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Effect of rotation and inclined load in a nonlocal magneto-thermoelastic solid with two temperature

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Abstract. This work deals with the two-dimensional deformation in a homogeneous isotropic nonlocal magneto-thermoelastic solid with two temperatures under the effects of inclined load at different inclinations. The mathematical model has been formulated by subjecting the bounding surface to a concentrated load. The Laplace and Fourier transform techniques have been used for obtaining the solution to the problem in transformed domain. The expressions for nonlocal thermal stresses, displacements and temperature are obtained in the physical domain using a numerical inversion technique. The effects of nonlocal parameter, rotation and inclined load in the physical domain are depicted and illustrated graphically. The results obtained in this paper can be useful for the people who are working in the field of nonlocal thermoelasticity, nonlocal material science, physicists and new material designers. It is found that there is a significant difference due to presence and absence of nonlocal parameter.

Keywords: concentrated load; Eringen model of nonlocal theories; inclined load; magnetic field; nonlocal theory of thermoelasticityl; nonlocality; rotation; thermoelasticity; two temperatures

1. Introduction

Thermoelasticity covers a broad area of developments. It deals with the theory of stresses and strains considering the heat transfer equation. The thermoelastic theories defining the heat flow and deformation have gained a lot of attention during last few years and the nonlocal theory of thermoelasticity, two temperature theory are some of those theories to be named. The concept of nonlocality has been well studied and documented till now. It considers the dependence of the various physical quantities defined at a point as not just a function of the values of independent constitutive variables at that point only but as a function of their values over the whole body.

Edelen *et al*. (1971) and Edelen and Law (1971) developed the concept of nonlocal continuum mechanics. Eringen and Edelen (1972) developed the nonlocal elasticity theory which contains information about long range forces of atoms and according to this theory the stress field at a particular point is impacted by the strain at all the other point of the body. Marin (1996) obtained solutions in micropolar bodies with voids. Marin (1997) find result for thermoelastic body with voids and proved their uniqueness. Eringen (2002) derived nonlocal continuum field theories.

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Ebrahimi and Shafiei (2016) applied Eringen's nonlocal elasticity theory for vibration analysis of FG nanobeams. Abouelregal (2019) studied the rotating magneto-thermoelastic rod due to moving heat sources via Eringen's nonlocal model. Balubaid *et al*. (2019) investigated nanoscale plate using nonlocal plate theory. Hussain *et al*. (2019) studied the nonlocality in thermoelastic materials. Lata and Singh (2019, 2020d) discussed the nonlocal effects due to inclined load and due to ramp type sources respectively for their research work and proved that nonlocal effects play a major role depicting the results graphically. Soleimani *et al*. (2019) also used nonlocal elasticity theory to prove his results. Asghar *et al*. (2020) assessed nonlocal natural frequencies of DWCNTs.

Thermoelasticity with two temperatures is a highly important non-classical theories of thermodynamics of elastic solids. This theory differs due to the consideration of thermal effects only. It was due to Chen and Gurtin (1968) that the theory gained importance. They suggested the dependence of deformable bodies upon two distinct temperatures, namely the thermodynamic temperature and the conductive temperature. Youssef (2005) gave the uniqueness theorem in case of two temperature generalized thermoelasticity. Youssef and Al-Lehaibi (2007) investigated various problems related to two temperature thermoelasticity and proved the reliability of the two temperatures generalized thermoelasticity. Othman and Abbas (2012) studied generalized thermoelasticity of thermal-shock problem using Green and Naghdi type II and type III theories. Marin *et al*. (2015) extended the thermoelasticity concepts to porous micropolar bodies and proved some relevant results. Abualnour *et al*. (2019) analyzed composite plates thermomechanically using a four variable trigonometric plate theory. Saeed *et al*. (2020) developed a GL model on thermoelastic interactions by using finite element method. Jahangir *et al*. (2020) studied reflection of photothermoelastic waves in a semiconducting medium and proved the existence of different type of waves.

The effect of magenetic field and rotation also has been discussed by many researchers. Othman *et al*. (2015) studied effect of rotation on plane waves in generalized thermo-microstretch elastic bodies in the context of Green and Naghdi theory. Kumar *et al*. (2016) described thermomechanical interactions with combined effects of rotation, vacuum and two temperatures. They proved the significance of thermodynamic temperature and conductive temperature. Lata and Singh (2020a, b) studied the deformation in a nonlocal magneto-thermoelastic solid with hall current due to normal force and time harmonic interactions in nonlocal thermoelastic solid with two temperatures respectively. Khan *et al*. (2019) studied third-grade magnetohydrodynamic fluid with variable thermal conductivity and chemical reaction over an exponentially stretching surface. Lata and Singh (2020c) investigated the thermomechanical interactions in a nonlocal thermoelastic model due to memory dependent derivatives. Lata and Singh (2020e) analyzed the propagation of plane waves in a nonlocal magneto-thermoelastic solid with Hall current.

Alzahrani and Abbas (2016) investigated the effects of magnetic field on a thermoelastic material using GN-III theory. Sharma *et al*. (2016) discussed the rotation effects in a transversely isotropic magnetothermoelastic medium with and without energy dissipiation due to two temperatures. Abouelregal (2019) used Eringen's nonlocal model to discuss the effects of moving heat sources on a rotating magneto-thermoelastic rod. Mondal (2020) used a novel mathematical model of generalized thermoelasticity to study the memory dependent response in a magneto-thermoelastic rod with a moving heat source using Eringen's theory of nonlocality. Zenkour (2020) gave a refined multi-phase-lag model to study the Magneto-thermal shock for a fiber-reinforced anisotropic body. Heidari *et al*. (2021) investigated the mechanics of nanocomposites reinforced by nanotubes. Matouk *et al*. (2020) used the integral Timoshenko beam theory to study the hygro-thermal vibration of nanobeam. Rouabhia *et al*. (2020) used the nonlocal integral first-order theory for their investigation of the physical stability response of a SLGS.

