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Stability analysis of integrated SWCNT reposed on Kerr medium under longitudinal magnetic field effect Via an NL-FSDT

Belkacem Selmoune[1,2], Abdelwahed Semmah[1,2], Mohammed L. Bouchareb[2], Fouad Bourada*[3,4], Abdelouahed Tounsi[3,5,6a] and Mohammed A. Al-Osta[6,7]

[1]*Department of Physics, University of Relizane, Algeria*
[2]*Multiscale Modeling and Simulation Laboratory, Department of Physics, Faculty of Exact Sciences, Department of Physics, University of Sidi Bel Abbés, Algeria*
[3]*Material and Hydrology Laboratory, Civil Engineering Department, Faculty of Technology, University of Sidi Bel Abbes, Algeria*
[4]*Science and Technology Department, Faculty of Science and Technology, Tissemsilt University, Algeria*
[5]*YFL (Yonsei Frontier Lab.), Yonsei University, Seoul, Korea*
[6]*Department of Civil and Environmental Engineering, King Fahd University of Petroleum & Minerals, 31261 Dhahran, Eastern Province, Saudi Arabia*
[7]*Interdisciplinary Research Center for Construction and Building Materials, KFUPM, 31261 Dhahran, Saudi Arabia*

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Abstract. This study aims to analyze the mechanical buckling behavior of a single-walled carbon nanotube (SWCNT) integrated with a one-parameter elastic medium and modeled as a Kerr-type foundation under a longitudinal magnetic field. The structure is considered homogeneous and therefore modeled utilizing the nonlocal first shear deformation theory (NL-FSDT). This model targets thin and thick structures and considers the effect of the transverse shear deformation and small-scale effect. The Kerr model describes the elastic matrix, which takes into account the transverse shear strain and normal pressure. Using the nonlocal elastic theory and taking into account the Lorentz magnetic force acquired from Maxwell relations, the stability equation for buckling analysis of a simply supported SWCNT under a longitudinal magnetic field is obtained. Moreover, the mechanical buckling load behavior with respect to the impacts of the magnetic field and the elastic medium parameters considering the nonlocal parameter, the rotary inertia, and transverse shear deformation was examined and discussed. This study showed useful results that can be used for the design of nano-transistors that use the buckling properties of single-wall carbon nanotubes (CNTs) due to the creation of the magnetic field effect.

Keywords: carbon nanotube; Kerr's medium; magnetic field; mechanical buckling; nonlocal theory; shear deformation theory

*Corresponding author, Ph.D., E-mail: bouradafouad@yahoo.fr
[a]Professor, E-mail: tou_abdel@yahoo.com

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1. Introduction

Nanotechnology is defined as the creation and use of materials, instruments, and systems relating to dimensions of the order of 1 to 100 nanometers (National Science and Technology Council). Suppose a new term, nanotechnologies, has been defined to refer to designate small objects. In that case, it is not only because they represent the final stage of miniaturization but also because, at the nanoscale, the behavior of matter gives rise to new physical, chemical, and even biological properties. Nanosciences are concerned with new phenomena at the level of nano-objects and interactions between nanometric objects.

Carbon nanotubes (CNTs) were found by Simio Iijima in 1991 (Iijima 1991). They are formed by the winding of one or more sheets of graphene. They are considered nano-objects containing a macroscopic dimension in one direction and nanometric in the other two. These nano-objects supreme and outstanding characteristics attracted many physicists, chemists, and biologists from all countries. For this, many applications in nanotechnology are already identified in various fields; electronics (Tsukagoshi *et al.* 2002), optics (Kempa *et al.* 2007), and other areas of materials science (Ma *et al.* 1998, Meyyappan 2004). Experimental studies have shown that the CNTs' physical properties are affected by the existence of buckling. Therefore, the alterable transformation between the normal and buckled states of CNTs can result in possible applications such as nano-fluid components "nano-valve" (Grujicic *et al.* 2005) and nano-electronic devices "nano-transistors" (Postma *et al.* 2001), and alterable elements in nano-electromechanical systems.

Two essentially one-of-a-kind approaches to be had for theoretical simulation of nanostructured materials are the continuum mechanics simulations and atomistic approaches. The atomistic approach consists of tight-binding molecular dynamics "TBMD" and density functional theory "DFT", and classical molecular dynamics "MD" (Iijima *et al.* 1996, Sánchez-Portal *et al.* 1999). These approaches are frequently computationally in-depth and expensive, particularly for large-scale CNTs with an excessive range of walls. Several researchers have studied different structures by nonlocal models that have proven satisfying results compared to atomic models. Thus, the study based on the continuum mechanics of CNTs is increasingly considered a way of opportunity to model substances at the nanometer scale.

