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Modified algorithmic LMI design with applications in aerospace vehicles

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Abstract. A modified fuzzy mechanical control of large-scale multiple time delayed dynamic systems in states is considered in this paper. To do this, at the first level, a two-step strategy is proposed to divide a large system into several interconnected subsystems. As a modified fuzzy control command, the next was received as feedback theory based on the energetic function and the LMI optimal stability criteria which allow researchers to solve this problem and have the whole system in asymptotically stability. Modeling the Fisher equation and the temperature gauge for high-speed aircraft and spacecraft shows that the calculation method is efficient.

Keywords: aerospace vehicles, LMI, smart control, nonlinear systems, stability analysis

1. Introduction

Mathematics seems to be a guide, appearing by the physicist at the right time, bringing light to the gloomy world of physics. However, the mutual influence of mathematics and physics is far more complicated than the story told. In most recorded history, physics and mathematics are not even separate subjects. The mathematics of ancient Greece, Egypt, and Babylon believed that we live in a world where distance, time, and gravity all operate in a certain way. The mathematical and statistical models for many physical, nature and technical systems are generally large or contain dynamic interaction phenomena and the cost for testing these models of control purposes are often too high. Therefore, it is natural to find a technique that can reduce the calculation costs. The large systems methodology provides this technique by manipulating the structure of the system in some way. Therefore, research on modeling, math, analysis, collection, optimization and control of large-scale systems has generated great interest. Recently, many of these methods have been proposed to verify the stability of the literature and the stability of large systems (Yang and Chang 1996, Bedirhanoglu 2014, Eswaran and Reddy 2016 and references included).

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In a computer network, because different communication subnets and network architectures adopt different transfer control methods, the transfer delay in the communication subnet is determined by the network status. The delay time caused by the electrical signal response is fixed. The smaller the response time, the smaller the delay, the larger the bandwidth, and the higher the transmission rate. Therefore, the larger the channel bandwidth, the smaller the delay. Delay time is the time it takes to get a packet from a specific point. Delay time is generally the sum of response delay and transmission delay. Delays usually occur in other technological systems. Computer control systems, for example, experience delays because computers take a long time to perform digital tasks. Also, there are remote operations, radar, power grid, transportation, metal delay and so on. The outputs of these systems do not respond to the input data until a certain amount of time has passed. The introduction of a delay factor usually causes instability and often complicates the analysis. Therefore, the analysis of the delay stability of the system on research (Mori 1985 and Trine Aldeen 1995) have published and executed by demonstrations.

In recent years, there has much been on the topic of a growing interest in system controls. There are already many successful applications. Despite that of its success, it is clear that a great of basic problems remain to be solved and the main problem with control systems is system design to ensure stability. Recently, there have been many studies on the stability (see Sugeno 1992, Wang *et al.* 1996, Tanaka *et al.* 1996, Feng *et al.* 1997 and references). However, studies in the literature have yet to solve the stability and non-stable problem of large systems with multiple delays.

Consequently, this study has a stability criterion based directly on the Lyapunov method to provide asymptotic stability to large systems with multiple delays. In accordance with this criterion and decentralized control schemes, fuzzy control groups are incorporated, stabilizing large-scale systems in multiple delays consisting of multiple interconnected subsystems. Furthermore, these subsystems are represented by a fuzzy Takagi-Sugeno model in multiple delays. In these models, each rule is represented by a linear system model, so linear feedback control can be used as feedback stability. Therefore, the kind of control design is based on the fuzzy model that uses a parallel distributed compensation (PDC) scheme. The ideas are those all linear local linear models control feedback share the same premises. The resulting controller is typically nonlinear with blend of each linear fuzzy rule.

In summary, we briefly introduce Takagi Sugeno’s fuzzy model with some delays and describe the system. The stability criterion is then derived and considered based on the Lyapunov method, ensuring asymptotic stability of systems with multiple delays. Finally, the results explain and draw conclusions for the numerical simulation examples they are referred to.

2. System description

The following we review a nonlinear parabolic PDE (Razavi and Sarkar 2018):

$$\begin{cases} \dfrac{\partial \bar{x}(z,t)}{\partial t} = \dfrac{\partial}{\partial z}\left(p(z)\dfrac{\partial \bar{x}(z,t)}{\partial z}\right) + f(\bar{x}(z,t)) \\ \qquad\qquad + \bar{b}_u(z)u(t) + \bar{b}_\omega(z)\omega(t) \\ y(t) = \int_\Omega \bar{c}(z)\bar{x}(z,t)dz, t = t_k \end{cases} \tag{1}$$

where $\Omega \triangleq [\alpha_1,\ \alpha_2] \subset \mathbb{R}$, $z \in \Omega$, $t \in [0, \infty)$, $\bar{x}(z,t) \in \mathbb{R}$, $\omega(t) \in \mathbb{R}^{n_\omega}$, and $y(t) \in \mathbb{R}^{n_y}$ based on

$\{t_k\} \in \mathcal{S}(h) \triangleq$, $\{t_k, k \in \mathbb{N} | \epsilon \leqslant h_k \leqslant h, \epsilon > 0\}$.

To simplify the construction of the equation Eq. (1), we consider a nonlinear J as interconnected in subsystems F_j, j=1,2,…,J. The jth as isolated subsystems (without any interconnection) of $\boldsymbol{F}$ are represented by the technique of IF-THEN delay control model of Takagi-Sugeno. The main feature of the Takagi-Sugeno fuzzy model with multiple delays is the expression of each of rule by means of a linear equation of state, and the model is as follows (Chen 2014, Chen *et al.* 2019, 2020):

Rule i : IF any $x_{1j}(t)$ is M_{i1j} and $\cdots$ and $x_{gj}(t)$ is M_{igj}

THEN $\dot{x}_j(t) = A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x(t-\tau_{kj}) + B_{ij}u_j(t)$

where $x_j^T(t) = [x_{1j}(t), x_{2j}(t), \cdots, x_{gj}(t)]$; $u_j^T(t) = [u_{1j}(t), u_{2j}(t), \cdots, u_{mj}(t)]$

r_j is the jth subsystem's IF-THEN rule umber. A_{ij}, A_{ikj} and B_{ij} are system matrices, state $x_j(t)$, input $u_j(t)$, delay τ_{kj} fuzzy set M_{ipj} (p=1,2,..,g), and premise $x_{1j}(t)$~$x_{gj}(t)$ are used to infer the fuzzy dynamic model:

$$\dot{x}_j(t) = \frac{\sum_{i=1}^{r_j} w_{ij}(t)\left\{A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x(t-\tau_{kj}) + B_{ij}u_j(t)\right\}}{\sum_{i=1}^{r_j} w_{ij}(t)} \tag{2}$$

$$= \sum_{i=1}^{r_j} h_{ij}(t)\left\{A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x(t-\tau_{kj}) + B_{ij}u_j(t)\right\}$$

with

$$w_{ij}(t) = \prod_{p=1}^{g} M_{ipj}(x_{pj}(t)), \qquad h_{ij}(t) = \frac{w_{ij}(t)}{\sum_{i=1}^{r_j} w_{ij}(t)} \tag{3}$$

in which $M_{ipj}(x_{pj}(t))$ is in the grade of any membership of $x_{pj}(t)$ in M_{ipj} if

$$w_{ij}(t) \geq 0, \quad i = 1, 2, \cdots, r_j \text{ and } \sum_{i=1}^{r_j} w_{ij}(t) > 0, \quad h_{ij}(t) \geq 0, \quad i = 1, 2, \cdots, r_j, \quad \sum_{i=1}^{r_j} h_{ij}(t) = 1.$$

