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Impact of frequency on nonlocal thermoelastic media with diffusion under a normal load

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Abstract. This study investigates the dynamic behaviour of nonlocal thermoelastic solid with diffusion subjected to a normal source with the focus on the effect of angular frequency on the medium. The governing equations are solved in the frequency domain. Numerical inversion technique is applied to find the solution in physical domain using Matlab. The obtained results are depicted graphically. As an application we use concentrated normal force, uniformly distributed and linearly distribued sources. We find that angular frequency significantly effects the components of normal stress, shear stress, mass concentration and temperature change. The results provide valuable insight for applications in advanced materials science, micro and nano-scale engineering, and dynamic load analysis.

Keywords: angular frequency; Fourier transformation; nonlocal; normal force; stress; thermoelastic

1. Introduction

The study of nonlocal thermoelastic materials diffusive solids has received a lot of attention in recent years because of its applications in advanced materials science, such as micro and nano materials and biomechanics. The frequency-dependent response of such materials under different loading is an important field of study. This study summarizes significant contributions to the area emphasizing theoretical and computational advances (Srinivasa and Reddy 2017). Thermoelastic diffusion theory assumes that irrespective of the heat and mass transfer mechanism, it must obey the classical Fourier and Fick's laws. In Fourier's law, in the governing equation the temperature gradient is not synchronized with the heat flow vector. Fourier law predicts that the heat propagates at an infinite speed (Li *et al.* 2021).

Nonlocal continuum field theories go with the physics of material bodies having properties of material that are influenced by the body's state at all places. Nonlocal theory generalises classical field theory in two ways, i.e. the energy balancing law is accepted globally (for the entire body) and the response functional explain the behaviour of the body at a given point in time (Zeng *et al.* 2006).

Eringen (1972) proposed the idea of nonlocal elasticity, which has been critical in understanding material behavior at tiny scales. Eringen's nonlocal theory states that stress at a given place is determined by the strain field at all other sites in the body. Recent research has expanded this theory

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to incorporate thermal and diffusive effects resulting in a more complete framework for studying coupled phenomena. Abbas (2015) investigated the thermoelastic response of a microscale beam exposed to a moving heat source based on the Green and Naghdi type III theory.

Abbas (2014) derived a general solution to the field equations of the two-temperature generalized thermoelastic theory for an infinite medium. Alzahrani and Abbas (2020) analysed the photo-thermoelastic interactions in an infinite semiconductor medium that includes a spherical cavity. Kumar *et al*. (2013) studied the deformation caused by a thermal source in a thermoelastic body governed by a fractional order derivative model. Abbas and Othman (2012) studied the propagation of plane waves in a thermomicrostretch elastic half-space using both the Lord-Shulman model and the classical coupled dynamical theory. A two-dimensional problem of a generalized thermoelastic diffusion material incorporating thermal and diffusion relaxation times is analyzed within the framework of the Lord-Shulman theory.

Abbas *et al*. (2012), Marin *et al*. (2014) established fundamental theorems for microstretch thermoelastic materials through the application of the Lagrange identity. Abouelregal *et al*. (2023) employs a modified Moore-Gibson-Thompson heat conduction model incorporating memory-dependent derivatives to analyse thermoelastic interactions generated by non-Gaussian laser heating in an infinite nonlocal elastic medium. Gopalakrishnan and Narendar (2013) studied how heat and mechanical forces affect materials, especially at very small scales.

Marin *et al*. (2014) studied the domain of influence consequence to the micropolar thermoelastic diffusion, El-Karamany and Ezzat (2016) developed mathematical model to describe how materials respond to heat, force and diffusion. They showed that how materials behave under complex conditions involving heat, forces and diffusion. (Ezzat *et al*. 2018) developed a mathematical model of two temperature thermoelasticity theories based on fractional derivative heat transfer. They used Laplace transform techniques on a half-space subjected to arbitrary time dependent heating together with free traction under two-dimensional temperature parameters.

Singh and Lata (2024) studied the thermomechanical deformations in a nonlocal, homogeneous, isotropic thick circular plate within the frequency domain, using the numerical inversion technique. Li *et al*. (2022) developed mathematical model of theory of nonlocal thermoelastic diffusive materials, nonlocal heat and mass transport effects based on Lord and Shulman model and studied the fluctuation of the temperature, stress, displacement, con centration and chemical potential with nonlocal parameters. Abouelregal *et al*. (2022) develop a mathematical model encompassing generalized thermoelastic diffusion characterized by four lags and incorporating higher-order time fractional derivatives. Alterations were made to the heat equations and Fick's law, with particular adjustments made to the Fourier law to accommodate a higher time fractional order of heat conduction. Singh and Lata (2023) studied the effect of two temperature parameters on the axisymmetric deformation in a two dimensional nonlocal homogeneous isotropic thick circular plate without energy dissipation.

Kumar *et al*. (2022) developed a mathematical model that incorporated nonlocal, phase-lag and temperature dependent features and they investigated the nonlocal phase-lag and temperature dependent behaviour within a modified coupled stress thermoelastic diffusive medium under the influence of thermomechanical sources. Kumar *et al*. (2025) developed a deformation model for a photo thermoelastic thick circular plate based on Moore-Gibson-Thompson thermoelasticity with a fractional order time derivative and solved by Laplace transformation and Hankel transforms.

Lata and Kaur (2024) investigated propagation of Stoneley waves at the interface of two dissimilar transversely isotropic thermoelastic solids under a new modified couple stress theory without energy loss and with two temperatures. Marin (2010) prepared coupled equations that characterize the pore's evolution with basic equations that describe the elastic deformations of the Cosserat body. The coupling is done using predetermined coefficients. Lata and Heena (2024) analysed two-dimensional deformation in a transversely isotropic thermoe lastic diffusion medium, exploring the influence of diffusion and thermal effects on such solids under an inclined load. Singh and Lata (2024) studied the thermomechanical deformations that appear in a nonlocal homogeneous isotropic thick circular plate through the frequency domain without energy dissipation and solved the problem through Hanker transformation techniques. Kumar *et al.* (2025) investigated the deformation of a thick circular photo thermoelastic plate utilizing Moore-Gibson Thompson thermoelasticity and fractional order time derivative, employing Laplace transformation, Fourier transformation and Hankel equations.

In above research review, we find that lot of work is done on nonlocal thermoelastic media with diffusion that focuses on time domain formulations. Most studies focus solely on mechanical and thermal interactions without fully integrating diffusive effects. This gap highlights the need for advanced mathematical modelling and analytical techniques to bridge the gap between classical and modern nonlocal thermoelastic diffusion theories in the frequency domain. In this study we analyse the effect of frequency on nonlocal thermoelastic media with diffusion. We study the effect of frequency on stress components, temperature change and mass concentration and depict it graphically.

