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Numerical investigation of the effects angles of attack on the flutter of a viscoelastic plate

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Abstract. As is shown in the paper, the Koltunov-Rzhanitsyn singular kernel of heredity (when constructing mathematical models of the dynamics problem of the hereditary theory of viscoelasticity) adequately describes real mechanical processes, best approximates experimental data for a long period of time. A mathematical model of the problem of the flutter of viscoelastic plates moving in a gas with a high supersonic velocity is given. Using the Bubnov-Galerkin method, discrete models of the problem of the flatter of viscoelastic plates flowed over by supersonic gas flow are obtained. A numerical method is developed to solve nonlinear integro-differential equations (IDE) for the problem of the hereditary theory of viscoelasticity with weakly singular kernels. A general computational algorithm and a system of application programs have been developed, which allow one to investigate the nonlinear dynamic problems of the hereditary theory of viscoelasticity with weakly singular kernels. On the basis of the proposed numerical method and algorithm, nonlinear problems of the flutter of viscoelastic plates flowed over in a gas flow at an arbitrary angle are investigated. In a wide range of changes in various parameters of the plate, the critical velocity of the flutter is determined. It is shown that the singularity parameter α affects not only the oscillations of viscoelastic systems, but the critical velocity of the flutter as well.

Keywords: mathematical model; viscoelasticity; integro-differential equations; flutter; angle of flow

1. Introduction

Mathematical and computer simulation of the flutter of viscoelastic elements and units of the aircraft structure is an actual scientific problem, the study of which is stimulated by the failure of aircraft structures, units of space and jet engines.

Due to the complexity of the flutter phenomenon in aircraft units, some simplifying assumptions have been used in the studies. However, these assumptions, as a rule, turn out to be so restrictive that the mathematical model no longer reflects the real state with sufficient accuracy. Therefore, the results of theoretical and experimental studies are in poor agreement.

One of the main difficulties in understanding the phenomenon of supersonic panel flutter is that the critical velocity of panel flutter depends on a large number of parameters. At present, the

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difficulty of identifying many of these factors in an experimental study does not allow one to obtain a satisfactory agreement between experimental and theoretical results. There are reviews of the tasks studied; extensive bibliography is found in publications by Fung (1960), Eisley and Luessen (1963), Dowell and Ventres (1970). The development of the plate flutter problems taking into account the angle of flow is reflected in the studies of Tang and Dowell (2016), Koreanschi *et al*. (2016), Xie *et al*. (2013), Yaman (2016), Saeed and Salman (2017), Yang *et al*. (2012), Sefat and Fernandes (2012, 2013), Bichiou *et al*. (2016), Attar *et al*. (2003) and others. It turns out that the sensitivity of the flutter velocity to such factors as the angle of flow is incomplete insufficiently studied. So, this paper provides a theoretical study of the nonlinear flutter of viscoelastic plates flowed over at an arbitrary angle. Emphasis is placed on comparing the results with previously obtained known results.

2. Statement and algorithm for solving the problem

Consider the problem of oscillations of a flexible hereditary-deformable rectangular plate with sides *a* and *b*, moving in a gas with a high supersonic velocity (Fig. 1). Assuming that the stress and strain relation for a plate material is linearly hereditary, a simplified model of flexible plates is used; the force of aerodynamic effect is written according to the new linearized piston theory Il'yushin *et al*. (1994). The Berger equation is obtained to describe nonlinear oscillations of a thin isotropic plate in the following form:

$$D\left(1-R^*\right)\Delta^2 w - D_1 \Delta w\left(1-R^*\right)J_1(t) + \rho h\frac{\partial^2 w}{\partial t^2} + B\frac{\partial w}{\partial t} + BV(\vec{n}_0 \cdot grad w) = 0 \tag{1}$$

where $D=\dfrac{Eh^3}{12(1-\mu^2)}$ is the cylindrical rigidity; $D_1=\dfrac{Eh}{2ab(1-\mu^2)}$; $\Delta^2=\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)^2$; ρ is the density of plate material; *h* is the plate thickness; *E* – is the elasticity modulus; μ is the Poisson ratio; *w* is the deflection of the plate; *V* – is the flow velocity; $B=\dfrac{\aleph p_\infty}{V_\infty}$; $\aleph$- is the polytropic gas index; p_∞, V_∞ are the pressure and sound velocities, respectively, in the undisturbed flow of gas; R^* is an integral operator with a relaxation kernel *R(t)* having a weakly singular Abel-type feature:

$$R^*\varphi(t)=\int_0^t R(t-\tau)\varphi(\tau)d\tau,$$

$$\left(R(t-\tau)=A\cdot\exp(-\beta\,(t-\tau))\cdot(t-\tau)^{\alpha-1},\ A>0, \beta>0, 0<\alpha<1\right);$$

$$J_1(t)=\int_0^a\int_0^b\left[\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right]dxdy \quad \vec{n}_0=(\cos\theta, \sin\theta);$$

θ is the angle of flow; *t* is the time; τ is the time preceding the moment of observation.