In recent years, several researchers have studied the behavior of nanostructures using different continuum models, Wang *et al.* (2006) conducted micro-and nano-rods/tubes buckling analysis based on nonlocal Timoshenko beam theory. Ahmad *et al.* (2021) showed a thermal buckling analysis of circular bilayer graphene sheets resting on an elastic matrix based on nonlocal continuum mechanics. Abouelregal *et al.* (2021) investigated on temperature-dependent physical characteristics of the rotating nonlocal nanobeams subject to a varying heat source and a dynamic load established in the context of nonlocal thermoelasticity theory. Mikhasev *et al.* (2022) carried out a pull-in instability Analysis of a nanocantilever based on the two-phase nonlocal theory of elasticity. Shariati *et al.* (2022) studied size effect on the axisymmetric vibrational response of functionally graded circular nano-plate based on the nonlocal stress-driven method. Abouelregal *et al.* (2022) presented a computational analysis of an infinite magneto-thermoelastic solid periodically dispersed with varying heat flow based on non-local Moore-Gibson-Thompson approach. In the literature there are many studies about carbon nanotubes that deal with their mechanical and vibrational response, some of them present them in a void medium, and others embedded in an elastic one. Sudack (2003) presented column buckling of multiwalled CNTs by utilizing the approach of nonlocal continuum mechanics. Salamat and Sedighi (2017) investigated vibrational behavior of single-walled CNTs at a small scale and a moving nanoparticle Bensattalah *et al.* (2019) established a free vibration analysis of chiral single-walled carbon nanotubes for a new nonlocal beam model. Malikan *et al.* (2018a) analyzed the dampened forced vibration of single-walled carbon nanotubes based on a viscoelastic foundation in a thermal environment using non-local strain gradient theory. Malikan *et al.* (2019b, 2020a, 2020b and 2020c) analyzed the buckling response of a différents carbon nanotubes based on a various continuum beam models. Koochi and Goharimanesh (2021) explored the nonlinear oscillations of the CNT nano-resonator. Yusufoglu, and Avey (2021) analyzed the nonlinear dynamic response of shells reinforced various distributions of CNTs. The distributions were hyperbolic paraboloidal. Gia Phi *et al.* (2022) studied nonlinear free vibration characteristics of functionally graded (FG) composite micro-beams reinforced by carbon nanotubes (CNTs) with piezoelectric layers in thermal environment. Civalek and Avcar (2022) utilized the discrete singular convolution method to analyze the free vibration and buckling of laminated non-rectangular plates reinforced with carbon nanotubes based on nonlocal elasticity.

In recent years, many researchers have been widely interested in CNTs embedded in an elastic medium due to their large applications in different fields such as nanotechnology, electronics, physics, chemistry, reinforced composite structures, and engineering. Winkler (1867) presented the first type of elastic foundation as the "one parameter" foundation model, given that it is characterized only by the vertical stiffness of the Winkler foundation springs. Pasternak (1954) improved the Winkler model by introducing a second parameter to describe the existence of shear stress inside the elastic medium. Several researchers have utilized the first type of elastic foundation and the second type (Pradhan and Reddy 2011, Shahsavari *et al.* 2018, Avcar *et al.* 2016 and 2018, She *et al.* 2018, Demir *et al.* 2018, Malikan *et al.* 2019a and 2019c Shanab *et al.* 2020, Benferhat *et al.* 2020, Rachedi *et al.* 2020, Timesli 2020a and 2022, and Jena *et al.* 2021.), they studied the responses of nano and microstructures embedded in the Winkler-Pasternak and visco-Pasternak medium In order to further improve the Pasternak model, Kerr (1965) has proposed the addition of a third parameter (Kerr type) to present greater flexibility in controlling the grade of foundation-surface continuity among the loaded and the unloaded area of the elastic beam system. Recently many researchers have adopted Kerr's foundation to model elastic mediums. Using Kerr elastic medium and the nonlocal Donnell shell theory under axial compression, Timesli (2020b) analyzed the stability of embedded double-walled CNTs. Bensattalah *et al.* (2018) conducted critical buckling loads analysis of SWCNT implanted in the Kerr medium. Zhang *et al.* (2019) studied the impact of the Kerr-type elastic foundation parameters on the buckling response of a beam.

The magnetic field effect on the mechanical and vibrational response of CNT embedded using the continuum model is not often referred to within the open literature. Narendar *et al.* (2012) utilized the approach of wave propagation and nonlocal elasticity to study the vibrations in SWCNTs under the longitudinal magnetic field. Their work includes circumferential vibration patterns in both axial and radial directions; it demonstrated that the vibration frequencies of SWCNTs drop significantly for various circumferential wave numbers and in the presence of the magnetic field. Such an effect is also observed for the SWCNT with diverse boundary conditions. Wei and Wang (2004) investigated the various wave patterns in the magnetic field with coupling in longitudinal or transverse. Wong *et al.* (2010) studied the influence of a longitudinal magnetic field on wave propagation in a CNT implanted in an elastic matrix. Using a new refined beam theory, Jena *et al.* (2020) recently studied the vibration and buckling responses of a nonlocal beam embedded in a magnetic field and resting on Winkler-Pasternak elastic foundation. They found that the critical buckling loads and natural frequency increase with increasing the Winkler modulus. Still, this rise is more important in the case of critical buckling load.