2. Basic equations

Following Eringen (2002) the stress tensor at arbitrary point x of a nano material body not only depends up on the stress tenssor at x, but also depend on all points of the body. The nonlocal stress tensor σ_{ij} for a homogeneous isotropic elastic material in the absence of body force can be expressed as

$$\sigma_{ij}(x) = \int_{v} \alpha(|x - x'|, \quad \xi) t_{ij}(x) dV\,(x'). \tag{1}$$

By employing Eringen's nonlocal formulation, the nonlocal stress tensor $\sigma_{ij}(x)$ can be expressed with components of stress tensor t_{ij} as

$$(1 - \xi^2 \nabla^2)\sigma_{ij}(x) = t_{ij}(x). \tag{2}$$

Constitutive equation for coupled thermoelastic diffusive medium while neglecting the body forces can be expressed as

$$(1 - \xi^2 \nabla^2)\sigma_{ij}(x) = 2\mu e_{ij} + \delta_{ij}(\lambda e_{kk} - \gamma_1 T - \gamma_2 C), \quad P = -\gamma_2 e_{kk} - aT + bC. \tag{3}$$

Following Ram *et al.* (2008) and Malik *et al.* (2023) the basic equations for isotropic nonlocal thermoelastic media with diffusion can be given by

$$(\lambda + \mu)\nabla.\nabla\mathbf{u} + \mu\nabla^2\mathbf{u} - \gamma_1\nabla\boldsymbol{T} - \gamma_2\nabla\boldsymbol{C} + \rho(1 - \xi^2\nabla^2)\vec{F} = \rho(1 - \xi^2\nabla^2)\ddot{\boldsymbol{u}} \tag{4}$$

$$\left(K\frac{\partial}{\partial t} + K^*\right)\nabla^2\boldsymbol{T} = \rho(1 - \xi^2\nabla^2)\frac{\partial^2}{\partial t^2}\left(\rho C_E\boldsymbol{T} + \gamma_1 T_0\mathbf{u}_{i,i} + aT_0\boldsymbol{C}\right) \tag{5}$$

$$(1-\xi^2\nabla^2)\dot{\boldsymbol{C}} = d\nabla^2(b\boldsymbol{C} - \gamma_2 e_{kk} - a\boldsymbol{T}) \tag{6}$$

$$\gamma_1 = (3\lambda + 2\mu)\alpha_t, \qquad \gamma_2 = (3\lambda + 2\mu)\alpha_c \tag{7}$$

where T is temperature change, ρ is density, α_t is the coefficient of linear thermal expansion, α_c is the coefficient of diffusion expansion, K is coefficient of thermal conductivity, K^* is the materialistic constant, a is coefficients of thermal effects, b is coefficients of diffusive effects, σ_{ij} are components of stress tensor, C is concentration distribution, u_i is displacement component, λ and μ are Lame's constants, C_E is specific heat at constant strain, T_0 is temperature of the medium in its natural state assumed, d is diffusion constant, ξ is nonlocal parameter, $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$; $i, j = 1, 2, 3$, ∇ is gradient and ∇^2 is Laplacian operator.

3. Formulation of problems

Considering a two dimensional homogeneous nonlocal isotropic thermoelastic body initially at undeformed state at initial temperature T_0. We take a rectangular coordinate system (x, y, z) having origin on the surface $z = 0$ with $z - axis$ pointing vertically downward into the medium is introduced. The surface of half space is subjected to normal force. For two dimensional problems in xz plane we take

$$\boldsymbol{u} = (u, 0, w), \qquad \boldsymbol{u} = u(x,z,t), \qquad \boldsymbol{w} = w(x,z,t), \qquad \boldsymbol{T} = T(x,z,t) \;\; and \;\; \boldsymbol{C} = C(x,z,t) \tag{8}$$

using (8) in Eqs. (4)-(6), the component form of the equations (4)-(6) is derived as

$$(\lambda+\mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u - \gamma_1\frac{\partial T}{\partial x} - \gamma_2\frac{\partial C}{\partial x} + \rho(1-\xi^2\nabla^2)F_1 = \rho(1-\xi^2\nabla^2)\,\ddot{u} \tag{9}$$

$$(\lambda+\mu)\frac{\partial e}{\partial x} + \mu\nabla^2 u - \gamma_1\frac{\partial T}{\partial x} - \gamma_2\frac{\partial C}{\partial x} + \rho(1-\xi^2\nabla^2)F_1 = \rho(1-\xi^2\nabla^2)\,\ddot{u} \tag{10}$$

$$\left(K\frac{\partial}{\partial t} + K^*\right)\nabla^2 T = \rho(1-\xi^2\nabla^2)\frac{\partial^2}{\partial t^2}\left(\rho C_E\frac{\partial T}{\partial t} + \gamma_1 T_0 e + aT_0 C\right) \tag{11}$$

$$(1-\xi^2\nabla^2)\frac{\partial C}{\partial t} = d\left(b\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial z^2}\right) - \gamma_2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) - a\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right)\right) \tag{12}$$

The initial conditions of field variables are given by

$$\begin{aligned}
&u(x,z,0) = 0 = \dot{u}(x,z,0), \qquad w(x,z,0) = 0 = \dot{w}(x,z,0),\\
&T(x,z,0) = 0 = \dot{T}(x,z,0), \qquad C(x,z,0) = 0 = \dot{C}(x,z,0),\\
&for \quad z \geq 0, \qquad -\infty < x < \infty,\\
&u(x,z,t) = w(x,z,t) = T(x,z,t) = C(x,z,t) = 0,\\
&for \quad t > 0, \qquad when, \qquad z \to \infty
\end{aligned} \tag{13}$$

In our calculation we use the following dimensionless quantities for simplification

$$
\begin{aligned}
&x' = \frac{\omega_1^*}{c_1}x, \quad z' = \frac{\omega_1^*}{c_1}z, t' = \omega_1^* t, \quad u' = \frac{\omega_1^*}{c_1}u, \quad a' = \frac{\omega_1^*}{c_1}a,\\
&w' = \frac{\omega_1^*}{c_1}w, \quad C' = \frac{\gamma_2 C}{\rho c_1^2}, \quad T' = \frac{\gamma_1}{\rho c_1^2}T, \quad F_1' = \frac{F_1}{\gamma_1 T_0},\\
&F_3' = \frac{F_3}{\gamma_1 T_0}, \quad e' = e, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad \omega_1^* = \frac{\rho C_E c_1^2}{K}
\end{aligned}
\tag{14}
$$

We express the displacement variables $u(x,z,t)$ and$w(x,z,t)$ in terms of dimensionless potential functions $\mathit{\Psi}_1$ and $\mathit{\Psi}_2$

$$
u = \frac{\partial \Psi_1}{\partial x} - \frac{\partial \Psi_2}{\partial z}, \qquad w = \frac{\partial \Psi_1}{\partial z} + \frac{\partial \Psi_2}{\partial x}. \tag{15}
$$