Introduce dimensionless coordinates to equation (1): $x=a\bar{x}$, $y=b\bar{y}$, $t=t_1\bar{t}$, $(t_1^2=\rho ha^4/D)$ and $w=h\bar{w}$, keeping the previous notations of the coordinates. In dimensionless coordinates, Eq. (1) has the form

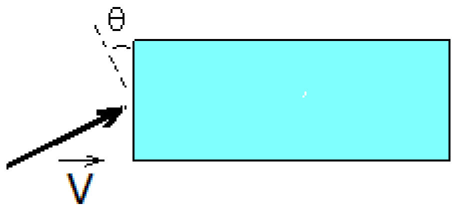

Fig. 1 Flowing over the plate at an arbitrary angle

$$(1-R^*)\Delta^2 w - \Delta w(1-R^*)\overline{J}(t) + a_2 M^*(\vec{n}_0 \cdot gradw) + a_1 \frac{\partial w}{\partial t} + \frac{\partial^2 w}{\partial t^2} = 0 \tag{2}$$

where $a_1 = p_\infty \aleph a^4/(V_\infty D t_1)$, $a_2 = p_\infty \aleph a^3/D$, grad$w=(w_x, \lambda w_y)$, M^* are the Mach numbers; $\lambda_1 = b/h$, $\lambda = a/b$, $\Delta w = w_{xx} + \lambda^2 w_{yy}$, $\Delta^2 = w_{xxxx} + 2\lambda^2 w_{xxyy} + \lambda^4 w_{yyyy}$, $\overline{J}(t) = \int_0^1\int_0^1 \left[(w_x)^2 + \lambda^2 (w_y)^2\right] dxdy$.

The solution of Eq. (2) is sought using the Bubnov-Galerkin method. Let $\{\varphi_{nm}(x,y)\}$ be a complete sequence of coordinate functions satisfying the boundary conditions. Substituting in (2) a series

$$w(x,y,t) = \sum_{n=1}^{N} \sum_{m=1}^{M} w_{nm}(t)\varphi_{nm}(x,y). \tag{3}$$

and conducting the known procedure of the Bubnov-Galerkin method, we get:

$$\begin{gathered} \sum_{n=1}^{N}\sum_{m=1}^{M}\left\{ L_{klnm}\left(\ddot{w}_{nm} + a_1 \dot{w}_{nm}\right) + F_{klnm}\left(1-R^*\right) w_{nm} + \right. \\ + \sum_{n=1}^{N}\sum_{m=1}^{M} G_{klnm} w_{nm} + \sum_{n,i,j=1}^{N}\sum_{m,r,s=1}^{M} Q_{klnmirjs} w_{nm}(1-R^*) w_{ir} w_{js} = 0, \quad k=\overline{1,N}; \quad l=\overline{1,M}; \end{gathered} \tag{4}$$

where

$$L_{klnm} = \int_0^1\int_0^1 \phi_{nm}\phi_{kl}\,dxdy, \quad F_{klnm} = \int_0^1\int_0^1 \left(\phi^{IV}_{nm,xxxx} + 2\lambda^2\phi^{IV}_{nm,xxyy} + \lambda^4\phi^{IV}_{nm,yyyy}\right)\phi_{kl}\,dxdy,$$

$$G_{klnm} = a_2 M^* \int_0^1\int_0^1 (\vec{n}_0 \cdot grad\varphi_{nm})\varphi_{kl}\,dxdy, \; Q_{klnmirjs} = -6\overline{J}_{irjs}\int_0^1\int_0^1 \Delta\phi_{nm}\phi_{kl}\,dxdy,$$

$$\overline{J}_{irjs} = \int_0^1\int_0^1 \left\{\left[\varphi_{ir,x}\varphi_{js,x}\right] + \lambda^2\left[\varphi_{ir,y}\varphi_{js,y}\right]\right\} dxdy.$$

Initial conditions for the system of Eq. (4) are:

$$w_{nm}(0) = \alpha_{nm}, \quad \dot{w}_{nm}(0) = \beta_{nm}, \tag{5}$$

where α_{nm}, β_{nm} are the known constants.

The systems of nonlinear integro-differential equations (IDE) (4) are solved numerically with the method based on the use of quadrature formulas Badalov (1987a, b, 2007a, b, 2008), Khudayarov *et al*. (2019a, d). To do this, this system is written in integral form and, using rational transformations, the weakly singular features of the integral operator R^* are eliminated. Assuming that $t=t_i$, $t_i=i\Delta t$, $i=1,2,\ldots$ ($\Delta t=const$) and replacing the integrals with some quadrature formulas to calculate $w_{nm}= w_{nm}(t)$, the following recurrence relation is obtained:

$$\sum_{n=1}^{N}\sum_{m=1}^{M} L_{k\ln m} w_{pnm} = \frac{1}{1+C_p a_1}\left\{\sum_{n=1}^{N}\sum_{m=1}^{M} L_{k\ln m}\left\langle w_{0nm}+\left(\dot{w}_{0nm}+a_1 w_{0nm}\right)t_p\right\rangle - \right.$$
$$-\sum_{j=0}^{p-1} C_j\left(a_1\sum_{n=1}^{N}\sum_{m=1}^{M} L_{k\ln m} w_{jnm} - \left(t_p-t_j\right)\left[\sum_{n=1}^{N}\sum_{m=1}^{M} G_{k\ln m} w_{jnm} + \right.\right.$$
$$+\sum_{n=1}^{N}\sum_{m=1}^{M} F_{k\ln m}\left(w_{jnm}-\frac{A}{\alpha}\sum_{s=0}^{j} B_s \exp(-\beta t_s) w_{j-s,nm}\right)+ \tag{6}$$
$$\left.\left.\left.+\sum_{n,i,j_1=1}^{N}\sum_{m,r,s_1=1}^{M} Q_{k\ln m} w_{jnm}\left(w_{jir}w_{jj_1s_1}-\frac{A}{\alpha}\sum_{s=0}^{j} B_s \exp(-\beta t_s) w_{j-s,ir} w_{j-s,j_1s_1}\right)\right]\right)\right\}$$

$$p=1,2,\ldots;\qquad n=\overline{1,N};\qquad m=\overline{1,M};$$

where C_j , B_s are the numerical coefficients applied to quadrature trapezoidal formulas (Khudayarov *et al*. 2019b, Khudayarov 2019):

$$C_0=\frac{\Delta t}{2};\quad C_j=\Delta t,\quad j=\overline{1,i-1};\quad C_i=\frac{\Delta t}{2};$$
$$B_0=\frac{\Delta t^{\alpha}}{2};\quad s=j,\quad B_j=\frac{\Delta t^{\alpha}(j^{\alpha}-(j-1)^{\alpha})}{2};$$
$$B_s=\frac{\Delta t^{\alpha}((s+1)^{\alpha}-(s-1)^{\alpha})}{2}.$$

Algorithm (6) is quite general and suits for flutter problems both for ideally elastic and for hereditary-deformable flexible plates under various boundary conditions.

As an example, consider the problem of the flutter of hingedly supported rectangular plates in a supersonic gas flow, taking into account geometric nonlinearity.

The solution of the simplified equation describing this process in a dimensionless form (2) at initial conditions:

$$w(x,y,0)=a_0\sin nx\sin my;\ \dot{w}(x,y,0)=0, \tag{7}$$

and at boundary conditions

$$w=w_{xx}=0 \text{ at } x=0 \text{ and } x=1;$$
$$w=w_{yy}=0 \text{ at } y=0 \text{ and } y=1; \tag{8}$$

is sought for in the form

$$w(x,y,t)=\sum_{n=1}^{N}\ \sum_{m=1}^{M} w_{nm}(t)\sin n\pi x \sin m\pi y\,. \tag{9}$$

The system of nonlinear IDE (4) in this case is simplified and takes the form

$$\ddot{w}_{kl}+a_1\dot{w}_{kl}+\omega_{kl}^2(1-R^*)w_{kl}+4\sum_{n=1}^{N}\sum_{m=1}^{M}G_{klnm}w_{nm}+\frac{3}{2}\omega_{kl}w_{kl}\left(1-R^*\right)\sum_{n=1}^{N}\sum_{m=1}^{M}\omega_{nm}w_{nm}^2=0 \tag{10}$$

where

$$\omega_{kl}=\pi^2(k^2+\lambda^2 l^2);$$

$$G_{klnm}=a_2M^*\int_0^1\int_0^1\left(\vec{n}_0\cdot grad\varphi_{nm}\right)\varphi_{kl}\,dxdy=$$

$$=\frac{1}{2}a_2M^*\left[n\cos\theta(\gamma_{n+k}-\gamma_{n-k})(\delta_{m-l}-\delta_{m+l})+\lambda m\sin\theta(\gamma_{m+l}-\gamma_{m-l})(\delta_{n-k}-\delta_{n+k})\right],$$

$$\delta_n=\begin{cases}1, & \text{at } n=0,\\ 0, & \text{at } n\neq 0,\end{cases}\qquad \gamma_n=\begin{cases}\dfrac{1}{n}, & n-\text{odd},\\ 0, & n=0 \quad \text{or} \quad n-\text{even}.\end{cases}$$

