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Thermal buckling of FGM nanoplates subjected to linear and nonlinear varying loads on Pasternak foundation

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Abstract. Thermo-mechanical buckling problem of functionally graded (FG) nanoplates supported by Pasternak elastic foundation subjected to linearly/non-linearly varying loadings is analyzed via the nonlocal elasticity theory. Two opposite edges of the nanoplate are subjected to the linear and nonlinear varying normal stresses. Elastic properties of nanoplate change in spatial coordinate based on a power-law form. Eringen's nonlocal elasticity theory is exploited to describe the size dependency of nanoplate. The equations of motion for an embedded FG nanoplate are derived by using Hamilton principle and Eringen's nonlocal elasticity theory. Navier's method is presented to explore the influences of elastic foundation parameters, various thermal environments, small scale parameter, material composition and the plate geometrical parameters on buckling characteristics of the FG nanoplate. According to the numerical results, it is revealed that the proposed modeling can provide accurate results of the FG nanoplates as compared some cases in the literature. Numerical examples show that the buckling characteristics of the FG nanoplate are related to the material composition, temperature distribution, elastic foundation parameters, nonlocality effects and the different loading conditions.

Keywords: nanoplates buckling; functionally graded material; linear and nonlinear varying loading; thermal loading; Pasternak foundation; nonlocal elasticity theory; Navier's method

1. Introduction

Nanoplates offer unprecedented opportunities for incorporating physics-based concepts for controlling the physical and chemical properties to develop novel devices and sensors because nanoplates have unique electrical, magnetic, thermal and mechanical properties. The structures at nanoscale such as nanobeams, nanoplates and nanotubes can be identified as the consequences of molecular manipulation that are recognized as the main parts of various nanosystems and nanodevices (Ansari *et al*. 2015). In this subject search's first is needed to dealing with this issue takes controlling the edge of nanoplates to eliminating electron scattering, then characterizing the spectral properties (metal nanoplates), next controlling assembly of nanoplates, and optimizing sub-monolayer deposition and Optimizing layer-by-layer deposition include Meta-materials

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(electronic), and finally energy absorbing materials (Mechanical) and magnetic properties. Nanoplates can be fabricated in such way to exploit the mechanical and electronic properties of hybrid structures (metal-metal, metal-semiconductor and metal-oxide), Layer-by-Layer deposition with soft materials for enhanced mechanical properties and are used in uncooled infrared sensors, photovoltaic, meta-materials, biosensors and receptor-free detection.

FG materials are advanced composites which have continuously varying material composition and properties through certain dimension of the structure to achieve the desired goals. As the fiber-reinforced composites have mismatch material properties across an interface of two discrete material bonded together, there could be the severe thermal stress concentration phenomena at the interface of the fiber-reinforced composites. However, by gradually varying the material properties of the functionally graded materials (FGMs), this problem can be avoided or reduced. Therefore, FGMs with a mixture of the ceramic and metal are applied to the thermal barrier structures for the space shuttle, combustion chamber and nuclear planets etc. (Ebrahimi *et al*. 2009, Ebrahimi and Rastgoo 2009, 2011, Aghelinejad *et al*. 2011). Some of researchers in recent years have analyzed mechanical behaviors of FGM nanoplates based on various plate shear deformation plate theories. (Ebrahimi and Barati 2016a, b, c, d, e, Ebrahimi *et al*. 2016a, Ebrahimi and Dabbagh 2016, Ebrahimi and Hosseini 2016a, b).

Classical plate theory and first order shear deformation plate theory are reformulated based on nonlocal elasticity theory by Pradhan and Phadikar (2011). In this work the equation of the motion are extracted based on Eringen's nonlocal theory. Analysis of nano-structure's mechanical behaviors is one of recent interesting research topics. (Ebrahimi and Barati 2016f, g, h, i, j, k, l, m, n, Ebrahimi and Barati 2017). Cheng and Chen (2015) presents a theoretical study of the resonance frequency and buckling load for nanoplates with high-order surface stress model. In another work, Wang *et al*. (2011) extracted the governing equations for the nanoscale plates with consideration of both surface effects and non-local elasticity theory. Thermal post buckling and vibration response of the FG plate are investigated by Park and Kim (2006). Sandwich plates is one of the most famous mechanical structures and recently a large number of researchers have analyzed the mechanical behavior of FGM sandwich plate's as (Tounsi *et al*. 2016). Numerical solution to linear bending behavior of the circular plates is obtained by the method of harmonic differential quadrature by Civalek *et al*. (2004). Farajpour *et al*. (2011) studied nano-scale buckling behavior of the rectangular plates under axial pressure due to non-uniformity in the thickness. More advanced research including buckling of plates in the different scales have been proposed by other researchers. The buckling response of orthotropic graphene sheets subjected to various linearly varying normal in-plane forces studied by Farajpour *et al*. (2012). The analytical solutions of natural frequencies in FGM nanoplate for different boundary conditions are presented by Zare *et al*. (2015). Thermal buckling and free vibration analysis of FG nanobeams subjected to temperature distribution have been exactly investigated by Ebrahimi and Salari (2015a, b, c) and Ebrahimi *et al*. (2015 a, b). Ebrahimi and Barati (2016o, p, q) investigated buckling behavior of smart piezoelectrically actuated higher-order size-dependent graded nanoscale beams and plates in thermal environment.

A computational method based on refined plate theory involving the effect of thickness stretching in conjunction is proposed for the size-dependent bending, free vibration and buckling analysis of FGM nanoplates are developed by Nguyan *et al*. (2015). Thermomechanical bending response of functionally graded plates resting on Winkler-Pasternak elastic foundations is described by Bouderba *et al*. (2013). Thermal buckling of FGM nanoplates is studied by Nami *et al*. (2015) via nonlocal third order shear deformation theory to show the effects of considering the higher order plate theory on the accuracy of the analysis. Application of the trigonometric four variable plate theory for free vibration analysis of laminated rectangular plate supporting a localized patch mass is reported by Draiche *et al*. (2014).

