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Exact solution for axial vibration of the power, exponential and sigmoid FG nonlocal nanobeam

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Abstract. The present study investigates axial vibration of a FG nanobeam using nonlocal elasticity theory under clamped-clamped and clamped-free boundary conditions. Power law, exponential law and sigmoid law are applied as grading laws to examine the effect of the material distribution on axial vibration of the FG nanobeam. A parametric study was done to examine the effect of length scale on the dynamic behavior of the structure and the results are presented. It was observed that consideration of the nonlocal length scale is essential when analyzing the free vibration of a FG nanobeam. The results of the present study can be used as benchmarks in future studies of FG nanostructures.

Keywords: nanobeam; functionally graded materials; axial vibration; nonlocal elasticity

1. Introduction

Nanostructures have been a focus of study for researchers in recent decades. Most recently, functionally graded (FG) materials that exhibit smooth variation in material properties have been examined for use in FG nanostructures (Reddy 2011, Reddy and Kim 2012, Mochida and Ilanko 2016, Petrolo *et al.* 2016, Ebrahimi and Farazmandnia 2018, Ebrahimi and Fardshad 2018, Ebrahimi and Heidar 2018, Sayyad and Ghugal 2018). A survey of the literature suggests that the properties of materials on the nano-scale are size dependent and the influence of small variations in length scale must be considered to improve modeling of the mechanical behavior of nanomaterials. Non-local continuum theories that consider the material length scale may provide detailed and accurate predictions of the behavior of nanostructures (Reddy 2007, 2010). Reddy (Reddy 2007, 2010) has presented non-local versions of various beam and plate theories, and derived numerical solutions of bending, vibration, and buckling of nanostructures. Reddy and El-Borgito (Reddy and El-Borgi 2014) formulated the governing equations of the Euler-Bernoulli and Timoshenko beams based on Eringen's nonlocal differential constitutive model and modified von

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Karman nonlinear strains. According to this aforementioned discussion, a number of studies have been presented in order to use the nonlocal elasticity theory in constitutive law of nanomaterials as Refs. (Bastanfar *et al.* 2019, Hamidi *et al.* 2019, Hosseini *et al.* 2019, Hamidi *et al.* 2020, Hosseini *et al.* 2020, Khosravi *et al.* 2020a, b, c, d).

Rahmani and Pedram (Rahmani and Pedram 2014) applied Navier's method to study analytically the vibration behavior of graded nonlocal Timoshenko nanobeams. Aydogdu modeled a nonlocal elastic rod by considering small-scale effects on the axial vibration of nanostructures (Aydogdu 2009). In another study, Aydogdu examined axial vibration of single-walled carbon nanotubes embedded in elastic medium using nonlocal elasticity theory (Aydogdu 2012). Eltaher *et al.* studied the transverse vibration of Euler-Bernoulli FG nanobeams using the finite element method (Eltaher *et al.* 2012). Simsek *et al.* presented an analytical solution for bending and buckling of FG nanobeams using nonlocal Timoshenko beam theory. They derived governing equations and boundary conditions using the principle of minimum potential energy and developed a Navier-type solution for simple-support boundary conditions (Şimşek and Yurtcu 2013). Eltaher *et al.* used static and stability analysis of FG nanobeams to examine the size-dependency of FG nanobeam behavior (Eltaher *et al.* 2013). Nazemnezhad *et al.* investigated the transverse vibration of FG nanoscale beams using von Karman type nonlinearity (Nazemnezhad and Hosseini-Hashemi 2014). Nazemnezhad and Hosseini-Hashemi analyzed nonlinear free vibration of simply-supported FG nanobeams with surface effects. They investigated the effects of surface elasticity, tension and density on the nonlinear free vibration of FG nanobeams using Euler-Bernoulli beam theory (Hosseini-Hashemi and Nazemnezhad 2013). Rahmani and Pedram analyzed the effect of size on vibration of FG nanobeams. They modeled FG nanobeams using nonlocal Timoshenko beam theory and varied the material properties of the FG nanobeams along the thickness of the beam using power law (Rahmani and Pedram 2014). They showed that the power law index had an important influence on the vibration response of FG nanobeams and that dynamic behavior can be increased by selecting appropriate values for the power law index.

Khosravi and Hosseini (2020) employed the nonlocal elasticity theory and viscoelastic mass nanosensor to study the torsional behaviour of the model; the finite difference method was established to prove the accuracy of obtained results. Khosravi *et al.* (2020) established the nonlocal model along with the Rayleigh-Ritz theory to investigate the small scale torsional behaviour of the single-walled carbon nanotubes for free case and for the state in which model is subjected to the linear and harmonic torques. Khosravi *et al.* (2020) conducted torsional vibration of a single-walled carbon nanotube embedded in an elastic medium to evaluate the effect of the medium, excitation frequency, time constant, geometry and type of loads on the responses; also, the resonance behaviour was evaluated. Hosseini and Khosravi (2020) established the nonlocal theory to assess the free and forced torsional vibration of single-walled carbon nanotubes under both type of loadings.

Several studies have examined the transverse vibration of FG nanostructures (Asgharifard Sharabiani and Haeri Yazdi 2013, Şimşek 2014, Rahmani *et al.* 2016, 2017), but no studies have been found on axial vibration of FG nanobeams. The present study examines the influence of nonlocal parameters on axial vibration of FG nanobeams and an analytical solution for clamped-clamped and clamped-free boundary conditions is proposed. A parametric study was employed to investigate the effect of the power law index and nonlocal parameters on the dynamic behavior of a nanostructure.

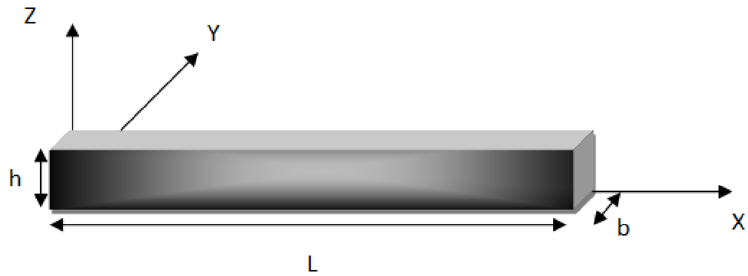


Fig. 1 Schematic illustration of FG nanobeam

2. Formulation

2.1 Materials properties

The nanobeam considered in this study was assumed to have length L, width b and thickness h (Fig. 1). The FG nanobeam was studied under clamped-clamped and clamped-free boundary conditions.