The general solution of the problem is given in the following form

$$
(\psi_1, \psi_2, T, C)(x,z,t) = (\psi_1, \psi_2, T, C)(x,z)e^{i\omega t}. \tag{16}
$$

The Fourier transformation is given as follow.

$$
f(\zeta, z, t) = \int_{-\infty}^{\infty} f(x,z,t)e^{-i\zeta x}\,dx. \tag{17}
$$

By using the dimensionless quantities defined by (14) on the equations (9)-(12) and supressing the primes for convenience and then applying Helmholtz decomposition defined by (15) on the resulting equations, thereafter applying (16)-(17), yields

$$
A_1(D^2 - \zeta^2) + \omega^2(1 - \xi'^2(D^2 - \zeta^2)))\widehat{\Psi}_1 - \hat{T} - \hat{C} = 0. \tag{18}
$$

$$
(A_2(D^2 - \zeta^2) + (1 - \xi'^2(D^2 - \zeta^2))\omega^2)\widehat{\Psi}_2 = 0 \tag{19}
$$

$$
\begin{aligned}
&\left(1 - \xi'^2(D^2 - \zeta^2)\right)(D^2 - \zeta^2)\widehat{\Psi}_1 + A_4\left(1 - \xi'^2(D^2 - \zeta^2)\right)\\
&-A_5(D^2 - \zeta^2))\hat{T} + A_6(1 - \xi'^2(D^2 - \zeta^2))\hat{C} = 0
\end{aligned}
\tag{20}
$$

$$
A_9(D^2 - \zeta^2)^2\widehat{\Psi}_1 + A_{10}(D^2 - \zeta^2)\hat{T} + (A_7(1 - \xi'^2(D^2 - \zeta^2)) - A_8(D^2 - \zeta^2))\hat{C} = 0 \tag{21}
$$

where

$$
\begin{aligned}
&A_1 = \frac{(\lambda + 2\mu)}{\rho c_1^2}, \quad A_2 = \frac{\mu}{\rho C_1^2}, \quad A_3 = -\rho\omega^2\gamma_1 T_0, \quad A_4 = -i\omega^3\frac{\rho^3 C_1^2\omega_1^*}{\gamma_1}C_E,\\
&A_5 = \frac{\rho c_1}{\gamma_1}(K\omega_1^* i\omega + K^*), \quad A_6 = -\omega^2\frac{\rho^2 C_1^3}{\omega_1^*\gamma_2}T_0 a' \; A_7 = \left(\frac{\rho\omega_1^* c_1^2}{\gamma_2}\right)i\omega, \quad A_8 = db\frac{\rho\omega_1^{*2}}{\gamma_2},\\
&A_9 = d\gamma_2\frac{\omega_1^{*2}}{c_1^2}, \quad A_{10} = ad\frac{\rho\omega_1^{*2}}{\gamma_1}, \quad A_{11} = \frac{\omega^{*2}\xi^2}{c_1^2}, \quad A_{12} = \gamma_1 T_0, \quad A_{13} = \frac{\rho\omega_1^* c_1}{\gamma_1},\\
&A_{14} = \gamma_1 T_0, \quad A_{15} = \frac{\rho\omega_1^* c_1}{\gamma_2}, \quad A_{16} = \frac{A_{12}}{A_{13}}, \quad A_{17} = \frac{A_{14}}{A_{13}}, \quad \xi'^2 = \frac{{\omega_1^*}^2}{{c_1}^2}\xi^2
\end{aligned}
$$

The Eqs. (18)-(21) possess a non-trivial solution if determinant of their coefficients vanishes. By simplifying the equation formed from them, we get following polynomial equations

$$(D^6R_1 + D^4R_2 + D^2R_3 + R_4)(\hat{\psi}_1, \hat{T}, \hat{C}) = 0 \tag{22}$$

$$(D^2 + r)\hat{\psi}_2 = 0, \;\; r = R_6/R_5 \tag{23}$$

where:

$$\begin{gathered} M_1 = -A_4A_7\omega^2, \;\; M_2 = (A_1A_4A_7 + A_4A_8\omega^2 - A_5A_7\omega^2 - A_6A_{10}\omega^2 + A_3A_7), \\ M_3 = (-A_1A_4A_8 + A_5A_8\omega^2), \\ M_4 = A_1A_5A_7 + A_1A_6A_{10} + A_3A_8 + A_9A_6 + A_3A_{10} - A_4A_9, \\ M_5 = A_1A_5A_8 - A_5A_9, \quad M_6 = M_2A_{11}^2 - A_{11}^3M_1 + M_4A_{11} + M_5\,, \\ M_7 = A_{11}^2M_1 - 2M_2A_{11} - M_3A_{11} - M_4, \; M_8 = M_2 + M_3 - 3A_{11}M_1, \;\; M_9 = M_1, R_1 = M_6, \\ R_2 = -3M_6\zeta^2 + M_7, \;\; R_3 = (3M_6\zeta^4 - 2M_7\zeta^2 + M_8), \;\; R_4 = M_7\zeta^4 + M_9 - M_6\zeta^6 - M_8\zeta^2 \\ R_5 = A_2 - A_{11}\omega^2, \;\; R_6 = -A_{11}\zeta^2\omega^2 - A_2\zeta^2 - 1 \end{gathered} \tag{24}$$

When we solve Eqs. (22) - (23) we get their roots $\pm r_i, \; i = 1,2,3$. We have also at infinity the solusions are vanishes or as $z \to \infty$, the solutions of these equations tends to zero, therefore general solutions of each variables are given in the following form.

$$\begin{gathered} \hat{\Psi}_1 = B_1e^{-r_1z} + B_2e^{-r_2z} + B_3e^{-r_3z}, \qquad \hat{T} = d_1B_1e^{-r_1z} + d_2B_2e^{-r_2z} + d_3B_3e^{-r_3z} \\ \hat{C} = l_1B_1e^{-r_1z} + l_2B_2e^{-r_2z} + l_3B_3e^{-r_3z}, \qquad \hat{\Psi}_2 = B_4e^{-r_4z} \end{gathered} \tag{25}$$

where:

$$d_i = \frac{r_i^6M_{13} + r_i^4M_{14} + r_i^2M_{15} + M_{16}}{r_i^4M_{10} + r_i^2M_{11} + M_{12}}, i = (1,2,3). \tag{26}$$

$$l_i = \frac{r_i^6M_{17} + r_i^4M_{18} + r_i^2M_{19} + M_{20}}{r_i^4M_{10} + r_i^2M_{11} + M_{12}}, i = (1,2,3). \tag{27}$$

where:

$$\begin{gathered} N_1 = A_1 - \omega^2A_{11}, \quad N_2 = (A_1 - \omega^2A_{11})\zeta^2 + \omega^2, \quad N_3 = -A_3A_{11}, \\ N_4 = A_3 + 2A_3A_{11}\zeta^2, \quad N_5 = -A_3A_{11}\zeta^4 + A_3\zeta^2, \;\; N_6 = -A_4A_{11} - A_5, \\ N_7 = A_4 + A_4A_{11}\zeta^2 + A_5\zeta^2, \;\; N_8 = A_6A_{11}, \quad N_9 = A_6A_{11}\zeta^2 + A_6, \quad N_{10} = A_9, \\ N_{11} = 2A_9\zeta^2, \;\; N_{12} = A_9\zeta^4, \;\; N_{13} = A_{10}, \;\; N_{14} = A_{10}\zeta^2, \quad N_{15} = -A_7A_{11} - A_8, \\ N_{16} = A_7A_{11} + A_8)\zeta^2 + A_7, \;\; N_{17} = (A_2 + \omega^2A_{11}), \quad N_{18} = (A_2 - \omega^2A_{11})\zeta^2 + \omega^2, \\ M_{10} = N_6N_{15} + N_8N_{13}, \;\; M_{11} = N_6N_{16} + N_7N_{15} - N_9N_{13} - N_8N_{14}, \\ M_{12} = N_7N_{16} + N_9N_{14}, \quad M_{13} = -N_3N_{15} - N_8N_{10}, \\ M_{14} = N_9N_{10} - N_3N_6 - N_4N_{15} + N_9N_{10} + N_8N_{11}, \\ M_{15} = N_4N_{16} - N_5N_{15} - N_9N_{11} - N_8N_{12}, \;\; M_{16} = N_5N_{16} + N_9N_{12}, \\ M_{17} = -N_3N_{13} - N_6N_{10}, \quad M_{18} = N_6N_{11} - N_7N_{10} + N_4N_{13} - N_3N_{14} \end{gathered} \tag{28}$$

4. Boundary conditions

On the half-space when normal force is applied at the origin of the plane, we define the boundary conditions below

$$(1-\xi^2\nabla^2)\sigma_{zz} = F_1\Psi_1(x)e^{i\omega t}. \tag{29}$$

$$(1-\xi^2\nabla^2)\sigma_{zx} = 0 \tag{30}$$

$$\frac{\partial}{\partial z}T(x,z,t) = 0. \tag{31}$$

$$\frac{\partial}{\partial z}C(x,z,t) = 0. \tag{32}$$

By using the dimensionless quantities defined by (14) on the equations (29)-(32) and supressing the primes for convenience and then applying (16)-(17) on the resulting equations yields

$$M_{21}B_1 + M_{22}B_2 + M_{23}B_3 + M_{24}B_4 = \hat{F}_1A_{12}\widehat{\Psi}_1(\zeta)e^{i\omega t}. \tag{33}$$

$$M_{25}B_1 + M_{26}B_2 + M_{27}B_3 + M_{28}B_4 = 0. \tag{34}$$

$$M_{29}B_1 + M_{30}B_2 + M_{31}B_3 = 0. \tag{35}$$

$$M_{32}B_1 + M_{33}B_2 + M_{34}B_3 = 0. \tag{36}$$

where:

$$\begin{gathered}
M_{21} = A_{11}\lambda(r_1^2-\zeta^2) - \lambda A_{11}^2(r_1^4 - 2\zeta^2r_1^2) + \zeta^4) - 2\mu r_1 + 2\mu A_{11}r_1^3 - 2\mu A_{11}\zeta^2r_1 \\
-A_3d_1 - A_{11}A_3d_1r_1^2 + A_{11}A_3\zeta^2d_1 - A_3l_1 - A_{11}A_3l_1r_1^2 + A_{11}A_3\zeta^2l_1, \\
M_{22} = A_{11}\lambda(r_2^2-\zeta^2) - \lambda A_{11}^2(r_2^4 - 2\zeta^2r_2^2 + \zeta^4) - 2\mu r_2 + 2\mu A_{11}r_2^3 - 2\mu A_{11}\zeta^2r_2 \\
-A_3d_2 - A_{11}A_3d_2r_2^2 + A_{11}A_3\zeta^2d_2 - A_3l_2 - A_{11}A_3l_2r_2^2 + A_{11}A_3\zeta^2l_2, \\
M_{23} = A_{11}\lambda(r_3^2-\zeta^2) - \lambda A_{11}^2(r_3^4 - 2\zeta^2r_3^2 + \zeta^4) - 2\mu r_3 + 2\mu A_{11}r_3^3 - 2\mu A_{11}\zeta^2r_3 \\
-A_3d_3 - A_{11}A_3d_3r_3^2 + A_{11}A_3\zeta^2d_3 - A_3l_3 - A_{11}A_3l_3r_3^2 + A_{11}A_3\zeta^2l_3, \\
M_{24} = i\zeta - A_{11}i\zeta r_4^2 + A_{11}i\zeta^3, M_{25} = \mu(-r_1 + A_{11}(r_1^3 - \zeta^2r_1)), \\
M_{26} = \mu(-r_2 + A_{11}(r_2^3 - \zeta^2r_2)), \quad M_{27} = \mu(-r_3 + A_{11}(r_3^3 - \zeta^2r_3)), \\
M_{29} = -r_1d_1, M_{30} = -r_2d_2, M_{31} = -r_3d_3, \quad M_{32} = r_1l_1, M_{33} = r_2l_2, M_{34} = r_3l_3
\end{gathered} \tag{37}$$

By solving the system of Eqs. (33)-(36), we calculate the nontrivial values of $B_i,\ i = 1,2,3,4$ given by

$$B_1 = \frac{-\hat{F}_1\widehat{\Psi}_1(\zeta)\beta_1e^{i\omega t}\Delta_{11}}{\Delta} \tag{38}$$

$$B_2 = \frac{-\hat{F}_1\widehat{\Psi}_1(\zeta)\beta_1e^{i\omega t}\Delta_{21}}{\Delta} \tag{39}$$

$$B_3 = \frac{-\hat{F}_1\widehat{\Psi}_1(\zeta)\beta_1e^{i\omega t}\Delta_{31}}{\Delta} \tag{40}$$

$$B_2 = \frac{-A_{12}\hat{F}_1\widehat{\Psi}_1(\zeta)\beta_1e^{i\omega t}\Delta_{41}}{\Delta} \tag{41}$$

where:

$$
\begin{aligned}
&\beta_1 = A_{12}M_{28}, \Delta_{11} = M_{30}M_{34} - M_{31}M_{33},\\
&\Delta_{12} = M_{28}M_{22}M_{34} - M_{28}M_{23}M_{33} + M_{24}M_{27}M_{33} - M_{24}M_{26}M_{34},\\
&\Delta_{21} = M_{31}M_{32} - M_{29}M_{34},\\
&\Delta_{22} = M_{24}M_{25}M_{34} - M_{24}M_{27}M_{32} + M_{28}M_{23}M_{32} - M_{21}M_{28}M_{34},\\
&\Delta_{31} = M_{29}M_{33} - M_{30}M_{32},\\
&\Delta_{32} = M_{24}M_{26}M_{32} - M_{24}M_{25}M_{33} + M_{28}M_{21}M_{33} - M_{22}M_{28}M_{32},\\
&\Delta_{41} = M_{27}M_{30}M_{32} - M_{26}M_{31}M_{32} - M_{27}M_{29}M_{33} + M_{25}M_{31}M_{33}\\
&+M_{26}M_{29}M_{34} - M_{25}M_{30}M_{34}\\
&\Delta_{42} = -M_{23}M_{26}M_{32} + M_{22}M_{27}M_{32} + M_{23}M_{25}M_{33} - M_{21}\,.
\end{aligned}
\tag{42}
$$

