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Numerical study of bonded composite patch repair in damaged laminate composites

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Abstract. The present study deals with the repair of composite structures by bonding composite patches. The composite structure is a carbon/epoxy laminate with stacking sequence $[45/\text{-}45/0/90]_S$. The damaged zone is simulated by a central crack and repaired by bonding symmetrical composite patches. The repair is carried out using composite patches laminated from the same elemental folds as those of the cracked specimen. Three-dimensional finite element method is used to determine the energy release rate along the front of repaired crack. The effects of the repair technique used single or double patch, the stacking sequence of the cracked composite patch and the adhesive properties were highlighted on the variations of the fracture energy in mode I and mixed mode I + II loading.

Keywords: composite patch; laminates; stacking sequence; energy release rate

1. Introduction

Composites are used in a wide range of applications in aerospace, marine, automotive, surface transport and sports equipment markets. From the family of composite materials are laminated materials consisting of a stacking sequence of layers (or plies) impregnated with resin (or matrix), each fold being oriented fiber component. However, heterogeneity and anisotropy, ensuring their excellent properties are also the causes of their damage including accidental damage which they may be subjected. In the case of impacts for example there may be a reduction in mechanical performance of structures including their fatigue resistance.

In case the damaged structure cannot be replaced systematically due to the lack of time or resources, the repair is considered a good economic alternative and mechanical. The performance of the repair depends strongly on the method of assembly between the structure and repair patches (Duong and Wang 2007, Russell and Bowers 1992, Pang and Bond 2005, Williams *et al.* 2007). The composite repair of composite structures is to replace all or part of the damaged area and pasting smaller or larger patches on the surface of the structure.

The analysis of the effects of the composite geometrical properties on the repair performance got great interest in the literature. Heller and Kaye (2002) used the genetic algorithm to optimize

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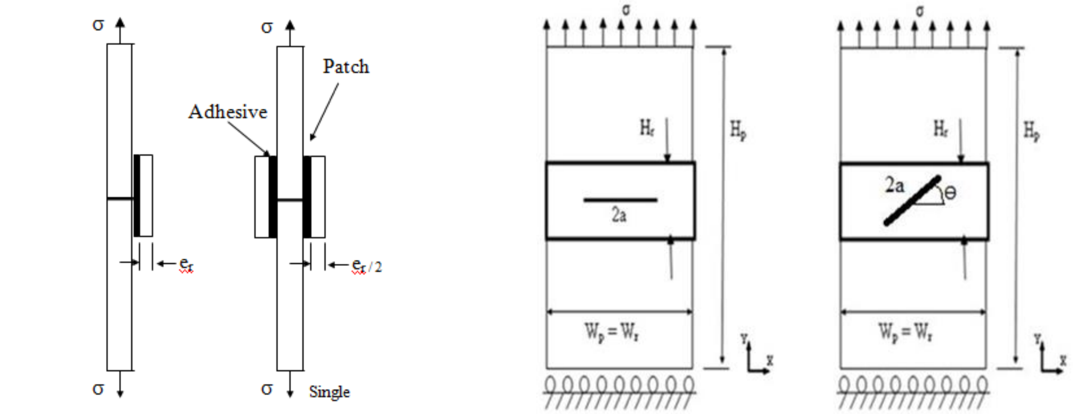
the patch shape. Mathias *et al.* (2006), coupled with the genetic algorithm method to a finite element code to optimize the stacking sequence of composite structures. The shape of the patch, numbers and directions of ply and the location of the patch are simultaneously optimized to reinforce notched structures. The authors minimize the average von Mises stress in a square area defined around the central hole in the structure to reinforce. Kim *et al.* (2010) and Deheeger *et al.* (2009) showed that the resistance of the patch to the same solicitation may be very different depending on the choice of material and the stacking sequence of the patch. (Duong and Wang 2007) shows that for patches carried out today maintenance workshop, additional plies are added, where feasible compared to the original stacking sequence, to obtain a safety margin more important on the mechanical properties of the repaired structure. Caminero *et al.* (2013) used different on-line monitoring techniques, such as Digital Image Correlation (DIC) and Lamb waves, in order to study the performance and damage detection in bonded composite repairs. Liu and Wang (2007) investigated the effect of the stacking ply sequence of the patch on the efficiency of the repair, by testing six different stacking ply sequences in two sided repair laminates subjected to tension. Their results indicated that the stacking sequence of the patches has little influence on the failure initiation strength and on the ultimate strength. The same applies to the failure mechanism of the repaired structures. However, recent studies like Cheng *et al.* (2014), Cheng *et al.* (2011) and Gong *et al.* (2015) has shown that the failure process of the parent laminate depends on the arrangement of the plies in the patch structure. Breitzman *et al.* (2009) and Fekih *et al.* 2018 developed a numerical and experimental study on scarf repairs with and without overlay plies loaded in tension. Their results indicated increased strength in repaired laminates with external ply due to additional load transfer in the over-ply. Wang *et al.* (2009) developed a numerical study with the aim of presenting an optimum scarf shape. Their results revealed that significant savings can be made to the amount of material removal (between 26% and 76%), by adopting optimum repairs over conventionally designed repairs. Brighenti *et al.* (2006) used the genetic algorithm to optimize the patch shape. By adopting the optimal shape patch, the stress-intensity factor can be reduced to about 50% with respect to that related to a simple shape (square or rectangular) patch. Mhamdia *et al.* (2012) found that the patch with arrow shape can reduce simultaneously the stress intensity factor at the crack tip and the adhesive shear stresses. They reported that the risk of adhesive failure is reduced by seven times, and that both the repair efficiency and durability are highly increased when the arrow shaped patch is used. Experimental studies show that adhesively bonded patch repairs can restore the strength of the repaired structure up to 80% of the original undamaged laminate strength (Cheng *et al.* 2014, Holzhüter and Sinapius 2011).