In this manuscript, thermal buckling of FGM nanoplates subjected to various linear and non-linear varying normal in-plane forces and resting on Pasternak foundation is investigated for the first time. Nonlocal elasticity theory and the Hamilton's principle are utilized to extract the governing equations and the Navier's method is applied to solve them. The presented shows a good agreement with the results available in literate. The small size effect on the buckling loads of functionally graded rectangular nanoplates are presented through considering various parameters such as FG-index, the length of nanoplates, numerical loading factor, nonlocal parameter, aspect ratio and the mode numbers.

2. Formulation

2.1 Nonlocal theory

The constitutive equation of classical elasticity is an algebraic relationship between the stress and strain tensors while that of Eringen's nonlocal elasticity involves spatial integrals which represent weighted averages of the contributions of strain tensors of all points in the body to the stress tensor at the given point (Eringen 1983). Though it is difficult mathematically to obtain the solution of nonlocal elasticity problems due to the spatial integrals in constitutive equations, these integro-partial constitutive differential equations can be converted to equivalent differential constitutive equations under certain conditions.

The theory of nonlocal elasticity, developed by Eringen and Edelen (1972) states that the nonlocal stress-tensor components σ_{ij} at any point x in a body can be expressed as

$$\sigma_{ij}(x) = \int_{\Omega} \Big(\alpha\left(|x' - x|, \tau \right) t_{ij}(x') \Big) d\Omega(x') \tag{1}$$

where $t_{ij}(x')$ are the components of the classical local stress tensor at point x, which are related to the components of the linear strain tensor ε_{kl} by the conventional constitutive relations for a Hookean material, i.e.

$$t_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

The meaning of Eq. (1) is that the nonlocal stress at point x is the weighted average of the local stress of all points in the neighborhood of x, the size of which is related to the nonlocal Kernel $\alpha(|x'-x|,\tau)$. Here $|x'-x|$ is the Euclidean distance and τ is a constant given by

$$\tau = \frac{e_0 a}{l} \tag{3}$$

which represents the ratio between a characteristic internal length, a (such as lattice parameter, C-C bond length and granular distance) and a characteristic external one, l (e.g., crack length, wavelength) through an adjusting constant, e_0, dependent on each material. The magnitude of e_0 is determined experimentally or approximated by matching the dispersion curves of plane waves with those of atomic lattice dynamics. According to (Eringen and Edelen 1972) for a class of physically admissible kernel $\alpha(|x'-x|,\tau)$ it is possible to represent the integral constitutive relations given by Eq. (1) in an equivalent differential form as

$$(1-(e_0a)\nabla^2)\sigma_{kl}=t_{kl} \tag{4}$$

In which ∇^2 is the Laplacian operator. Thus, the scale length e_0a takes into account the size effect on the response of nanostructures. For an elastic material in the one dimensional case, the nonlocal constitutive relations may be simplified as (Miller and Shenoy 2000)

$$\sigma_{xx}-\mu\frac{\partial^2\sigma_{xx}}{\partial x^2}=E\,\varepsilon_{xx} \tag{5}$$

Where σ and ε are the nonlocal stress and strain respectively, $\mu=(e_0a)^2$ is the nonlocal parameter and E is the elasticity modulus (Ebrahimi *et al*. 2016).

2.2 Functionally graded nanoplate

As depicted in Fig. 1, an FGM nanoplates of length l_x, width l_y and thickness h that is made of a mixture of ceramics and metals is considered. It is assumed that the materials at bottom surface $(Z=-h/2)$ and top surface $(Z=h/2)$ of the nanoplate are metals and ceramics, respectively. The local effective material properties of an FGM nanoplates can be calculated using homogenization method that is based on the Mori-Tanaka scheme. According to the Mori-Tanaka homogenization technique, the effective material properties of the FGM nanoplates such as Young's modulus(E), Poisson's ratio(ν), mass density $(\rho)$and thermal extension coefficient (α) can be determined as follows (Mori and Tanaka 1973)

$$E(z)=E_cV_c(z)+E_mV_m \tag{6a}$$

$$\rho(z)=\rho_cV_c(z)+\rho_mV_m \tag{6b}$$

$$\alpha(z)=\alpha_cV_c(z)+\alpha_mV_m \tag{6c}$$

$$\nu(z)=\nu_cV_c(z)+\nu_mV_m \tag{6d}$$

Here, the subscripts m and c refer to metal and ceramic phases. The volume fraction of the ceramic and metal phases can be defined by the power-law function as

$$V_f(z)=\left(\frac{1}{2}+\frac{z}{h}\right)^k \tag{7}$$

Where k represent the power-law index .Additionally ,the neutral axis of FGM nanoplates where the end supports are located on ,can be determined by the following relation