Integrating system (10) twice with respect to t, it can be written in integral form; after rational transformation we eliminate singular features of the integral operator R^*. Then, assuming that $t=t_i$, $t_i=i\cdot\Delta t$, $i=1,2,\ldots$ (Δt =const) and replacing the integrals by quadrature formulas of trapezoid to calculate $w_{ikl}=w_{kl}(t_i)$, recurrence formulas for the Koltunov-Rzhanitsyn kernel are obtained

$$\begin{aligned}
w_{pnm}=\frac{1}{1+C_pa_1}\Bigg\{&w_{0nm}+\left(\dot{w}_{0nm}+a_1w_{0nm}\right)t_p-\\
-\sum_{j=0}^{p-1}C_j\Bigg\langle\ &a_1w_{jnm}-\left(t_p-t_j\right)\Bigg[4\sum_{n=1}^{N}\sum_{m=1}^{M}G_{k\ln m}w_{jnm}+\\
&+\omega_{kl}^2\left(w_{jkl}-\frac{A}{\alpha}\sum_{s=0}^{j}B_s\exp(-\beta t_s)w_{j-s,kl}\right)+\\
&+\frac{3}{2}\omega_{kl}w_{jkl}\sum_{n=1}^{N}\sum_{m=1}^{M}\omega_{nm}\left(w_{jnm}^2-\frac{A}{\alpha}\sum_{s=0}^{j}B_s\exp(-\beta t_s)w_{j-s,nm}^2\right)\Bigg]\Bigg\rangle\Bigg\}
\end{aligned} \tag{11}$$

$$p=1,2,\ldots;\qquad n=\overline{1,N};\qquad m=\overline{1,M}\,;$$

The results of the calculations are presented in Tables 1, 2 and 3 and shown in graphs 2-7. Figs. 2-7 show the results of calculations in the field of modeling of viscoelastic plates in supersonic flow. Calculations have shown that when solving these problems in the Bubnov-Galerkin method expansion it is sufficient to retain the first 5 harmonics (N = 5, M =1), since a further increase in the number of terms does not significantly affect the oscillation amplitude of a viscoelastic plate.

3. Solution of the test cases

Verification of efficiency of the proposed numerical method and programs, based on the solution of test cases, is a necessary stage to confirm the reliability of research results obtained in solving specific problems. The problems for which an exact solution is known Badalov (1987a), Badalov *et al*. (2007a), Khudayarov *et al*. (2019b) have been considered as test cases. Table 1 show a satisfactory agreement of approximate solutions with exact ones; this shows the reliability and high accuracy of calculation results.

Consider a non-linear integro-differential equation of the form

$$\ddot{w}+\lambda_0\dot{w}+\omega^2 w=q-\lambda_1\int_0^t R(t-\tau)w(\tau)d\tau-\lambda_2 w\int_0^t R(t-\tau)w(\tau)d\tau-\lambda_3\int_0^t R(t-\tau)w^2(\tau)d\tau; \qquad (12)$$

$$w(0)=1,\quad \dot{w}(0)=-\beta,$$

where

$$R(t)=A\exp(-\beta t)t^{\alpha-1},\ 0<\alpha<1;\quad q=\left[\beta^2+\omega^2-\lambda_0\beta-\frac{At^\alpha}{\alpha}\left(\lambda_1+\left[\lambda_2+\lambda_3\right]\exp(-\beta t)\right)\right]\exp(-\beta t).$$

Eq. (12) has an exact solution $w=\exp(-\beta t)$, which satisfies the initial conditions.

According to (11), the approximate values $w_n=w(t_n)$ ($t=t_n=n\Delta t$, $n=0, 1, 2, \dots$) are found from the relationships

$$w_n=\frac{1}{1+\lambda_0 C_n}\left\{1-(\beta-\lambda_0)t_n-\sum_{i=0}^{n-1}C_i\left\langle\lambda_0 w_i-(t_n-t_i)\left[q(t_i)-\omega^2 w_i+\frac{A\lambda_2}{\alpha}w_i\times\right.\right.\right.$$
$$\left.\left.\left.\times\sum_{s=0}^{i}B_s\exp(-\beta t_s)w_{i-s}+\frac{A}{\alpha}\sum_{s=0}^{i}B_s\exp(-\beta t_s)w_{i-s}\left(\lambda_1+\lambda_3\exp(-\beta\ t_s)w_{i-s}\right)\right]\right\rangle\right\} \qquad (13)$$

Table 1 Comparison of exact and approximate solutions of the IDE

t	Solution		Δ
0	**Exact**	Approximate	-
1	1.000000	1.000000	$0{,}7\cdot10^{-4}$
2	0.970445	0.970373	$1{,}4\cdot10^{-4}$
3	0.941764	0.941622	$2{,}8\cdot10^{-4}$
4	0.913931	0.913644	$3{,}5\cdot10^{-4}$
5	0.886920	0.886569	$4{,}3\cdot10^{-4}$
6	0.860707	0.860271	$4{,}1\cdot10^{-4}$
7	0.835270	0.834855	$3\cdot10^{-4}$
8	0.810584	0.810278	$5{,}1\cdot10^{-4}$
9	0.786627	0.786113	$2{,}5\cdot10^{-4}$
10	0.763379	0.763126	$3\cdot10^{-4}$

n=1,2,…,; where C_i, B_s – are the coefficients of the quadrature formula of trapezoids.