In the present paper, three-dimensional finite element method is used to determine the energy release rate along the front of repaired crack. The effects of the repair technique used single or double patch , the stacking sequence of the cracked composite and the patch and the adhesive properties were highlighted on the variations of the fracture energy in mode I and mixed mode I + II loading.

2. Geometrical and finite element models in mode I and mixed mode

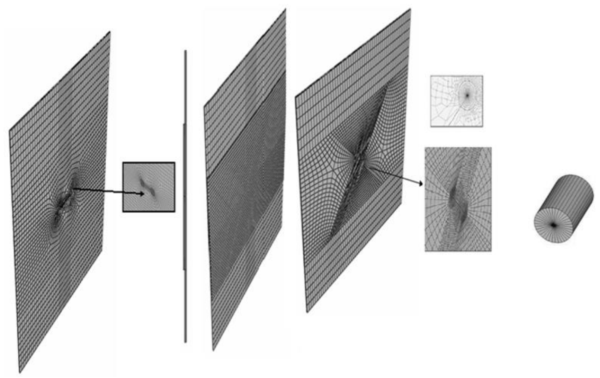
The basic geometry of the cracked structure in mode I considered in this study is shown in Fig. 1(a).

Consider a carbon/epoxy composite stratified plate with the following dimensions: height H_p= 304 mm, width w_p = 304 mm, thickness e_p = 1.6 mm. The plate is subjected to uniaxial



Single patch Double patch (a) Mode I (b) Mixed mode I+II

Fig. 1 Geometric model of plate with single and double patch for the pure mode I and mixed mode I+II



(a) Mode I (b) Mixed mode I+II

Fig. 2 Typical mesh model of the plate, patch and near the crack tip in pure mode I and mixed mode I+II

tensile load giving a remote stress state of σ_0= 100 MPa. A central crack of length 2a = 50 mm perpendicular to the loading axis is supposed to exist in the plate. This crack is repaired with unidirectional carbon / epoxy composite patch. The ply orientation is parallel to the loading axis. The dimensions of the patch are: height H_r = 50 mm, width w_r= 304 mm and thickness e_r= 1.6 mm. The composite patch is bonded to the cracked plate with the FM 73 adhesive having a thickness e_a = 0.2 mm and shear modulus G_a = 0.42 GPa. The geometric model of the plate in mixed mode I+ II is shown in Fig. 1(b). The stratified composite plate present a central crack inclined at an angle θ and length 2a = 50 mm repaired by single and double patch. The geometrical and mechanical properties of the plate and adhesive as well as the loading conditions are the same as those considered in mode I.

Calculations were carried out by the finite element ABAQUS code (2007) The finite element mesh used in the computation is shown in Fig. 2(a), where 20-node iso-parametrical block elements were sed. The finite element model consists of three subsections to model the cracked plate, the adhesive, and the composite patch. The plate has eight layers of elements in the thickness direction, the adhesive has only one layer of elements through thickness and the patch has four

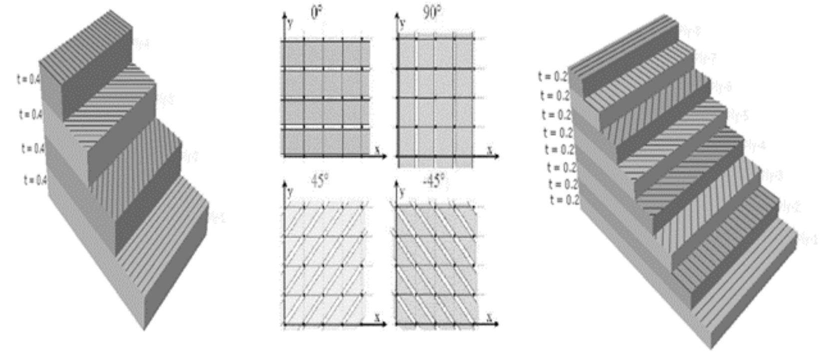


Fig. 3 Plies orientation in the patch and plate

layers of elements through thickness. To generate crack front some brick elements are replaced by “crack block”. These crack-blocks are meshes of brick elements which are mapped into the original element space and merged with surrounding mesh.

A crack blocks are used to introduce one or more crack fronts into the un-cracked meshed model ABAQUS (2007). The term crack-block refers to a collection of brick elements stored as a unit cube. These crack-blocks contain either a quarter circular or through crack front elements on one face. Part of this face is allowed to open up under loading giving the opening crack face within the crack-block. The meshing procedure involves replacement of one or more 8 or 20 noded brick elements in a user supplied un-cracked mesh by crack-blocks. During the mapping process to introduce the crack-blocks the user can control the size and shape of the generated crack front section for each crack-block. Crack-blocks can be connected together to form distinct crack fronts of the required size in the cracked mesh.

Boundary conditions and loads are transferred to the crack-block elements. The mesh was refined near the crack tip area with an element dimension of 0.060 mm using at least fifteen such fine elements in the front and back of the crack tip.

Numerical computation of stress and SIF’s with conventional elements near the crack tip was carried out by several researchers, (Sethuraman and Maiti (1989), Kashfuddoja and Ramji (2013)). Investigations on the accuracy of computation of SIF’s using quarter point elements around the crack yielded a reasonably accurate result but, later on it is proved that very fine mesh size around the crack tip gives $1/\sqrt{r}$ singularity precisely. Nakmura and Parks (1988), have modeled crack tip, like a circular disk with super fine meshing around the crack tip in order to capture very high stress gradient. Same procedure is adopted in this work. Initially individual areas are created around the crack tip and meshed with plane element having eight nodes. The crack tip mesh has a total of 7100 elements (36 circumferential, 33 radial; 6 elements through the thickness around the crack tip region). Later, all the areas are extruded in thickness direction to generate volume. Finally, all the generated volumes are meshed with 20 iso-parametrical elements.