The material properties of the FG nanobeam, e.g., Young modulus (E) and mass density (ρ) changed across the thickness of the nanobeam as shown in Tables 1, 2 and 3 and was assumed to follow the rule of mixtures (Chi and Chung 2006) modified to achieve to a symmetric distribution (Mahi *et al.* 2010).

In these tables E_1 is the material property at the upper and lower surfaces, and E_2 is the material property in the middle of the FG nanobeam. In this relation, n is a non-negative number that demonstrates the variation profile through the thickness of the nanobeam. Tables 1 to 3 show that the upper and lower surfaces of the FG nanobeam (z=+h/2, -h/2) are purely metallic and the middle of the nanobeam (z=0) is purely ceramic.

2.2 Kinematic equations

The displacement components can be expressed in the following relation:

$$u(x,z,t)=u_0(x,t)-zv_{0,x} \tag{1}$$

$$v(x,z,t)=v_0(x,t) \tag{2}$$

where v_0 and u_0 are the transverse and the axial displacements of each point located on the mid-plane, and t is time variable. Based on EBT (Euler-Bernoulli beam theory), nonzero strain is defined as:

$$\varepsilon_{xx}=u_{0,x}(x,t)-zv_{0,xx}(x,t)=\varepsilon_{xx}^0-zk^0 \tag{3}$$

where k^0 is the strain related to bending and $\varepsilon_{xx}{}^0$ is extensional strain.

2.3 Equations of motion

Hamilton's principle (Tauchert 1974) is defined as:

Table 1 Power law

	$\frac{-h}{2} \le z \le 0$	$0 \le z \le h/2$
$\mathrm{E}(z)$	$(E_2 - E_1)\left(\frac{-2z}{h}\right)^n + E_1$	$(E_2 - E_1)\left(\frac{2z}{h}\right)^n + E_1$
$\rho(z)$	$(\rho_2 - \rho_1)\left(\frac{-2z}{h}\right)^n + \rho_1$	$(\rho_2 - \rho_1)\left(\frac{2z}{h}\right)^n + \rho_1$

Table 2 Exponential law

	$\frac{-h}{2} \le z \le 0$	$0 \le z \le h/2$
$\mathrm{E}(z)$	$E(z) = E_1 e^{\frac{-2z}{h}\ln\left(\frac{E_2}{E_1}\right)}$	$E(z) = E_1 e^{\frac{2z}{h}\ln\left(\frac{E_2}{E_1}\right)}$
$\rho(z)$	$\rho(z) = \rho_1 e^{\frac{-2z}{h}\ln\left(\frac{\rho_2}{\rho_1}\right)}$	$\rho(z) = \rho_1 e^{\frac{2z}{h}\ln\left(\frac{\rho_2}{\rho_1}\right)}$

Table 3 Sigmoid law

	$-\frac{h}{2} \le z \le -h/4$	$-\frac{h}{4} \le z \le 0$	$0 \le z \le h/4$	$\frac{h}{4} \le z \le h/2$
$\mathrm{E}(z)$	$E_1 + (E_1 - E_2)\left(1 - 0.5\left(2 + \frac{4z}{h}\right)^n\right)$	$E_1 + (E_1 - E_2)\left(-0.5\left(-\frac{4z}{h}\right)^n\right)$	$E_1 + (E_2 - E_1)\left(0.5\left(\frac{4z}{h}\right)^n\right)$	$E_1 + (E_2 - E_1)\left(1 - 0.5\left(2 + \frac{4z}{h}\right)^n\right)$
$\rho(z)$	$\rho_1 + (\rho_1 - \rho_2)\left(1 - 0.5\left(2 + \frac{4z}{h}\right)^n\right)$	$\rho_1 + (\rho_1 - \rho_2)\left(-0.5\left(-\frac{4z}{h}\right)^n\right)$	$\rho_1 + (\rho_2 - \rho_1)\left(0.5\left(\frac{4z}{h}\right)^n\right)$	$\rho_1 + (\rho_2 - \rho_1)\left(1 - 0.5\left(2 + \frac{4z}{h}\right)^n\right)$

Table 4 Material properties of FGM

Properties	Steel	Alumina (Al_2O_3)
E	210 (GPa)	390 (GPa)
ρ	7800 ($\frac{kg}{m^3}$)	3960 ($\frac{kg}{m^3}$)

$$\int_{t_1}^{t_2} (\delta U - \delta T - \delta V)\, dt = 0 \tag{4}$$

where δT is the virtual kinetic energy , δU is the virtual strain energy and δV is the virtual potential of external load.

$$\delta U = b \int \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx}(z)\, \delta\varepsilon_{xx}\, dz dx = b \int \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_{xx}(z) \left(\delta\varepsilon_{xx}^0 - z k^0\right) dz dx = b \int (N\, \delta\varepsilon_{xx}^0 - M\, \delta k^0) dx \tag{5}$$

$$\delta T = b\int\int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(z)[(u_{0,x} - zv_{0,xt})\left((\delta u_0)_{,x} - z(\delta v_0)_{,xt}\right) + v_{0,t}\delta v_{0,t}]dzdx$$
$$= b\int[I_0(u_{0,t}(\delta u_0)_{,t} + v_{0,t}(\delta v_0)_{,t}) - I_1\left(u_{0,t}(\delta v_0)_{,xt} + v_{0,xt}(\delta u_0)_{,t}\right) + I_2 v_{0,xt}(\delta v_0)_{,xt}]dx \tag{6}$$

$$\delta V = -b\int[f\,\delta u_0 + q\,\delta u_0 + \bar{N}v_{0,x}(\delta v_0)_{,x}]dx \tag{7}$$