Table 1 gives approximate results of calculations by formulas (13) within the interval from 0 to 1 with Δt=0.01 step, and exact solutions. The following initial data have been used: λ_0=1.1; λ_1=1.2; λ_2=1.3; λ_3=1.4; A=0.01; β=0.03; α=0.01. It follows from the table that the maximum error Δ of calculations performed by described method represents the value $const\cdot\Delta t^2$. The efficiency of this numerical method and programs is shown in other test cases as well.

4. Numerical results and discussion

4.1 The criterion of instability

As a criterion determining the critical velocity V_{cr}, the condition proposed in Khudayarov *et al*. (2016, 2019a, b, c), Verlan *et al*. (2004), Khudayarov (2008), Badalov *et al*. (2007a, b) is assumed.

Here the main task is to find the critical flutter speed V_{cr}. To do so, various criteria are used. As a criterion determining the critical flutter speed, we assume the condition that at this speed the amplitude of the oscillation varies according to a harmonic law. At a speed $V> V_{cr}$, an oscillatory motion occurs with intensively increasing amplitudes, which can lead to the destruction of the structure. In the case $V< V_{cr}$ the flow rate is less than the critical one, the amplitude of the viscoelastic plates oscillations attenuates.

4.2 The procedure for finding the critical speed

To determine $V=V_{cr,}$ consider the values V_1 and V_2 located on the interval (V_0,V_n) so that $V_0<V_1<V_2<V_n$. Comparing the law of variation of w at $V=V_1$ and $V=V_2$, the following conclusions can be drawn:

a) if, at $V<V_1$, the law of variation of the function w is close to a harmonic one, then $V_{cr,}$ cannot be in the interval (V_0,V_1); that is V_{cr} lies in the interval ($V_{1,}$ V_n);

b) if, at $V>V_1$, a rapid growth of the function w with time is observed, then V_{kp} lies in the interval (V_0, V_1).

Processes a) and b), i.e., the processes of excluding the intervals that do not generate undesirable phenomena is repeated for (V_0,V_1) or (V_1, V_n), etc. The search ends when the remaining sub-interval is reduced to a sufficiently small size.

4.3 Discussion of the results

Longitudinal flow. The influence of viscoelastic properties of plate material on the critical values of time and velocity of the flutter is studied. Calculation results presented in Table 2 show that if the exponential kernel (α=1) is used

$$R(t) = A\cdot \exp(-\beta\, t) \tag{14}$$

the flutter velocity decreases by approximately 1%, and when the Koltunov-Rzhanitsyn kernel is used

$$R(t) = A\cdot \exp(-\beta\, t)\cdot t^{\alpha-1} \tag{15}$$

Table 2 Dependencies of flutter velocity on parameters A and α

A	α	β	λ	λ_1	θ	$V_{cr\ (m/s)}$
0.0						1003
0.01	0.25	0.05	1	220	0	853.4
0.1						537.2
	1					993.14
0.1	0.5	0.05	1	220	0	697.34
	0.15					412.42

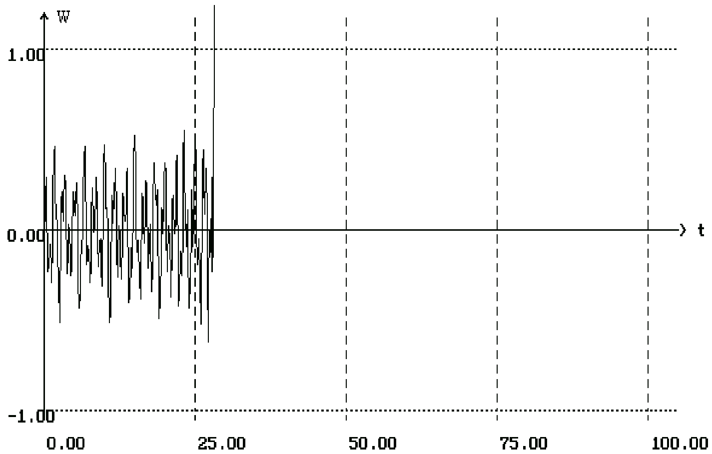

Fig. 2 Dependence of the deflection of the ideal-elastic plate on time t at V_{cr} =1003 m/s

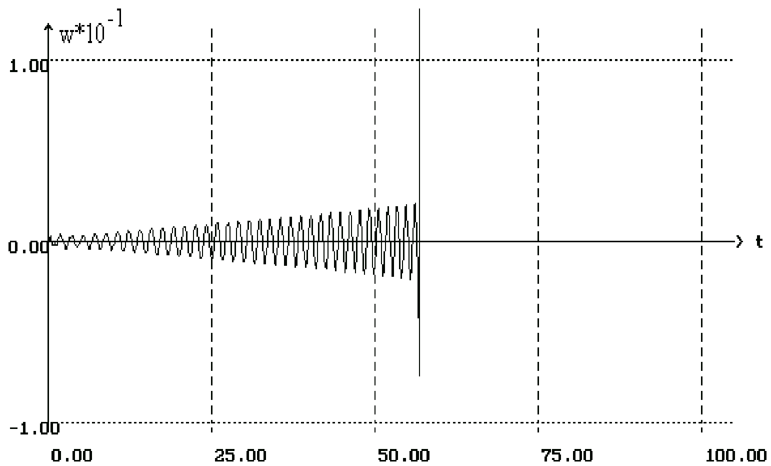

Fig. 3 Dependence of the deflection of a viscoelastic plate on time t at V_{cr} =537.2 m/s

the velocity decreases by 46.4%, relative to the critical velocity of the flutter of perfectly elastic plates, and the critical time is almost doubled (Figs. 2 and 3). Therefore, when using the exponential kernels, the flutter velocity of a viscoelastic plate practically coincides with the critical flutter velocity for ideal elastic plates. These conclusions and results fully agree with the