The total number of elements of the structure repaired by single patch is equal to 23500 (22280 for the plate, 400 for the adhesive and 820 for the patch) the number of elements depends on the shape of the patch and it changes to the case of the double patch (24700 items). Fig. 2(b) shows the overall mesh of the composite plate in mixed mode and mesh refinement in the crack tip region.

The stratification of plate has eight plies stacking sequence $[45/\text{-}45/0/90]_S$, the thickness of an

Table 1 Mechanical properties for elementary ply composite carbon/epoxy

E_1 (MPa)	E_2 (MPa)	E_3 (MPa)	G_{12} (MPa)	G_{23} (MPa)	G_{31} (MPa)	ν_{12}	ν_{23}	ν_{31}
103000	7000	7000	3150	2750	3150	0.34	0.25	0.34

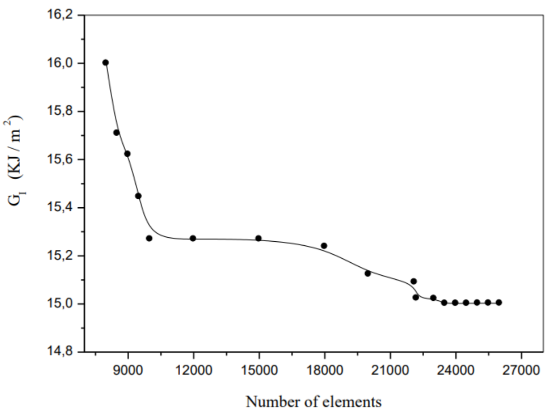


Fig. 4 Strain energy release rate Vs number of elements of cracked plate (a = 25 mm)

elementary ply is taken equal 0.2 mm. The patches are four sequence ply [45/-45/0/90]. The thickness of elementary ply is 0.4 mm (Fig. 3).

The adhesive is modeled as a third layer. It is assumed that the adhesive layer is homogeneous and isotropic. In the finite element model the nodes are common between the laminate composite plate, the adhesive and the patch so that there remains a continuity of deformation and stress. This technique of the three layers was employed by Naboulsi and Mall (1996). It employs the two-dimensional finite element analysis, composed of three layers, to model the composite plate, patch and the adhesive. The advantage of modeling the adhesive as a continuous body is to be able to capture the characteristics of the adhesive which will be required to model heating effects, a nonlinear behaviour of material, progressive damage, etc... (Naboulsi and Mall (1996)).

The composite cracked plate and the patch and the adhesive have a linear elastic behavior. A linear elastic static analysis was carried out. The model with nonlinear behaviour of adhesive was chosen for further study. The mechanical properties of elementary ply of the carbon / epoxy composite are illustrated in Table 1. The mechanical properties indicated in Table 1, are those of a unidirectional UD elementary ply, for stratification to 8 plies of the sequence [45 / -45 / 0/ 90]S; these properties are introduced into the Abaqus model according to the orientation angle.

The procedure used in the finite element analysis involved the following step: the tensile stress was applied to the gripped specimen. General static "STEP"-option was used for analysis with ABAQUS. Automatic increment of step was used with maximum number of increments of 100. Minimum increment size was 10-5. Maximum increment size was 1. Nevertheless, the ABAQUS solver code could override matrix solver choice according to the "STEP"-option.

A convergence study is performed on the cracked plate, to quantify the number of elements to get a converged value. The energy release rate G_I is evaluated; and its variation with respect to the number of elements is plotted as depicted in Fig. 4. It is observed that there is not much effect in the energy release rate G_I value with increasing the number of elements.

3. Results and analysis

3.1 Effect of stacking sequences

This study aims to find the parameters inherent in composite patch for optimal use in the repair of cracks. This optimal use is obtained when we get to select a judicious type of plate stacking sequence and the patch to minimize the energy release rate at the crack tip.

The influence of various parameters on the performance of the repair is very complex. For this study only the orientations of plies of the plate and the patch are optimized. Adhesive properties and its thickness and the dimensions of the patch and the plate are assumed to be constant.

To study the influence of the orientation of the plies of the plate and the patch on the level of the energy release rate G_I, We chose thereafter the stacking sequence of the plate minimizes the fracture energy at the crack tip. The procedure is the same for the patch. The stacking sequence of the patch and the plate will be chosen optimal.

The effect of stacking sequences of the unrepaired plate on the variation of the energy at the crack tip is illustrated in Fig. 5. The level of the higher fracture energy is obtained when the eight plies of the plate are oriented in the same direction $[0]_{2s}$ where $[90]_{2s}$. The use of orientation 45 or -45 in the stratified composite led to almost constant levels of fracture energy. The stacking sequence minimizing the value of the energy release rate G_I is [90/45 / -45 / 0]s.

The effect of orientation of the plies on the cracked composite repair performance is shown in Fig. 6. This latter shows the variation of the energy release rate at the crack tip repaired by a patch whose stacking sequence is [90/45/0/-45]. For this configuration, the 90 ply 1 of the patch is adjacent to the bonded joint of the plate. The orientation of the plies of the plate has a significant influence on the repair performance. Indeed; the lowest level of the fracture energy is obtained when adjacent plies in the plate (8 ply) and the patch (ply 1) are oriented at 90. An orientation of 0, 45 -45 or the ply 8 of the plate leads to practically the same variations of fracture energy, whose gap ranges from 3 to 12%. The repaired plate stacking sequence that produces the lowest level of the energy release rate is $[90\ 45\ 0\ -45]_s$.