In this work, the longitudinal magnetic field and the transverse shear deformation effects on the buckling loads of implanted SWCNTs in an elastic medium modeled by Kerr are discussed for the first time via a continuum model which targets thin and slightly thick structures. However, it is not recommended for high-thickness structures. The influence of a longitudinal magnetic field on buckling loads of SWCNTs integrated with an elastic medium modeled by Kerr, considering the transverse shear deformation effects, is analyzed for the first time via a nonlocal first shear deformation theory (NL-FSDT). The results were validated by comparing the published results acquired by other researchers. The effects of the transverse shear deformation, nonlocal parameter, radius, and length of SWCNT, and the foundation parameters on buckling of SWCNT implanted in an elastic medium with the influence of a magnetic field are investigated.

2. Methodology

2.1 Assumptions

Many beam theories suggested over the years to study the behaviour of unidimensional nanostructures modeled on isotropic nanobeams, NL-FSDT is fundamentally easier to adopt for modeling shear deformation effect. NL-FSDT is still in widespread use today thanks to their simplicity. It is now well known that in the analysis of nanobeams, shear deformation effects become important not only for thick beams, but even for thin beams. Since classical non-local beam theory (NL-CBT) does not take into account shear effects, many theories have developed to fill this gap. According to the N-FSDT, the following displacement field can be expressed as follows (Malikan 2017)

$$\begin{aligned} u(x,z) &= -z\emptyset(x) \\ w(x,z) &= w(x) \end{aligned} \tag{1}$$

where w is a transverse component of the dis-placement in the mid-plane of the beam. Moreover, ϕ denotes the rotation of the cross-sectional area of the beam.

In the proposed theory, the displacement field is selected with the subsequent assumptions:

(i) The displacements are small compared to the thickness of nanobeam, and, hence, the strains affected are infinitesimal.

(ii) The transverse displacement w consists of two-part, one for bending w_b, and the other for shear w_s. These parts are considered to be functions of coordinate x only

$$w(x,z) = w_s(x) + w_b(x) \tag{2a}$$

(iii) The transverse normal stress σ_z is insignificant compared to in-plane stresses σ_x.

(iv) The rotation variable in the S-FSDT is expressed in terms of the bending component only:

$$\emptyset = \frac{dw_b}{dx} \tag{2b}$$

2.2 Kinematics

$$u(x,z) = -z\frac{dw_b}{dx} \tag{3a}$$

$$w(x,z) = w_b + w_s \tag{3b}$$

where w is displacement in the transverse direction at point (x,z). On the middle plane, it is (i.e., z=0) of the beam. The strains related to the displacements in Eq. (3) are

$$\varepsilon_x = -z\frac{d^2 w_b}{dx^2} \text{ and } \gamma_{xz} = \frac{dw_s}{dx} \tag{4}$$

2.3 Constitutive relations

Eringen (1972) was the first considered nonlocal elasticity. The reference point's stress is considered to be a function of the strain field at every continuum point. Eringen developed a form of nonlocal constitutive relation as (Eringen 1972)

$$\sigma_x - \mu\frac{d^2\sigma_x}{dx^2} = E\varepsilon_x \tag{5a}$$

$$\tau_{xz} - \mu\frac{d^2\tau_{xz}}{dx^2} = G\gamma_{xz} \tag{5b}$$

where σ_x, τ_{xz}, are the axial and shear stresses in the nanobeam, respectively. E and G are the Young and shear modulus of the nanobeam, respectively. $\mu=(e_0 a)^2$ is the nonlocal parameter, e_0 is a constant factor used for each material, and a is a characteristic internal length (Demir and Civalek 2013, Ahmed *et al.* 2020, Bouhadra *et al.* 2021, Lata and Singh 2022).

2.4 Stability equation

The virtual work principle (VWP) is utilized here to acquire the equations of equilibrium. The principle can be analytically formulated in the following form (Ahmed *et al.* 2019, Mehar and Panda 2019, Hosseini *et al.* 2020)

$$\delta\int_V (U+V)dV = 0 \tag{6}$$

where δU represents the strain energy, virtual variation; δV is the potential energy virtual variation. The strain energy variation of the beam is given as

$$\delta U = -\int_0^L M_b\left(\frac{d^2\delta w_b}{dx^2}\right) + Q\left(\frac{d\delta w_s}{dx}\right)dx \tag{7}$$

where δw_b, δw_s, and (M_b, Q) are respectively the variation of bending displacement, the variation of de shear displacement, and the stress resultants defined as

$$M_b = \int_A z\,\sigma_x dA \text{ and } Q = \int_A \tau_{xz} dA \tag{8}$$

The potential energy variation with applied loads can be found as

$$\delta\, V = \int_0^L q\delta(w_b + w_s)dx - \int_0^L P_0\frac{dw}{dx}\frac{d\delta w}{dx}dx \tag{9}$$

where P_0 and q are the axial and elastic foundation effects, respectively.