From above discussion, it has been evaluated and observed that a lot of research has been carried out in recent years on nonlocal effects using two temperature theories. But not much attention has been given to the study of magneto-thermoelastic transversely isotropic nonlocal material with combined effects of rotation and two temperatures. So, in this paper an effort has been made to study the rotation and inclined load on a magneto-thermoelastic medium under the effect of local and non-local parameters. The analytic expressions for the displacements, stresses and temperature distribution have been obtained in two-dimensional transversely isotropic magneto-thermoelastic solid.

2. Basic equations

Following Eringen (2002) and Abouelregal (2019), the equation of motion for a homogeneous nonlocal magneto-thermoelastic solid rotating with a uniform angular velocity Ω = Ωn, where n is a unit vector demonstrating the direction of the rotation axis and taking into account of Lorentz force is

$$(\lambda + 2\mu)\nabla(\nabla.\boldsymbol{u}) - \mu\,(\nabla \times \nabla \times \boldsymbol{u}) - \beta\nabla\theta \;= (1 - \epsilon^2\nabla^2)\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2}. \tag{1}$$

where, $\boldsymbol{F} = \mu_0\,(\boldsymbol{J} \times \boldsymbol{H_0})$ denotes the Lorentz force, $\boldsymbol{H_0}$ is the external applied magnetic field intensity vector, $\boldsymbol{J}$ is the current density vector, $\boldsymbol{u}$ is the displacement vector, μ_0 and ε_0 are the magnetic and electric permeabilities respectively.

The above equations are supplemented by generalized Ohm's law for media with finite conductivity and including the hall current effect (from Kumar *et al.* (2017))

$$\boldsymbol{J} = \tfrac{\sigma_0}{1+m^2}\left(\boldsymbol{E} + \mu_0\left(\dot{\boldsymbol{u}} \times \boldsymbol{H} - \tfrac{1}{en_e}\boldsymbol{J} \times \boldsymbol{H_0}\right)\right). \tag{2}$$

where, **E** is the intensity vector of the electric field, **H** is the magnetic strength, $\dot{\boldsymbol{u}}$ is the velocity vector, $m\,(=\omega_e t_e)$ is the Hall parameter, ω_e is the electronic frequency, t_e is the electron collision time, e is the charge of an electron, n_e is the number of density of electrons.

Following Zenkour (2020), the heat conduction equation with multi-dual-phase-lag heat transfer is given as

$$\mathrm{K}^*\mathcal{L}_v\theta_{,ij} \;=\; \mathcal{L}_q\frac{\partial}{\partial t}\big(\rho C^*\theta + \beta\theta_0 u_{i,j}\big), \tag{3}$$

where $\mathcal{L}_v = 1 + \sum_{r=1}^{R_1}\frac{\tau_v^r}{r!}\frac{\partial^r}{\partial t^r}$, and $\mathcal{L}_q = \varrho + \tau_0\frac{\partial}{\partial t} + \sum_{r=2}^{R_2}\frac{\tau_q^r}{r!}\frac{\partial^r}{\partial t^r}$.

Here τ_v, τ_q and τ_0 are thermal memories in which τ_v is the phase lag of the temperature gradient while τ_q is the phase lag of the heat flux ($0 \le \tau_v < \tau_q$). Generally, the value of $R_1 = R_2 = R$ may reach 5 or more according as refined multi-dual-phase-lag theory required while ϱ is a non-dimension parameter (= 0 or 1 according to the thermoelasticity theory).

The constitutive relations are given by

$$(1 - \epsilon^2\nabla^2)t_{ij} = \lambda u_{k,k}\delta_{ij} + \mu\big(u_{i,j} + u_{j,i}\big) - \;\beta\theta\delta_{ij}. \tag{4}$$

where λ, μ are material constants, ϵ is the nonlocal parameter, ρ is the mass density, θ is absolute temperature and θ_0 is reference temperature, K^* is the coefficient of the thermal conductivity, C^* the specific heat at constant strain, $\beta = (3\lambda + 2\mu)\alpha$ where α is coefficient of linear thermal expansion, Ω is the angular velocity of the solid, e_{ij} are components of strain tensor, δ_{ij} is the Kronecker delta, t_{ij} are the components of stress tensor.

3. Formulation of the problem

We consider a homogeneous non local isotropic magneto-thermoelastic medium, permeated by an initial magnetic field $\boldsymbol{H_0} = (0, H_0, 0)$ acting along y-axis body in an initially undeformed state at temperature θ_0. The rectangular Cartesian coordinate system (x, y, z) is introduced, having origin on the surface (z = 0) with z-axis pointing normally into the half space. The surface of the medium is subjected to an inclined load acting at z = 0. We take a rectangular Cartesian co-ordinate system (x_1, x_2, x_3) with x_3 axis pointing normally into the half space. We assume that $\boldsymbol{\Omega} = (0, \Omega, 0)$. Also, the current density components (using generalized Ohm's law) are given as

$$J_1 = -\varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2}, \tag{5}$$

$$J_2 = 0, \tag{6}$$

$$J_3 = \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2}. \tag{7}$$

We restrict our analysis to two-dimensional problem by using

$$\boldsymbol{u} = (u, 0, w). \tag{8}$$

Using Eq. (8) in Eqs. (1)-(3), yields

$$\begin{aligned}(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 w}{\partial x \partial z} + \mu\frac{\partial^2 u}{\partial z^2} - \beta\frac{\partial \theta}{\partial x} - (1 - \epsilon^2\nabla^2)\mu_0 J_3 H_0 \\ = \rho(1 - \epsilon^2\nabla^2)\left\{\frac{\partial^2 u}{\partial t^2} - \Omega^2 \mathrm{u} + 2\Omega\frac{\partial w}{\partial t}\right\},\end{aligned} \tag{9}$$