The effect of the patch stacking sequences on the variation of the fracture energy at the crack

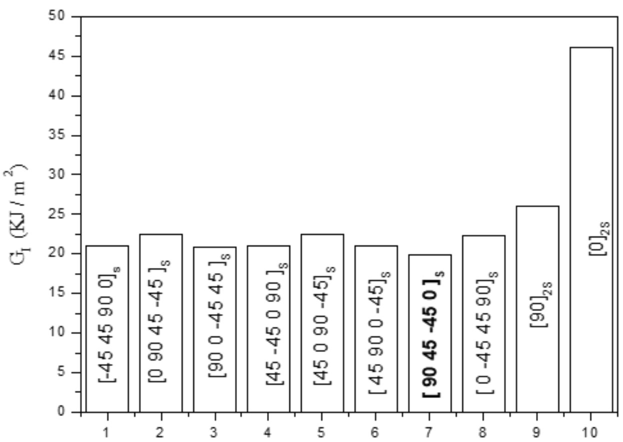


Fig. 5 Effect of plies orientation of the unrepaired plate on the GI values

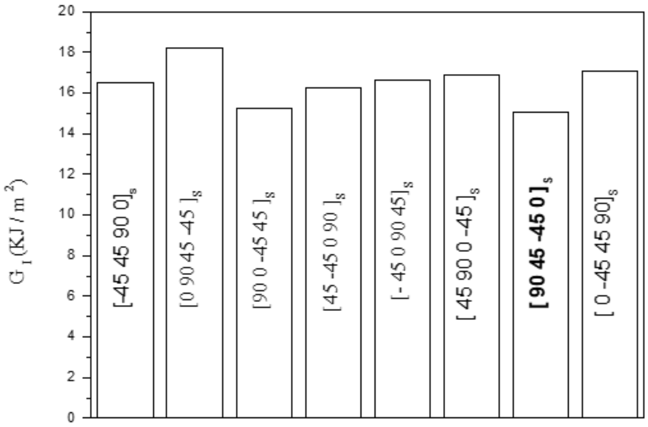


Fig. 6 Effect of plies orientation of the repaired plate on the GI values

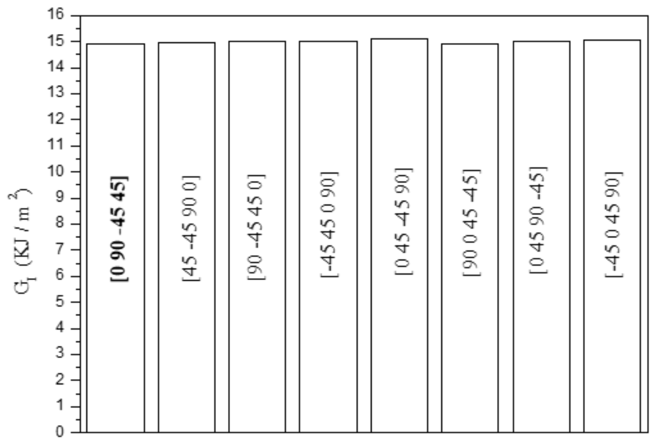


Fig. 7 Effect of plies orientation of the patch on the GI values

tip is shown in Fig. 7. The variations of energy release rates are a function of the orientation of the patch of the plies are nearly constant. The sequence leading to a minimum fracture energy is [0 90 -45 45]. In most of repairs, the use of unidirectional patch is optimal because this provides the highest reinforcing efficacy in the loading direction. However, in some cases under load biaxial high and a change of the orientation probable of crack (mixed mode), it is indispensable to provide the transverse reinforcement and / or shear. This can be achieved by using a laminate with an appropriate number of plies of + 45 and 90.

For this study we consider the sequence of crack plate [90 45 -45 0]s repaired by composite patch whose orientation of the plies is of the form [0 90 -45 45]. For this configuration the ply 8 of the plate oriented at 90 and in contact with the ply 1 of the patch also oriented at 0.

3.2 Comparison between patched and unpatched crack

Fig. 8 presents the variation of the energy release rate according to the crack length for a crack repaired by single and double patch and an unrepaired crack. The thickness of the single patch is

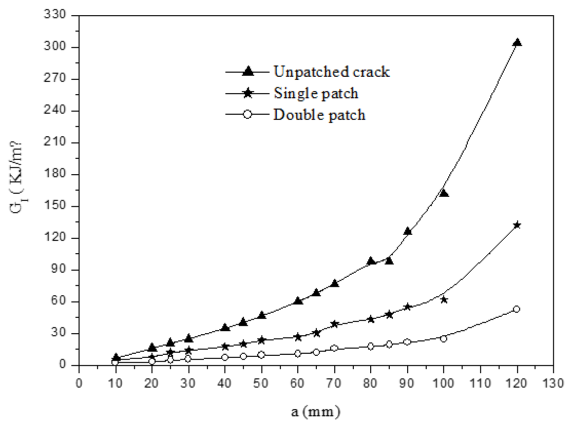


Fig. 8 Comparison of the fracture energy between patched and unpatched cracks

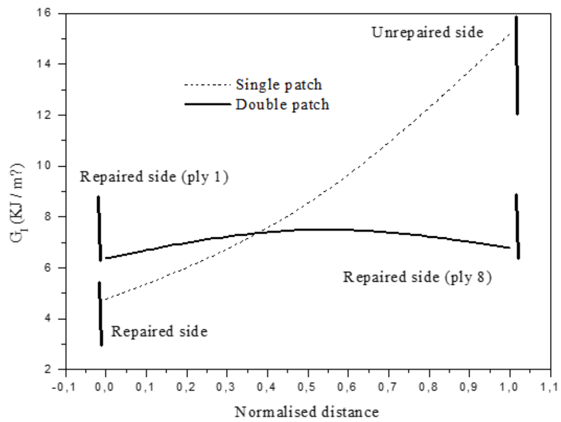


Fig. 9 Variation of the fracture energy GI along the crack front

twice that of the double patch.