$$z_0=\frac{\int_{-(h/2)}^{(h/2)}zE(z)dz}{\int_{-(h/2)}^{(h/2)}E(z)dz} \tag{8}$$

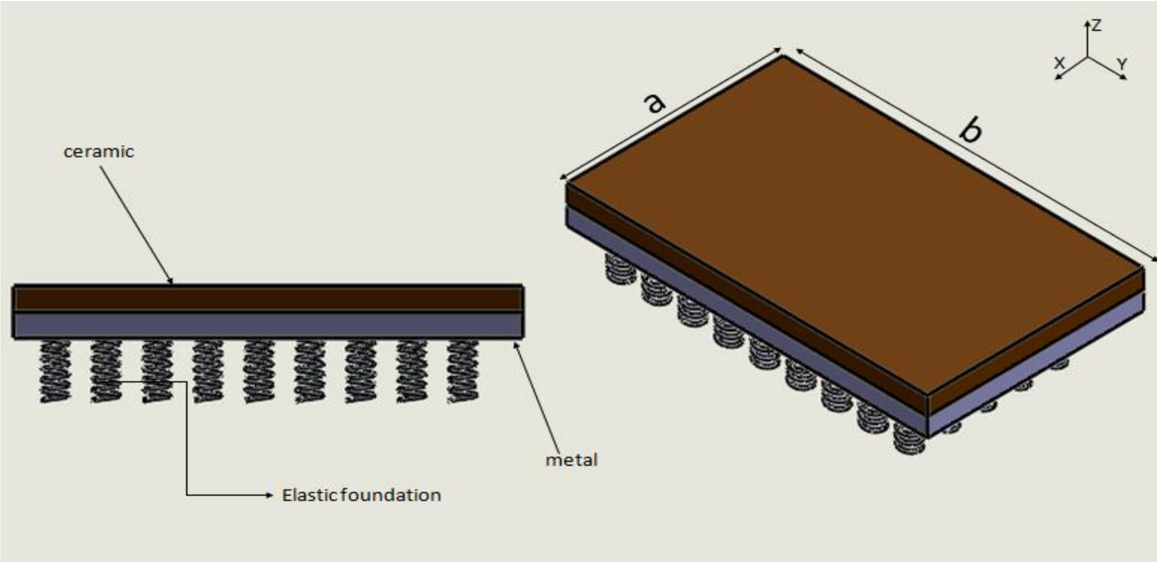

Fig. 1 Schematic view of functionally graded nanoplates

2.3 Governing equation

u, *v* and *w*, are displacements component of an arbitrary point in the mid-plane along the *x*, *y* and *z* directions, respectively. According to the classical theory of plate (CPT), the displacement field can be presented as

$$U = u(x,y) - z\frac{\partial W}{\partial x} \quad , \quad V = v(x,y) - z\frac{\partial W}{\partial y} \quad , \quad W = w(x,y) \tag{9}$$

U, *V* and *W*, are the displacement components of an arbitrary point (*x*, *y*, *z*) at a distance z from the middle of the plane thickness in the x, y and z directions, respectively. The strain-displacement relationships are presented following strain field. These equations are independent from constitutive equations. The tensorial strain field can be written as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} - z\frac{\partial^2 W}{\partial x^2}, \quad \varepsilon_{yy} = \frac{\partial v}{\partial y} - z\frac{\partial^2 W}{\partial y^2}, \quad \varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - z\frac{\partial^2 W}{\partial x \partial y} \tag{10}$$

It is important that the transverse shear deformation is negligible in the classical theory of plates. Force and moment of nonlocal elasticity are used in the obtained formulation can be presented as

$$\begin{aligned} \{N_{xx}, N_{yy}, N_{xy}\}^T &= \int_{-(h/2)}^{(h/2)} \{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\}^T dz \\ \{M_{xx}, M_{yy}, M_{xy}\}^T &= \int_{-(h/2)}^{(h/2)} \{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\}^T z dz \end{aligned} \tag{11}$$

The strain energy of the nanoplates in the presence of surface stress on the basis of the continuum surface elasticity theory can be introduced as

$$U=\frac{1}{2}\int_A\int_{-(h/2)}^{(h/2)}\sigma_{ij}\varepsilon_{ij}dzdA \tag{12}$$

The work done by the external force can be represented as follows

$$W_{ext}=\int_{-(h/2)}^{-(h/2)}qwdz \tag{13}$$

Now, by using Hamilton's principle

$$\int_{t_1}^{t_2}(\delta U-\delta W^{ext})\,dt=0 \tag{14}$$

And taking the variation of *w* and integrating by parts as follows

$$\delta U=\int_V\sigma_{xx}\delta\varepsilon_{xx}dv+\int_V\sigma_{yy}\delta\varepsilon_{yy}dv+\int_V\sigma_{xy}\delta\varepsilon_{xy}dv \tag{15}$$

And for external work can be revealed

$$\delta W^{ext}=\int_A q\delta wdA \tag{16}$$

Where q is the transverse force per unit area. Thermal loading and Pasternak foundation can be considered as external forces. So, external forces can be rewritten for thermal loading as

$$\begin{aligned}W_{ext}{}^{T}&=\int_A\left[N_{Tx}\left(\frac{\partial W}{\partial x}\right)^2+N_{Ty}\left(\frac{\partial W}{\partial y}\right)^2\right]dA\rightarrow\delta W_{ext}{}^{T}\\&=N_{Tx}\frac{\partial w}{\partial x}\delta[w]_0^a-\int_A N_{Tx}\frac{\partial^2 w}{\partial x^2}\delta wdA+N_{Ty}\frac{\partial w}{\partial y}\delta[w]_0^b-\int_A N_{Ty}\frac{\partial^2 w}{\partial y^2}\delta wdA\end{aligned} \tag{17}$$

In which, the distribution of thermal loading factor along the *X* and *Y* direction including N_{Tx} and N_{Ty} can be shown as (Ansari *et al*. 2015)

$$N_{Tx}=\int_{-(h/2)}^{(h/2)}\sigma_{xx}^{T}dz\ ,\qquad N_{Ty}=\int_{-(h/2)}^{(h/2)}\sigma_{yy}^{T}dz \tag{18}$$

where $\sigma_{xx}^T$and σ_{yy}^T are defined as follows

$$\sigma_{xx,yy}^{T}=-\frac{E\alpha(T-T_0)}{1-\nu} \tag{20}$$

T_0 is the initial uniform temperature of a stress free state that it is assumed as 300°K. Also, external forces can be rewritten for Pasternak foundation as

$$\begin{aligned}W_{ext}{}^{P}&=\int_A\left[K_G\left(\frac{\partial W}{\partial x}\right)^2+K_G\left(\frac{\partial W}{\partial y}\right)^2\right]dA\rightarrow\ \delta W_{ext}{}^{P}\\&=K_G\frac{\partial w}{\partial x}\delta[w]_0^a-\int_A K_G\frac{\partial^2 w}{\partial x^2}\delta wdA+K_G\frac{\partial w}{\partial y}\delta[w]_0^b-\int_A K_G\frac{\partial^2 w}{\partial y^2}\delta wdA\end{aligned} \tag{21}$$