The force resultant can be represented as:

$$N = \int_{-h/2}^{h/2} \sigma_{xx}(z)dz \tag{8}$$

The moment resultant and mass moment of inertia can be represented as:

$$M = \int_{-h/2}^{h/2} z\,\sigma_{xx}(z)dz \tag{9}$$

$$\begin{Bmatrix} I_0 \\ I_1 \\ I_2 \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} 1 \\ z \\ z^2 \end{Bmatrix} \rho(z)dz \tag{10}$$

where $\bar{N}$ is the applied axial force (compressive), $f(x,t)$ is the axial distributed force, and $q(x, t)$ is the transverse distributed force. Substituting Eqs. (5), (6) and (7) into Eq. (4), produces the Euler-Lagrange equation:

$$N_{,x} + f(x,t) = I_0 u_{0,tt} - I_1 v_{0,xtt} \tag{11}$$

$$M_{,xx} + q(x,t) - \frac{\partial}{\partial x}\left(\bar{N}v_{0,x}\right) = I_0 v_{0,tt} + I_1 u_{0,xtt} - I_2 v_{0,xxtt} \tag{12}$$

Based on the distribution of the material, I_1 and I_2 are parameters that equal zero. Thus:

$$N_{,x} + f(x,t) = I_0 u_{0,tt} \tag{13}$$

2.4 Nonlocal elasticity theory

Despite the elastic continuum theory that the stress field at point *X* depends only on the strain at the same point, based on Eringen nonlocal theory stress field at point *X* not only depends on strain at that point but also depends to all other points of the body. Thus the nonlocal stress tensor at point *X* can be obtained as follows:

$$\sigma^{nl}(X) = \int_V \mathrm{k}(|X' - X|, \tau) T(X') dV(X') \tag{14a}$$

$$T(X) = C(X) : \varepsilon(X) \tag{14b}$$

$T(X)$ represents the classical macroscopic stress tensor at point X, the kernel function $K\,|(X' - X), \tau|$ denotes the nonlocal modulus, $(X' - X)$ indicates the distance and τ is the material constant which depends on type of material. The macroscopic stress tensor at a point X in a Hookean solid is represented by T and is depends to the strain at the same point which is based on the generalized Hook's law. C is the fourth-order elasticity tensor which represents the double-dot product. A simplified equation of differential form is utilized due to the complicated solution of the integral constitutive equation, which is as follows:

$$\begin{gathered} L\sigma^{nl}(X) = (1 - \mu\nabla^2)\sigma^{nl} = T \\ \mu = \tau^2 l^2 \end{gathered} \tag{15}$$

$L = (1 - \mu\nabla^2)$ and ∇^2 indicates nonlocal differential and the Laplacian operator, respectively. τ is determined by $\tau = e_0\,\alpha/l$ where e_0 is a constant which varies based on each material and α and l represents the internal and external characteristic length. The nonlocal parameter which is represented by μ varies in accordance with different materials.

For Euler-Bernoulli nonlocal FG beam written as:

$$\sigma_{xx} - \mu \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E(z)\varepsilon_{xx} \tag{16}$$

where σ and ε are the nonlocal stress and strain, respectively and E is the Young's modulus.

Integrating Eq. (16) into the FG beam cross-sectional area obtains an axial force-strain of:

$$N - \mu N_{,xx} = b\left[A\right]\varepsilon_{xx}^0 - bk^0\left[B\right] \tag{17}$$

$$\begin{Bmatrix} A \\ B \\ D \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} 1 \\ z \\ z^2 \end{Bmatrix} E(z)\,dz \tag{18}$$

Eq. (17) results in:

$$N = b\left[A\right]\mathrm{u}_{0,\mathrm{x}} + \mu^2\left[-\left(f_{,x}\right) + I_0 u_{0,xtt}\right] \tag{19}$$

Substituting Eq. (19) into Eq. (11) obtains the displacement equation for free axial vibration as:

$$b\left[A\right]u_{0,xx} + \mu^2\left(I_0 u_{0,xxtt}\right) = I_0 u_{0,tt} \tag{20}$$

Hence

$$b\left[A\right]u_{0,xx}=\left(1-\mu^2\frac{\partial^2}{\partial x^2}\right)I_0u_{0,tt} \tag{21}$$

Eq. (21) is the consistent fundamental equation of the nonlocal FG nanobeam model for axial vibration. Assuming that nonlocal parameter μ is equal to zero produces the classic equation for a FG nanobeam model. Assuming the homogeneity of the material and nonlocal continuum, the equation obtained will be same that obtained by Ref. (Aydogdu 2009).

By solving Eq. (21) considering harmonic vibration and using the separation of variables method, u_0 can be defined as:

$$u_0\left(x,t\right)=F\left(x\right)\sin\left(\omega t\right) \tag{22}$$

Substituting Eq. (22) into Eq. (21) obtains:

$$F\left(x\right)_{,xx}+\beta^2F\left(x\right)=0 \tag{23}$$

where β is defined as:

$$\beta^2=\frac{I_0\omega^2}{b\left[A\right]-(\mu)^2I_0\omega^2} \tag{24}$$

Non-dimensional parameter Ω^2 is defined as:

$$\Omega^2=\frac{I_0\omega^2L^2}{b\left[A\right]} \tag{25}$$

Thus:

$$\beta^2=\frac{\Omega^2}{1-(\frac{\mu}{L})^2\Omega^2} \tag{26}$$

The solution for Eq. (23) takes the form:

$$F\left(x\right)=C_1cos\left(\beta x\right)+C_2sin\left(\beta x\right) \tag{27}$$

Clamped-clamped and clamped free boundary conditions are assumed to determine the frequency parameter and mode shapes:

$$\text{Clamped-clamped}\quad u\left(0,t\right)=u\left(L,t\right)=0 \tag{28}$$

$$\text{Clamped-free}\quad u\left(0,t\right)=u_{,x}\left(L,t\right)=0 \tag{29}$$

The non-dimensional form of Eq. (28) can be applied for clamped-clamped boundary conditions in Eq. (27) as:

$$C_1=0 \tag{30}$$

$$C_2 sin\left(\beta\right)=0 \tag{31}$$

To satisfy Eq. (31), $\beta=k\pi$, k=1,2,… and k is the mode number. Using Eq. (25), the frequency parameter for clamped-clamped boundary conditions can be defined as:

$$\Omega^2=\frac{\left(k\,\pi\right)^2}{1+(\frac{\mu}{L})^2(k\,\pi)^2} \tag{32}$$