According to the above mentioned analysis, these jth F_j could be

$$\dot{x}_j(t) = \sum_{i=1}^{r_j} h_{ij}(t)\left\{A_{ij}x_j(t) + \sum_{k=1}^{N_j} A_{ikj}x(t-\tau_{kj}) + B_{ij}u_j(t)\right\} + \sum_{\substack{n=1 \\ n\neq j}}^{J} C_{nj}x_n(t) \tag{4}$$

where C_{nj} is the interconnection.

3. Parallel distributed compensation (PDC)

The fuzzy control, according to a distributed control system using PDC technology, is used to stabilize a large number of synthetic design. The concept of the PDC scheme is to provide a way to handle each distribution rule for the relevant rules of the Takagi Sugenno model with multiple delays. Each rule in the model is described linear, so you can use linear control theory to develop controllers.

The fuzzy controller of the jth subsystem of rule i is derived as follows.

IF any of $x_{1j}(t)$ is M_{i1j} and… $x_{gj}(t)$ is M_{igj}; THEN one

$$u_j(t)=-K_{ij}x_j(t), \tag{5}$$

in which i =1, 2,…, r_j. Hence, these final outputs of the fuzzy controllers are

$$u_j(t)=-\frac{\sum_{i=1}^{r_j} w_{ij}(t)K_{ij}x_j(t)}{\sum_{i=1}^{r_j} w_{ij}(t)}=-\sum_{i=1}^{r_j} h_{ij}(t)K_{ij}x_j(t). \tag{6}$$

Combine Eqs. (6) and (4), the subsystem becomes

$$\dot{x}_j(t)=\sum_{i=1}^{r_j}\sum_{f=1}^{r_j} h_{ij}(t)h_{fj}(t)\left\{(A_{ij}-B_{ij}K_{fj})x_j(t)+\sum_{k=1}^{N_j} A_{ikj}x(t-\tau_{kj})\right\}+\phi_j(t). \tag{7}$$

Theorem 1: These multiple time-delay fuzzy large-scale systems $\boldsymbol{F}$ are asymptotically regarded as stable condition, if the feedback gain (K_{ij}) is chosen in satisfying at least one of the following conditions, j=1,2,…,J:

$$\bar{\lambda}_j \equiv \max_k \lambda_M(\bar{Q}_{kj})<0 \qquad \text{for } k=1,2,\cdots,N_j \tag{8a}$$

$$\lambda_{ij} \equiv \lambda_m(Q_{ij})>0 \quad \text{for } i=1,2,\cdots,r_j \tag{8b}$$

$$\lambda_{ifj} \equiv \lambda_m(Q_{ifj})>0 \quad \text{for } i<f\le r_j \tag{8c}$$

or

$$\Lambda_j \equiv \begin{bmatrix} -\bar{\lambda}_j & 0 & 0 & \cdots & 0 \\ 0 & \lambda_{1j} & 1/2\lambda_{12j} & \cdots & 1/2\lambda_{1r_jj} \\ 0 & 1/2\lambda_{12j} & \lambda_{2j} & \cdots & 1/2\lambda_{2r_jj} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1/2\lambda_{1r_jj} & 1/2\lambda_{2r_jj} & \cdots & \lambda_{r_jj} \end{bmatrix}>0 \tag{9}$$

where

$$\bar{Q}_{kj} \equiv I-P_{kj}, \quad k=1,2,\cdots,N_j \tag{10}$$

$$Q_{ij} = -\left\{ (A_{ij} - B_{ij}K_{ij})^T P_j + P_j (A_{ij} - B_{ij}K_{ij}) + \overline{P}_j + P_j \overline{A}_{ij} P_j + \sum_{n=1}^{J} \left[(\frac{J-1}{J}) I + P_j C_{nj} C_{nj}^T P_j \right] \right\} \quad (11)$$

$$Q_{ifj} = -2\left\{ G^T{}_{ifj} P_j + P_j G_{ifj} + \overline{P}_j + P_j \overline{A}_{ij} P_j + \sum_{n=1}^{J} \left[(\frac{J-1}{J}) I + P_j C_{nj} C_{nj}^T P_j) \right] \right\}, \quad i < f \le r_j \quad (12)$$

with

$$\overline{P}_j = \sum_{k=1}^{N_j} P_{kj}, \quad \overline{A}_{ij} = \sum_{k=1}^{N_j} A_{ikj} A_{ikj}^T, \quad G_{ifj} = \frac{(A_{ij} - B_{ij}K_{fj}) + (A_{fj} - B_{fj}K_{ij})}{2},$$

$$P_j = P_j^T > 0, \quad P_{kj} = P_{kj}^T > 0, \quad k = 1, 2, \cdots, N_j$$

and $\lambda_M(.)$ is the maximum eigenvalues; $\lambda_m(.)$ is minimal eigenvalues.

Proof: See Appendix.

Remark 1: Both conditions are met by default. The stability of systems F with delays can be verified using the equations in (8) and (9). Therefore, it is advisable to check for asymptotic stability under certain conditions. If this fails, another condition is required.

An evolutionary bat algorithm (EBA) based on a complex system of bats in the wild is proposed. Unlike other cluster reconnaissance algorithms, the strength of EBA lies in the fact that only one of the parameter (called the environment) is determined. Therefore, it is necessary to use an algorithm to solve the problem (Yan *et al.* 1998, Tsai *et al.* 2015, Zandi *et al.* 2018). During the evolution, the choice of support determines the different phases of the study. In this study, we chose air because it is the original natural habitat in which bats live. The capabilities of the EBA could be summarized by four steps:

Initialization: random assignment of artificial reagents, diffusion in the solution area.

Movement: An artificial example is movement. Generate a random number and make sure it doesn't exceed a fixed heart rate intensity. If positive, a random walking process is used to move the artificial specimen. $x_i^t = x_i^{t-1} + D$, in which these x^t_i indicate the coordinate in these i-th artificial agents in thees t-th iteration, then x^{t-1}_i is the last iteration i-th artificial agents, and D moving distance as follows.

$$D = \gamma \cdot \Delta T$$

where y=0.17, $\Delta T \in [-1,1]$ random number when the chosen mediums are air.

$$x_i^{tR} = \beta\left(x_{\text{best}} - x_i^t\right), \quad \beta \in [0,\ 1]$$

where random β; x_{best} is almost best solution are found so long throughout all of the artificial agents; and x^{tR}_i new coordinates in these artificial mean upon each walking movement.