Table 3 Dependence of flutter velocity on the physico-mechanical and geometrical parameters of the plate

A	α	β	λ	λ_1	θ	V_{cr} (m/s)
0.0						1211.76
0.01	0.25	0.05	1	220	π/6	965.6
0.05						721.48
0.1						646.34
0.1	0.15	0.05	1	220	π/6	483.14
	0.5					930.98
0.1	0.25	0.1	1	220	π/6	649.4
		0.01				645.66
			1.2			805.8
0.1	0.25	0.05	1.4	220	π/6	1016.26
			1.5			1135.6
				180		1164.5
0.1	0.25	0.05	1	200	π/6	870.4
				250		442.34
					π/9	567.04
0.1	0.25	0.05	1	220	π/5	685.78
					π/4	789.14

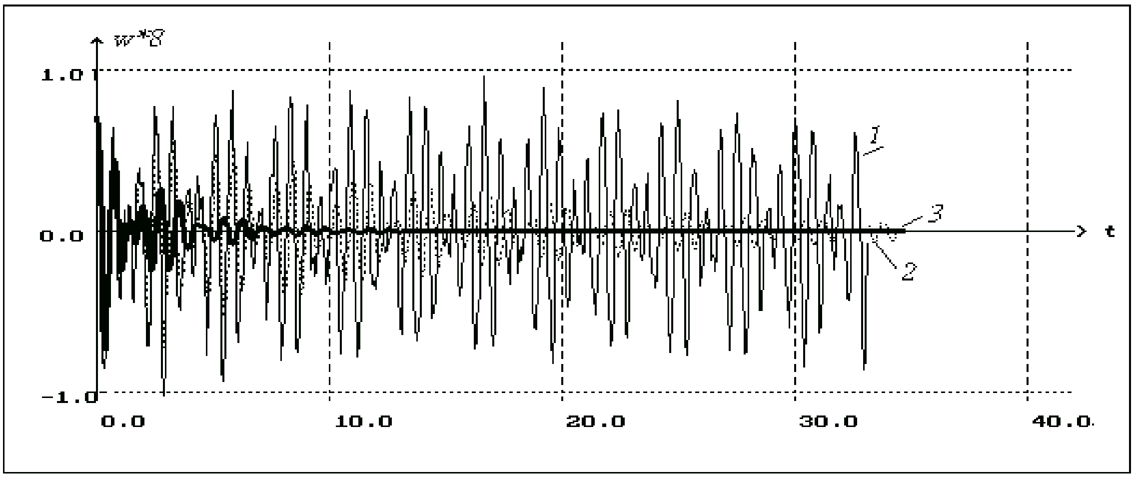

Fig. 4 A=0.001 (1); A=0.02 (2); A=0.1 (3); α=0.25 β=0.05; θ=0; λ=1; λ_1=220

conclusions and results given in Kiyko *et al*. (2005), where the critical velocity of the flutter is determined by a numerical-analytical method.

Flow at an arbitrary angle. Consider a viscoelastic plate with sides *a, b* and thickness *h*, flowed over by a supersonic gas flow at an arbitrary angle.

Table 3 shows the critical values of the flutter velocity depending on the physico-mechanical and geometrical nature of the plate, taking into account the angle θ. From Table 3 it is seen that the