$$\begin{aligned}(\lambda + 2\mu)\frac{\partial^2 w}{\partial z^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial z} + \mu\frac{\partial^2 w}{\partial z^2} - \beta\frac{\partial \theta}{\partial z} - (1 - \epsilon^2\nabla^2)\mu_0 J_1 H_0 \\ = \rho(1 - \epsilon^2\nabla^2)\left\{\frac{\partial^2 w}{\partial t^2} - \Omega^2 \mathrm{w} - 2\Omega\frac{\partial w}{\partial t}\right\},\end{aligned} \tag{10}$$

$$K^*\mathcal{L}_v\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial z^2}\right) = \mathcal{L}_q \frac{\partial}{\partial t}\left[\rho C^* \theta + \beta\theta_0\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right]. \tag{11}$$

we define the following dimensionless quantities

$$(x',z',u',w') = \frac{\omega_1}{c_1}(x,z,u,w), \quad t'_{ij} = \frac{t_{ij}}{\beta\theta_0}, \quad t' = \omega_1 t,$$
$$a' = \frac{\omega_1^2}{c_1^2}a, \quad \theta' = \frac{\theta}{\theta_0}, \quad \Omega' = \frac{\Omega}{\omega_1}, \quad \tau'_v = \omega_1\tau_v, \quad \tau'_0 = \omega_1\tau_0, \quad \tau'_q = \omega_1\tau_q, \tag{12}$$
$$F_1{}' = \frac{F_1}{\beta\theta_0} \quad \text{and} \quad F_2{}' = \frac{F_2}{\beta\theta_0}.$$

where

$$c_1^2 = \frac{\mu}{\rho} \quad \text{and} \quad \omega_1 = \frac{\rho\, C^* c_1^2}{K^*}.$$

The relations between non-dimensional displacement components u, w and the dimensionless potential functions q, ψ can be expressed as potential functions defined by

$$u = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z}, \qquad w = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{13}$$

Upon introducing the quantities defined by Eqs. (12)-(13) in Eqs. (9)-(11), and suppressing the primes, yields

$$\left\{(1+a_1)\nabla^2 - (1-\epsilon^2\nabla^2)\left[\frac{M}{1+m^2}\frac{\partial}{\partial t} + \frac{\partial^2}{\partial t^2} - a_3\Omega^2\right]\right\} q - a_2\theta = 0, \tag{14}$$

$$\left\{\nabla^2 - (1-\epsilon^2\nabla^2)\left(\frac{\partial^2}{\partial t^2} - a_3\Omega^2\right)\right\}\psi = 0 \tag{15}$$

$$a_2\nabla^2 q - \left[\mathcal{L}_v a_4 \nabla^2 - \mathcal{L}_q a_5 \frac{\partial}{\partial t}\right]\theta = 0 \tag{16}$$

where, $a_1 = \frac{\lambda+\mu}{\mu}$, $a_2 = \frac{\beta T_0}{\mu}$, $a_3 = \frac{{\omega_1}^2}{{c_1}^2}$, $a_4 = \frac{\mathrm{K}\omega_1}{c_1}$, $a_5 = \rho C^*$ and $M = \frac{\sigma_0{\mu_0}^2{H_0}^2}{\rho}$.

The initial and regularity conditions are given by

$$u(x,z,0) = 0 = \dot{u}(x,z,0),$$
$$w(x,z,0) = 0 = \dot{w}(x,z,0),$$
$$\theta(x,z,0) = 0 = \dot{\theta}(x,z,0 \quad \text{for} \quad z \geq 0, -\infty < x < \infty,$$
$$u(x,z,t) = w(x,z,t) = \theta(x,z,t) = 0 \quad \text{for} \quad t > 0 \quad \text{when} \quad z \to \infty.$$

Applying Laplace and Fourier Transform defined by

$$\bar{f}(\mathrm{x},\mathrm{z},\mathrm{s}) = \int_0^\infty f(x,z,t)\, e^{-st} dt, \tag{17}$$

$$\hat{f}(\xi,z,s) = \int_{-\infty}^\infty \bar{f}(x,z,s)\, e^{i\xi x} dx. \tag{18}$$

on Eqs. (14)-(16), we obtain a system of equations

$$\left[(1+a_1)(-\boldsymbol{\xi}^2+D^2)-(1+\epsilon^2\xi^2-\epsilon^2\mathrm{D}^2)(\frac{Ms}{1+m^2}+s^2-a_3\Omega^2)\right]\tilde{q}-[a_2]\tilde{\theta}=0, \tag{19}$$

$$[a_2\mathrm{D}^2-\xi^2]\tilde{q}-\left[\mathcal{L}_v a_4(D^2-\xi^2)-\mathcal{L}_q a_5 s\right]\tilde{\theta}=0, \tag{20}$$

$$[\,D^2-\xi^2-(1+\epsilon^2\xi^2-\epsilon^2D^2)(s^2-a_3\Omega^2]\tilde{\psi}=0. \tag{21}$$

where $\mathcal{L}_v=1+\sum_{r=1}^{R_1}\frac{\tau_v{}^r}{r!}s^r$ and $\mathcal{L}_q=\varrho+\tau_0 s+\sum_{r=2}^{R_2}\frac{\tau_q^r}{r!}s^r$.