The motion equation and the boundary conditions will be obtained by setting the coefficients of δw equal to zero as, motion equation obtained from the above relationships are as follows

$$\frac{\partial(N_{xx})}{\partial x}+\frac{\partial(N_{xy})}{\partial y}=0 \tag{22a}$$

$$\frac{\partial(N_{xy})}{\partial x}+\frac{\partial(N_{yy})}{\partial y}=0 \tag{22b}$$

$$\frac{\partial^2(\mathrm{M_{xx}})}{\partial \mathrm{x}^2}+2\frac{\partial^2(\mathrm{M_{xy}})}{\partial \mathrm{x}\,\partial \mathrm{y}}+\frac{\partial^2(\mathrm{M_{yy}})}{\partial \mathrm{y}^2}+\mathrm{q}+\frac{\partial}{\partial \mathrm{x}}\left(\mathrm{N_{xx}}\frac{\partial \mathrm{w}}{\partial \mathrm{x}}+\mathrm{N_{xy}}\frac{\partial \mathrm{w}}{\partial \mathrm{y}}\right)+\frac{\partial}{\partial \mathrm{y}}\left(\mathrm{N_{yy}}\frac{\partial \mathrm{w}}{\partial \mathrm{y}}+\mathrm{N_{xy}}\frac{\partial \mathrm{w}}{\partial \mathrm{x}}\right)+(\mathrm{K_G}+\mathrm{N_{Tx}})\frac{\partial^2 \mathrm{w}}{\partial \mathrm{x}^2}+(\mathrm{K_G}+\mathrm{N_{Ty}})\frac{\partial^2 \mathrm{w}}{\partial \mathrm{y}^2}=0 \tag{22c}$$

To obtain the equation of motion should be nonlocal effect must give effect to the above equation.

According to the generalized Hook's law

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{bmatrix}=[Q]\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{bmatrix} \tag{23}$$

Where $[\sigma]$ and $[\varepsilon]$ are stress and strain matrix, and $[Q]$ represent fourth order elasticity matrix as follows

$$[Q]=\begin{bmatrix}\frac{E(z)}{1-\nu^2} & \frac{E(z)\nu}{1-\nu^2} & 0\\ \frac{E(z)\nu}{1-\nu^2} & \frac{E(z)}{1-\nu^2} & 0\\ 0 & 0 & \frac{E(z)}{2(1+\nu)}\end{bmatrix} \tag{24}$$

By definition that we had in the previous section article, Should we change the above equation as follows

$$\begin{bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{bmatrix}=[C]\begin{bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{bmatrix} \tag{25}$$

$$\begin{bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{bmatrix}=[D]\begin{bmatrix}\frac{\partial^2 W}{\partial x^2}\\ \frac{\partial^2 W}{\partial y^2}\\ 2\frac{\partial^2 W}{\partial x\partial y}\end{bmatrix} \tag{26}$$

The matrix of $[C]$and $[D]$'s component in the above equations are defined as follows

$$C_{ij} = \int_{-(h/2)}^{(h/2)} Q_{ij} dz \tag{27}$$

$$D_{ij} = -\int_{-(h/2)}^{(h/2)} Q_{ij} z dz \tag{28}$$

Eringen's equations as follows spreads

$$N_{xx} - \mu\left(\frac{\partial^2 N_{xx}}{\partial x^2} + \frac{\partial^2 N_{xx}}{\partial y^2}\right) = C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y} \tag{29a}$$

$$N_{yy} - \mu\left(\frac{\partial^2 N_{yy}}{\partial x^2} + \frac{\partial^2 N_{yy}}{\partial y^2}\right) = C_{21}\frac{\partial u}{\partial x} + C_{22}\frac{\partial v}{\partial y} \tag{29b}$$

$$N_{xy} - \mu\left(\frac{\partial^2 N_{xy}}{\partial x^2} + \frac{\partial^2 N_{xy}}{\partial y^2}\right) = C_{33}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \tag{29c}$$

$$M_{xx} - \mu\left(\frac{\partial^2 M_{xx}}{\partial x^2} + \frac{\partial^2 M_{xx}}{\partial y^2}\right) = -D_{11}\frac{\partial^2 W}{\partial x^2} + D_{12}\frac{\partial^2 W}{\partial y^2} \tag{29d}$$

$$M_{yy} - \mu\left(\frac{\partial^2 M_{yy}}{\partial x^2} + \frac{\partial^2 M_{yy}}{\partial y^2}\right) = -D_{21}\frac{\partial^2 W}{\partial x^2} + D_{22}\frac{\partial^2 W}{\partial y^2} \tag{29e}$$

$$M_{xy} - \mu\left(\frac{\partial^2 M_{xy}}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial y^2}\right) = -2D_{33}\frac{\partial^2 W}{\partial x \partial y} \tag{29f}$$

By inserting motion equation in the nonlocal equation, the governing equation of the nonlocal theory of plate for buckling in terms of *w* can be obtained as follows

$$\begin{gathered} \mathrm{D}_{11}\frac{\partial^4 \mathrm{w}}{\partial \mathrm{x}^4} + 2(\mathrm{D}_{12} + \mathrm{D}_{33})\frac{\partial^4 \mathrm{w}}{\partial \mathrm{x}^2 \partial \mathrm{y}^2} + \mathrm{D}_{22}\frac{\partial^4 \mathrm{w}}{\partial \mathrm{y}^4} + \left(\mu\nabla^2\text{-}1\right)\left(\mathrm{N}_{\mathrm{xx}}\frac{\partial^2 \mathrm{W}}{\partial \mathrm{x}^2} + 2\mathrm{N}_{\mathrm{xy}}\frac{\partial^2 \mathrm{W}}{\partial \mathrm{x}\,\partial \mathrm{y}} + \mathrm{N}_{\mathrm{yy}}\frac{\partial^2 \mathrm{W}}{\partial \mathrm{y}^2}\right) + \\ (\mathrm{K}_{\mathrm{G}} + \mathrm{N}_{\mathrm{Tx}})\frac{\partial^2 \mathrm{w}}{\partial \mathrm{x}^2} + (\mathrm{K}_{\mathrm{G}} + \mathrm{N}_{\mathrm{Ty}})\frac{\partial^2 \mathrm{w}}{\partial \mathrm{y}^2} = 0 \end{gathered} \tag{30}$$