Under clamped-free boundary conditions, the non-dimensional form of Eq. (29) can be substituted into Eq. (27) as follows:

$$C_1=0 \tag{33}$$

$$cos\left(\beta\right)=0 \tag{34}$$

If $\beta=\frac{\left(2k-1\right)\pi}{2}, k=1,2,\ldots$, Eq. (34) satisfies and the frequency parameter for a clamped-free nanobeam becomes:

$$\Omega^2=\frac{\left(\frac{\left(2k-1\right)\pi}{2}\right)^2}{1+(\frac{\mu}{L})^2(\frac{\left(2k-1\right)\pi}{2})^2} \tag{35}$$

Assuming μ=0 makes it arrive at the classic form of the frequency parameter.

3. Results and discussion

3.1 Physical properties

The FG nanobeam is composed of steel and alumina and the properties vary throughout the thickness of the beam according to the three functions as described. The bottom and top surfaces of the beam are pure steel. The middle of the beam is pure alumina. Table 4 lists the pure properties of the steel and alumina (h=0.5 nm and b=1 nm).

To demonstrate the effect of small length scale on the axial vibration of a FG nanobeam, the ratio of local frequency to nonlocal frequency was studied under clamped-clamped and clamped-free boundary conditions for different modes, scale coefficients and lengths. Frequency is non-dimensional in the following equation:

$$Nondimensioal\ frequency=\frac{\Omega^2\left[A\right]\rho_{Alumina}}{I_0 E_{Alumina}} \tag{36}$$

To compare the results of this study with the results of previous studies, n in the power law is assumed to equal zero. This allows comparison with the results of Ref. (Aydogdu 2009), who also

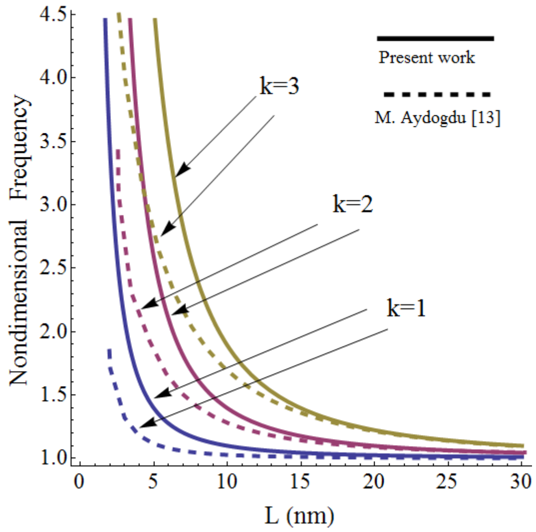


Fig. 2 Nondimensional frequency for clamped-clamped boundary condition

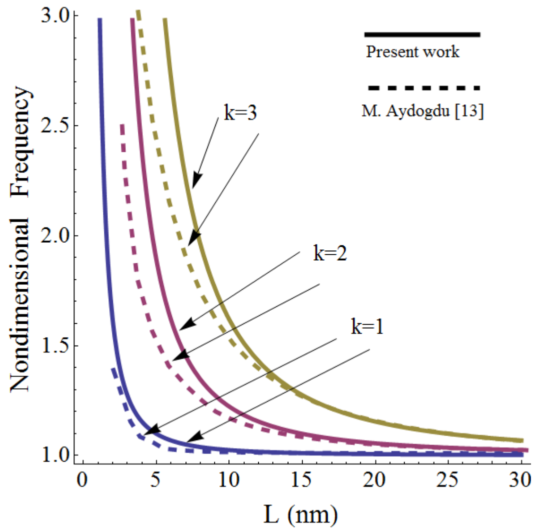


Fig. 3 Nondimensional frequency for clamped-free boundary condition

assumed clamped-clamped and clamped-free boundary conditions. The small scale parameter in the study was μ=1 nm. Figs. 2 and 3 are plotted for the first three frequencies. Figs. 2 and 3 indicate that the non-dimensional frequency decreased as the length increased.

The results of Ref. (Aydogdu 2009) are also shown for comparison in Figs. 2 and 3. When n=0, the results of the present study are consistent with the previous analytical results.

Figs. 4(a), 4(b) and 4(c) show the clamped-clamped boundary conditions for power law. Clamped-free boundary conditions are shown in Figs. 4(d), 4(e) and 4(f). Parameter n is assumed for power law 0.1 in Figs. 4(a) and 4(d) and n = 1 was assumed in Figs. 4(b) and 4(e). In Figs. 4(c) and 4(f), n=10 was assumed. All parts of Fig. 4 incorporate the first three frequencies (k=1, 2, 3).