From Eqs. (18), (19) and (20), we obtain a set of homogeneous equations which will have a nontrivial solution if determinant of coefficient $[\tilde{q},\tilde{\theta},\tilde{\psi}]^T$ vanishes so as to give a characteristic equation as

$$[D^6+\mathrm{Q}D^4+RD^2+S](\tilde{q},\tilde{\theta},\tilde{\psi})=0. \tag{22}$$

where $Q=\frac{\zeta_6}{P}\{\zeta_6\zeta_{11}+\zeta_{13}+\zeta_8\zeta_{12}+\zeta_8\zeta_9\}$, $R=\frac{-1}{P}\{\zeta_6\zeta_{11}\zeta_{12}+\zeta_6\zeta_{14}+\zeta_6\zeta_9\zeta_{11}+\zeta_9\zeta_{13}+\zeta_8\zeta_9\zeta_{12}\}$, $S=\frac{\zeta_9}{P}\{\zeta_{11}\zeta_{12}+\zeta_{14}\}$, $P=-\zeta_6{}^2\zeta_8$.

$D=\dfrac{d}{dz}$, $\zeta_1=1+a_1$, $\zeta_2=1+\epsilon^2\xi^2$, $\zeta_3=\dfrac{Ms}{1+m^2}+s^2-a_3\Omega^2$,

$\zeta_4=1+a\xi^2\zeta_5=s^2-a_3\Omega^2$, $\zeta_6=1+\zeta_5\epsilon^2$, $\zeta_7=\zeta_1+\zeta_3\epsilon^2$, $\zeta_8=\mathcal{L}_v a_4$, $\zeta_9=\xi^2+\zeta_2\zeta_5$,

$\zeta_{10}=a_2{}^2\xi^2$, $\zeta_{11}=\mathcal{L}_v a_4\xi^2+\mathcal{L}_q a_5 s$, $\zeta_{12}=\zeta_1\xi^2+\zeta_2\zeta_3$, $\zeta_{13}=a_2\zeta_1$.

The roots of the Eq. (22) are $\pm\lambda_i$ $(i=1,2,3)$ satisfying the radiation condition that $\tilde{q},\tilde{\theta},\tilde{\psi}\to 0$ as $z\to\infty$, the solutions of equation can be written as

$$\tilde{q}=A_1e^{-\lambda_1 z}+A_2e^{-\lambda_2 z}+A_3e^{-\lambda_3 z}, \tag{23}$$

$$\tilde{\theta}=d_1A_1e^{-\lambda_1 z}+d_2A_2e^{-\lambda_2 z}+d_3A_3e^{-\lambda_3 z}, \tag{24}$$

$$\tilde{\psi}=l_1A_1e^{-\lambda_1 z}+l_2A_2e^{-\lambda_2 z}+l_3A_3e^{-\lambda_3 z}, \tag{25}$$

$$d_i=\frac{P^*\lambda_i{}^4+Q^*\lambda_i{}^2+R^*}{T^*\lambda_i{}^4+U^*\lambda_i{}^2+V^*}\qquad i=1,2,3, \tag{26}$$

$$l_i=\frac{P^{**}\lambda_i{}^4+Q^{**}\lambda_i{}^2+R^{**}}{T^*\lambda_i{}^4+U^*\lambda_i{}^2+V^*}\qquad i=1,2,3, \tag{27}$$

where

$P^*=\zeta_6{}^2$, $Q^*=-\zeta_6(\zeta_9+\zeta_{12})$, $R^*=\zeta_9\zeta_{12}$,
$T^*=-\zeta_6\zeta_8$, $U^*=\zeta_8\zeta_9+\zeta_6\zeta_{11}$, $V^*=-\zeta_9\zeta_{11}$,
$P^{**}=-\zeta_6\zeta_8$, $Q^{**}=\zeta_8\zeta_{12}+\zeta_6\zeta_{11}+\zeta_{13}$, $R^{**}=-(\zeta_{11}\zeta_{12}+\zeta_{14})$.

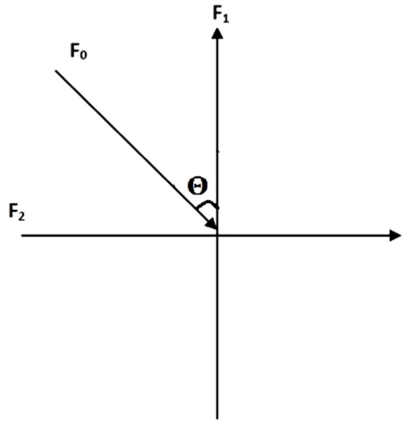

Fig. 1 Inclined load over a nonlocal isotropic magneto-thermoelastic solid

4. Applications

We consider a normal line load F_1 per unit length acting in the positive z-axis on the plane boundary $z = 0$ along the y-axis and a tangential load F_2 per unit length, acting at the origin in the positive x-axis. The boundary conditions are

$$t_{zz}(x,z,t) = -F_1\psi_1(x)H(t), \tag{28}$$

$$t_{xz}(x,z,t) = -F_2\psi_2(x)H(t), \tag{29}$$

$$\frac{\partial}{\partial x_3}\varphi(x_1,x_3,t) = 0. \tag{30}$$

where, F_1 and F_2 are the magnitude of the forces applied, $\psi_1(x)$ and $\psi_2(x)$ specify the vertical and horizontal load distribution function along x axis and H(t) is the Heaviside unit step function given by