Now, we assume that the plate under the following linearly and non-linearly varying normal loads

$$N_{xx} = -P_0\left(1 - \chi\left(\frac{y}{l_y}\right)^{1 or 2}\right), N_{yy} = 0\,, N_{xy} = 0\,, q = 0 \tag{31}$$

χ Specifies the amount of numerical loading factor, If *y* is of the first order loading factor is in linearly phase and if *y* is of the second order loading factor is in non-linearly condition. P_0 is the compressive force per unit length at *y*=0. This in-plane force distribution is seen at the two nanoplates opposite edges (x=0, $x=l_x$). χ's change, shows different form of in-plane loadings. If χ=0, then the situation of uniform compressive force is investigated. If χ=1, the force decrease

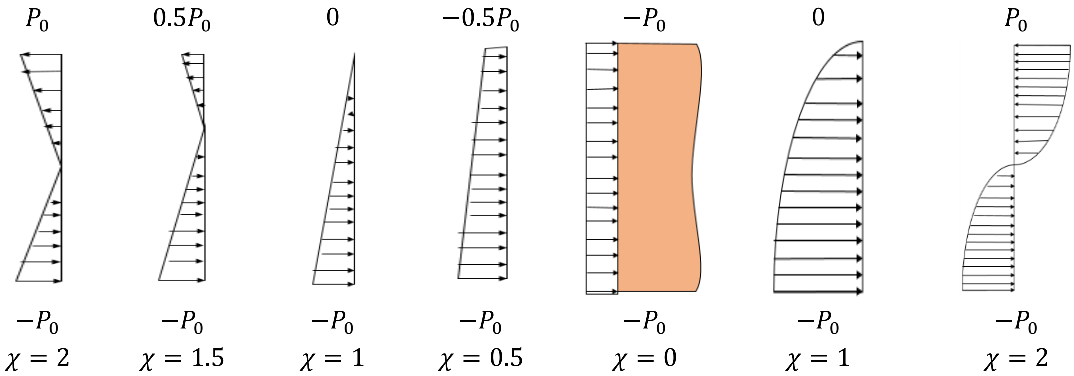

Fig. 2 Different load factor

from **-**P_0 at **y=0**, to zero at $y=l_y$ and if χ=2, we'll see pure bending. This type of modeling allows the full range of loads from pure compression to pure bending considered only on the basis of unit modeling. These different situations of loadings condition are shown in Fig. 2. Substituting N_{xx} defined variable into equation of motion yields the below fourth-order PDE of the nonlocal theory of the plate for buckling of functionally graded nanoplates

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+D_{33})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+(K_G+N_{Tx})\frac{\partial^2 w}{\partial x^2}+(K_G+N_{Ty})\frac{\partial^2 w}{\partial y^2}- \\ P_0\left(\mu\left[\left(1-\chi\frac{y}{l_y}\right)\frac{\partial^4 w}{\partial x^4}+\left(1-\chi\frac{y}{l_y}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}\right]-\left(1-\chi\frac{y}{l_y}\right)\frac{\partial^2 w}{\partial x^2}\right)=0 \tag{31}$$

For create non-dimensional state of above equation, define the following parameters

$$W=\frac{w}{l_x}\ ,\ \xi=\frac{x}{l_x},\ \eta=\frac{y}{l_y},\ \psi=\frac{D_{12}+D_{33}}{D_{11}},\ \lambda=\frac{D_{22}}{D_{11}},\ \gamma=\frac{\mu}{l_x},\ \beta=\frac{l_x}{l_y}, \tag{32}$$

3. Solution procedure

In order to predict solution of Eq. (32) analytical approach can be applied for simply supported boundary condition. In this paper, governing equation is solved by using the Navier's approach.

3.1 Navier's method

For the simply supported boundary condition, according to (Aksencer and Aydogdu 2011) it can be shown that the shape function can be written in following statement (double Fourier series). This approach is a simple method to find displacement and stress changing partial equation to numerical equation by inserting following shape function to governing equation. As being an analytical approach, Navier's method is more accurate than the numerical ones like differential transform method or differentiae quadrature method. Here the idea is to express the applied load in terms of Fourier components and then finding the solution for a sinusoidal load (a single Fourier component), and finally superimposing the Fourier components to get the solution for an arbitrary load as follows

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}sin\left(\frac{m\pi x}{l_x}\right)sin\left(\frac{m\pi y}{l_y}\right)e^{i\omega t} \tag{30}$$

Because the subject matter is buckling article, time-dependent terms are deleted. And *m* and *n* are the half wave numbers. For this purpose, incorporating Eq. (30) into Eq. (28), will have

$$\begin{aligned}&D_{11}\left(\frac{m\pi}{l_x}\right)^4+2(D_{12}+D_{33})\left(\frac{m\pi}{l_x}\right)^2\left(\frac{n\pi}{l_y}\right)^2+D_{22}\left(\frac{n\pi}{l_y}\right)^4+(K_G+N_{Tx})\left(\frac{m\pi}{l_x}\right)^2+(K_G+N_{Ty})\left(\frac{n\pi}{l_y}\right)^2\\&\quad-P_0\left(\mu\left[\left(\left(1-\chi\frac{y}{l_y}\right)\left(\frac{m\pi}{l_x}\right)^4\right)+\left(\left(1-\chi\frac{y}{l_y}\right)\left(\frac{m\pi}{l_x}\right)^2\left(\frac{n\pi}{l_y}\right)^2\right)\right.\right.\\&\quad\left.\left.-\left(\left(1-\chi\frac{y}{l_y}\right)\left(\frac{n\pi}{l_x}\right)^2\right)\right]\right)=0\end{aligned} \tag{31}$$