The growth the crack length leads to an increase of the parameter G_I. We note that there is a critical crack size equal to 85 mm, beyond the fracture energy at the crack tip grows significantly. This increase is more marked when the crack propagates in the unrepaired composite. By against, in the plate repaired by a single patch the fracture energy increases slowly this may be due essentially to the delay of the propagation rate caused by the patch. Our results show clearly the beneficial effect of the patch. Indeed, the values of the magnitude G_I declines sharply, this is because the patch absorbing efforts transferred by the plate across the adhesive that the crack length increases. The highest values of G_I are obtained in the unrepaired structure, whose the difference would expect 60% compared to the failure energy obtained for repaired cracks.

The presence of the double patch reduces again the fracture energy at the crack tip. For a small crack size the use of both types of patch practically leads to the same values of the energy release rate. The effect of double patch appears beyond a = 40 mm. From this crack length we notice a difference between the values of fracture energy. This difference is almost constant it is of the order of 40%. The reduction of energy at the crack tip is due to the fact that the stresses are doubly absorbed by the patch.

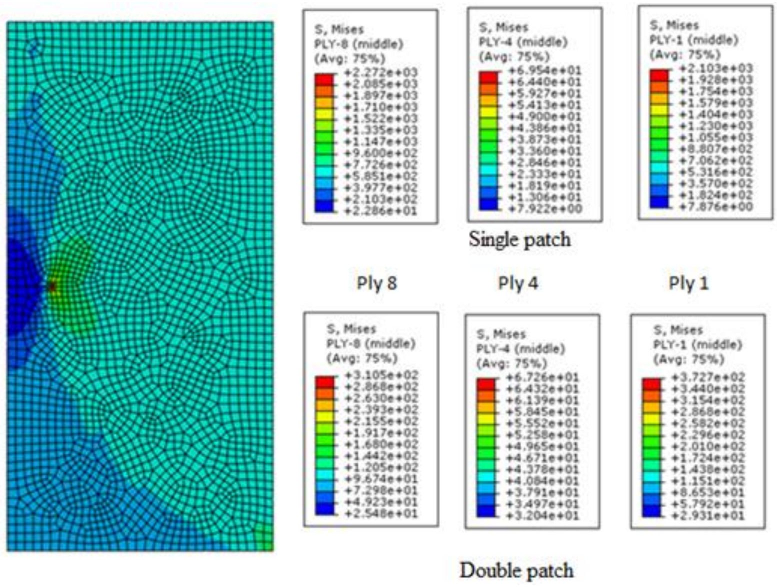


Fig. 10 Contour of Von Mises stresses (MPa) in the various layers of the plate

Analysis of preceding results shows that the repair patch dramatically reduces the fracture in crack tip. To confirm this finding we represent in Fig. 9 the variation of fracture energy along the front crack; this distance is normalized by the plate thickness. The G_I parameter is determined for a crack length 2a = 50 mm repaired by single and double patch.

The fracture energy of the crack tip do not only depends on the size of the crack and the stacking sequence but also the position along the thickness. Indeed, the most important values of this energy are obtained from the unrepaired side of the crack. Energy release rate reaches its maximum at the free edge of the plate and then decreases on the repaired side. The fracture energy of single patch increases linearly along the crack front. This energy decreases by 70% on both sides of the plate repaired and unrepaired.

The behavior of the fracture energy of the double patch is different from that single patch. Indeed, on both sides of the plate repaired the values of this parameter are almost constant. The highest value of the energy release rate is obtained at the middle of the plate.

The effect of double patch on the distribution of the Von Misses equivalent stresses is illustrated in Fig. 10. The stress distribution is determined on both top faces of the plate ply 1, ply 8 and the ply 4 in the middle of the plate. The single patch repairs the ply 8; on the other hand the double patch repairs two plies 1 and 8.

The stress distribution in the different plies depends on the repair technique. Indeed; a single patch leads to reduced stress between the ply 8 and 1 of the order of 8%. This difference increases to almost 16% for double patch. The beneficial effect of a double patch is clearly visible because the stress at the crack tip decreases sharply, this is because the double patch absorbs the forces transferred by the two sides of the plate across the adhesive. Comparing the equivalent stresses shows both the single patch gives a higher maximum stress than seven times that of the a double patch . On the other hand the patch same minimizes the stresses in the intermediate plies (ply 4) relative to other plies. The two types of repair practically generate the same distribution of stresses in the ply 4.

3.3 Effect of repair adhesive

The behavior of a repaired crack depends on number of factors including the mechanical and geometrical properties of the structure itself, of the adhesive and the patch. In the bonded repair, the adhesive is the weak point of the reinforcement of composite materials. Indeed, 53% of the deficiencies found in aerospace structures and repaired due to the adhesive (Davis and Bond 1999). These failings are mainly due to the transfer of effort from the substrate to the composite patch. In the bonded repair, the mechanicals properties of the adhesive largely determine the efficiency of load transfer into the patch (Davis and Bond 1999). If the adhesive is considered as an elastic material, its elastic properties and particularly the shear modulus have an important role on the performances of the bonded composite repair.

In this paragraph, the effect the adhesive properties on the energy release rate variation is analysed. Five kind of adhesive were chosen for calculations: FM73, FM43, Epon 422 J Redux K-6 and Adekit A140. The elastic properties of these adhesives are given in Table 2. Fig. 11 presents the variation of the energy release rate according to the crack length for the different adhesives. It can be seen according to this last figure that, the effect of the adhesive shear modulus Ga on the energy release rates variation is almost constant. The energy release rate decreases as the shear modulus of the adhesive increases as shown in Fig. 12, but the decrement of G_I according to Ga is not significant. Actually, an increase of the adhesive shear modulus reduces the adhesive strength, which can generate the adhesion failure.