Then we use the custom fitness function to calculate the artificial treatment fit and update it using the best stored solution.

4. Example

In this section, we will examine Fisher's equations and temperature control of high-speed aircraft cooling coils to demonstrate about this effectiveness of these proposed method in design. Fisher's equations have been used as the basis for various models of spatial gene spread of populations, chemical wave propagation, flame propagation, branched brown motion processes, and reactor theory. A full description of the Fisher equation is provided in (Zhang 2015, Zhou *et al.* 2015).

$$\begin{cases} \dfrac{\partial \bar{x}}{\partial t} = \alpha \dfrac{\partial^2 \bar{x}}{\partial z^2} + \beta_1 \bar{x} - \beta_2 \bar{x}^2 \\ \qquad + \bar{b}_u(z)u(t) + \bar{b}_\omega(z)\omega(t) \\ y(t) = \int_0^\pi \delta_\epsilon \left(z - \dfrac{\pi}{2}\right) \bar{x}(z,t)dz, t = t_k \end{cases}$$

subject to the Dirichlet boundary conditions

$$\bar{x}(0,t) = 0,\ \bar{x}(\pi,t) = 0$$

The purpose of this example is to create the fuzzy controller based on the system stabilization model (4.1).

J=1, then

Rule No. 1: If $x_{11}(t)$ is about 4; Rule No. 2: If the $x_{11}(t)$ is about -4

Then we say

$$\dot{x}_1(t) = A_{11}x_1(t) + \sum_{k=1}^{3} A_{1k1}x(t-\tau_{k1}) + B_{11}u_1(t);\quad \dot{x}_1(t) = A_{21}x_1(t) + \sum_{k=1}^{3} A_{2k1}x(t-\tau_{k1}) + B_{21}u_1(t)$$

in

$$A_{11}=\begin{bmatrix} -9 & 1 \\ 3 & 2 \end{bmatrix},\ A_{21}=\begin{bmatrix} -35 & -4 \\ 5 & -34 \end{bmatrix},\ A_{111}=\begin{bmatrix} -6 & -3 \\ 2 & 2 \end{bmatrix},\ A_{121}=\begin{bmatrix} -5 & -4 \\ 0.5 & 3 \end{bmatrix},$$

$$A_{131}=\begin{bmatrix} -8 & -4 \\ 3 & 5 \end{bmatrix},\ A_{211}=\begin{bmatrix} 7 & 3 \\ -2 & -1 \end{bmatrix},\ A_{221}=\begin{bmatrix} 6 & 4 \\ -0.5 & -2 \end{bmatrix},\ A_{231}=\begin{bmatrix} 9 & 4 \\ -3 & -4 \end{bmatrix}, \tag{13}$$

$$B_{11} = \begin{bmatrix} 0.55 \\ -2.1 \end{bmatrix},\ B_{21} = \begin{bmatrix} 0.3 \\ 1.1 \end{bmatrix},\ \tau_{11} = 1.2 \text{ (sec)},\ \tau_{21} = 1.8 \text{ (sec)},\ \tau_{31} = 2.5 \text{ (sec)}$$

and membership functions for 2 rules are $\dfrac{1}{1+\exp[-2x_{11}(t)]}$ and $1 - M_{111}(x_{11}(t))$.

J=2, then

Rule 1: If $x_{12}(t)$ is about 0; Rule 2: If $x_{12}(t)$ is about 4

Then $\dot{x}_2(t) = A_{12}x_2(t) + \sum_{k=1}^{3} A_{1k2}x(t-\tau_{k2}) + B_{12}u_2(t)$; $\dot{x}_2(t) = A_{22}x_2(t) + \sum_{k=1}^{3} A_{2k2}x(t-\tau_{k2}) + B_{22}u_2(t)$

With

$$A_{12}=\begin{bmatrix}-10 & 1\\ 1 & 3\end{bmatrix},\ A_{22}=\begin{bmatrix}-34 & -4\\ 3 & -33\end{bmatrix},\ A_{112}=\begin{bmatrix}-2 & -3\\ 1 & 2\end{bmatrix},\ A_{121}=\begin{bmatrix}-1.5 & -2\\ 0.8 & 1.3\end{bmatrix},\ A_{132}=\begin{bmatrix}-4 & -1\\ 3.5 & 5\end{bmatrix},$$

$$A_{132}=\begin{bmatrix}-4 & -1\\ 3.5 & 5\end{bmatrix},\ A_{212}=\begin{bmatrix}3 & 3\\ -1 & -1\end{bmatrix},\ A_{222}=\begin{bmatrix}2.5 & 2\\ -0.8 & -0.3\end{bmatrix},\ A_{232}=\begin{bmatrix}5 & 1\\ -3.5 & -4\end{bmatrix},\ B_{12}=\begin{bmatrix}0.5\\ -2\end{bmatrix}, \tag{14}$$

$$B_{22}=\begin{bmatrix}0.36\\ 1\end{bmatrix},\ \tau_{12}=1.5\ \text{(sec)},\ \tau_{22}=2.2\ \text{(sec)},\ \tau_{32}=3.1\ \text{(sec)}$$

and membership functions for 2 rules are $\exp[-x^2_{12}(t)]$ and $1-M_{112}(x_{12}(t))$.
J=3, then

Rule No. 1: If $x_{13}(t)$ is about 3; Rule No. 2: If $x_{13}(t)$ is about -3

Then $\dot{x}_3(t)=A_{13}x_3(t)+\sum_{k=1}^{3}A_{1k3}x(t-\tau_{k3})+B_{13}u_3(t)$; $\dot{x}_3(t)=A_{23}x_3(t)+\sum_{k=1}^{3}A_{2k3}x(t-\tau_{k3})+B_{23}u_3(t)$.

With

$$A_{13}=\begin{bmatrix}-12.5 & 1\\ 3.2 & 2\end{bmatrix},\ A_{23}=\begin{bmatrix}-33 & -4\\ 6.7 & -32\end{bmatrix},\ A_{113}=\begin{bmatrix}-6.3 & -3\\ 3 & 2\end{bmatrix},\ A_{123}=\begin{bmatrix}-9 & -4.5\\ 0.5 & 7\end{bmatrix},$$

$$A_{133}=\begin{bmatrix}-18 & -0.4\\ 3 & 15\end{bmatrix},\ A_{213}=\begin{bmatrix}7.3 & 3\\ -3 & -1\end{bmatrix},\ A_{223}=\begin{bmatrix}10 & 4.5\\ -0.5 & -6\end{bmatrix},\ A_{233}=\begin{bmatrix}19 & 0.4\\ -3 & -14\end{bmatrix},\ B_{13}=\begin{bmatrix}0.5\\ -2.2\end{bmatrix}, \tag{15}$$

$$B_{23}=\begin{bmatrix}0.3\\ 1.4\end{bmatrix},\ \tau_{13}=1.3\ \text{(sec)},\ \tau_{23}=1.9\ \text{(sec)},\ \tau_{33}=3.4\ \text{(sec)}$$

and membership functions for 2 rules are $\frac{1}{1+\exp[-4x_{13}(t)]}$ and $1-M_{113}(x_{13}(t))$.