By applying Fourier transform on Eq. (15), substitute from Eq. (25) using Eqs. (38)-(41) we get the components of displacement, stress, temperature and concentration as follow.

$$\hat{u} = \frac{-\hat{F}_1\widehat{\Psi}_1(\zeta)e^{i\omega t}}{\Delta}\left(i\zeta\beta_1\sum_{n=1}^{3}\Delta_{n1}e^{-r_n z} + r_4A_{12}\Delta_{41}e^{-r_4 z}\right) \tag{43}$$

$$\hat{w} = \frac{-\hat{F}_1\widehat{\Psi}_1(\zeta)e^{i\omega t}}{\Delta}\left(\beta_1\sum_{n=1}^{3}r_n\Delta_{n1}e^{-r_n z} - A_{12}i\zeta\Delta_{41}e^{-r_4 z}\right) \tag{44}$$

$$\hat{T} = \frac{-\hat{F}_1\widehat{\Psi}_1(\zeta)\beta_1e^{i\omega t}}{\Delta}\sum_{n=1}^{3}\Delta_{n1}d_n\,e^{-r_n z} \tag{45}$$

$$\hat{C} = \frac{-\hat{F}_1\widehat{\Psi}_1(\zeta)\beta_1e^{i\omega t}}{\Delta}\sum_{n=1}^{3}\Delta_{n1}l_ne^{-r_n z} \tag{46}$$

$$\hat{\sigma}_{zz} = \frac{\hat{F}_1\widehat{\Psi}_1(\zeta)e^{i\omega t}}{\Delta}\left(\beta_1\sum_{n=1}^{3}\Delta_{n1}(\lambda r_n^2 - \lambda\zeta^2 - 2\mu r_n - A_3d_n - A_3l_n) + A_{12}\Delta_{41}\right) \tag{47}$$

$$\hat{\sigma}_{xz} = \frac{\hat{F}_1\widehat{\Psi}_1(\zeta)e^{i\omega t}}{\Delta}\left(-2\beta_1\sum_{n=1}^{3}\Delta_{n1}r_n\mu i\zeta - A_{12}\Delta_{41}(\mu r_4^2 + \mu\zeta^2)\right) \tag{48}$$

5. Applications

5.1 Concentrated load3.1 Review of Kinematics of the deformation field in HSDT

A concentrated mechanical load is a force or pressure applied at a single point, or over an area that is very small. In mechanical analysis, this kind of load is commonly modelled as a point force (such as a load on the center of a beam or a pin). In case a concentrated load is applied , we consider $\psi_1(x) = \delta(x)$ and $\psi_2(x) = \delta(x)$. By Fourier transformation on both of these equations, we get function as $\hat{\psi}_1(\zeta) = 1$ and $\hat{\psi}_2(\zeta) = 1$.

5.2 Uniformly distributed load

A uniform mechanical load means applying a constant load (force or pressure) uniformly across the surface or length of the structure. For a uniformly distributed load $\psi_1(x), \psi_2(x)$ assume the values as given below

$$\left(\psi_1(x), \psi_2(x)\right) = \begin{cases} 1 & if \ |x_1| \le a \\ 0 & if \ |x_1| > a \end{cases} \tag{49}$$

The Fourier transformation of Eq. (49) is calculated and equal to

$$(\hat{\psi}_1(\zeta), \hat{\psi}_2(\zeta)) = \left(\frac{2}{\zeta} sin\left(\frac{\zeta C_1 a}{\omega}\right)\right) \zeta \neq 0 \tag{50}$$

5.3 linearly distributed load

A load for which the loading intensity varies linearly along a specific direction is referred to as a linear load. In thermomechanical analysis, a uniform thermal load means that the temperature varies linearly in a stepwise manner throughout the structure, that is, the temperature variation from one end of a beam to the other. A linear mechanical load could be a force or pressure that varies at a constant rate along the length or area of the structure. It generally produces a linear variation in the response of the system, such as stress or displacement, with position. For linearly distributed force $\psi_1(x), \psi_2(x)$ are taken as

$$(\psi_1(x), \psi_2(x)) = \begin{cases} 1 - \frac{|x|}{a}, & if \ |x| \le a \\ 0, & if \ |x| > a \end{cases} \tag{51}$$

By taking of Fourier transformation on Eq. (51)

$$(\hat{\psi}_1(\zeta), \hat{\psi}_2(\zeta)) = \frac{2\left(1 - cos\left(\frac{\zeta C_1 a}{\omega_1^*}\right)\right)}{\frac{\zeta^2 C_1 a}{\omega_1^*}} \tag{52}$$

6. Inversion of the transforms

To obtain the solution in a physical domain we should inverse the transformed domain from the Fourier transformation domain to the real or physical domain. The inverse Fourier transform from Eqs. (43)- (48) given as follow.

$$f(x, z, t) = \frac{1}{\pi} \int_0^{\infty} cos(\zeta x) f_e - sin(\zeta x) f_o) d\zeta \tag{53}$$

Table 1 copper parameters and their values

Constant notation	Values	Constant notation	Values
λ	$7.76\ \times\ 10^{10} NM^{-2}$	μ	$3.86\ \times 1010 NM^{-2}$
α_t	$1.78 \times\ 10^{-5} K^{-1}$	α_c	$2.65\ \times\ 10^{-4} K^{-1}$
ρ	8954	γ_1	0.02s
γ_2	0.2s	K	$386 J(msk)^{-1}$
a	$1.2 \times 10^4 \mathrm{m}^2 \mathrm{KS}^2$	b	$0.9 \times 10^6 \mathrm{kgm}^5 \mathrm{s}^2$
T_0	$293K$	C_E	$383.1 J(KgK)^{-1}$
d	$8.5\ \times 10^{-9}$	ξ	$3.95\ \times\ 10^{-10}$
ζ	$2\ \times 10^{-10}$	K^*	1.2
S	1		