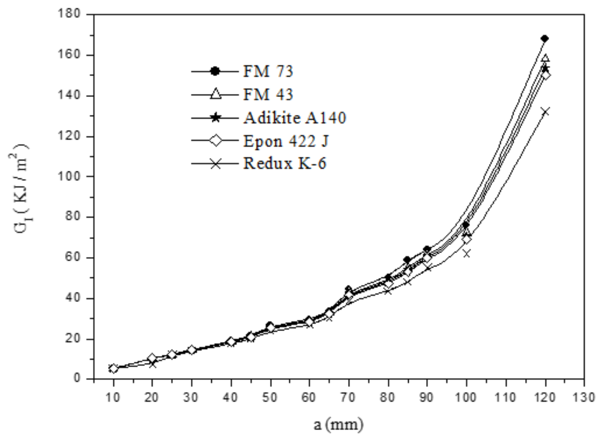


Fig. 11 GI vs crack length for different adhesives

Table 2 Mechanical properties of adhesives

Adhesive	Shear modulus 'Ga' (MPa)	Poisson ratio ‘ν’
Adekit A140	1070	0.30
FM-73	420	0.30
FM-43	810	0.38
Epon 422 J	1100	0.29
Redux K-6	1270	0.36

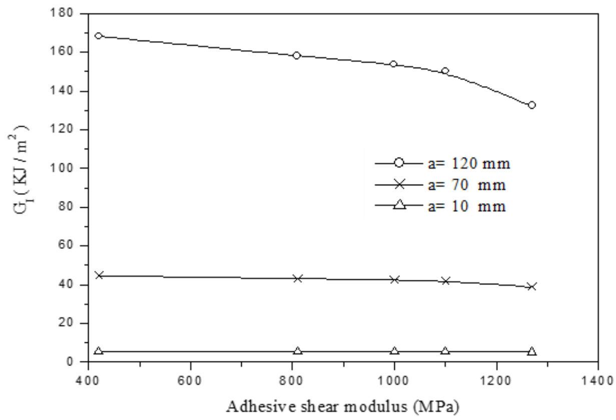


Fig. 12 Variation of the GI according to the adhesive shear modulus

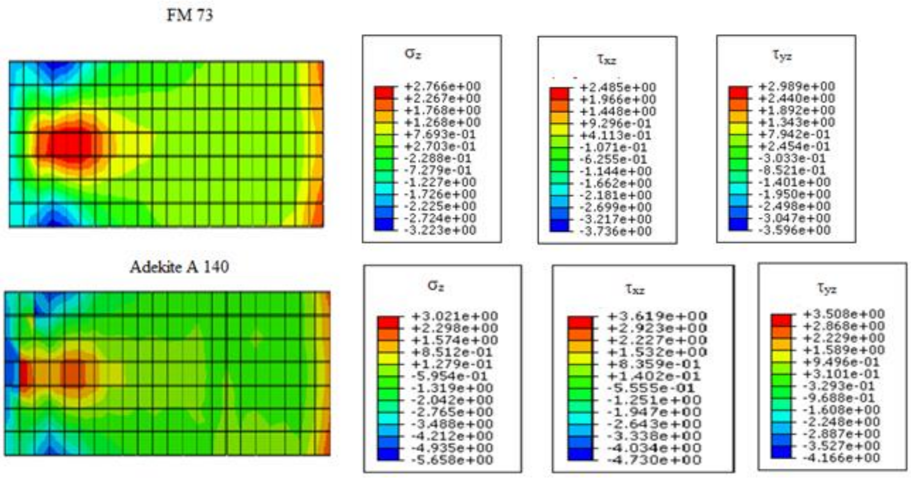


Fig. 13 Contour of peeling and shear stress in the adhesive layers

The increase in the size of the crack leads to an increase of the energy release rate. This effect is marked more for cracks lengths greater than 70 mm. For lower cracks a < 70 mm the difference of the energy release rate between the different adhesives is almost negligible. Repairing cracks by adhesives FM-43, Epon 422 J and the Adekit A 140 led the same levels of fracture energy. The repair by Redux K-6 engenders the lowest fracture energy, by against that the FM 73 leads to higher fracture energy. The difference observed between the values of G_I parameter at the crack tip repaired by the two adhesives increases with the size of the crack. This one does not exceed 20%; it decreased to 8% for cracks less than 70 mm.

The adhesive is a fundamental element, its main role is to ensure good adhesion and minimizing the transfer of stresses of the structure to the composite patch. The knowledge of intensities of stresses and their distribution in the adhesive layer is of great importance for predicting the lifetime of the repaired structure. The contours of the peeling stress σ_z and shear stress τ_{xz} and τ_{yz} in plans xz and yz are illustrated in Fig. 13. These contours are determined for a crack length 2a = 50 mm repaired by the Adekit A 140 and the FM 73.

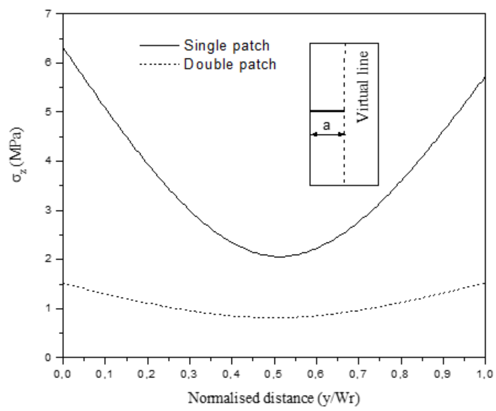


Fig. 14 Distribution of the peeling stress σz along virtual line in adhesive layer