In addition, the interconnection are described as

$$C_{21}=\begin{bmatrix}2 & 0.1\\ -0.2 & 4\end{bmatrix},\ C_{31}=\begin{bmatrix}1.5 & 0.1\\ 1.4 & 1.8\end{bmatrix},\ C_{12}=\begin{bmatrix}2 & -2.4\\ 1.6 & 3\end{bmatrix},$$
$$C_{32}=\begin{bmatrix}1.3 & 0.2\\ 1.7 & -3.8\end{bmatrix},\ C_{13}=\begin{bmatrix}2 & 0.1\\ 0.3 & 3\end{bmatrix},\ C_{23}=\begin{bmatrix}2.2 & 0.4\\ 1.4 & 1.6\end{bmatrix}. \tag{16}$$

Therefore, the multiple time-delay fuzzy large-scale system can be summarized as

$$F:\begin{cases}\dot{x}_1(t)=\sum_{i=1}^{2}h_{i1}(t)[A_{i1}x_1(t)+B_{i1}u_1(t)+\sum_{k=1}^{3}A_{ik1}x_1(t-\tau_{k1})]+\phi_1(t) & \text{(17a)}\\ \dot{x}_2(t)=\sum_{i=1}^{2}h_{i2}(t)[A_{i2}x_2(t)+B_{i2}u_2(t)+\sum_{k=1}^{3}A_{ik2}x_2(t-\tau_{k2})]+\phi_2(t) & \text{(17b)}\\ \dot{x}_3(t)=\sum_{i=1}^{2}h_{i3}(t)[A_{i3}x_3(t)+B_{i3}u_3(t)+\sum_{k=1}^{3}A_{ik3}x_3(t-\tau_{k3})]+\phi_3(t) & \text{(17c)}\\ \phi_j(t)=\sum_{\substack{n=1\\ n\neq j}}^{3}C_{nj}x_n(t), & \text{(17d)}\end{cases}$$

where $x_1^T(t)=[x_{11}(t)\ \ x_{21}(t)]$, $x_2^T(t)=[x_{12}(t)\ \ x_{22}(t)]$, $x_3^T(t)=[x_{13}(t)\ \ x_{23}(t)]$ are in and the matrices A_{ij}, B_{ij} and τ_{kj} are shown in Eqs. (17a)-(17c).

The pairs (A_{ij}, B_{ij}), i=1,2; then j=1,2,3 are all locally controllable. Then we need to control (4.5) where three designed fuzzy controllers by these PDC schemes are described as the following.

Fuzzy controller j=1:

Rule No. 1: If $x_{11}(t)$ is about 4; Rule No. 2: If $x_{11}(t)$ is about -4

Then one of

$$u_1(t)=-K_{11}x_1(t)\text{ ; or }\ u_1(t)=-K_{21}x_1(t) \tag{18}$$

Choosing these closed-loop eigenvalues (-27.8, -8) for an $A_{11}-B_{11}K_{11}$ and these closed-loop eigenvalues at (-17, -28) are $A_{11}-B_{11}K_{11}$, we would get K_{11}=[-0.8848 -13.946] and then K_{21}=[-27.153 -14.4128].

Fuzzy controller j=2:

Rule No. 1: If $x_{12}(t)$ is considered about 0; Rule No. 2: If $x_{12}(t)$ is considered about 4

Then we get

$$u_2(t)=-K_{12}x_2(t),\ \text{ and we get }\ u_2(t)=-K_{22}x_2(t)\ , \tag{19}$$

In another way, we choose these closed-loop eigenvalues (-27.5, -8.5) in $A_{12}-B_{12}K_{12}$ and these closed-loop eigenvalues (-16.5, -29) in $A_{22}-B_{22}K_{22}$, we have K_{12}=[2.1493 -13.9627] and K_{22}=[-20.7884 -14.0162].

Fuzzy controller j=3:

Rule No. 1: If $x_{13}(t)$ is about 3; Rule No. 2: If $x_{13}(t)$ is about -3

Then the

$$u_3(t)=-K_{13}x_3(t)\text{ ; and Then }\ u_3(t)=-K_{23}x_3(t)\ . \tag{20}$$

Choosing one of the closed-loop eigenvalues saying (-28, -10) for $A_{13}-B_{13}K_{13}$ and one of these closed-loop eigenvalues (-20, -23) for $A_{23}-B_{23}K_{23}$, then some get K_{13}=[-1.7114 -12.1111] and K23=[-21.2908 -11.152].

The choice and dominance of an appropriate matrix to satisfy Theorem 1 will be key issues to address. In this article, we will use EBA to find the right solution. The solution obtained in this case can be divided into two categories: possible and impossible. That said, developing an adaptive function with binary arithmetic is an easier way to meet the needs of this application. In this article, we use the Lyapunov function method to construct a fitness function according to stability criteria derived from the LMI state. The logical AND operation is used in the fit function to validate a solution and generate a binary classification result for the solution found. The formula for the training function is:

$$F=\begin{cases}1, & \text{if } \Theta<0 \text{ and } P=P^T>0.\\ 0, & \text{otherwise.}\end{cases}$$