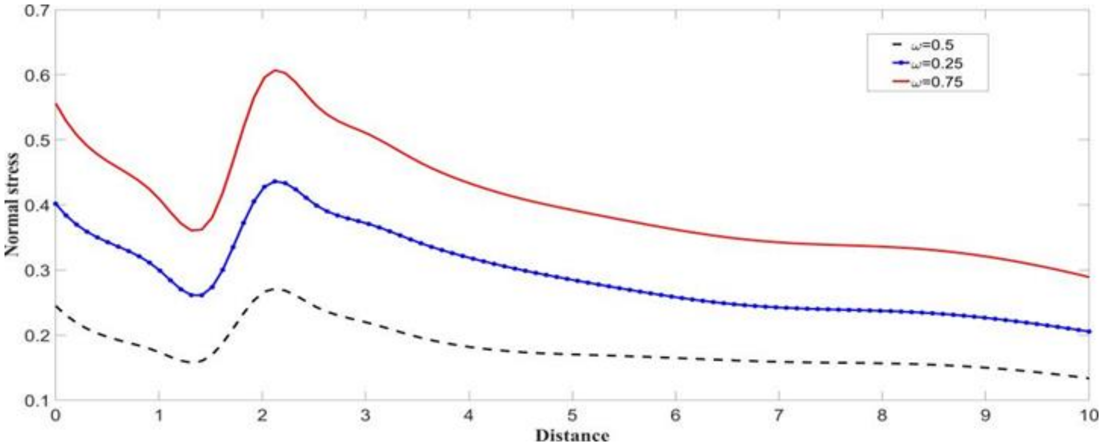

Fig. 1 Variation in normal stress σ_{zz} due to angular frequency (concentrated Load)

where f_e and f_0 atre even function and odd function of $f(\zeta, x, s)$ respectively. The inverse Laplace transform is calculated use of Honig and Hirdes (1984) methods and we get equations in the $f(x, z, t)$ form.

7. Numerical solutions

In order to investigate angular frequency effects on nonlocal thermoelastic media with diffusion, the material considered is copper. As mentioned in the studies of (Malik *et al*. 2023), the material constants of copper metal are given by the following table.

8. Discussion

8.1 Concentrated load

These Figs. 1-4 above are shows the impact of angular frequency on the normal stress, shear

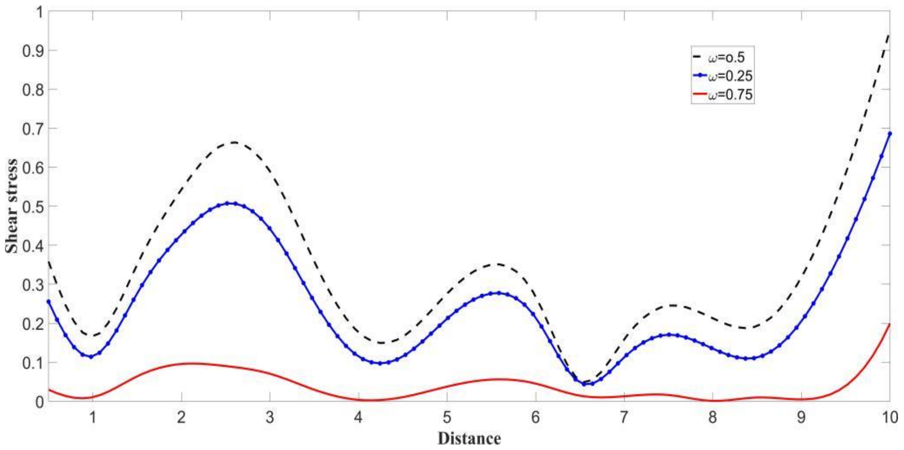

Fig. 2 Variation in shear stress σ_{zz} due to angular frequency (concentrated Load)

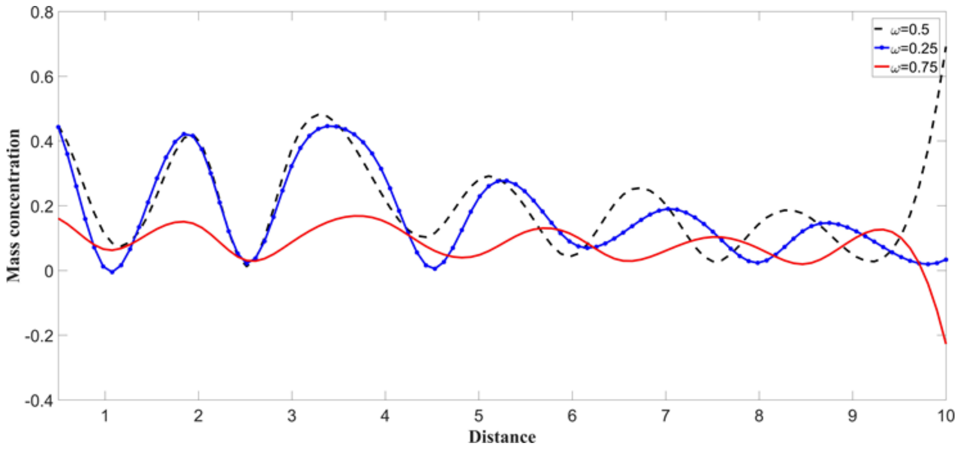

Fig. 3 Variation in mass concentration C due to angular frequency (concentrated Load)

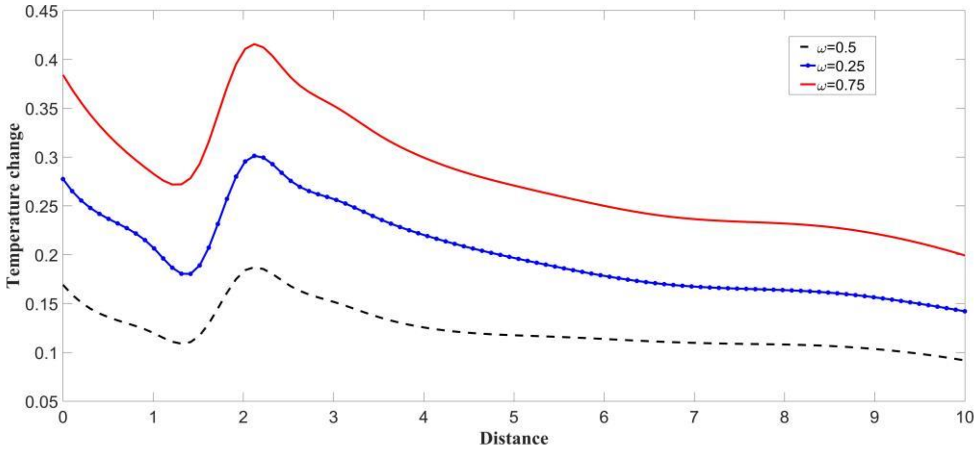

Fig. 4 Variation in temperature change T due to angular frequency (concentrated Load)