That P_0 shows as follows

$$P_0=\left(D_{11}\left(\frac{m\pi}{l_x}\right)^4+2(D_{12}+D_{33})\left(\frac{m\pi}{l_x}\right)^2\left(\frac{n\pi}{l_y}\right)^2+D_{22}\left(\frac{n\pi}{l_y}\right)^4+(K_G+N_{Tx})\left(\frac{m\pi}{l_x}\right)^2+(K_G+N_{Ty})\left(\frac{n\pi}{l_y}\right)^2\right)\Big/\left(\mu\left[\left(\left(1-\chi\frac{y}{l_y}\right)\left(\frac{m\pi}{l_x}\right)^4\right)+\left(\left(1-\chi\frac{y}{l_y}\right)\left(\frac{m\pi}{l_x}\right)^2\left(\frac{n\pi}{l_y}\right)^2\right)\right]-\left(\left(1-\chi\frac{y}{l_y}\right)\left(\frac{n\pi}{l_x}\right)^2\right)\right) \tag{32}$$

It is assumed that simply supported boundary conditions are in all directions but for clamped-clamped boundary condition following equation and condition must be applied to the governing equations instead of Eq. (30) (Sobhy 2015)

$$w(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}F(x)sin\left(\frac{m\pi y}{l_y}\right) \tag{33}$$

$$F(x)=\sin(\lambda_m x)-\sinh(\lambda_m x)-\tau(\cos(\lambda_m x)-\cosh(\lambda_m x)) \tag{34}$$

$$\tau=\frac{\sin(\lambda_m a)-\sinh(\lambda_m a)}{\cos(\lambda_m a)-\cosh(\lambda_m a)} \tag{35}$$

$$\lambda_m=\frac{(m+0.5)\pi}{a} \tag{36}$$

4. Results and discussion

According to the previous studies in buckling of size dependent plates we will define the

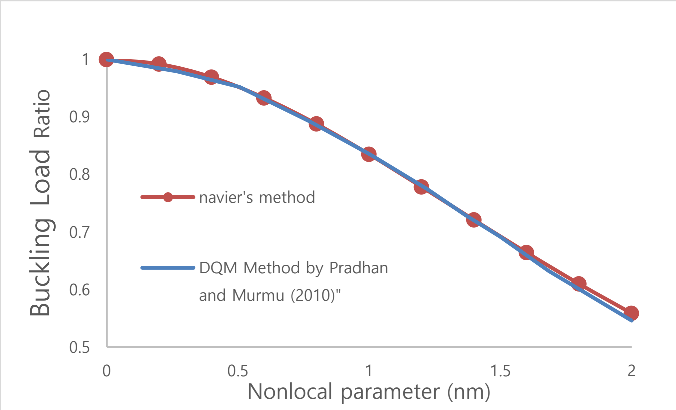

Fig. 3 Change of buckling load ratio with the nonlocal parameter for different load factors ($l_x = 10\ nm$)

following non-dimensional parameter

$$Buckling\ load\ factor = \frac{nonlocal\ buckling\ load}{local\ buckling\ load} \tag{37}$$

Above parameter is the name of the famous buckling load ratio and it has been used in many articles (Pradhan and Murmu 2009, Pradhan and Murmu 2010). The present results are compared with the results presented by Pradhan and Murmu (2010) for the buckling response of square single-layered grapheme sheet. The comparison results can be seen in Fig. 3 which shows a good agreement and verifies the presented approach. The FG nanoplate is supposed to be made of aluminum (E=70 GPa) and alumina (E=380 GPa) and subjected a uniformity distributed normal load from $x = 0$ to $x = l_x$. In this case, the numerical loading factor is equal to zero at $(\chi = 0)$. The plate is rectangular with simply supported boundary condition along four edges and made of

Effectiveness of the different loading conditions on nanoplate's buckling characteristics and the changes of the non-dimensional buckling load with respect to the nonlocal parameter (μ) for various linear loading factors are shown in Fig. 4. This figure shows the buckling load increases by increase in the value of the loading factor from *0* to *2*, especially for the local case. The effects of various loading factor in buckling increases for lower value of nonlocal parameter. For more description, the nonlocal parameter effect on the buckling load is more important in the pure bending ($\chi = 2$). However, the difference between local and nonlocal buckling load increases by increase in value the load factor.

The nonlocal parameter (μ) effect on non-dimensional buckling loads for Simply Supported boundary condition in various nonlinear loading factor shown in Fig. 5. This figure illustrates, the buckling increases by increase in value of loading factor from *1* to *2*. The effects of various loading factor in buckling increases by decrease in value of nonlocal parameter. The amount of buckling load in the nonlinear in-plane loads have a large difference with linear in-plane loads. When the nanoplates is subjected to in-plane nonlinear loading, plate's stability disappears.

The effect of various nonlocal parameter on non-dimensional buckling load in different linear and nonlinear loading factor can be seen in Table 1. In this table, the amount of nonlocal parameter is changed from 0 to 4. According to the table, the non-dimensional buckling loads calculated based on the nonlocal theory, are lesser than the non-dimensional buckling loads calculated based on the local theory in various load factors and in all cases. In addition, non-dimensional buckling decreased by increase in value of nonlocal parameter, and also it causes the stiffness of structure decreases in fixed side length. The amount of non-dimensional buckling of functionally graded nanoplates when $\chi\ is\ 2$ is more than other states. In addition, the amount of buckling increases when loading factor increased from 0 to 2.

To illustrate the effect of aspect ratio (β) on non-dimensional buckling load of nanoplates, the variation of non-dimensional buckling load with aspect ratio in various linear and nonlinear load factors are presented in Table 2. According to these table, it can be concluded that, the aspect ratio effects are lost after a special length in all cases and for various load factor. This is predictable because the effect of nonlocal parameter decreases by increase in structure's size. Furthermore, the gap between the amounts in different loading factor piecemeal reduces for higher dimension of nanoplates. Anyway, it is realized that, the gap between pure bending with other states doesn't disappear. The amount of non-dimensional buckling load decreased for higher value of nanoplate's side length that is subjected to in-plane loading.