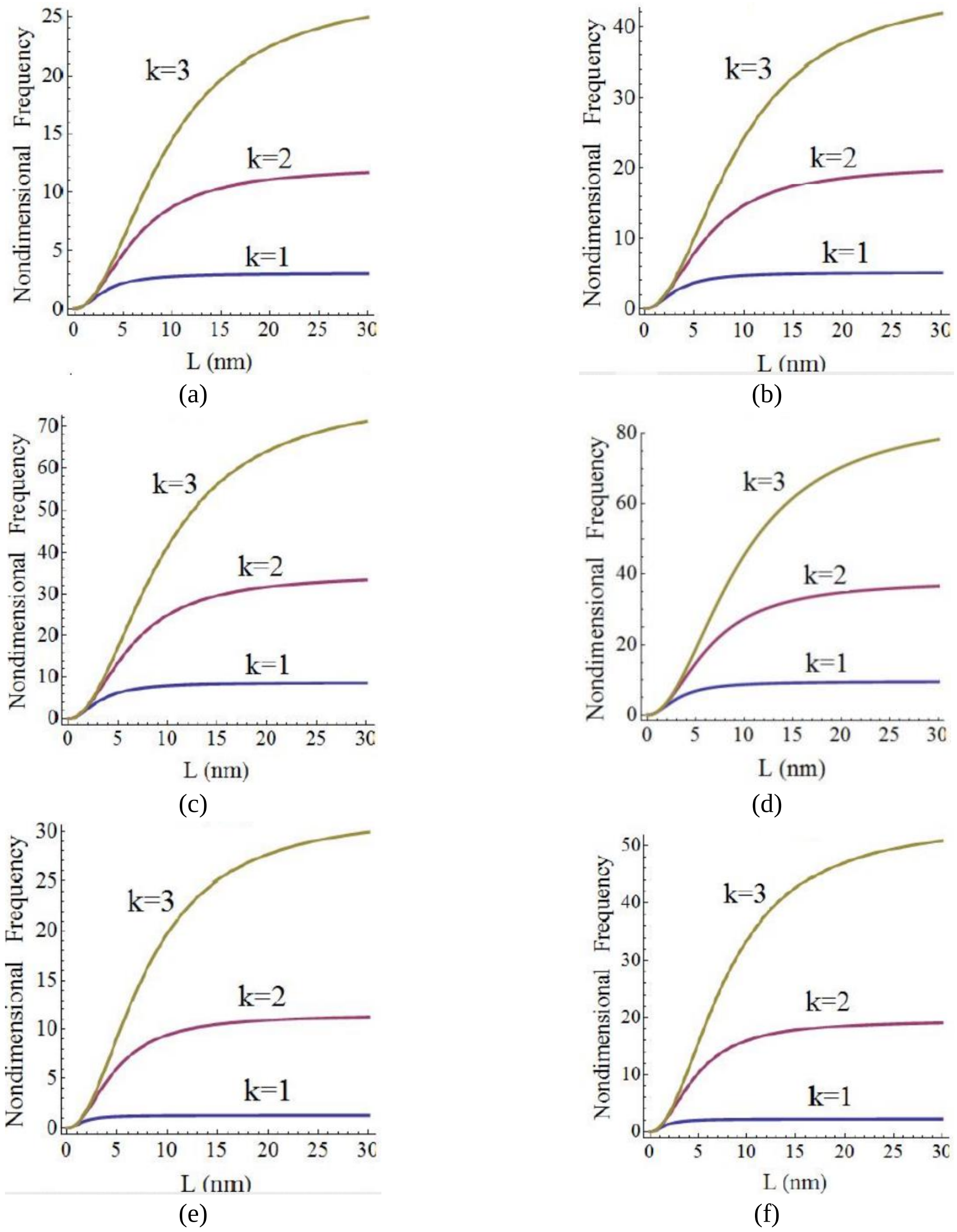


Fig. 4 Nondimensional frequency versus length for three frequencies modes in which scale coefficient

In these figures, it is evident that non-dimensional frequency increases as length increases.

As shown, non-dimensional frequency for the clamped-clamped condition was greater than that for the clamped-free condition. As the non-dimensional frequency for a short length significantly increased, the rate of increase in the non-dimensional frequency slowed.

Fig. 5 shows the exponential function. Figs. 5(a) and 5(b) show that as mode frequency increased, the non-dimensional frequency parameter increased. It can be seen in Table 2 that

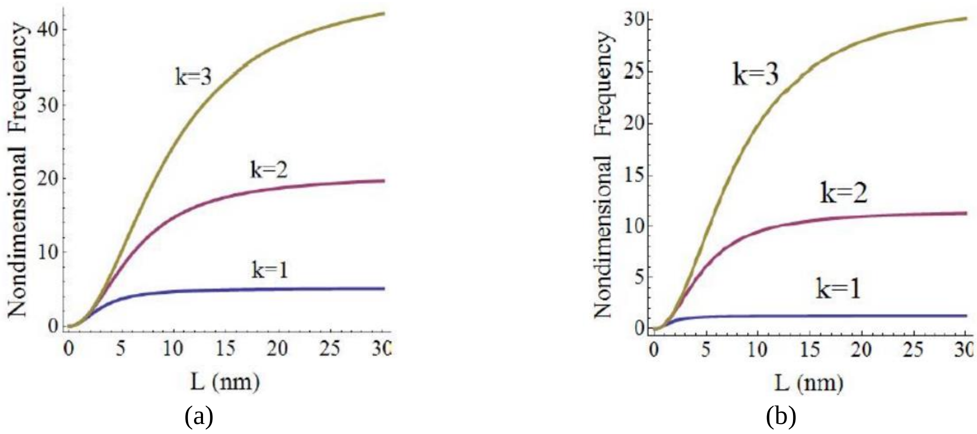


Fig. 5 Nondimensional frequency versus length for two boundary conditions containing (a) clamped-clamped and (b) clamped-free with the first three frequency modes for exponential law (nonlocal parameter 1 nm)