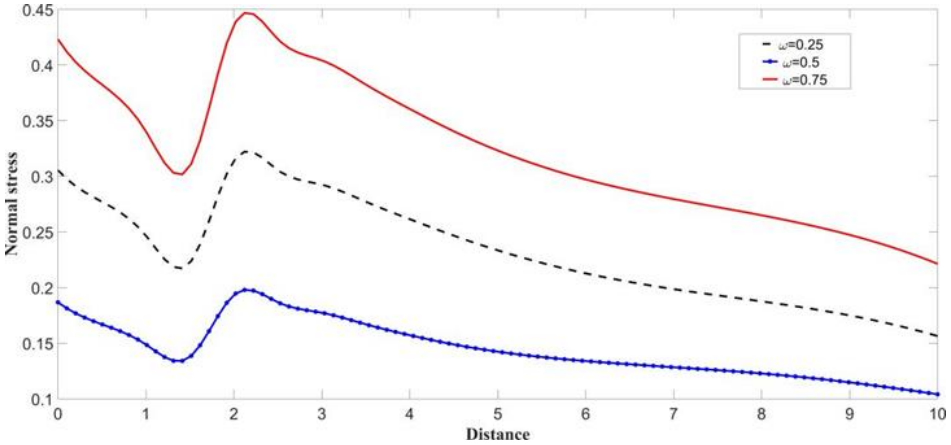

Fig. 5 Variation in normal stress σ_{zz} due to angular frequency (linearly distributed Load)

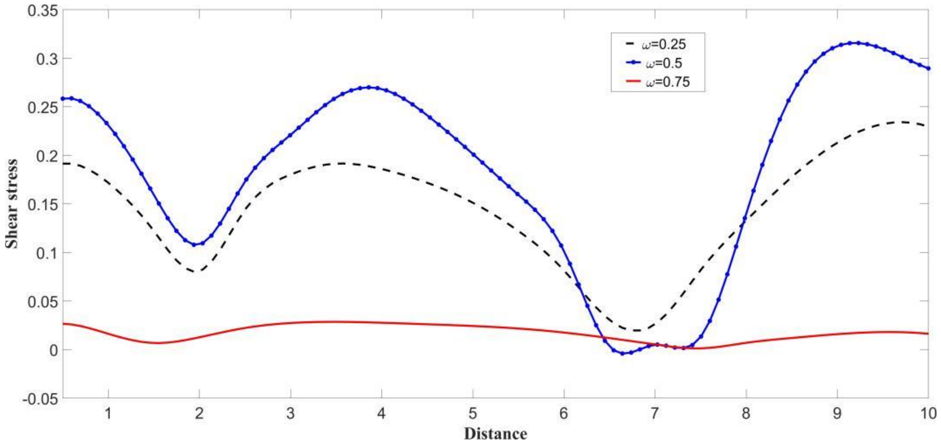

Fig. 6 Variation in shear stress σ_{zx} due to angular frequency (linearly distributed Load)

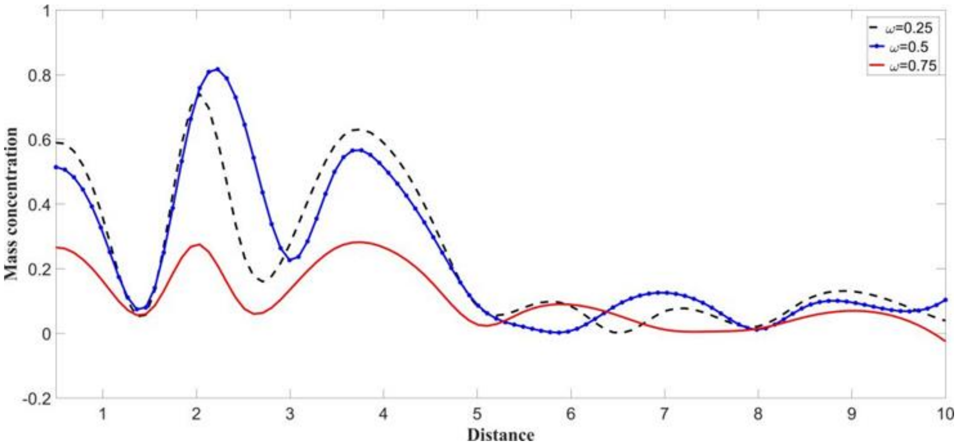

Fig. 7 Variation in mass concentration C due to angular frequency (linearly distributed Load)

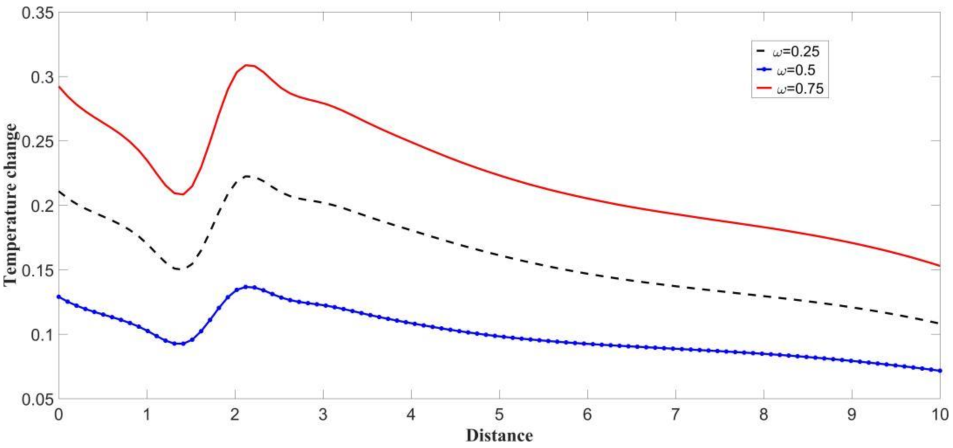

Fig. 8 Variation in temperature change T due to angular frequency (linearly distributed Load)

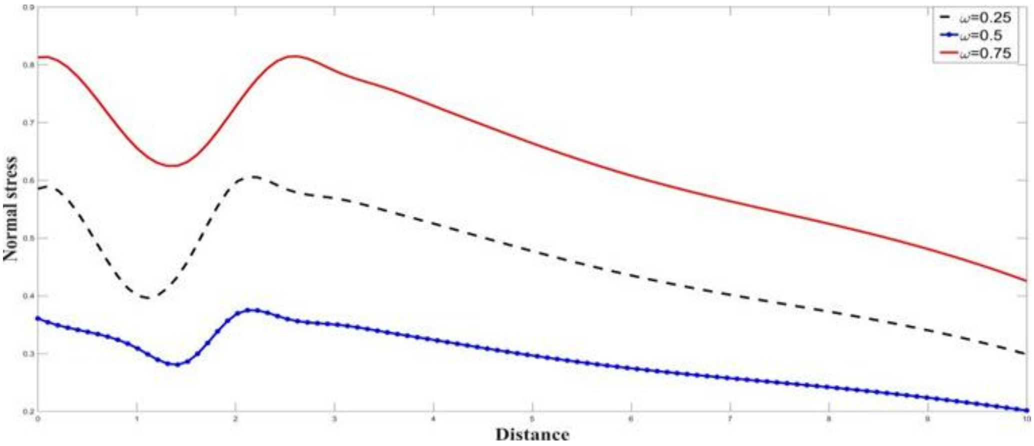

Fig. 9 Variation in normal stress σ_{zz} due to angular frequency (uniformly distributed load)