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}.$$

Applying the Laplace and Fourier transform defined by Eqs. (17) and (18) on the boundary conditions (28)-(30) and then using the dimensionless quantities defined by Eq. (12) and using Eqs. (4), (8), (13) and substituting values of $\hat{q}, \hat{\theta}, \hat{\psi}$ from Eqs. (23)-(25), and solving, we obtain the components of displacement, normal stress, tangential stress and conductive temperature as

$$\tilde{u} = \frac{F_1\widetilde{\psi_1}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} M_{1i}e^{-\lambda_i z}\right\} + \frac{F_2\widetilde{\psi_2}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} M_{2i}e^{-\lambda_i z}\right\}, \tag{31}$$

$$\widetilde{w} = \frac{F_1\widetilde{\psi_1}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} d_i M_{1i}e^{-\lambda_i z}\right\} + \frac{F_2\widetilde{\psi_2}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} d_i M_{2i}e^{-\lambda_i z}\right\}, \tag{32}$$

$$\hat{\theta} = \frac{F_1\widetilde{\psi_1}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} l_i M_{1i} e^{-\lambda_i z}\right\} + \frac{F_2\widetilde{\psi_2}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} l_i M_{2i} e^{-\lambda_i z}\right\}. \tag{33}$$

$$\widetilde{t_{zz}} = \frac{F_1\widetilde{\psi_1}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} d_i M_{1i} e^{-\lambda_i z}\right\} + \frac{F_2\widetilde{\psi_2}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} d_i M_{2i} e^{-\lambda_i z}\right\}, \tag{34}$$

$$\widetilde{t_{zx}} = \frac{F_1\widetilde{\psi_1}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} \Delta_{2i} M_{1i} e^{-\lambda_i z}\right\} + \frac{F_2\widetilde{\psi_2}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} \Delta_{2i} M_{2i} e^{-\lambda_i z}\right\}, \tag{35}$$

$$\widetilde{t_{xx}} = \frac{F_1\widetilde{\psi_1}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} S_i M_{1i} e^{-\lambda_i z}\right\} + \frac{F_2\widetilde{\psi_2}(\xi)}{s\Delta}\left\{\sum_{i=1}^{3} S_i M_{2i} e^{-\lambda_i z}\right\}. \tag{36}$$

where

$$\Delta = \sum_{i=1}^{3} M_{3i} \mathrm{T}_i ,$$
$$M_{11} = \Delta_{22}\Delta_{33} - \Delta_{32}\Delta_{23}, \quad M_{12} = \Delta_{21}\Delta_{33} - \Delta_{31}\Delta_{23}, \quad M_{13} = \Delta_{32}\Delta_{21} - \Delta_{31}\Delta_{22},$$
$$M_{21} = N_{23}, \quad M_{22} = N_{13}, \quad M_{23} = N_{12},$$
$$M_{31} = \Delta_{22}\Delta_{33} + \Delta_{32}\Delta_{23}, \quad M_{32} = \Delta_{21}\Delta_{33} + \Delta_{31}\Delta_{23}, \quad M_{33} = \Delta_{32}\Delta_{21} + \Delta_{31}\Delta_{22},$$
$$\Delta_{2j} = \iota\xi d_j - \lambda_j, \qquad \Delta_{3j} = l_j\lambda_j, \qquad N_{ij} = \lambda_i d_j(\lambda + 2\mu) + \beta l_i l_j(\lambda_j - \lambda_i).$$

5. Special case

5.1 Concentrated force

The solution due to concentrated normal force on the half space is obtained by setting

$$\psi_1(x) = \delta(x), \qquad \psi_2(x) = \delta(x),$$

where $\delta(x)$ is dirac delta function.

Applying Laplace and Fourier transform, we obtain

$$\widehat{\psi_1}(\xi) = 1, \qquad \widehat{\psi_2}(\xi) = 1. \tag{37}$$

Using Eq. (37) in Eqs. (31)-(36), the components of displacement, stress and temperature are obtained.

6. Particular cases

Suppose an inclined load, F_0 per unit length is acting on the y-axis and its inclination with z-axis is Θ, i.e.

$$F_1 = F_0 \cos\Theta \quad \text{and} \quad F_2 = F_0 \sin\Theta \tag{38}$$

Using Eq. (38) in Eqs. (31)-(36), we obtain the expressions for displacement components, stress components and temperature for concentrated force on the surface of a homogeneous isotropic magneto-thermoelastic solid.

- If $\epsilon = 0$, then from Eqs. (31)-(36), the corresponding expressions for displacements, stresses and conductive temperature for isotropic solid with local effects are obtained.
- If $\Omega = 0$, then from Eqs. (31)-(36), the corresponding expressions for displacements, stresses and conductive temperature for isotropic solid without rotation and with nonlocal effects are obtained.
- If $\Omega = \epsilon = 0$, then from Eqs. (31)-(36), the corresponding expressions for displacements, stresses and conductive temperature for local isotropic solid without rotation are obtained.

7. Inversion of the transformation

To obtain the solution of the problem in physical domain, we must invert the transforms in Eqs. (31)-(36). Here the displacement components, normal and tangential stresses and conductive temperature are functions of z and the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f(\xi, z, s)$. To obtain the function $f(x, z, t)$ in the physical domain, we first invert the Fourier transform using the formula

$$\tilde{f}(x,z,s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\xi x}\,\hat{f}(\xi,z,s)d\xi = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\cos(\xi x)\, f_e - i\sin(\xi x)\, f_0|\, d\xi. \tag{39}$$

where, f_e and f_0 are respectively the even and odd parts of $\hat{f}(\xi, z, s)$. Thus the expression (39) gives the Laplace transform $\tilde{f}(x, z, s)$ of the function $f(x, z, t)$. Following Honig and Hirdes, the Laplace transform function $\tilde{f}(x, z, s)$ can be inverted to $f(x, z, t)$.