in which F is one of these fitness values and Θ is regarded as follows

$$\Theta=(A_i-B_iK)^T P+P(A_i-B_iK)$$

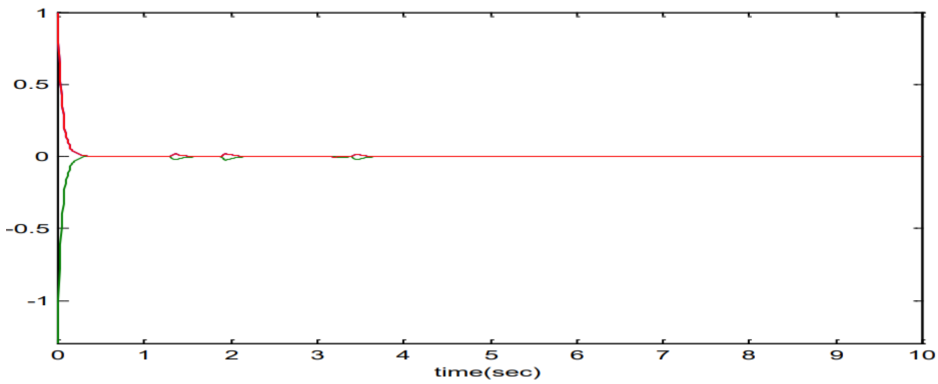

Fig. 1 The states of the nonlinear systems

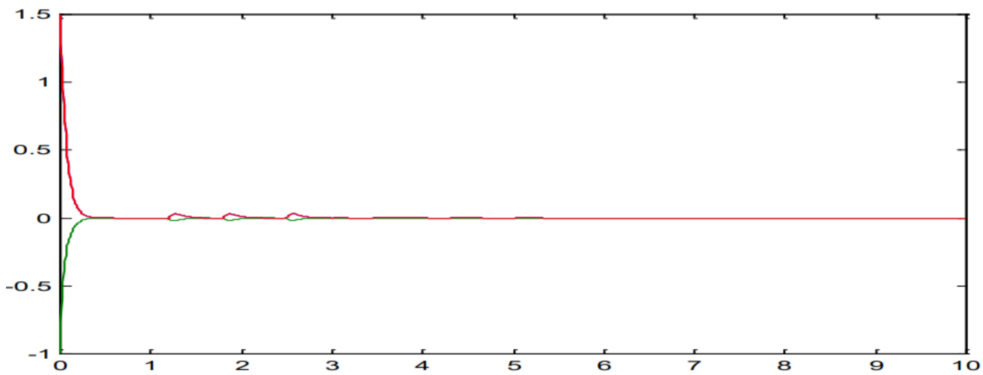

Fig. 2 The states of the nonlinear systems

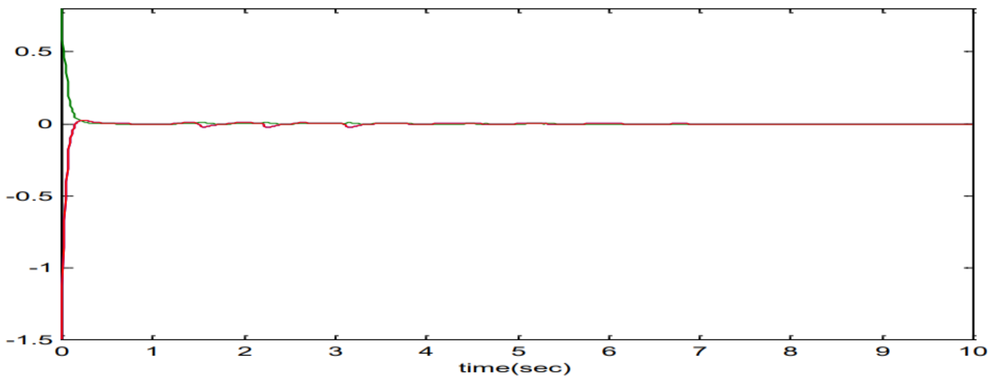

Fig. 3 The states of the nonlinear systems

If the elements of the array must always be symmetric, EBA is modified using the array. Also, the matrix and boundary conditions are used in the initialization process. Using the LMI optimization algorithm when the matrix is affected by the constraints of the same region, a workable solution is provided in Eq. (21).

$$P_1 = \begin{bmatrix} 0.78 & 0.22 \\ 0.22 & 0.25 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.7 & 0.22 \\ 0.22 & 0.3 \end{bmatrix}, \quad P_3 = \begin{bmatrix} 0.86 & 0.22 \\ 0.22 & 0.27 \end{bmatrix}. \tag{21}$$

Combining Eqs. (13)-(15), (21), (18)-(20) and (10)-(12) yields

$$\begin{gathered}
Q_{11} = \begin{bmatrix} 1.8532 & -1.9714 \\ -1.9714 & 2.6466 \end{bmatrix}, \quad Q_{21} = \begin{bmatrix} 15.6251 & -1.8688 \\ -1.8688 & 1.7487 \end{bmatrix}, \quad Q_{121} = \begin{bmatrix} 44.2194 & 15.9673 \\ 15.9673 & 5.0908 \end{bmatrix}, \\
Q_{12} = \begin{bmatrix} 1.6302 & -2.2738 \\ -2.2738 & 3.3238 \end{bmatrix}, \quad Q_{22} = \begin{bmatrix} 15.1128 & -0.8759 \\ -0.8759 & 2.8105 \end{bmatrix}, \quad Q_{122} = \begin{bmatrix} 42.5249 & 16.8178 \\ 16.8178 & 6.2273 \end{bmatrix}, \\
Q_{13} = \begin{bmatrix} 5.5364 & -3.3978 \\ -3.3978 & 2.9264 \end{bmatrix}, \quad Q_{23} = \begin{bmatrix} 15.3401 & -3.6436 \\ -3.6436 & 1.8599 \end{bmatrix}, \quad Q_{123} = \begin{bmatrix} 49.3992 & 13.7099 \\ 13.7099 & 3.0063 \end{bmatrix}, \quad \overline{Q}_{11} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \\
\overline{Q}_{21} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad \overline{Q}_{31} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad \overline{Q}_{12} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad \overline{Q}_{22} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \\
\overline{Q}_{32} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad \overline{Q}_{13} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \quad \overline{Q}_{23} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix} \overline{Q}_{33} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}
\end{gathered} \tag{22}$$

From Eq. (9), we have

$$\begin{gathered}
\Lambda_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.239 & -0.5979 \\ 0 & -0.5979 & 1.5014 \end{bmatrix}, \quad \Lambda_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.0506 & -0.3669 \\ 0 & -0.3669 & 2.7485 \end{bmatrix}, \\
\Lambda_3 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.5916 & -0.7423 \\ 0 & -0.7423 & 0.9381 \end{bmatrix}
\end{gathered} \tag{23}$$

and the eigenvalues of them are given below:

$$\lambda(\Lambda_1) = 0.1,\ 0.0008,\ 1.7396 > 0, \tag{24}$$

$$\lambda(\Lambda_2) = 0.1,\ 0.0016, 2.7975 > 0, \tag{25}$$

$$\lambda(\Lambda_3) = 0.1,\ 0.0026,\ 1.5272 > 0. \tag{26}$$

These math matrices in Λ_j (j=1,2,3) are considered with positive definite, which means Theorem 1 then let these fuzzy controllers (4.6-4.8) asymptotically in stabilization of the system (4.5). Simulation in results are all illustrated in the Figs. 1-3 with random initial conditions.

5. Conclusions

In this article, we propose a criterion for ensuring the asymptotic stability of large multiple delays, based on the direct Lyapunov method. Based on this criterion and distributed control scheme, the controllers are synthesized by the PDC to stabilize these large-scale systems with multiple delays. Finally, the numerical simulations confirmed the effectiveness of the method.

References

Bedirhanoglu, I. (2014), "A practical neuro-fuzzy model for estimating modulus of elasticity of concrete", *Struct. Eng. Mech.*, **51**(2), 249-265. https://doi.org/10.12989/sem.2014.51.2.249.

Chen, C.W. (2014a), "Interconnected TS fuzzy technique for nonlinear time-delay structural systems", *Nonlinear Dynam.*, **76**(1), 13-22. https://doi.org/10.1007/s11071-013-0841-8.

Chen, C.W. (2014b), "A criterion of robustness intelligent nonlinear control for multiple time-delay systems based on fuzzy lyapunov methods", *Nonlinear Dynam.*, **76**(1), 23-31.

Chen, T. (2020), "An intelligent algorithm optimum for building design of fuzzy structures", *Iran J. Sci. Technol. Trans Civ. Eng.*, **44**(2), 523-531.

Chen, T. (2020), "Evolved fuzzy NN control for discrete-time nonlinear systems", *J. Circuits Syst. Comput.*, **29**(1), 2050015. https://doi.org/10.1142/S0218126620500152.