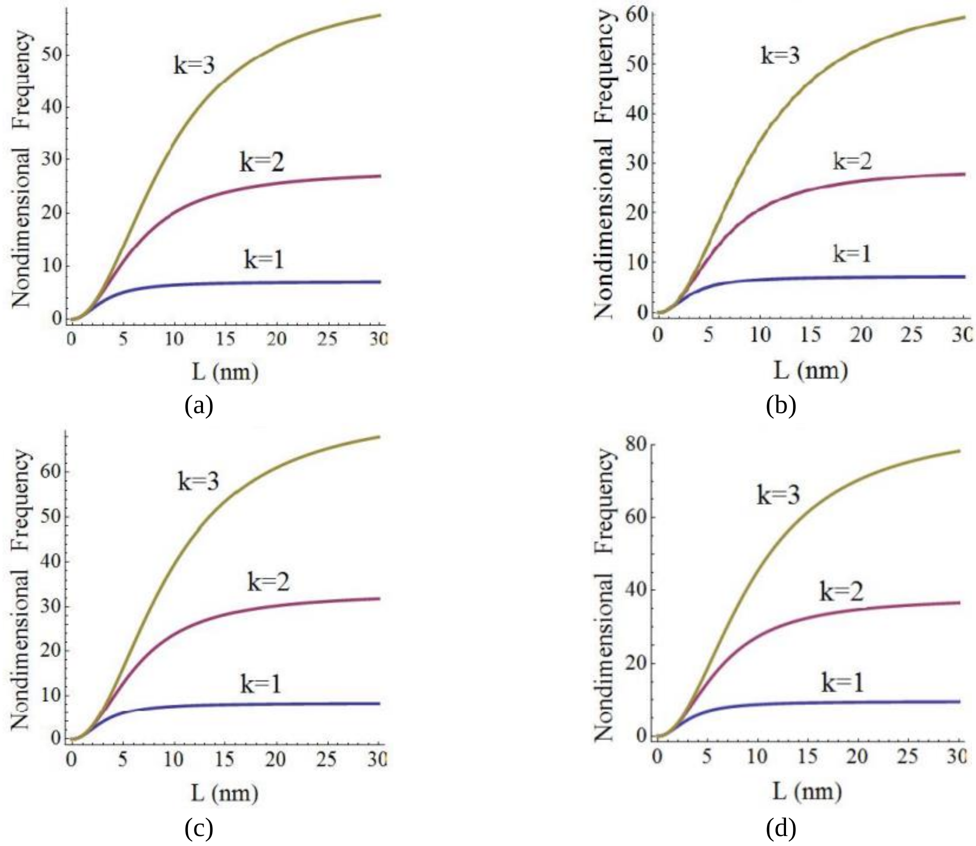
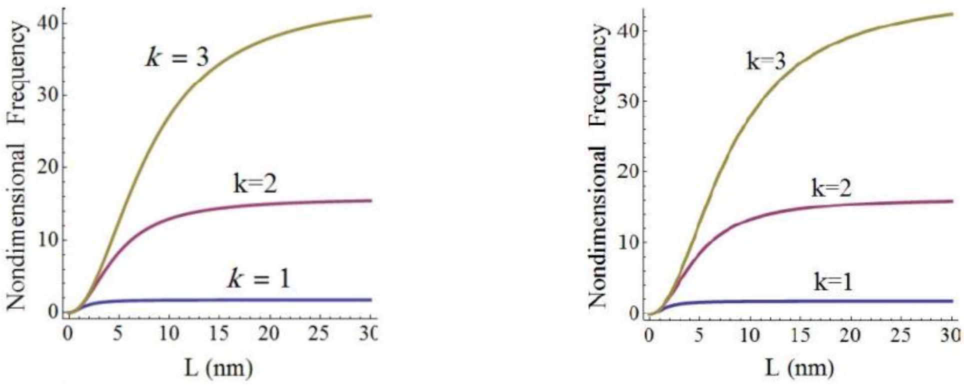
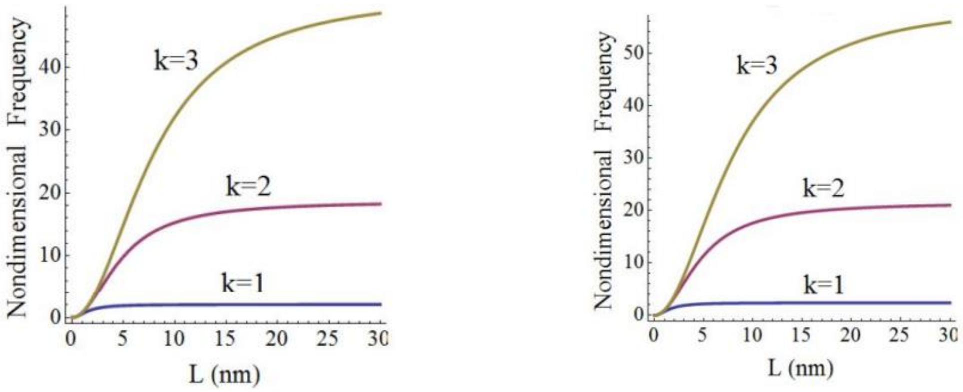


Fig. 6 Nondimensional frequency versus length for clamped-clamped boundary condition (a) n=0, (b) n=0.1, (c) n=1 and (d) n=10 for clamped-free boundary condition (e) n=0, (f) n=0.1, (g) n=1 and (h) n=20 based on sigmoid law



(e) (f)



(g) (h)

Fig. 6 Continued

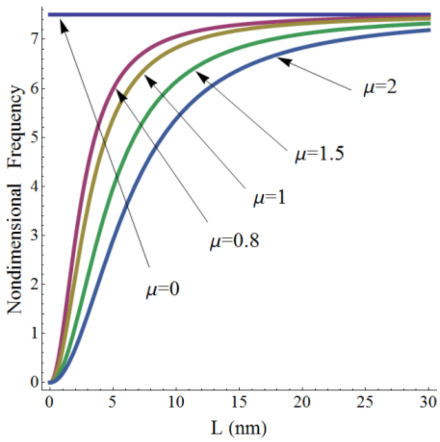


Fig. 7 Nondimensional frequency versus length with regard to different values for nonlocal parameter assumed for clamped-clamped FG nanobeam for power law

exponential law was independent of n.

Fig. 6 shows the sigmoid law under two boundary conditions. Parameter n is assumed to be

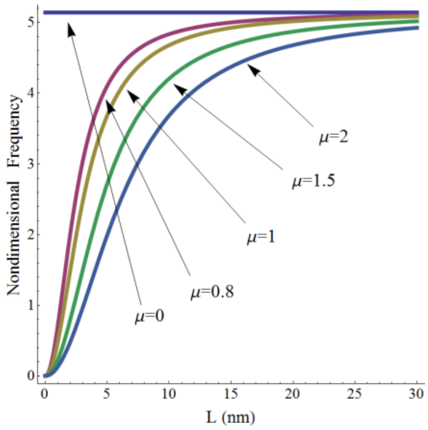


Fig. 8 Nondimensional frequency versus length with regard to different value for nonlocal parameter assumed for clamped-clamped FG nanobeam for exponential law