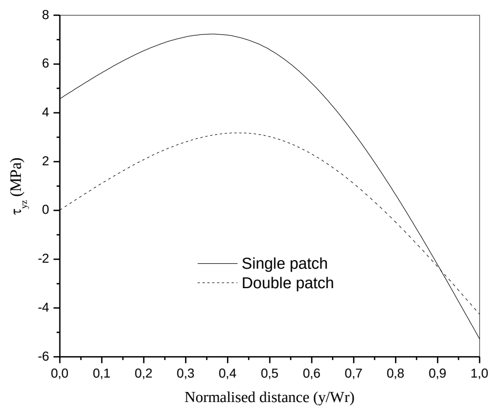


Fig. 15 Distribution of the shear stress τyz along virtual line in adhesive layer

Reparation by both adhesives engenders hight levels of shear stress and nearly identical peeling. A maximum difference of the order of 15% is found between the stresses of the two adhesives. The shear stress and of peeling are concentrated in the vicinity of the crack tip and the edge of the patch (plate-patch interface). Then they drop intensity to the middle of the adhesive layer to reach their minimum. These stresses are positive in the vicinity of the crack, and then they change their sign by increasing in absolute value away from this point. These stresses can provoke shear failure of the adhesive or delaminating of the composite patch.

In conclusion the analysis of our results shows that the intensity of the peel and shear stress is relatively low. These stresses are no immediate risk of damage to the adhesive layer. But long-term, to fatigue these stresses may lead to rupture of the adhesive. This behavior can be accelerated par the presence of defects in the adhesive layer or by aging due to the environment.

To complete this study, we determine the variations of the peeling and shear stress in the FM 73 adhesive layer. These stresses are plotted following a line virtual in the adhesive layer. This line passes through the crack tip and traversing the width of the composite patch,

Fig. 14 and 15 compare respectively peeling stress σ_z and shear τ_{yz} of the adhesive FM 73. This comparison is performed for a single and double patch. The comparison of the stresses shows the beneficial effect of the dual patch on the absorption of stress transfer of the composite plate to the patch through the adhesive layer. Indeed; double patch led to an average reduction of stresses on the order of 60% compared to single patch. The intensity of the shear stress and peeling stresses are relatively low compared to the failure stress of shearing of FM73 (Cytec Engineered Materials 2009).

3.4 Repaired and unrepaired crack in mixed mode

The fracture of the materials by opening (Mode I) is not only responsible for the propagation of the crack, several industrial examples show the presence of the mode II and mixed mode. Mixed mode most commonly encountered and most dangerous, is the result of the modes of openings (mode I) and slip (mode II). Many experimental studies and numerical simulations were performed on fracture in mixed mode. It is well known that the fracture resistance in mixed mode depends on the real mode crack propagation and fracture parameter used (Fekih *et al.* 2012, Ramji and Srilakshmi 2012, Ramji *et al.* 2013, Floros *et al.* 2015). The calculation of the direction of crack propagation in mixed mode can be determined by mechanical criteria which are based on the stress fields, deformations of or displacements. However, in real applications, the structures are subjected to various forms of loading conditions. Crack propagation and fracture mode occur due the combinations such loads.

The behavior of an inclined crack repaired by single composite patch is shown in Fig. 16. This latter presents the variation of the energy release rate as a function of the inclination θ.

The increase of the crack angle orientation leads to reduction of the fracture energy. A crack solicited in opening leads to higher energy levels than those obtained for inclined cracks. This may be due to the fact that the crack is solicited in opening mode (pure mode I), the fields of strains and stresses are maximal thus causing an increase of the G_I parameter. When the inclination of the crack increases it propagates in mixed mode I + II where the mode II dominates the mode I. This orientation leads to a change in the stress field and deformation close to the crack tip. These stresses and strains are divided in turn into stresses and deformations opening and shear thus leading to a decrease in the G_I parameter.

Our results clearly show the effect of the performance of the repair patch on the absorption of stress at the crack tip. Indeed, the values of the highest energy release rates are obtained in the unrepaired structure. This energy decreases by 70% compared to the energies of repaired cracks. The inclination θ of the crack (from 40° to 60°) leads to a difference almost constant failure energies.

The results of Fig. 16 show that for the two repair techniques, the fracture energy decreases asymptotically. It tends to stabilize as the inclination θ of the crack increases. This effect is marked more for the double patch. Beyond an angle of 15°, the values of the Griffith grandeur are nearly identical. The beneficial effect of dual patch appears for a crack solicited in mode I where we note a 60% gain, this gain decreases with increasing the angle of inclination of the crack.

Fig. 17 illustrates the variation in fracture energy along the front of a crack (2a = 50 mm) and inclined at θ = 45°. Whatever the mode of propagation of the crack, the fracture energy of single patch reaches its maximum at the free edge of the specimen and then decreases on the repaired side. We register a difference in energy on both sides of 33%. A propagation in mode I (Fig. 9) leads to a linear variation in fracture energy along the crack front. On the other hand the propagation in mixed mode modifies completely this behavior.

A crack oriented perpendicularly at the loading leads to the most important fracture energy. The

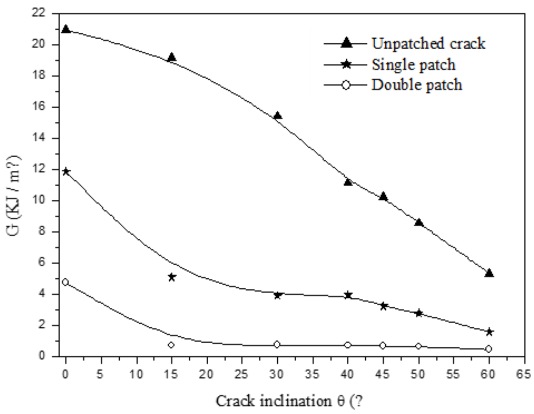


Fig. 16 Comparison of the fracture energy between patched and unpatched cracks in mixed mode