Substituting the relations of δU and δV from Eqs. (7) and (9) into Eq. (6) and applying the integration by parts, and finding coefficients of $\delta\, w_b$ and $\delta\, w_s$, the equilibrium equations of the proposed beam theory can be determined

$$\delta w_b : -\frac{d^2 M_b}{dx^2} + q - P_0\frac{d^2}{dx^2}(w_b + w_s) = 0 \tag{10a}$$

$$\delta w_s:-\frac{dQ}{dx}+q-P_0\frac{d^2}{dx^2}(w_b+w_s)=0 \tag{10b}$$

By considering Eq. (8) into Eq. (5), the nonlocal moment resultant is found as

$$M_b-\mu\frac{d^2M_b}{dx^2}=-D\frac{d^2w_b}{dx^2} \tag{11a}$$

$$Q-\mu\frac{d^2M_b}{dx^2}=B\frac{dw_s}{dx} \tag{11b}$$

where

$$D=\int_A z^2EdA=EI \text{ and } B=k_s\int_A G\,dA=k_sGA \tag{12}$$

where E, G, I, A, and k_s are respectively Young's and shear modulus, the inertia moment of area, cross-area of the tube, and the shear correction factor that can be utilized to balance the error due to the constant shear stress assumption. To derive the nonlocal governing equations, first, we put Eqs. (10) into (11) to derive the resultants M and Q in the nonlocal forms. Then, the obtained resultants will be put into Eq. (10) to derive the final equations; the nonlocal equilibrium equation which describe the buckling load of a SWCNT embedded in an elastic medium and subjected to a longitudinal magnetic field effect, can be stated in terms of displacements (w) as

$$D\frac{d^4w_b}{dx^4}+\left(1-\mu\frac{d^2}{dx^2}\right)\left[q_{kerr}+f(x)-P_0\frac{d^2}{dx^2}(w_b+w_s)\right]=0 \tag{12a}$$

$$-B\frac{d^2w_s}{dx^2}+\left(1-\mu\frac{d^2}{dx^2}\right)\left[q_{kerr}+f(x)-P_0\frac{d^2}{dx^2}(w_b+w_s)\right]=0 \tag{12b}$$

The current theory is utilized to analyse the buckling behaviour of the SWCNTs under an axial compressive load P_0 and embedded in a longitudinal magnetic field. The elastic medium is represented as Kerr type of foundation. It contains three parameters in the elastic model consisting of an independent upper (with stiffness k_c), shear layer (with stiffness k_G), and lower (with stiffness k_w) elastic layers (represented by distributed springs) q_{Kerr} denotes the Kerr foundation model distributed reaction described by (Van Cauwelaert *et al.* 2002)

$$q_{kerr}=\frac{1}{1+\frac{k_w}{k_c}}k_ww-k_G\frac{d^2w}{dx^2}-\frac{Dk_G}{k_c}\frac{d^6w}{dx^6} \tag{12c}$$

It is assumed that the SWCNT is under a longitudinal magnetic field effect in this problem.

Here $f(x)$ represents the force per length. Therefore, it can be expressed as (Narendar *et al.* 2012)

$$f(x)=f_zA \tag{13}$$

where f_z is the Lorentz magnetic force that can be found from Maxwell's relations (Narendar *et al.* 2012, Jena *et al.* 2020) and A the cross area.

$$f_z=\eta H_x^2\frac{d^2w}{dx^2} \tag{14}$$

in which f_z represents the body force, η is the magnetic permeability, and H_x is an axial magnetic field (Narendar *et al.* 2012).

2.5 Analytical solutions

In this work, the analytical solutions for buckling were given for simply supported isotropic

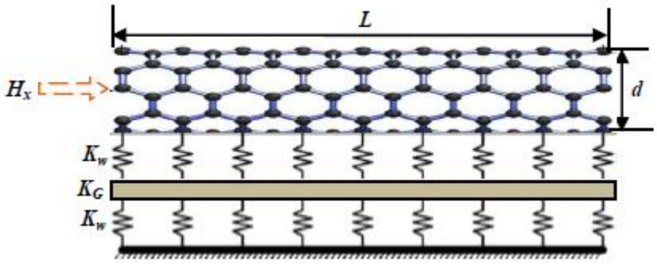

Fig. 1 The SWCNT in a longitudinal magnetic field embedded in Kerr medium (Bouchareb *et al.* 2022)

beams. The boundary conditions of beams used are

$$w = 0, M = 0 \text{ at } x = 0, L \tag{15}$$

Based on Navier's procedure (Malikan and Dastjerdi 2018, Hadji *et al.* 2019, Hadji and Bernard 2020). The displacement field satisfies governing equations, and boundary conditions can be given as follows

$$\begin{Bmatrix} w_b \\ w_s \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} W_{bn} \sin(\alpha x)\, e^{i\omega t} \\ W_{sn} \sin(\alpha x)\, e^{i\omega t} \end{Bmatrix} \tag{16}$$

where W_{bn} and W_{sn}, are arbitrary parameters to be obtained, ω is the eigenfrequency related to m, the eigenmode, and α=.$m\pi/L$.

Considering Eq. (16) into Eqs. (11) the closed-form solutions can be expressed as follows

$$P_{cr} = -\frac{S_{11}S_{22}}{\lambda(S_{11}-S_{22})} + \eta A H_x^2 + \frac{\frac{\alpha^2 S_{11} k_G}{k_c} + \frac{k_W}{\alpha^2} + k_G}{\left(1+\frac{k_W}{k_C}\right)} \tag{17}$$

where $S_{11} = D\alpha^2$, $S_{22} = k_s GA$ and $\lambda = 1 + \mu\alpha^2$

And by putting only H_x=0, the corresponding buckling load via NFSDT of the CNT embedded in Kerr's medium can be obtained.