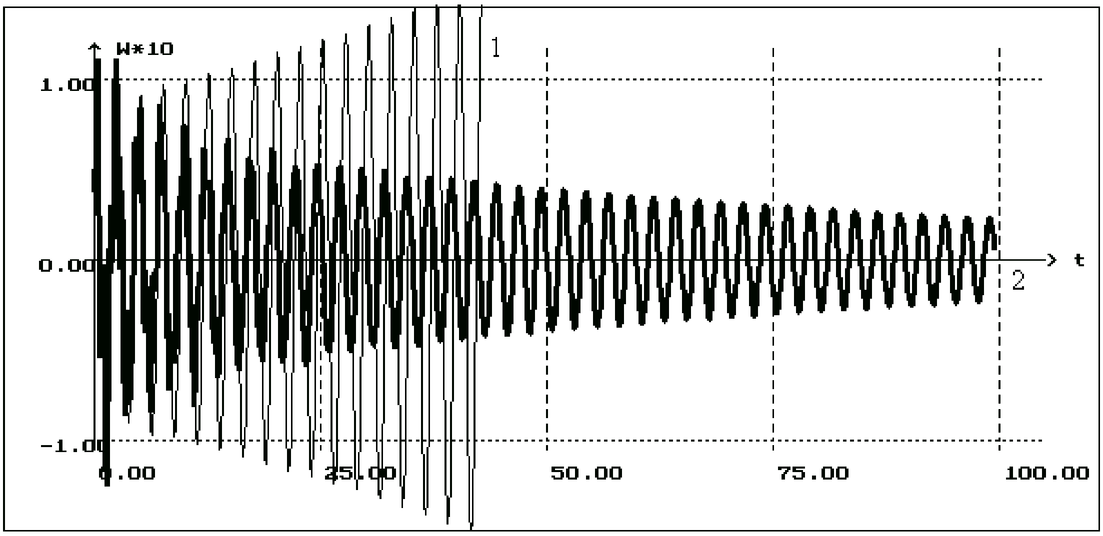

Fig. 5 α=0.1 (1); α=0.4 (2); A=0.1; β=0.05; θ=π/6; λ=1; λ_1=220; V=510 m/s

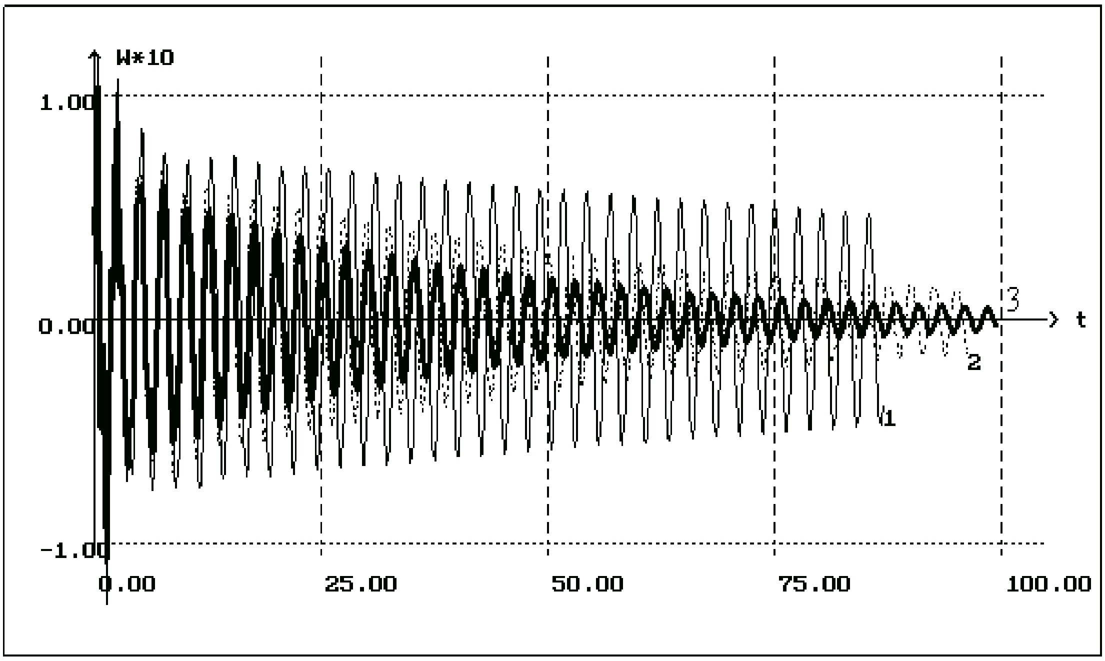

Fig. 6 θ=0 (1); θ=π/6 (2); θ=π/4; A=0.1; α=0.25; β=0.05; λ=1; λ_1=220; V=510 m/s

value of the critical number V_{cr} for a plate at θ> 0 is obviously greater than the one for a plate at θ=0.

For example, at θ=20^0, the critical number V_{cr} of the plate increases by 11.14% compared with the corresponding values of V_{cr} at θ=0; at θ=36^0 - by 27.66%, and at θ=45^0 - by 46.9%. Note that the influence of the parameter of an angle of flow θ, agrees perfectly with the results given in Kiyko *et al*. (2005).