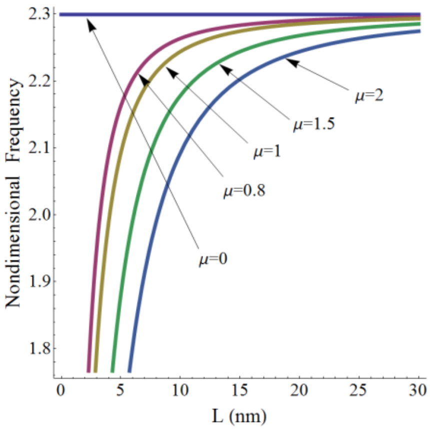


Fig. 9 Nondimensional frequency versus length with regard to different value for nonlocal parameter assumed for clamped-clamped FG nanobeam for sigmoid law

zero in sigmoid law, as shown in Figs. 6(a) and 6(e). Figs. 6(b), 6(c) and 6(d) show the clamped-clamped boundary condition in which n is equal to 0.1, 1, and 10, respectively. Figs. 6(f), 6(g) and 6(h) show the clamped-free boundary condition. The value of n is constant in Figs. 6(a), 6(b), 6(c) and 6(d).

Figs. 6(a) to 6(d) show that the maximum non-dimensional frequency for the third mode was 60 to 80. The maximum non-dimensional frequency for the third mode for the variation in non-dimensional frequency in Figs. 6(e) to 6(h) was 40 to 46. This confirms that the variation of the ratio in the clamped-clamped condition was greater than for the clamped-free condition.

Figs. 7, 8 and 9 assume clamped-clamped boundary conditions. Figs. 7 to 9 show the variation in non-dimensional frequency for the clamped-clamped boundary conditions for power, exponential and sigmoid law, respectively. In these figures, n = 4 and the first frequency mode was assumed. These figures contain five values for the small scale parameter (μ=0, 0.8,1,1.5,2).

Figs. 7 to 9 indicate that, when the small scale parameter is neglected, the non-dimensional frequency was constant. As shown, as the small length scale increases, μ, the non-dimensional

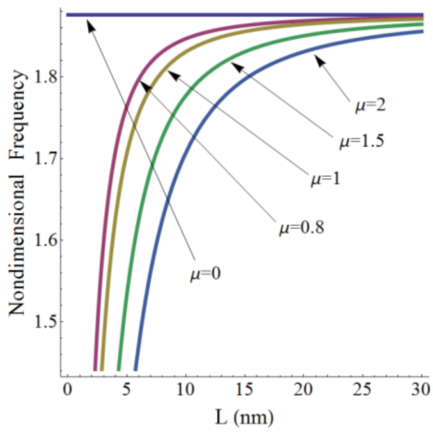


Fig. 10 Nondimensional frequency versus length with regard to different value for nonlocal parameter assumed for clamped-free FG nanobeam for power law

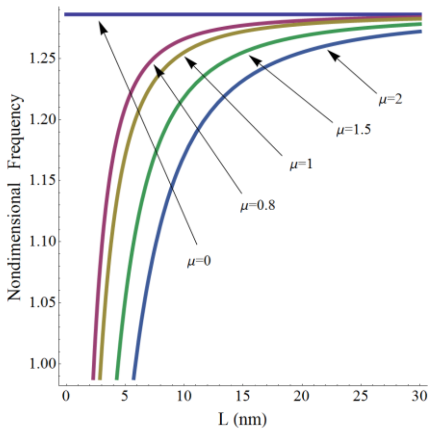


Fig. 11 Nondimensional frequency versus length with regard to different values for nonlocal parameter assumed for clamped-free FG nanobeam for exponential law

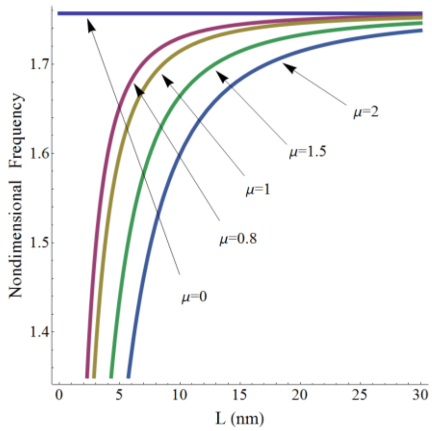


Fig. 12 Nondimensional frequency versus length with regard to different value for nonlocal parameter assumed for clamped-free FG nanobeam for sigmoid law

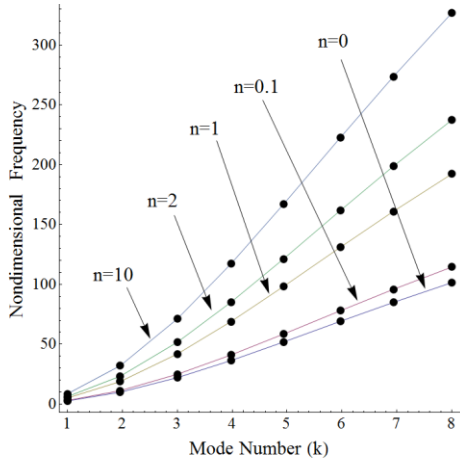


Fig. 13 Nondimensional frequency versus mode number containing different *n* value for clamped-clamped case (power law)

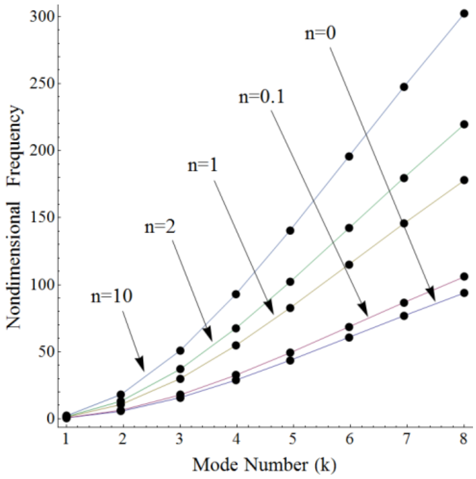


Fig. 14 Nondimensional frequency versus mode number containing different n value for clamped-free case (power law)

parameter, decreases.

