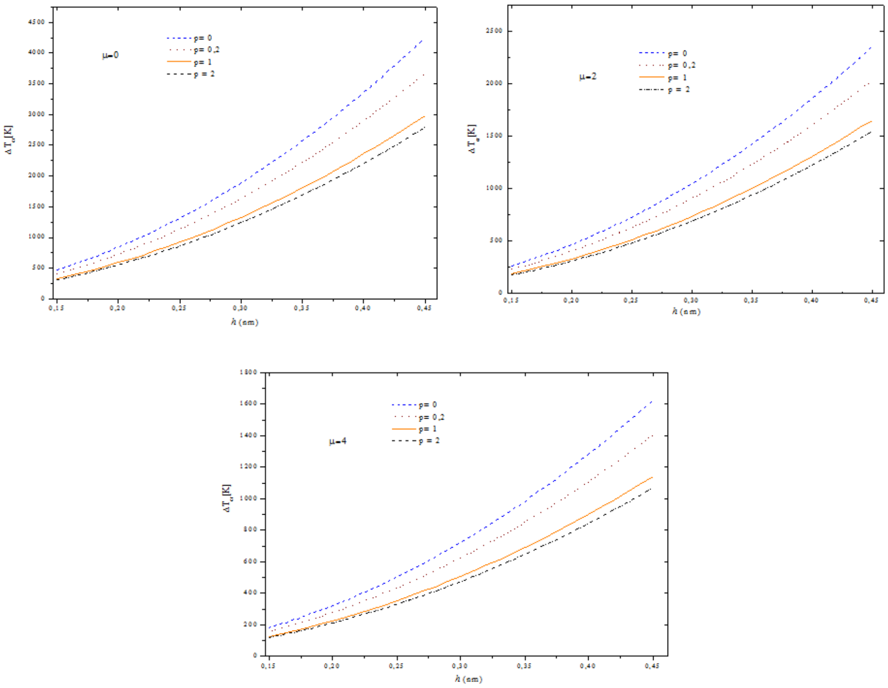


Fig. 3 Variation of thermal bifurcation load of the C-C FGP nanobeam versus nanobeam thickness under uniform temperature rise for numerous values of power index and nonlocal parameters.

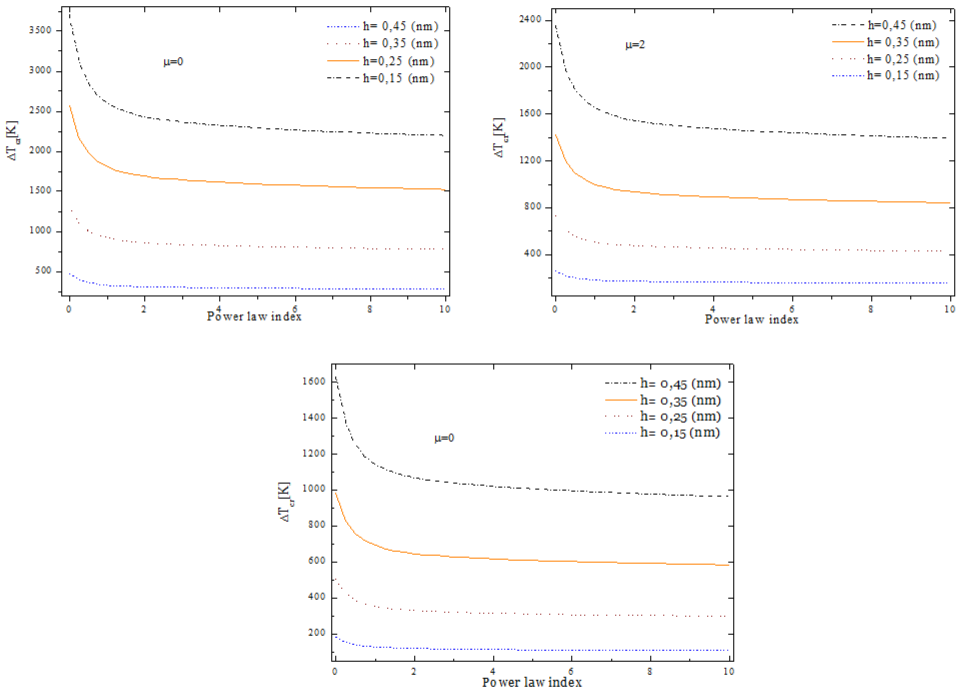


Fig. 4 Variation of thermal bifurcation load of C-C FGP nanobeam as function of power law index and nanobeam thickness for numerous values of nonlocal parameters