Chen, T. (2020), "LMI based criterion for reinforced concrete frame structures", *Adv. Concr. Constr.*, **9**(4), 407-412. https://doi.org/10.12989/acc.2020.9.4.407.

Chen, T. (2020), "On the algorithmic stability of optimal control with derivative operators", *Circuits Syst. Signal Process.*, **39**(12), 5863-5881. https://doi.org/10.1007/s00034-020-01447-1.

Chen, T., Khurram, S. and Cheng, C. (2019), "A relaxed structural mechanics and fuzzy control for fluid-structure dynamic analysis", *Eng. Comput.*, **36**(7), 2200-2219. https://doi.org/10.1108/EC-11-2018-0522.

Chen, T., Khurram, S. and Cheng, C. (2019), "Prediction and control of buildings with sensor actuators of fuzzy EB algorithm", *Earthq. Struct.*, **17**(3), 307-315. https://doi.org/10.12989/eas.2019.17.3.307.

Eswaran, M. and Reddy, G.R. (2016), "Numerical simulation of tuned liquid tank-structure systems through sigma-transformation based fluid-structure coupled solver", *Wind Struct.*, **23**(5), 421-447. https://doi.org/10.12989/was.2016.23.5.421.

Feng, G., Cao, S.G., Rees, N.W. and Chak, C.K. (1997), "Design of fuzzy control systems with guaranteed stability", *Fuzzy Sets Syst.*, **85**, 1-10. https://doi.org/10.1016/0165-0114(95)00375-4.

Mori, T. (1985), "Criteria for asymptotic stability of linear time delay systems", *IEEE T. Automat. Contr.*, **30**(2), 158-162. https://doi.org/10.1109/TAC.1985.1103901.

Razavi, A. and Sarkar, P.P. (2018), "Laboratory investigation of the effects of translation on the near-ground tornado flow field", *Wind Struct.*, **26**(3), 179-190. https://doi.org/10.12989/was.2018.26.3.179.

Tanaka, K. and Sugeno, M. (1992), "Stability analysis and design of fuzzy control system", *Fuzzy Set. Syst.*, **45**(2), 135-156.

Tanaka, K., Ikeda, T. and Wang, H.O. (1996), "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability, H/sup/spl infin//control theory, and linear matrix inequalities", *IEEE T. Fuzzy Syst.*, **4**(1), 1-13. https://doi.org/10.1109/91.481840.

Trinh, H. and Aldeen, M. (1995), "A comment on decentralized stabilization of the large scale interconnected systems with delays", *IEEE T. Automat. Contr.*, **40**(5), 914-916. https://doi.org/10.1109/9.273363.

Tsai, P.W., Hayat, T., Ahmad, B. and Chen, C.W. (2015), "Structural system simulation and control via NN based fuzzy model", *Struct. Eng. Mech.*, **56**(3), 385-407. https://doi.org/10.12989/sem.2015.56.3.385.

Wang, H.O., Tanaka, K. and Griffin, M.F. (1996), "An approach to fuzzy control of nonlinear systems: Stability and design issues", *IEEE T. Fuzzy Syst.*, **4**(1), 14-23. https://doi.org/10.1109/91.481841.

Yan, X.G. and Dai, G.Z. (1998), "Decentralized output feedback robust control for nonlinear large-scale systems", *Automatica*, **34**(11), 1469-1472. https://doi.org/10.1016/S0005-1098(98)00090-9.

Yang, G.H. and Zhang, S.Y. (1996), "Decentralized control of a class of large-scale systems with symmetrically interconnected subsystems", *IEEE T. Automat. Contr.*, **41**(5), 710-713.

https://doi.org/10.1109/9.489207.

Zandi, Y., Shariati, M., Marto, A., Wei, X., Karaca, Z., Dao, D., Toghroli, A., Hashemi, M.H., Sedghi, Y., Wakil, K. and Khorami, M. (2018), "Computational investigation of the comparative analysis of cylindrical barns subjected to earthquake", *Steel Compos*. *Struct*., **28**(4), 439-447. http://doi.org/10.12989/scs.2018.28.4.439.

Zhang, Y. (2015), "A fuzzy residual strength based fatigue life prediction method", *Struct*. *Eng*. *Mech*., **56**(2), 201-221. https://doi.org/10.12989/sem.2015.56.2.201

Zhou, X., Lin, Y. and Gu, M. (2015), "Optimization of multiple tuned mass dampers for large-span roof structures subjected to wind loads", *Wind Struct*., **20**(3), 363-388. https://doi.org/10.12989/was.2015.20.3.363.

GY

Appendix: Proof of Theorem 1

(I): Let these Lyapunov function in these multiple time-delay fuzzy large-scale systems ***F*** are defined as

$$V=\sum_{j=1}^{J}v_j(t)=\sum_{j=1}^{J}\left\{x_j^T(t)P_jx_j(t)+\sum_{k=1}^{N_j}\int_0^{\tau_{kj}}x_j^T(t-\tau)P_{kj}x_j(t-\tau)d\tau\right\} \tag{A1}$$

where $P_j=P_j^T>0$ and $P_{kj}=P_{kj}^T>0$, $k=1,2,\cdots,N_j$. We therefore evaluate these time derivatives of V in the trajectories of Eq. (3.3), so we have

$$\dot V=\sum_{j=1}^{J}\dot v_j(t)$$

$$=\sum_{j=1}^{J}[\dot x_j^T(t)P_jx_j(t)+x_j^T(t)P_j\dot x_j(t)+\sum_{k=1}^{N_j}(x_j^T(t)P_{kj}x_j(t)-x_j^T(t-\tau_{kj})P_{kj}x_j(t-\tau_{kj}))]$$

$$=\sum_{j=1}^{J}\{[\sum_{i=1}^{r_j}\sum_{f=1}^{r_j}h_{ij}(t)h_{fj}(t)((A_{ij}-B_{ij}K_{fj})x_j(t)+\sum_{k=1}^{N_j}A_{ikj}x(t-\tau_{kj})+\phi_j(t))]^TP_jx_j(t)$$

$$+x_j^T(t)P_j[\sum_{i=1}^{r_j}\sum_{f=1}^{r_j}h_{ij}(t)h_{fj}(t)((A_{ij}-B_{ij}K_{fj})x_j(t)+\sum_{k=1}^{N_j}A_{ikj}x(t-\tau_{kj})+\phi_j(t))]$$

$$+\sum_{k=1}^{N_j}(x_j^T(t)P_{kj}x_j(t)-x_j^T(t-\tau_{kj})P_{kj}x_j(t-\tau_{kj}))\}$$

$$=\sum_{j=1}^{J}\sum_{i=f=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^TP_j+P_j(A_{ij}-B_{ij}K_{ij})+\bar P_j]x_j(t) \tag{A2}$$

$$+\sum_{j=1}^{J}\sum_{i\neq f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[(A_{ij}-B_{ij}K_{fj})^TP_j+P_j(A_{ij}-B_{ij}K_{fj})+\bar P_j]x_j(t)$$