The Last step is to calculate the integral in Eq. (39). The method for evaluating this integral is described in Press at al. It involves the use of Romberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion

Magnesium material is chosen for the purpose of numerical calculation which is isotropic and according to Dhaliwal and Singh (1980), physical data for which is given as

$$\begin{array}{lll} \lambda = 9.4 \times 10^{10} Nm^{-2}, & \mu = 3.278 \times 10^{10} Nm^{-2}, & K^* = 1.7 \times 10^{2} Wm^{-1}K^{-1}, \\ \rho = 1.74 \times\ 10^{3} Kgm^{-3}, & \theta_0 = 298\, K, & C^* = 10.4 \times 10^{2} JKg^{-1}deg^{-1}, \\ \mu_0 = 4\pi \times 10^{-7} Hm^{-1}, & \varepsilon_0 = \dfrac{10^{-9}}{36\pi} Fm^{-1}, & H_0 = 1\, Jm^{-1}n\, b^{-1}. \end{array}$$

Using the above values, a comparison of values of displacement components u and w, stress components t_{zz}, t_{xx}, t_{zx} and temperature θ for a homogeneous isotropic nonlocal magneto-thermoelastic solid with distance x has been made and the effects of rotation and inclination has been depicted for local parameter $(\epsilon = 0)$ and non local parameter $(\epsilon = 2)$, for $\Omega = 0$ and $\Omega =$

0.5 and angle of inclination $\Theta = 30^0$ and $\Theta = 45^0$.

1) The solid black colored line with center symbol square corresponds to local parameter $(\epsilon = 0)$ and $\Omega = 0$.
2) The dashed reddish colored line with center symbol circle represents local parameter $(\epsilon = 0)$ and $\Omega = 0.5$.
3) The dotted blue colored line with center symbol upward triangle corresponds to nonlocal parameter $(\epsilon = 2)$ and $\Omega = 0$.
4) The dashed-dotted purplish colored line with center symbol downward triangle represents nonlocal parameter $(\epsilon = 2)$ and $\Omega = 0.5$.

Fig. 2, shows the variations of the displacement component u for isotropic magneto-thermoelastic medium with rotation and nonlocal effects at $\Theta = 30^0$. It is clear that the values of u follow oscillatory pattern. For $\epsilon = 0$ and $\Omega = 0$, the variations are increasing rapidly for $0 < x < 2$ while later on it follows oscillatory path. Same way all other values for different ϵ and Ω

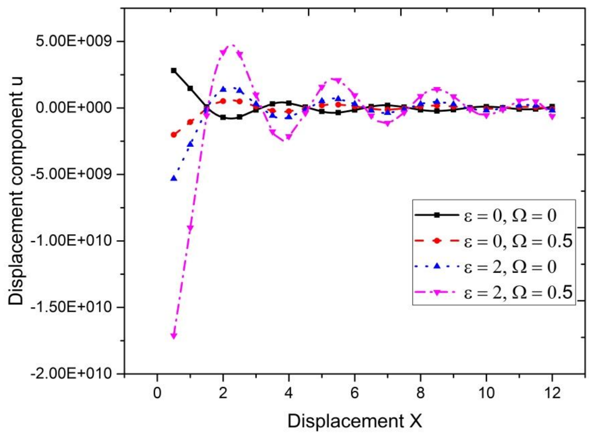

Fig. 2 Variation of displacement component u with displacement x at $\Theta = 30^0$

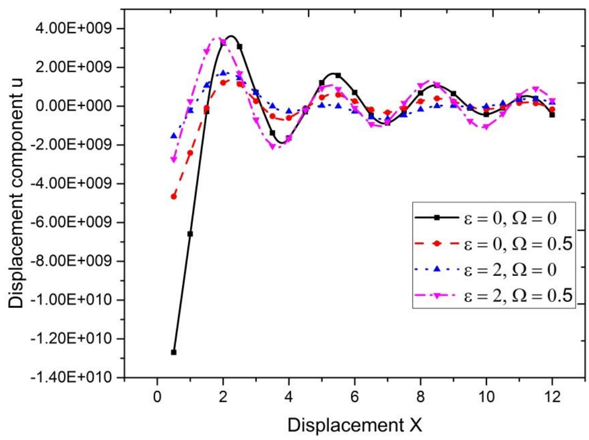

Fig. 3 Variation of displacement component u with displacement x at $\Theta = 45^0$

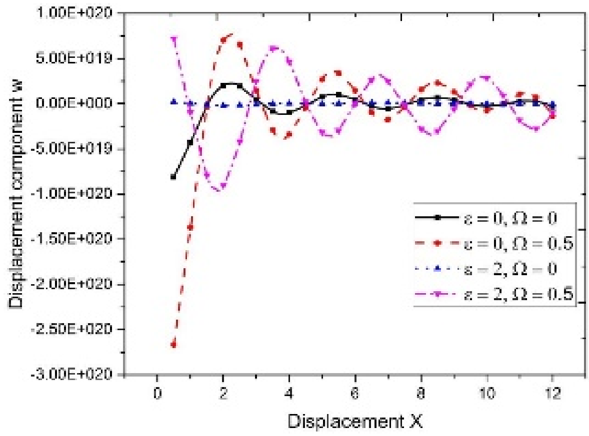

Fig. 4 Variation of displacement component w with displacement x at $\Theta = 30^0$

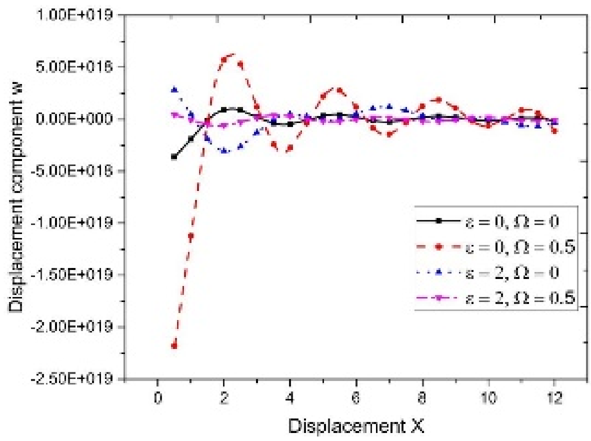

Fig. 5 Variation of displacement component w with displacement x at $\Theta = 45^0$