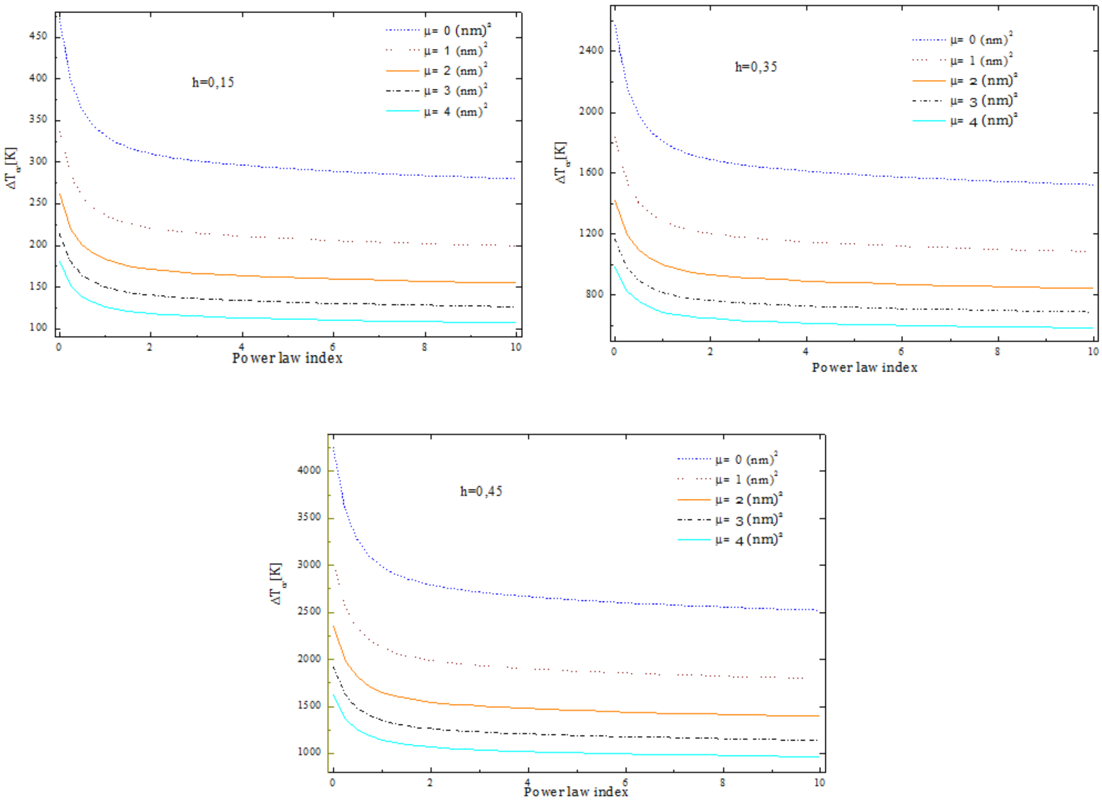


Fig. 5 Variation of thermal bifurcation load of C-C FGP nanobeam as function of power law index and nonlocal parameters for different values of nanobeam thickness

## 5. Conclusions

In this work, we have analyzed thermal buckling bifurcation characteristic of temperature dependent FG nanobeams subjected to uniform temperature rise through the thickness direction by using Euler-Bernoulli beam theory and the concept of neutral surface position. Eringen's nonlocal elasticity model is deployed to account for small scale effects. Material properties are supposed to be temperature dependent and graded in the thickness direction via a power-law model. The governing equations of motion are derived by using principle of minimum total potential energy which are resolved by applying an analytical approach for various boundary edges. Accuracy of the results is validated with available data existing in the literature and a good agreement was shown. With the consideration of clamped boundary condition, thermal buckling bifurcation type can occur, due to eliminating the extra thermal bending moment at the edges and lead to more accurate results, which has not been considered in previous studies. It is deduced through the parametric that various parameters such as small-scale parameter, material-distribution index, temperature increment change and nanobeam thickness have significant impact on the thermal buckling response of the FG nanobeams. Finally, the present model can be improved to consider influences of transverse shear deformation effect (Li *et al*. 2016, Bouremana *et al*. 2013, Bourada *et al*. 2015, Chaht *et al*. 2015, Zemri *et al*. 2015, Ahouel *et al*. 2015, Bensaid *et al*. 2017).

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