$$+\sum_{j=1}^{J}[\phi_j^T(t)P_jx_j(t)+x_j^T(t)P_j\phi_j(t)]-\sum_{j=1}^{J}\sum_{k=1}^{N_j}[x_j^T(t-\tau_{kj})P_{kj}x_j(t-\tau_{kj})]$$

$$+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{k=1}^{N_j}h_{ij}(t)[x_j^T(t-\tau_{kj})A_{ikj}^TP_jx_j(t)+x_j^T(t)P_jA_{ikj}x_j(t-\tau_{kj})]$$

$$\le\sum_{j=1}^{J}\sum_{i=f=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^TP_j+P_j(A_{ij}-B_{ij}K_{ij})+\bar P_j]x_j(\mathrm{t})$$

$$+\sum_{j=1}^{J}\sum_{i\neq f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[(A_{ij}-B_{ij}K_{fj})^TP_j+P_j(A_{ij}-B_{ij}K_{fj})+\bar P_j]x_j(t)$$

$$
\begin{aligned}
&+\sum_{j=1}^{J}[\phi_j^T(t)P_j x_j(t)+x_j^T(t)P_j\phi_j(t)]-\sum_{j=1}^{J}\sum_{k=1}^{N_j}[x_j^T(t-\tau_{kj})P_{kj}x_j(t-\tau_{kj})]\\
&+\sum_{j=1}^{J}\sum_{i=1}^{r_j}\sum_{f=1}^{r_j}h_{fj}(t)\sum_{k=1}^{N_j}h_{ij}(t)[x_j^T(t)P_j A_{ikj}A_{ikj}^T P_j x_j(t)+x_j^T(t-\tau_{kj})x_j(t-\tau_{kj})]\\
&=\sum_{j=1}^{J}\sum_{i=f=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^T P_j+P_j(A_{ij}-B_{ij}K_{ij})+\overline{P}_j]x_j(t)\\
&+\sum_{j=1}^{J}\sum_{i\neq f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[(A_{ij}-B_{ij}K_{fj})^T P_j+P_j(A_{ij}-B_{ij}K_{fj})+\overline{P}_j]x_j(t)\\
&+\sum_{j=1}^{J}[\phi_j^T(t)P_j x_j(t)+x_j^T(t)P_j\phi_j(t)]-\sum_{j=1}^{J}\sum_{k=1}^{N_j}[x_j{}^T(t-\tau_{k_j})P_{kj}x_j(t-\tau_{kj})]\\
&+\sum_{j=1}^{J}\sum_{i=f=1}^{r_j}h_{ij}{}^2(t)[x_j^T(t)P_j\overline{A}_{ij}P_j x_j(t)]\\
&+\sum_{j=1}^{J}\sum_{\substack{i=1\\ i\neq f}}^{r_j}h_{ij}(t)h_{fj}(t)[x_j^T(t)P_j\overline{A}_{ij}P_j x_j(t)]+\sum_{j=1}^{J}\sum_{k=1}^{N_j}x_j^T(t-\tau_{kj})Ix_j(t-\tau_{kj})
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
&=\sum_{j=1}^{J}\sum_{i=f=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^T P_j+P_j(A_{ij}-B_{ij}K_{ij})+\overline{P}_j+P_j\overline{A}_{ij}P_j]x_j(t)\\
&+\sum_{j=1}^{J}\sum_{\substack{i=1\\ i\neq f}}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[(A_{ij}-B_{ij}K_{fj})^T P_j+P_j(A_{ij}-B_{ij}K_{fj})\\
&+(\overline{P}_j+P_j\overline{A}_{ij}P_j)]x_j(t)+\sum_{j=1}^{J}[\phi_j^T(t)P_j x_j(t)+x_j^T(t)P_j\phi_j(t)]\\
&-\sum_{j=1}^{J}\sum_{k=1}^{N_j}\left[x_j^T(t-\tau_{kj})P_{kj}x_j(t-\tau_{kj})\right]+\sum_{j=1}^{J}\sum_{k=1}^{N_j}x_j^T(t-\tau_{kj})Ix_j(t-\tau_{kj})\\
&=\sum_{j=1}^{J}\sum_{i=f=1}^{r_j}h_{ij}^2(t)x_j^T(t)[(A_{ij}-B_{ij}K_{ij})^T P_j+P_j(A_{ij}-B_{ij}K_{ij})+\overline{P}_j+P_j\overline{A}_{ij}P_j]x_j(t)\\
&+\sum_{j=1}^{J}\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)[2(G_{ifj}^T P_j+P_jG_{ifj}+\overline{P}_j+P_j\overline{A}_{ij}P_j)]x_j(t)\\
&+\sum_{j=1}^{J}[\phi_j^T(t)P_j x_j(t)+x_j^T(t)P_j\phi_j(t)]+\sum_{j=1}^{J}\sum_{k=1}^{N_j}\left\{x_j^T(t-\tau_{kj})[I-P_{kj}]x_j(t-\tau_{kj})\right\}\\
&=D_1+D_2+D_3+D_4,
\end{aligned}
$$

where

$$D_1 \equiv \sum_{j=1}^{J} \sum_{i=f=1}^{r_j} h_{ij}^2(t) x_j^T(t)[(A_{ij} - B_{ij}K_{ij})^T P_j + P_j(A_{ij} - B_{ij}K_{ij}) + \overline{P}_j + P_j \overline{A}_{ij} P_j] x_j(t) \tag{A3}$$

$$D_2 \equiv \sum_{j=1}^{J} \sum_{i<f}^{r_j} h_{ij}(t) h_{fj}(t) x_j^T(t)[2(G^T{}_{ifj} P_j + P_j G_{ifj} + \overline{P}_j + P_j \overline{A}_{ij} P_j)] x_j(t) \tag{A4}$$,

$$\begin{aligned}
D_3 \equiv & \sum_{j=1}^{J} [\phi_j^T(t) P_j x_j(t) + x_j^T(t) P_j \phi_j(t)] \\
& = \sum_{j=1}^{J} \sum_{n\neq j}^{J} [x_n^T(t) C_{nj}^T P_j x_j(t) + x_j^T(t) P_j C_{nj} x_n(t)] \\
& \le \sum_{j=1}^{J} \sum_{n\neq j}^{J} [x_n^T(t) x_n(t) + x_j^T(t) P_j C_{nj} C_{nj}^T P_j x_j(t)] \\
& = \sum_{n=1}^{J} \sum_{j=1}^{J} (\frac{J-1}{J}) x_j^T(t) x_j(t) + \sum_{j=1}^{J} \sum_{n=1}^{J} x_j^T(t) P_j C_{nj} C_{nj}^T P_j x_j(t) \qquad \text{(A5)} \\
& = \sum_{j=1}^{J} \sum_{i=1}^{r_j} \sum_{f=1}^{r_j} \sum_{n=1}^{J} h_{ij}(t) h_{fj}(t) x_j^T(t) \left[(\frac{J-1}{J}) I + P_j C_{nj} C_{nj}^T P_j \right] x_j(t) \\
& = \sum_{j=1}^{J} \sum_{i=f}^{r_j} \sum_{n=1}^{J} h_{ij}^2(t) x_j^T(t) \left[(\frac{J-1}{J}) I + P_j C_{nj} C_{nj}^T P_j \right] x_j(t) \\
& + \sum_{j=1}^{J} \sum_{i\neq f}^{r_j} \sum_{n=1}^{J} h_{ij}(t) h_{fj}(t) x_j^T(t) \left[(\frac{J-1}{J}) I + P_j C_{nj} C_{nj}^T P_j \right] x_j(t)
\end{aligned}$$,

$$D_4 \equiv \sum_{j=1}^{J} \sum_{k=1}^{N_j} \{x_j{}^T(t-\tau_{kj})[I - P_{kj}] x_j(t-\tau_{kj})\} \le \sum_{j=1}^{J} \sum_{k=1}^{N_j} \lambda_M(I - P_{kj}) \left\| x_j(t-\tau_{kj}) \right\|^2 . \tag{A6}$$