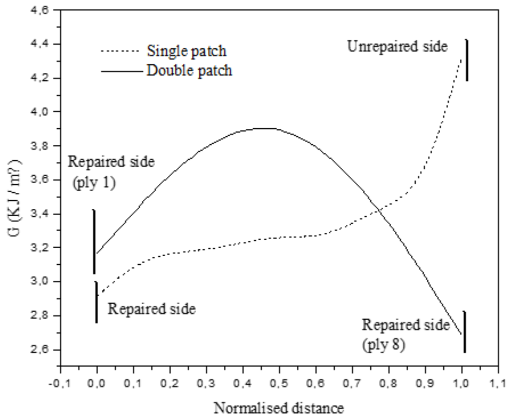


Fig. 17 Variation of the fracture energy in mixed mode along the crack front

inclination of the crack changes the distribution of the stresses absorbed by the patch of the plate through the adhesive. These stresses are divided in their turn in stress of opening and shear thus leading to a decrease in energy at the crack tip. The crack propagation mode does not affect the variation of the fracture energy along the crack front. Indeed, this energy is minimal at the edges repaired; it increases in the middle of the repaired composite plate. A propagation mode I lead to an increase of about 50% of this energy. This rate is obtained on the side of the ply 8 of the plate.

The effect of the fiber orientation of the adjacent ply to the bonded joint is not negligible. It has a significant influence on the repair performance. This parameter can change the damage scenario and therefore it needs to be optimized (Benkheira *et al.* 2018). For this study the stratified composite plate present a central crack inclined at θ = 45° and length 2a = 50 mm.

Fig.18 shows the variation of the energy release rate at the inclined crack tip repaired by a single patch. The effect of the patch stacking sequences on the variation of the fracture energy at the crack tip is shown in Fig. 19.

In mixed mode we consider the optimal sequence of crack plate [90 0 45 -45]s repaired by single composite patch whose orientation of the plies is of the form [-45 90 45 0]. For this configuration the ply 8 of the plate oriented at 90 and in contact with the ply 1 of the patch also

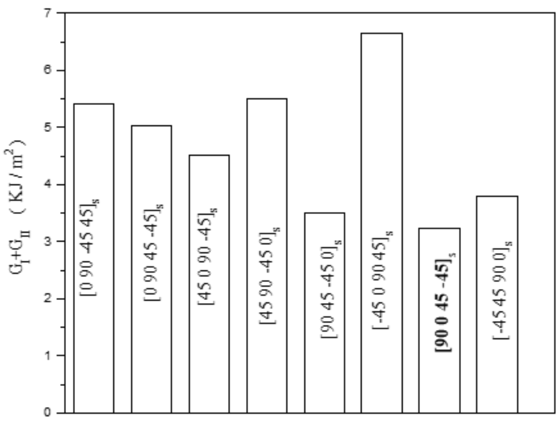


Fig. 18 Effect of plies orientation of the repaired plate on the GI+GII values (θ = 45°, 2a = 50 mm)

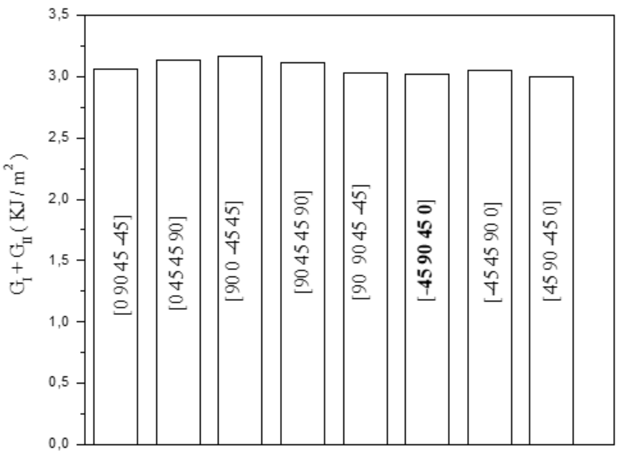


Fig. 19 Effect of plies orientation of the single patch on the GI+GII values (θ = 45°, 2a = 50 mm)

oriented at -45. The optimum of the stacking sequences of patch and composite plate in mixed mode differs from that of pure mode I.

4. Conclusions

The purpose of this study is to analyze the repair of cracked composite laminate structures by bonding single and double composite patches. The obtained results allow us to deduce the following conclusions:

The stiffness of the patch has a significant influence on the repair performance. There is an optimal stacking sequence leading to the minimization of fracture energy at the crack tip. The stacking sequence $[90\ 45\ \text{-}45\ 0]_s$ of the cracked composite plate and that of the patch [0 90 -45 45] are the sequences retained by our analysis. These two sequences generate the lowest level of the energy release rate.

The increase of the crack length leads to the increase of the fracture energy at the crack tip. There is a critical crack size a= 85 mm beyond it the fracture energy increases substantially. This increase is marked more when the crack propagates in the unrepaired composite.

The single patch reduces the intensity of the stress field and the fracture energy at the crack tip. This reduction can exceed 60%. The use of the symmetrical double patch also improves the repair performance due to the double transfer of stress between the repaired plate and the patch.

The choice of the adhesive properties for repairing crack, with the bonded composite patch, must be optimized. Indeed, the use of FM 73 generates the lowest energy release rate. The difference of the energy release rate between the different adhesives depends on the size of the crack.

Propagation in mixed mode modifies completely the failure behavior of repaired cracks. The increasing of the inclination of the crack leads to the decrease of its energy release rate. An open-mode bias produces the highest energy levels compared to the mixed I + II mode. The fracture energy decreases with the inclination angle. There is a critical angle ($\theta = 15°$) beyond which this fracture energy decreases asymptotically.

Whatever the propagation mode of crack, the fracture energy of the single patch reaches its maximum at the free edge of the specimen and then decreases on the repaired side.

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