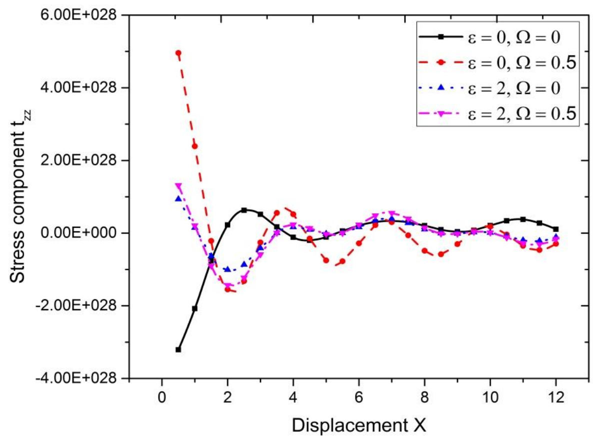

Fig. 6 Variation of stress component t_{zz} with displacement x at $\Theta = 30^0$

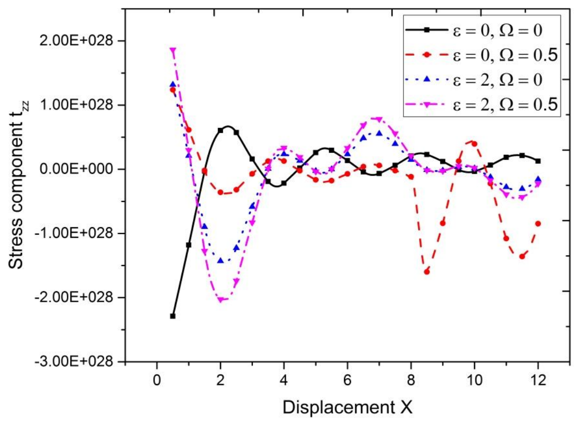

Fig. 7 Variation of stress component t_{zz} with displacement x at $\Theta = 45^0$

follows perfectly oscillatory path from beginning to end. Fig. 3 depicts the variation of values of displacement component u at $\Theta = 45^0$. The pattern is oscillatory with a clear difference between values for local and non-local parameters. the values of u follow oscillatory pattern. For $\epsilon = 2$ and $\Omega = 0.5$, the variations are increasing rapidly for $0 < x < 2$ while later on it followsoscillatory path. Figs. 4 and 5 describe the variations of the displacement component w for isotropic magneto-thermoelastic medium with rotation and nonlocal effects at $\Theta = 30^0$ and $\Theta = 45^0$ respectively. It is clear that the values of w follow oscillatory pattern. For $\epsilon = 0$ and $\Omega = 0.5$, the variations are increasing rapidly for $0 < x < 2$ while later on it follows oscillatory path with more clearly visible oscillations for $\Theta = 45^0$, but the nonlocality effects are dominant for all values. Figs. 6 and 7 shows the variation of stress component t_{zz} for $\Theta = 30^0$ and $\Theta = 45^0$ respectively. Here too the behavior followed is oscillatory with more variations for $\epsilon = 0$ and $\Omega = 0.5$ for $x > 8$ at $\Theta = 30^0$ while the values decrease rapidly for $0 < x < 2$ at $\Theta = 45^0$. Figs. 8 and 9 shows the variation of stress component t_{zx} for $\Theta = 30^0$ and $\Theta = 45^0$ respectively. Here too the behavior followed is oscillatory with nonlocality effects clearly visible

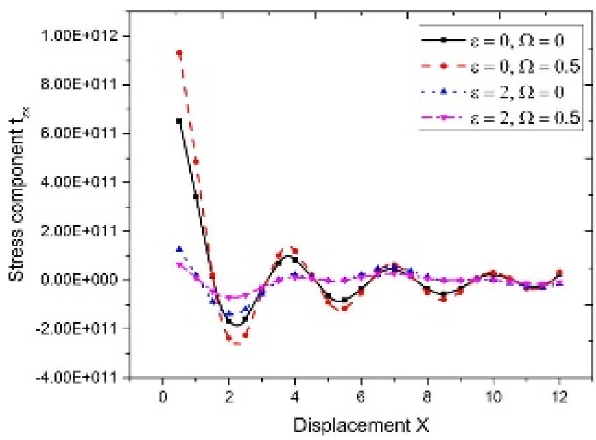

Fig. 8 Variation of stress component t_{zx} with displacement x at $\Theta = 30^0$

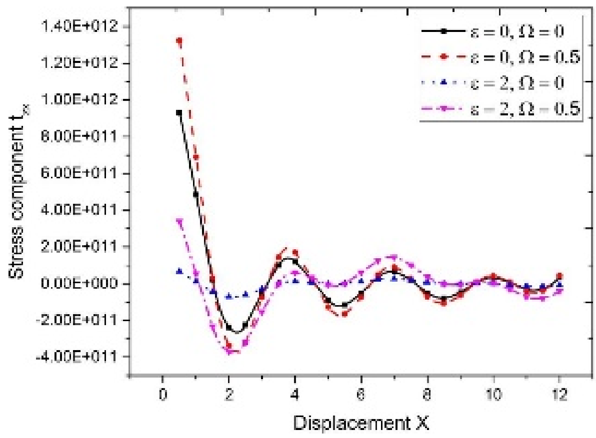

Fig. 9 Variation of stress component t_{zx} with displacement x at $\Theta = 45^0$