Fig. 6 shows the effect of Pasternak foundation (K_G) on dimensional buckling load in various linear loading factor. This figure shows, the buckling decreases when the nanoplates embedded on Pasternak foundation, especially for pure bending ($\chi = 2$). The effects of Pasternak foundation on dimensional buckling load increases for higher value of linear load factors. For more information, the difference between buckling load with foundation and without foundation increases by increase in value of loading factor. Fig. 7 shows the effect of aspect ratio (β) on the dimensional buckling load ratio of simply supported nanoplates in different linear load factors. According

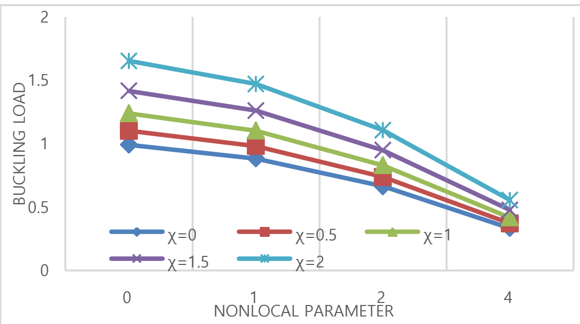

Fig. 4 The effect of different nonlocal parameter on the buckling load in various linear loading factor (β=0.5, l_x=10 *nm*, k=1, K_G=5&T=200℃)

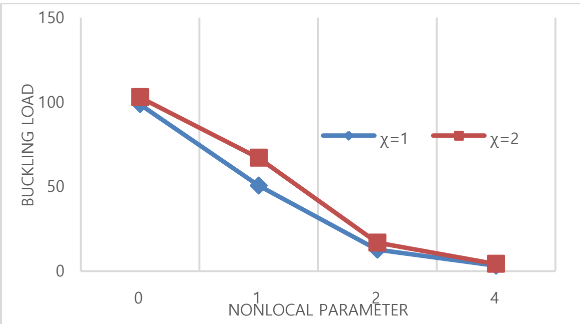

Fig. 5 The effect of different nonlocal parameter on the buckling load in various nonlinear loading factor(β=0.5, l_x=10 *nm*, k=1, K_G=5&T=200℃)

Table 1 Change of non-dimensional buckling with nonlocal parameter for different load factors (β=0.5, l_x=10 *nm*, k=1, K_G=5&T=200℃)

μ	Linear loading factor χ					Nonlinear loading factor χ	
	0	0.5	1	1.5	2	1	2
0	0.9912	1.1013	1.2390	1.4160	1.6520	98.586	102.739
1	0.8823	0.9804	1.1029	1.2605	1.4706	50.471	66.9562
2	0.6637	0.7374	0.8296	0.9481	1.1061	12.5543	16.739
4	0.3333	0.3703	0.4166	0.4761	0.5555	3.1385	4.1847

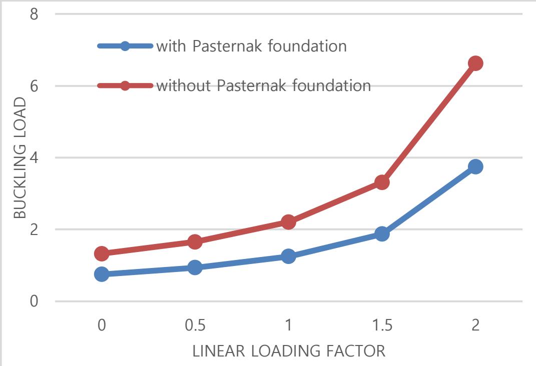

Fig. 6 The effect of Pasternak foundation on the dimensional buckling load in different linear load factor (β=1, l_x=10 *nm*, k=1, K_G=5&T=200℃)

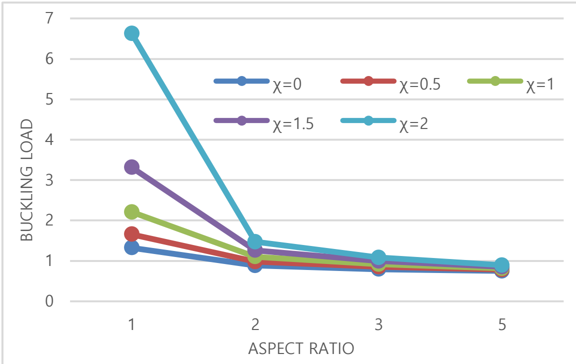

Fig. 7 The effect of different aspect ratio on the dimensional buckling load in different linear load factor (μ=1, l_x=10 *nm*, k=1, K_G=5&T=200℃)

Table 3 Change of dimensional buckling in different linear and nonlinear loading factor and temperature (β=1, l_x=10 nm, k=1, K_G=5&T=200℃)

Temperature	Linear load factor χ					Nonlinear load factor χ	
	0	0.5	1	1.5	2	1	2
100℃	2.0739	2.5924	3.4565	5.1848	10.3697	37.0018	68.8923
200℃	1.3245	1.6556	2.2075	3.3113	6.6227	23.6316	44.6375
250℃	0.9498	1.1873	1.583	2.2374	4.7492	16.9465	32.01
273℃	0.7774	0.9718	1.2958	1.9437	3.8874	13.8713	26.2014
300℃	0.5751	0.7189	0.9585	1.4378	2.8757	10.2614	19.3826

Table 4 Change of dimensional buckling load ratio in different load ratio and FG index (μ=1, l_x=10 nm, k=1, K_G=5&T=200℃)

FG Index	Linear load factor χ					Nonlinear load factors χ	
	0	0.5	1	1.5	2	1	2
0	2.6065	2.8961	3.2582	3.7236	4.3442	148.341	197.789
2	0.8002	0.8892	1.0003	1.1432	1.3338	45.5445	60.7261
4	0.7562	0.8402	0.9452	1.0803	1.2603	43.0373	57.383
6	0.7410	0.8234	0.9263	1.0586	1.2351	42.1742	56.2332
10	0.7287	0.8097	0.9109	1.0410	1.2145	41.4734	55.2978

to the figure, dimensional bucking loading decrease by increase in aspect ratio. It is predictable because increase in length of plate that subjected to the in-plane loading causes increased structural stability. The effects of aspect ratio on dimensional buckling load increases for higher value of linear loading factors.