3. Results and discussions

In this part, the numerical calculations are carried out for the mechanic buckling characteristics of embedded SWCNTs in the Kerr medium under a longitudinal magnetic field (see Fig. 1).

3.1 Validation

By putting (k_w, k_G)=0 and k_c>> and H_x=0, we obtain the corresponding analytical solutions for SWCNT without any elastic medium or magnetic field effect. Then the results are compared with those published by Malikan and Dastjerdi (2018), Jena *et al.* (2020) for different beam lengths and nonlocal parameters, which can be demonstrated in Tables 1 and 2. For the calculations purpose, the values of E=1 TPa, Poisson's ratio (υ)=0.18, and diameter (d)=1 nm were used.

In Table 1, the results of the critical buckling load (P_{cr}) via the current method are compared with those shown by Malikan and Dastjerdi (2018b) carried out by the Timoshenko beam theory

Table 1 Validation of critical buckling load (P_{cr}) with (Malikan and Dastjerdi 2018)

		e_0a=0	e_0a=0.5	e_0a=1	e_0a=1.5	e_0a=2
Malikan and Dastjerdi (2018)	L=10	4.7609	4.6462	4.3332	3.8957	3.4133
Present		4.7609	4.6462	4.3332	3.8957	3.4133
Malikan and Dastjerdi (2018)	L=12	3.3237	3.2677	3.1105	2.8797	2.6086
Present		3.3237	3.2677	3.1105	2.8797	2.6086
Malikan and Dastjerdi (2018)	L=14	2.4498	2.4193	2.3323	2.2005	2.0391
Present		2.4498	2.4193	2.3323	2.2005	2.0391
Malikan and Dastjerdi (2018)	L=16	1.8795	1.8616	1.8098	1.7295	1.6284
Present		1.8795	1.8616	1.8098	1.7295	1.6284
Malikan and Dastjerdi (2018)	L=18	1.4872	1.4760	1.4432	1.3918	1.3257
Present		1.4872	1.4760	1.4432	1.3918	1.3257
Malikan and Dastjerdi (2018)	L=20	1.2059	1.1985	1.1768	1.1424	1.0976
Present		1.2059	1.1985	1.1768	1.1424	1.0976

Table 2 Validation of critical buckling load with Ref. (Malikan and Dastjerdi 2018, Jena *et al.* 2020) (P_{cr} in nN)

		e_0a=0	e_0a=0.5	e_0a=1	e_0a=1.5	e_0a=2
Malikan and Dastjerdi (2018), Jena *et al.* (2020)	L=10	4.7609	4.6462	4.3332	3.8957	3.4133
Present*		4.7609	4.6462	4.3332	3.8957	3.4133
Malikan and Dastjerdi (2018), Jena *et al.* (2020)	L=12	3.3991	3.3418	3.1810	2.9449	2.6677
Present*		3.3991	3.3418	3.1810	2.9449	2.6677
Malikan and Dastjerdi (2018), Jena *et al.* (2020)	L=14	2.4905	2.4595	2.3711	2.2370	2.0729
Present*		2.4905	2.4595	2.3711	2.2370	2.0729
Malikan and Dastjerdi (2018), Jena *et al.* (2020)	L=16	1.9034	1.8852	1.8327	1.7515	1.6494
Present*		1.9034	1.8852	1.8327	1.7515	1.6494
Malikan and Dastjerdi (2018), Jena *et al.* (2020)	L=18	1.5021	1.4907	1.4577	1.4057	1.3389
Present*		1.5021	1.4907	1.4577	1.4057	1.3389
Malikan and Dastjerdi (2018), Jena *et al.* (2020)	L=20	1.2156	1.2082	1.1864	1.1517	1.1064
Present*		1.2156	1.2082	1.1864	1.1517	1.1064

(TBT), it is noted that there is an excellent matching between the results.

In Table 2, the results of the critical buckling load (P_{cr}) obtained from the current study are compared with those presented by Malikan and Dastjerdi (2018b), Jena *et al.* (2020) analyzed with two models, consecutively, a simple first shear theory (S-FSDT) and a one variable shear deformation theory (OVFSDT). It can be observed that the results shown for the two theories are an excellent match with the proposed method results.

It can be noted that the shear correction factor found in Timoshenko's theory could be a severe defect in light of its approximate quantity (k_s=5/6). Even though this value has been applied in the case of moderately thick models, it is found that it cannot be considered an exact value to analyze several cases, especially nanostructures (Malikan and Dastjerdi 2018, Jena *et al.* 2020). This is why we made the shear correction factor k_s disappear in the current method (present *).