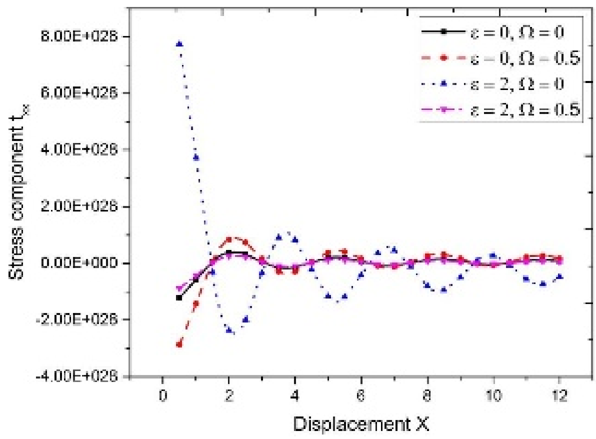

Fig. 10 Variation of stress component t_{xx} with displacement x at $\Theta = 30^0$

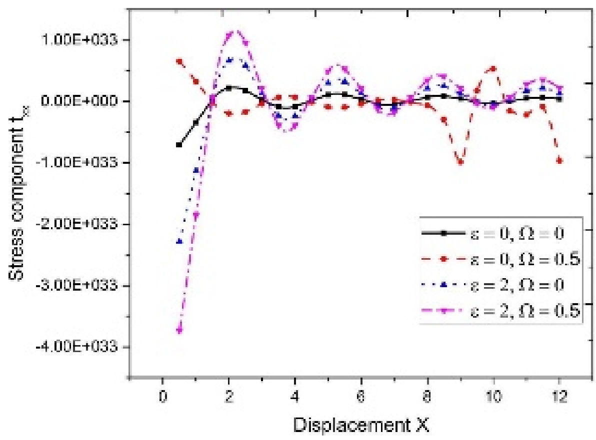

Fig. 11 Variation of stress component t_{xx} with displacement x at $\Theta = 45^0$

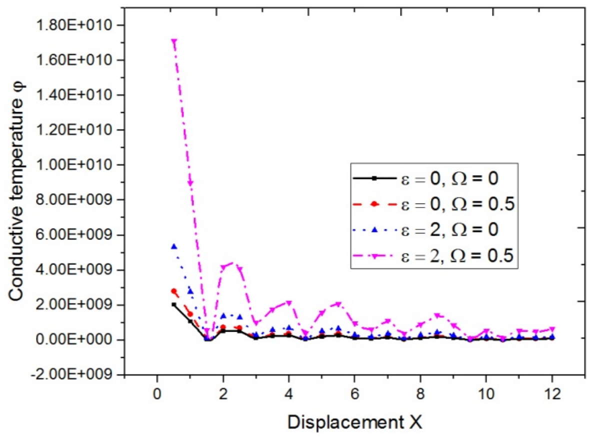

Fig. 12 Variation of conductive temperature φ with displacement x at $\Theta = 30^0$

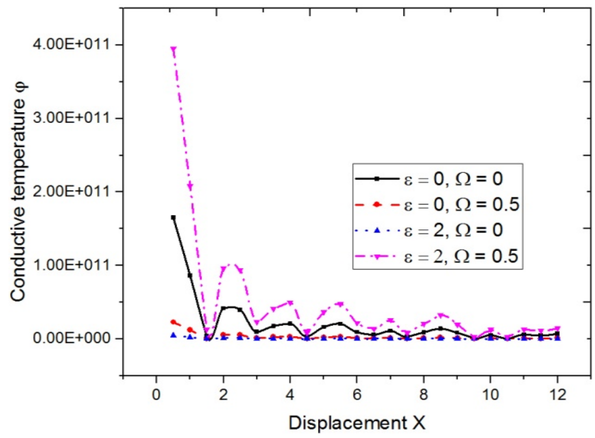

Fig. 13 Variation of conductive temperature φ with displacement x at $\Theta = 45^0$

for both angles of inclination. Figs. 10 and 11 shows the variation of stress component t_{xx} for $\Theta = 30^0$ and $\Theta = 45^0$ respectively. The behavior followed is oscillatory with a rapid decrease in values for $\epsilon = 2$ and $\Omega = 0$ for $0 < x < 2$ at $\Theta = 45^0$ while there is a rapid increase for the values for $\epsilon = 2$ and $\Omega = 0$ and $\epsilon = 2$ and $\Omega = 0.5$ for $0 < x < 2$ at $\Theta = 30^0$. Figs. 12 and 13 shows the variation of temperature θ for $\Theta = 30^0$ and $\Theta = 45^0$ respectively. The pattern followed is oscillatory for all the values with only exception being a slight and rapid decrease in values for $\epsilon = 2$ and $\Omega = 0.5$ for $0 < x < 2$ at both the inclination angles. Also, the effects for nonlocality and rotation are clearly visible from the graphs.

9. Conclusions

In the above discussion the combined effects of nonlocal parameter, rotation and inclined load on the components of displacements, stresses and temperature have been examined in a nonlocal magneto-thermoelastic solid with Hall current and rotation. It is observed that nonlocality is playing a significant effect on displacement components, stress components and temperature and the effects of magnetic rotation and the angle of inclination of the applied load are clearly visible along with local and nonlocal parameters. It is observed from the Figs. 2~13 that the trends in the variations of the characteristics mentioned are similar with difference in their magnitude when the concentrated forces are applied. Under the combined effects of nonlocality, rotation and inclined load; all the components are following an oscillatory path with respect to variations in x. There are differences in the magnitude for both local and nonlocal parameters and the magnitude increases as rotational effect and nonlocal effect are introduced in most of the cases. The results obtained give an inspiration to study nonlocality further in magneto-thermoelastic materials. The results of this paper can be helpful for the researchers working in the field of material engineering, geophysics, marine engineering, acoustics etc., for analysis of deformation field around mining tremors.

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