A comparison of Figs. 7 to 9 based on the distribution of the FG nanobeam shows that non-dimensional frequency for the power law is greater than that for exponential law and the non-dimensional frequency for exponential law was greater than for sigmoid law.

Figs. 10 to 12 show clamped-free boundary conditions (n = 4) in which the length was increased to 30 nm for the FG nanobeam and the non-dimensional frequency was 1.86, 1.28 and 1.76, respectively.

Figs. 13 and 14 show the non-dimensional frequency versus mode numbers for power law under the clamped-clamped and clamped-free boundary conditions, respectively. As shown, the mode number increased as the non-dimensional frequency increased. Figs. 13 and 14 indicate that the mode number increased as the difference in non-dimensional frequency for different values of *n* increased.

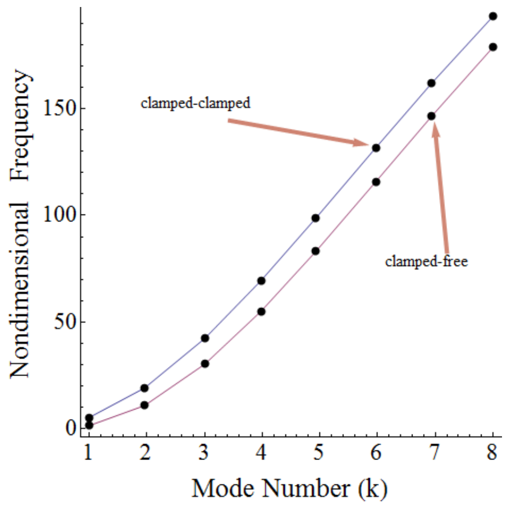


Fig. 15 Nondimensional frequency versus mode number for two boundary conditions demonstrated for exponential law

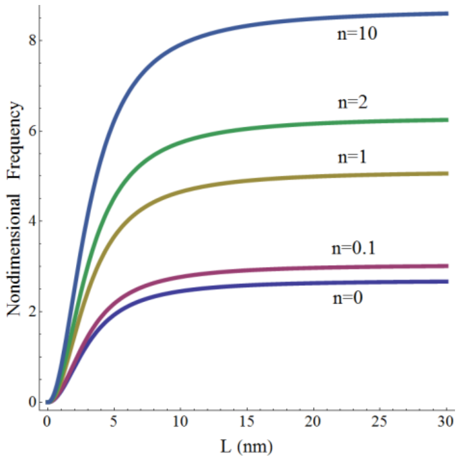


Fig. 16 Nondimensional frequency versus length for clamped-clamped boundary condition for the selected values of the n in power law function (k=1 and μ=1 nm)

Fig. 15 indicates the non-dimensional frequency with respect to the mode number for clamped-clamped and clamped-free boundary conditions. As shown in the figure, the non-dimensional frequency increased as the mode numbers increased. The figure also demonstrates that, for the same mode, the non-dimensional frequency of the clamped-clamped condition was greater than that for the clamped-free condition.

Figs 16 and 17 show the variation in non-dimensional frequency versus length for. n = (0, 0.1, 1, 2, and 10) in the clamped-clamped and clamped-free boundary conditions, respectively. As seen, when L was greater than 10 nm, the non-dimensional frequency was constant. It should be noted that n was assumed to be constant here.

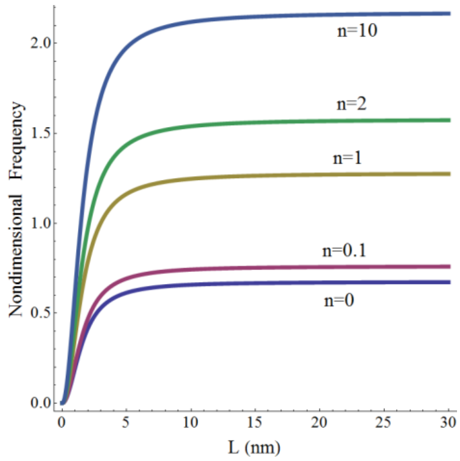


Fig. 17 Nondimensional frequency versus length for clamped-free boundary condition considering the 5 values for n parameter in power law function (k=1 and μ=1 nm)

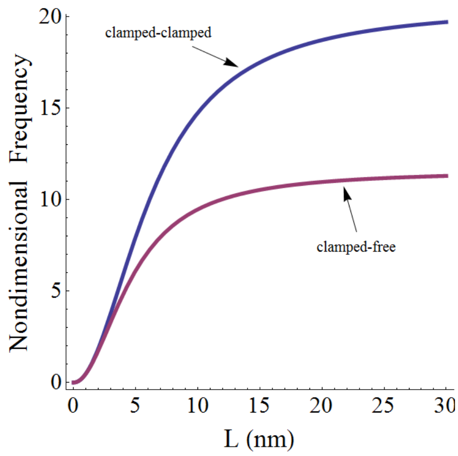


Fig. 18 Nondimensional frequency versus length for clamped-clamped and clamped-free conditions for exponential law (k=1)

Fig 18 shows the two boundary condition for exponential law, where L is less than 5 nm; the non-dimensional frequencies for the clamped-clamped and clamped-free boundary conditions were not identical.

4. Conclusions

An analytical solution and model for axial vibrations of a nanobeam were developed using non-local elasticity. The underlying solution assumed two boundary conditions, different frequency modes, power law, exponential law, and sigmoid law were considered. The main conclusions are:

• Increasing length increased the non-dimensional frequency. This increase becomes significant when the length is less than 10 nm.

• Increasing the nonlocal parameter decreased the non-dimensional frequency.

• The non-dimensional frequency for power law was higher than that for exponential law and sigmoid law.

• The frequency ratio for the clamped-clamped condition was greater that for the clamped-free condition.

• When the frequency modes increased, the non-dimensional frequency increased; this increase was greater for the clamped-clamped condition than the clamped-free condition.

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EC

Nomenclature

E	Young modulus
ρ	mass density
L	length
b	width
h	thickness
v_0	Transverse displacements
u_0	axial displacements
M	moment resultant
N	force resultant
$f(x,t)$	axial distributed force
$q(x,t)$	transverse distributed force