Substituting Eqs. (A3)-(A6) into Eq. (A2) yields

$$\begin{aligned}
\dot{V} \le & \sum_{j=1}^{J} \sum_{i=f=1}^{r_j} h_{ij}^2(t) x_j^T(t)[(A_{ij} - B_{ij}K_{ij})^T P_j + P_j(A_{ij} - B_{ij}K_{ij}) + \overline{P}_j + P_j \overline{A}_{ij} P_j] x_j(t) \\
& + \sum_{j=1}^{J} \sum_{i<f}^{r_j} h_{ij}(t) h_{fj}(t) x_j^T(t)[2(G_{ifj}^T P_j + P_j G_{ifj} + \overline{P}_j + P_j \overline{A}_{ij} P_j)] x_j(t) \qquad \text{(A7)} \\
& + \sum_{j=1}^{J} \sum_{i=f}^{r_j} \sum_{n=1}^{J} h_{ij}^2(t) x_j^T(t) \left[(\frac{J-1}{J}) I + P_j C_{nj} C_{nj}^T P_j \right] x_j(t)
\end{aligned}$$

$$+\sum_{j=1}^{J}\sum_{i\neq f}^{r_j}\sum_{n=1}^{J}h_{ij}(t)h_{fj}(t)x_j^T(t)\left[(\frac{J-1}{J})I+P_jC_{nj}C_{nj}^TP_j\right]x_j(t)$$
$$+\sum_{j=1}^{J}\sum_{k=1}^{N_j}\lambda_M(I-P_{kj})\left\|x_j(t-\tau_{kj})\right\|^2$$
$$=\sum_{j=1}^{J}\sum_{i=f=1}^{r_j}h_{ij}^2(t)x_j^T(t)\Big\{(A_{ij}-B_{ij}K_{ij})^TP_j+P_j(A_{ij}-B_{ij}K_{ij})+\overline{P}_j+P_j\overline{A}_{ij}P_j$$
$$+\sum_{n=1}^{J}\left[(\frac{J-1}{J})I+P_jC_{nj}C_{nj}^TP_j\right]\Big\}x_j(t)$$
$$+\sum_{j=1}^{J}\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)\Big\{2(G^T{}_{ifj}P_j+P_jG_{ifj}+\overline{P}_j+P_j\overline{A}_{ij}P_j)$$
$$+2\sum_{n=1}^{J}\left[(\frac{J-1}{J})I+P_jC_{nj}C_{nj}^TP_j\right]\Big\}x_j(t)+\sum_{j=1}^{J}\sum_{k=1}^{N_j}\lambda_M(I-P_{kj})\left\|x_j(t-\tau_{kj})\right\|^2 \tag{A7}$$
$$=-\sum_{j=1}^{J}\left\{\sum_{i=f=1}^{r_j}h_{ij}^2(t)x_j^T(t)Q_{ij}x_j(t)+\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)x_j^T(t)Q_{ifj}x_j(t)-\sum_{k=1}^{N_j}\lambda_M(I-P_{kj})\left\|x_j(t-\tau_{kj})\right\|^2\right\}$$
$$\leq-\sum_{j=1}^{J}\left\{[\sum_{i=1}^{r_j}h_{ij}^2(t)\lambda_m(Q_{ij})+\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)\lambda_m(Q_{ifj})]\left\|x_j(t)\right\|^2-\overline{\lambda}_j\sum_{k=1}^{N_j}\left\|x_j(t-\tau_{kj})\right\|^2\right\}$$

According to these Eq. (8), we therefore get $\dot{V}<0$ as well as the proof in condition (I) is then satisfied.

(II): Based in Eq. (A7), we then get

$$\dot{V}\leq-\sum_{j=1}^{J}\left\{[\sum_{i=1}^{r_j}h_{ij}^2(t)\lambda_m(Q_{ij})+\sum_{i<f}^{r_j}h_{ij}(t)h_{fj}(t)\lambda_m(Q_{ifj})]\left\|x_j(t)\right\|^2-\overline{\lambda}_j\sum_{k=1}^{N_j}\left\|x_j(t-\tau_{kj})\right\|^2\right\}$$
$$=-\sum_{j=1}^{J}\{\begin{bmatrix}\sqrt{\sum_{k=1}^{N_j}\left\|x_j(t-\tau_{kj})\right\|^2} & h_{1j}(t)\left\|x_j(t)\right\| & h_{2j}(t)\left\|x_j(t)\right\| & \cdots & h_{r_jj}(t)\left\|x_j(t)\right\|\end{bmatrix}$$
$$.\begin{bmatrix}-\overline{\lambda}_j & 0 & 0 & \cdots & 0\\ 0 & \lambda_{1j} & 1/2\lambda_{12j} & \cdots & 1/2\lambda_{1r_jj}\\ 0 & 1/2\lambda_{12j} & \lambda_{2j} & \cdots & 1/2\lambda_{2r_jj}\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 1/2\lambda_{1r_jj} & 1/2\lambda_{2r_jj} & \cdots & \lambda_{r_jj}\end{bmatrix}.\begin{bmatrix}\sqrt{\sum_{k=1}^{N_j}\left\|x_j(t-\tau_{kj})\right\|^2}\\ h_{1j}(t)\left\|x_j(t)\right\|\\ h_{2j}(t)\left\|x_j(t)\right\|\\ \vdots\\ h_{r_jj}(t)\left\|x_j(t)\right\|\end{bmatrix}\}$$
$$=-\sum_{j=1}^{J}H_j^T\Lambda_jH_j,$$

in which $H_j^T \equiv \left[\sqrt{\sum_{k=1}^{N_j} \left\| x_j(t-\tau_{kj}) \right\|^2} \quad h_{1j}(t)\left\| x_j(t) \right\| \quad h_{2j}(t)\left\| x_j(t) \right\| \quad \cdots \quad h_{r_j j}(t)\left\| x_j(t) \right\| \right]$. The Lyapunov math derivatives are negative if one of these matrices Λ_j (j=1,2..,J) is positive digit, which accomplish one of the proof in condition (II).