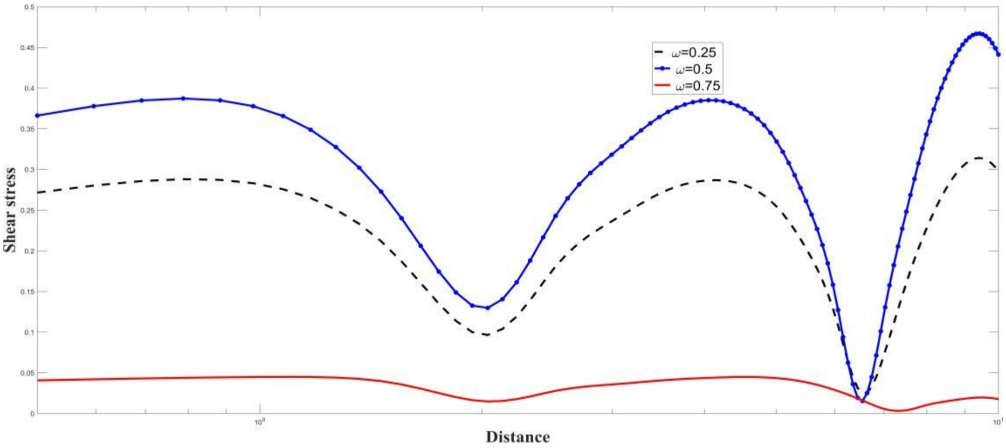

Fig. 10 Variation in shear stress σ_{zx} due to angular frequency (uniformly distributed load)

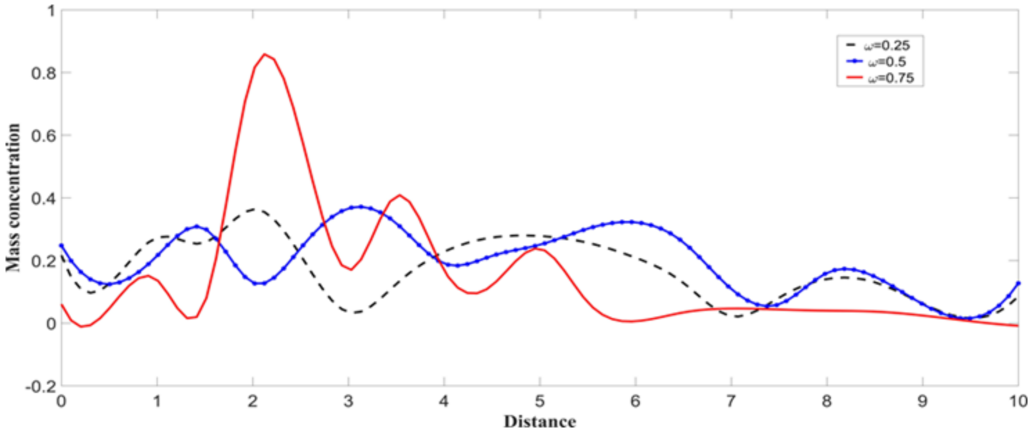

Fig. 11 Variation in mass concentration C due to angular frequency (uniformly distributed load)

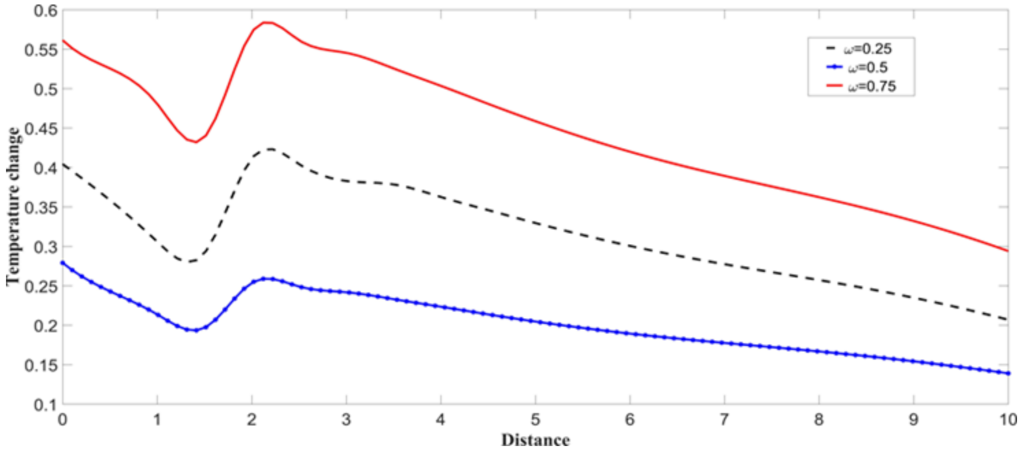

Fig. 12 Variation in temperature change T due to angular frequency (uniformly distributed load)

stress, mass concentration and temperature reveals how they influence the overall behavior of the graphs of nonlocal thermoelastic media with diffusion subjected to concentrated load.

8.2 Linearly distributed load

From Figs. 5-8 show the effect of change of angular frequency on the normal stress, shear stress, mass concentration and temperature under the linearly distributed load. Fig. 5 shows the change of normal stress (σ_{zz}), Fig. 6 is the graph of shear stress (σ_{zx}), Fig. 7 illustrates the mass concentration (C) and Fig. 8, is graph of temperature change (T) of thermoelastic media with diffusion due to change of angular frequency domain with linearly distributed load.

8.3 Uniformly distributed load

Fig. 9 is graph of normal stress (σ_{zz}), Fig. 10 represent the graph of shear stress(σ_{zx}), Fig. 11

illustrates the mass concentration (C) and From Fig. 12 is graph of the temperature change (T) to show the effect of change of angular frequency domain in uniformily distributed force.

9. Conclusions

In this paper, we analyzed the effects of angular frequency on normal stress, shear stress, mass concentration and temperature change in thermoelastic media with diffusion subjected to various loading including concentrated loads, linearly distributed loads and uniformly distributed loads. In thermoelastic media with diffusion under the angular frequency domain, the coupled governing equations for normal stress, shear stress, mass concentration and temperature change are solved in the frequency domain, providing insights into the dynamic behavior of the material. The interaction between thermoelastic and diffusive fields significantly influences the material's response. From all the graphs change of angular frequency all ways make changes in nonlocal thermoelastic media with diffusion. The understanding of frequency-dependent behavior in thermoelastic media with diffusion has wide ranging applications in fields such as geophysics, materials science, and engineering. For instance, it is essential for designing materials and structures subjected to dynamic loads, optimizing thermal management systems in complex media like porous or composite materials.

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