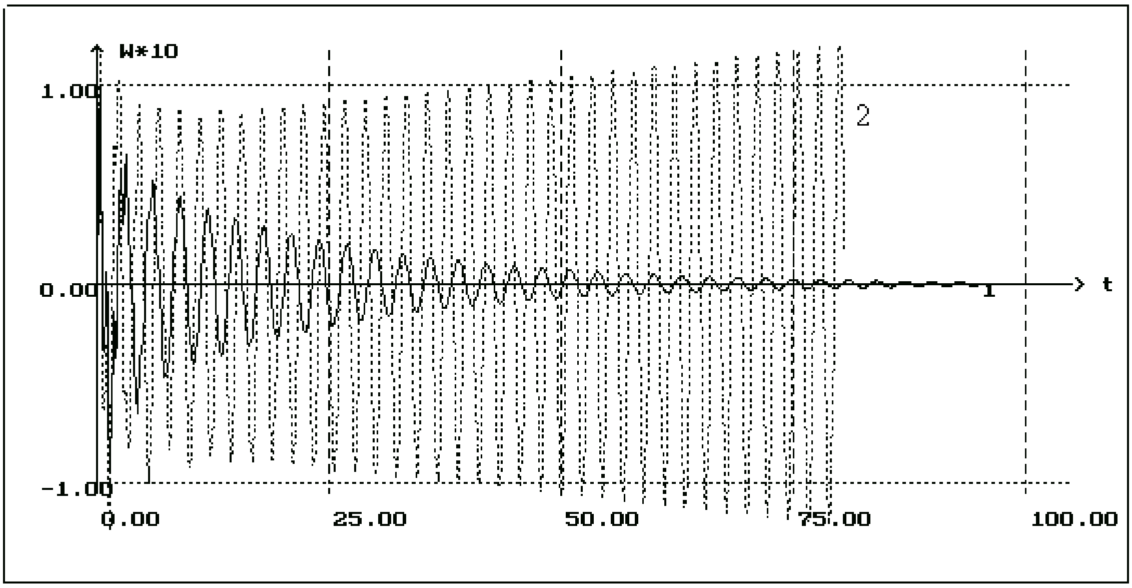

Fig. 7 λ_1=180 (1); λ_1=240 (2); A=0.1; α=0.25; β=0.05; θ=π/6; λ=1; V=510 m/s

Table 3 shows the effect of viscoelastic properties of plate material on the flutter velocity at $\theta\neq0$. For an elastic plate (A=0) at $\theta = \pi/6$, the critical velocity V_{cr} is 1211.76, and for a viscoelastic plate (A=0.1) at $\theta = \pi/6$, the critical velocity V_{cr} is 646.34. The difference is 46.4%.

Computational experiments show that a slight increase in the singularity parameter α leads to a significant increase in the critical velocity of the flutter.

The table below shows that the effect of the attenuation parameter β of the hereditary kernel on the flutter velocity of the plate is insignificant compared to the viscosity parameter A and the singularity parameter α; this once again confirms the well-known conclusion that the exponential kernel of relaxation is unable to fully describe the hereditary properties of the construction material.

The effect of the viscoelastic properties of material on the plate behavior is investigated. Fig. 4 shows the law of distribution of the pipeline deflection with account of viscoelastic properties of material and its development over time. As we see, an account of viscoelastic material properties of the structure sharply decreases the amplitude of oscillations. Meanwhile, the effect of the viscoelastic properties of pipeline material on the amplitude of its oscillations at the beginning of the process (part of the curve w(t) in the range of $0\leq t\leq1.5$) is manifested to a much lesser extent. Beginning from t >1.5, the viscoelastic properties of material significantly affect the oscillatory process of the pipeline. Analysis of the results shows that an increase in the value of the viscosity parameter A leads to a damping of the oscillatory process.

The effect of the parameter α (Fig. 5) was investigated for the following values: 0.15 (curve 1); 0.4 (curve 2). The flow velocity is 510 m/s. At the first values of the parameter α, the amplitude of oscillations increases rapidly over time. Flow rate is higher than the critical one. At α = 0.4, oscillations damp.

Fig. 6 shows the variation of the amplitude of oscillations of the plate at different values of the angle of flow θ. With an increase in the parameter θ, the amplitude and frequency of oscillations decrease.

Fig. 7 shows the curves of change in the dimensionless deflection depending on time t at various values of the parameter of relative thickness, λ_1= 180 (curve 1); λ_1= 240 (curve 2). With an increase in the parameter λ_1 (a decrease in thickness), the flutter velocity decreases.

5. Conclusions

As is shown in the paper, the Koltunov-Rzhanitsyn singular kernel of heredity (when constructing mathematical models of the dynamics problem of the hereditary theory of viscoelasticity) adequately describes real mechanical processes, best approximates experimental data for a long period of time. In a wide range of changes in various parameters of the plate, the critical velocity of the flutter is determined. It is shown that the singularity parameter α affects not only the oscillations of viscoelastic systems, but the critical velocity of the flutter as well.

Consequently, an account of this effect when designing aircraft structures is important, since the less the singularity parameter of the structure material, the more intensive dissipative processes occur in these structures. When modeling the nonlinear problems of the flutter of viscoelastic plates, a number of new mechanical effects were obtained:

• it was found that an account of the viscoelastic properties of the plate material leads to a decrease in the critical velocity of the flutter by 40-60%; and to an increase in the critical time by 70-90%;

• it was found that the angle of flow over the plate contributes to a noticeable increase in the velocity of the flutter.

The developed models, algorithms and application programs can be used in the study of dynamic behavior, design and testing of structural elements of aircraft built from composite viscoelastic materials, and of other technical structures in various fields of aircraft and machine engineering.

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