In Table 3, the results of the critical buckling load by the current model (present*) are compared with the results displayed by Jena *et al.* (2020) via OVFSDT for Winkler modulus and Pasternak

Table 3 Effect of Winkler, Pasternak, and Kerr modulus on the critical buckling load: (P_{cr} in nN with L=10 nm)

			e_0a=0.5	e_0a=1	e_0a=1.5	e_0a=2
Effect of Winkler modulus (k_w in GPa)	k_w=0	Jena *et al.* (2020)	700.7004	700.2334	699.5807	698.8610
		Present*	698.1328	697.6573	696.9929	696.2601
	k_w=1	Jena *et al.* (2020)	711.2901	710.8230	710.1704	709.4506
		Present*	708.2649	707.7894	707.1250	706.3922
	k_w=2	Jena *et al.* (2020)	721.8797	721.4127	720.7600	720.0402
		Present*	718.3970	717.9215	717.2571	716.5243
	k_w=3	Jena *et al.* (2020)	732.4693	732.0023	731.3496	730.6299
		Present*	728.5291	728.0536	727.3892	726.6565
	k_w=4	Jena *et al.* (2020)	743.0590	742.5919	741.9393	741.2195
		Present	738.6612	738.1858	737.5213	736.7886
	k_w=5	Jena *et al.* (2020)	753.6486	753.1816	752.5289	751.8091
		Present*	748.7933	748.3179	747.6534	746.9207
Effect of shear modulus (k_G in nN)	k_G=0	Jena *et al.* (2020)	188.7128	188.2458	187.5931	186.8733
		Present*	208.2649	207.7894	207.1250	206.3922
	k_G=100	Jena *et al.* (2020)	293.2283	292.7612	292.1086	291.3888
		Present*	308.2649	307.7894	307.1250	306.3922
	k_G=200	Jena *et al.* (2020)	397.7437	397.2767	396.6240	395.9042
		Present*	408.2649	407.7894	407.1250	406.3922
	k_G=300	Jena *et al.* (2020)	502.2592	501.7921	501.1395	500.4197
		Present*	508.2649	507.7894	507.1250	506.3922
	k_G=400	Jena *et al.* (2020)	606.7746	606.3076	605.6549	604.9351
		Present	608.2649	607.7894	607.1250	606.3922
	k_G =500	Jena *et al.* (2020)	711.2901	710.8230	710.1704	709.4506
		Present*	708.2649	707.7894	707.1250	706.3922
Effect of Kerr modulus (kc in GPa)	kc =10	Present*	693.7105	693.2350	692.5706	691.8378
	kc =20		700.6411	700.1657	699.5012	698.7685
	kc =30		703.1004	702.6250	701.9605	701.2278
	kc =40		704.3600	703.8846	703.2201	702.4874
	kc =50		705.1257	704.6502	703.9858	703.2530

modulus. It can be observed that there is excellent matching between the results. The adopted data in generating these results are:

To calculate the effect of the Winkler modulus, k_G=500 nN, k_c>>, and H_x=0 with L=10 nm were considered.

To calculate the effect of the Pasternak modulus, k_W=1 GPa, k_c>>, and H_x=0 with L=10 nm were used.

For this analysis's interest, the Kerr modulus was calculated by setting k_W=1 GPa, k_G=500 nN, and H_x=0 with L=10 nn.

On the other hand, it is clearly observed from Table 3 that the trend of the critical buckling load is the same for the variation of the three parameters of the elastic medium, it increases with the increased modulus, and we also notice that the nonlocal parameter drops the critical buckling load.

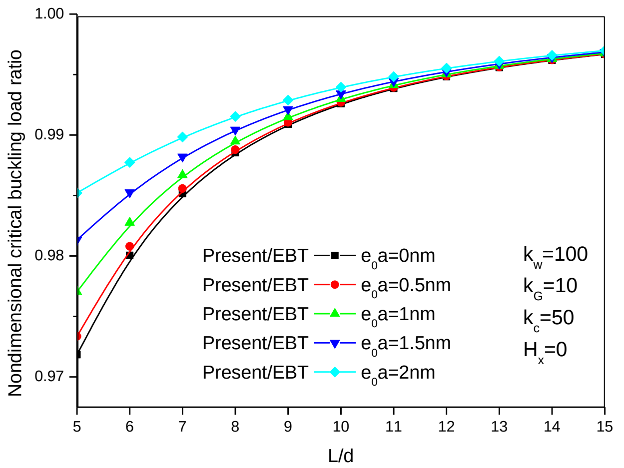

Fig. 2 Ratio of the critical buckling load of SWCNT by NL-FSDT to the nonlocal EBT (Bensattalah *et al.* 2018) and the length to diameter ratio (*L/d*) for different nonlocal parameters values (e_0a)

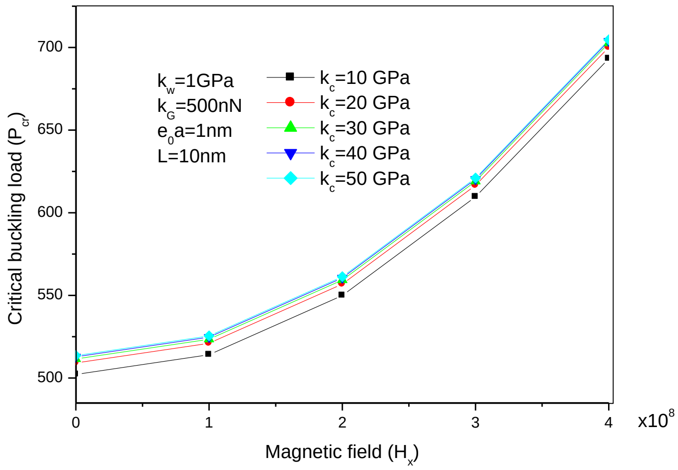

Fig. 3 Magnetic field effect (*Hx*) on the critical buckling loads with various values of Kerr modulus (*kc*)