Table 3 shows the effect of temperature (T) and various loading factors (χ) whether linear or nonlinear on dimensional buckling of FG nanoplates. The results shows that the buckling load decreases by increase in plate's temperature. This behavior is visible for all of cases and also, the plate's buckling increases for higher value of the load factors whether linear or nonlinear. The

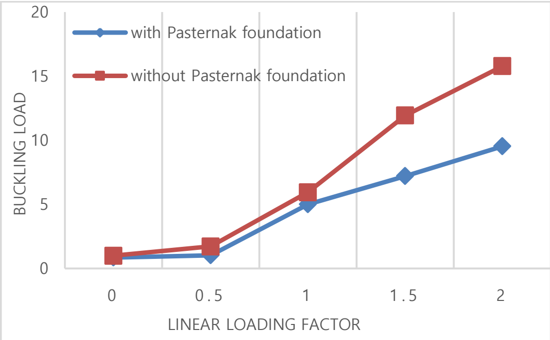

Fig. 8 The effect of Pasternak foundation on the dimensional buckling load in different linear load factor and clamped-clamped boundary condition (β=1, l_x=10 nm, k=1, K_G=5&T=200℃)

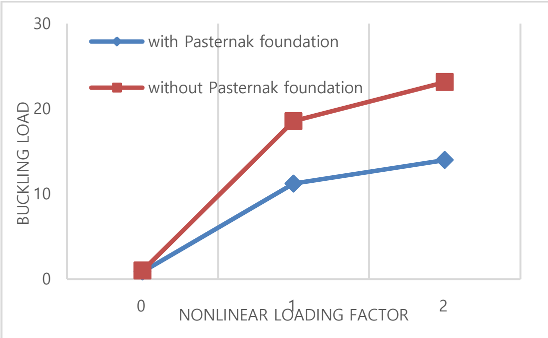

Fig. 9 The effect of Pasternak foundation on the dimensional buckling load in different nonlinear load factor and clamped-clamped boundary condition (β=1, l_x=10 nm, k=1, K_G=5&T=200℃)

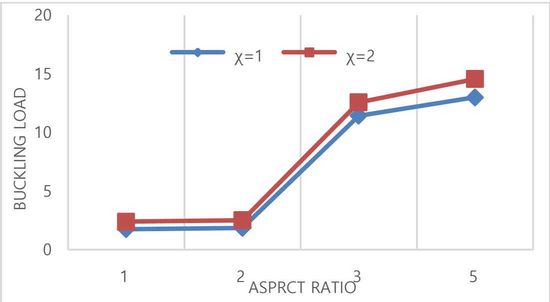

Fig. 10 The effect of different aspect ratio on the dimensional buckling load in different linear load factor and clamped-clamped boundary condition (μ=1, l_x=10 nm, k=1, K_G=5&T=200℃)

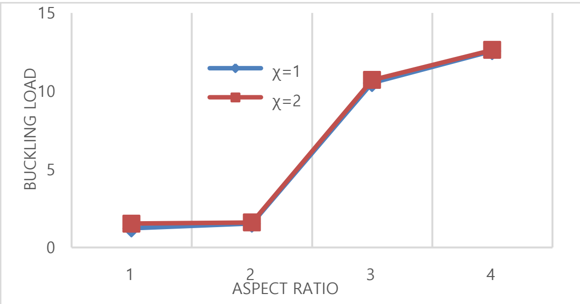

Fig. 11 The effect of different aspect ratio on the dimensional buckling load in different nonlinear load factor and clamped-clamped boundary condition (μ=1, l_x=10 nm, k=1, K_G=5&T=200℃)

important note in this table is the difference in value of buckling between pure bending cases with other cases.

Table 4 shows the effect of FG index and various linear and nonlinear loading factor on buckling of plate. Dimensional bucking loading ratio in the FGM plate is higher than pure metallic plate. Fundamental frequency and bucking loading increase significantly for higher value of k. These behavior are occurs on the plates because pure metallic plate has lower stiffness than FGM plate. Similarly, Figs. 8-11 shows these details for nonlinear different loading factors.

5. Conclusions

A general view of the article contains a lot of content, including the importance of temperature raises, Pasternak foundation and size dependent effect on the buckling response of functionally graded nanoplates that subjected to linear and nonlinear in-plane load based on elasticity nonlocal theory. The governing equations are solved by using Navier's method. The results are compared with the differential quadrature method (DQM) and shows good agreement with them. Simply supported boundary condition is considered for the FGM plate. According to the results it can be seen that, Pasternak foundation causes decrease in buckling for various loading condition. The buckling load ratio decreases for higher value of nanoplate's temperature. Responses of nonlocal buckling are lesser than local states for various loading condition in all cases. In addition, the buckling load ratio decreases for higher value of nonlocal parameter. The non-dimensional buckling increases by increase in value of FG index. For the special state, when loading condition make pure in-plane bending, the effect of nonlocal is most importantly than other states. Anyway, at the huge plates the difference in the small scale effect is negligible, however there is difference between bending state and another states even for large dimension. It can be seen that, buckling load ratio decreases by increase in aspect ratio. Further, the effect of small scale effect increases for higher value of mode number. By increase in the FG index at both of linear and nonlinear loading factor, the amount of plates buckling increases by increase in value of nonlocal parameter or aspect ratio, the amount of plates buckling reduced whether in linear load factor or nonlinear load factor.

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