The parameters used in calculations for Fig. 2 are: E=1 TPa, $G=E/[2(1+v)]$, v=0.19, rod diameter d=1 nm and $I=\pi d^4/64$. And the following dimensionless variables are introduced for the lower spring parameter k_w, the upper spring parameter k_c and the intermediate shear layer parameter k_G: (Bensattalah *et al.* 2018)

$$\bar{k}_w = \frac{L^4}{D} k_w, \bar{k}_G = \frac{L^2}{D} k_G, \bar{k}_c = \frac{L^4}{D} k_c$$

From Fig. 2, it can be found that for various nonlocal parameter values, all ratios are smaller than 1.0. This can be attributed to the effects of the critical buckling load, and the transverse shear deformation of the nonlocal NL-FSDT is lower than that of the nonlocal Euler-Bernoulli beam model (Bensattalah *et al.* 2018). This phenomenon is evident for smaller nonlocal parameter values and slenderness ratios. It means that the effects of the transverse shear deformation can be used, and the nonlocal NL-FSDT is more precise for short CNT.

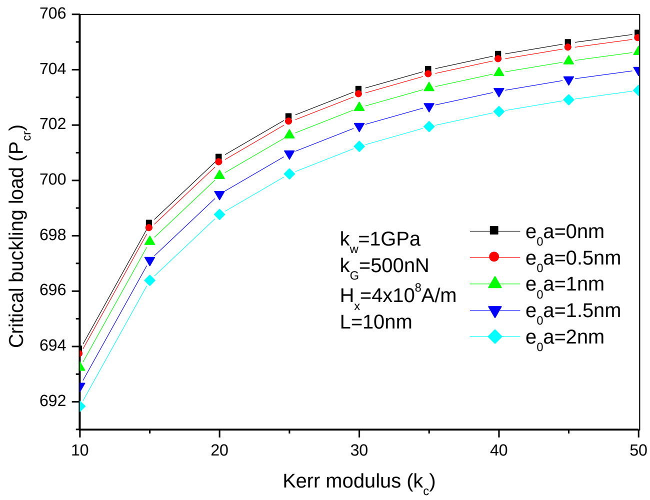

Fig. 4 Critical buckling load variation of SWCNT versus the Kerr modulus (k_c) for different values of nonlocal parameters (e_0a)

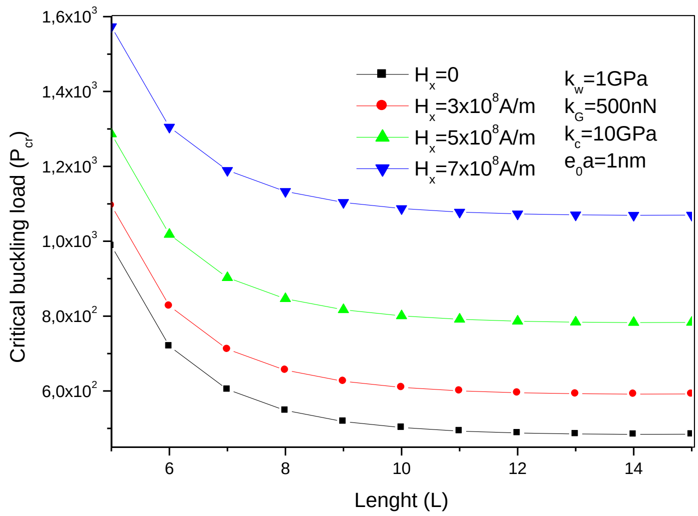

Fig. 5 Critical buckling load variation of SWCNT according to the Lenght of the nanotube (L) for various magnetic field values (H_x)

For different values of H_x and k_c, the influence of the intensity in the magnetic field on critical buckling loads is presented in Fig. 3. It can be observed from the results that the critical buckling loads increased very quickly in response to the change in the value of H_x and for different values of k_c (10, 20, 30, 40, and 50 GPa), the obtained curves for the critical buckling loads keep the same trend as for H_x. We can deduce from this analysis that the magnetic field plays the same role as the elastic medium, i.e. increases the hardness of the material.

Fig. 4 shows the variation of the critical buckling load of SWCNT as a function of Kerr modulus for different nonlocal parameter values. Five different nonlocal parameters values (e_0a=0, 0.5, 1, 1.5 and 2 nm.) are considered. This present computation uses a constant value of magnetic field

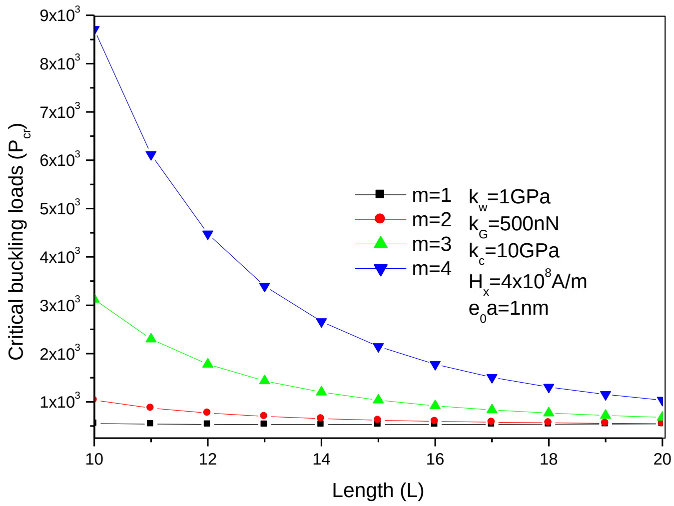

Fig. 6 Relation between the critical buckling loads and the length (L) with different values of the mode number (m)

(H_x=4×108 A/m) and Kerr's parameters (k_w=1 GPa, k_G=500 nN). The figure shows that the critical buckling load decreases as the nonlocal parameter increases. It can also be seen from the obtained curves that the critical buckling loads directly correlate with Kerr modulus values (k_c). From this analysis, it is obvious that the elastic medium hardens our material contrary to the scale parameter, Hence the need to take that into account when manipulating these kinds of structures.

Fig. 5 shows the critical buckling loads variation of SWCNT with the length for different magnetic field values. Four different magnetic field values are considered for the study, viz. H_x=0, 3×10^8, 5×10^8 and 7×10^8 A/m. The figure shows that as the length (L) increases, the critical buckling loads decrease until they become constant for higher values of L. This can be explained by the disappearance of the shear effect which becomes negligible for longer structures. On the other hand, the magnetic field effect keeps the trend as for the other figures.

Fig. 6 demonstrates the relation between the critical buckling loads and the length and axial mode number. The most observed characteristic is that the mode number (m) influence increases the critical buckling loads, unlike the length (L), which decreases the critical buckling loads. However, the difference becomes insignificant with increasing lengths; this can be justified by the weakness of the structure due to the increase in its length

5. Conclusions

In this study and using nonlocal first shear deformation beam theory (NL-FSDT), the critical buckling features of SWCNTs implanted in an elastic medium in a longitudinal magnetic field were predicted. The size effect is considered in the mathematical formulation with the help of Eringen's nonlocal model. The governing equations of the system were determined via a virtual work model and resolved by Navier's method. The influences of the small scale, the length, the mode number, the stiffness of the surrounding elastic medium, the transverse shear deformation, and the magnetic field of the critical buckling properties are investigated.

It is observed that the critical buckling load of NL-FSDT is lower than that of the nonlocal Euler-Bernoulli beam model (Bensattalah *et al.* 2018) for smaller aspect ratio values because of the presence of transverse shear deformation that decreases the critical buckling loads. It means that the effects of the transverse shear deformation could be considered, and the nonlocal NFSDT is more accurate for short CNT.

The critical buckling loads decrease with increasing nonlocal parameter values. Therefore, it can be observed that the classical elastic (i.e., the local) model that doesn't take the nonlocal parameter influences will provide a higher estimation for the critical buckling load. However, the theory of the nonlocal continuum will demonstrate a precise and reliable result. Moreover, an attractive characteristic that can be presumed is that as Kerr's parameters increase, the critical buckling load value decreases regardless of the magnetic field values. The critical buckling load follows an increasing pattern with the rise of mode number, unlike the length, which decreases the critical buckling loads but becomes low in slender nanotubes.

These results are important in the mechanical design considerations of the next generation of nano-devices using carbon nanotubes under a magnetic field (e.g., in electronic applications; the design of nano-transistors that use the buckling properties of single-wall CNTs due to the creation of a magnetic field effect, etc.). In fact, the embedded CNTs may not give researchers a good performance if the mechanical response of the nanotubes in a magnetic surround is not understood. Similar work will be started in the near future concerning other nano-objects, based on other continuums models.

The current model can be extended to examine others types of materials as used in (Cuong-Le *et al.* 2019a, b, 2020a, b, Khatir *et al.* 2019, Zenzen *et al.*2020, Khatir *et al.* 2021, Akbas 2022, Alimoradzadeh and Akbas 2022, Azandariani *et al.* 2022, Chinnapandi *et al.* 2022, Cho 2022a, b, Choi *et al.* 2022, Cuong-Le *et al.* 2022a, b, Du *et al.* 2022, Bochkareva and Lekomtsev 2022, Ding *et al.* 2022, Fan *et al.* 2022, Huang *et al.* 2022, Hagos *et al.* 2022, Kumar and Kattimani 2022, Liu *et al.* 2022, Mula *et al.* 2022, Man 2022, Rezaiee-Pajand *et al.* 2022, Tran and Cuong-Le 2022, Polat and Kaya 2022, Yaylaci *et al.* 2022c, Wu and Fang 2022, Zhu *et al.* 2022).

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