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Photo-thermo-elastic interaction in a semiconductor material with two relaxation times by a focused laser beam

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Abstract. The effect of relaxation times is studied on plane waves propagating through semiconductor half-space medium by using the eigen value approach. The bounding surface of the half-space is subjected to a heat flux with an exponentially decaying pulse and taken to be traction free. Solution of the field variables are obtained in the form of series for a general semiconductor medium. For numerical values, Silicon is considered as a semiconducting material. The results are represented graphically to assess the influences of the thermal relaxations times on the plasma, thermal, and elastic waves.

Keywords: semiconductor material; eigen value approach; elastic waves; focused laser beam; generalized thermoelastic theories

1. Introduction

Semiconductors with a band gap energy are considered by means of a laser beam with an energy E greater than that of E_g, then an excitation process occurs. The electrons from the valence range are transferred to the energy level and their energy $(E - E_g)$ is greater than the edge of the conductivity band. These free carriers will relax to an unfilled level near the lower of the conduction band. After the relaxation process, the electron and hole plasma are found, followed by pairs of electron holes formed by the recombination process. Local deformation can cause local tensions in the sample, which provides a plasma wave like the thermal wave resulting from local periodic elastic deformation. The semiconductors materials are useful in radar and microwave similarly semiconductors are also used in very-high-speed SiGe devices.

In the last 40 years, the thermal theory has been very interested in the finite speed of thermal signals. These theories are called general thermoelastic theory. Lord and Shulman (1967) both proposed the first general theory of thermoelasticity which involved one relaxation time, while Green and Lindsay (1972) had a second general theory of thermal heat with two relaxation times. On the contrary, the coupled thermoelastic theory is associated with the parabolic heat equation.

Experimental and theoretical analysis of plasma, thermal and micromechanical in one dimension was made by Todorović *et al.* (2003a, b) to deal with the properties of carrier

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recombination and transport in the semiconducting material. In addition, such study includes the propagation of changes in heat and plasma waves due to the linear coupling between heat and mass transfer. Opsal and Rosencwaig (1985) and Rosencwaig *et al.* (1983) studied the depth of thermal and plasma waves in silicon. As an important branch of the mechanical properties of solids, the literature addresses various problems through numerical and analytical methods. On the other side, Song *et al.* (2010) studied the thermoelastic vibration produced by the optically excited semiconducting microcantilevers. They concluded that, the wave reflection in a semiconducting plane under photo thermal and theories of generalized thermoelasticity. Kumar and Vohra (2017) studied the vibration analysis of thermoelastic double porous microbeam subjected to laser pulse. They used the Laplace and Fourier's transformation to find the solution of the problem. Kumar et. al. (2016a, b), Sharma and Sharma (2014) are containing some important results related to wave propagation phenomena.

The photothermal waves in a one-dimensional semiconductor medium is studied by Abbas *et al.* (2017) and Hobiny and Abbas (2017). In Laplace, the eigen value method gives an analytical solution without any supposed restriction on the actual physical quantities. Lotfy (2017) presented photothermal waves for two-temperature model with a semiconductor medium due to a dual-phase-lag theory and hydrostatic initial stress. In other recent articles, Zenkour (2018a, b, 2019a, b) presented a multi dual-phase-lag theory to treat the thermomechanical response of microbeams, the micro-temperatures for plane wave propagation in thermoelastic medium, and the photothermal waves of a gravitated semiconducting half-space.

In this article, the heat waves propagating through the body are analyzed by using heat conduction equation with two relaxation times by Green and Lindsay (1972) (see also, Sharma *et al.* 2008). An analytical technique of eigen value approach is used to study their effects on the waves.

2. Mathematical formulation

The efforts are made to study the plasma, thermal and elastic waves generated by a focused laser beam in an elastic medium. For simplicity, the surface of the medium $\Im$ is supposed to be half space and y-axis is pointing vertically into the medium $\Im = \{(x, y, z)\colon -\infty \le x \le \infty, y \ge 0, -\infty \le z \le \infty\}$, with z-axis taken along the symmetry such that effects and changing do not appear along this axis. System of governing equations for the two-dimensional semiconductor under the influence of laser beam with radius r (Todorović 2005, Mandelis *et al.* 1997) is represented as

$$(\lambda+\mu)u_{j,ij} + \mu u_{i,jj} - \gamma_n N_{,i} - \gamma_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta_{,i} = \rho\frac{\partial^2 u_i}{\partial t^2}, \tag{1}$$

$$D_e N_{,jj} = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \vartheta\frac{\theta}{\tau} + Q, \tag{2}$$

$$K\theta_{,jj} = -\frac{E_g}{\tau}N + \rho c_e\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t} + \gamma_t T_0\left(1+m\tau_0\frac{\partial}{\partial t}\right)\frac{\partial u_{j,j}}{\partial t} + \delta_E Q, \tag{3}$$

and the stress-strain relations are presented as

$$\sigma_{ij} = \mu\left(u_{i,j}+u_{j,i}\right) + \left[\lambda u_{k,k} - \gamma_n N - \gamma_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\theta\right]\delta_{ij}, \tag{4}$$

where ρ denotes the density of medium, σ_{ij} are the stress components, u_i are the displacement components, τ_0, τ_1 are the thermal relaxation times (for semiconductor $10^{-12} \le \tau_0 \le 10^{-10}$ s, $10^{-12} \le \tau_1 \le 10^{-10}$ s), $\theta = T - T_0$, T_0 is the reference temperature, $N = n - n_0$, n_0 is the equilibrium carrier concentration, K denotes the thermal conductivity, $\delta_E = E - E_g$, E represents the excitation energy, E_g denotes the energy gap of the semiconductor, λ, μ represent Lame's constants, c_e denotes the specific heat at constant strain, D_e denotes the carrier diffusion coefficient, $\gamma_n = (3\lambda + 2\mu)d_n$, d_n represents the coefficient of electronic deformation, $\gamma_t = (3\lambda + 2\mu)\alpha_t$, α_t represents the linear thermal expansion coefficient, τ denotes the photogenerated carrier lifetime, $\vartheta = \frac{\partial n_0}{\partial \theta}$ denotes the coupling parameter of thermal activation, and t is the time.

The plate surface is illuminated by a laser pulse (Othman *et al.* 2015, Zenkour and Abouelregal 2015, Abouelregal and Zenkour 2019) given as

$$Q(x, y, t) = \frac{I_0 \gamma^* t}{2\pi r^2 t_0^2} \exp\left(-\frac{x^2}{r^2} - \frac{t}{t_0}\right) \exp(-\gamma^* y), \tag{5}$$

where I_0 denotes the energy absorbed, t_0 represents the pulse rise time, r represents the beam radius and γ^* denotes the absorption depth of heating energy.

Let us consider the state of plane strain in the present 2D problem of a semiconductor half-space. The variable components are defined by $u_i \equiv (u, v, 0)$, $u \equiv u(x, y, t)$, $v \equiv v(x, y, t)$, $\theta \equiv \theta(x, y, t)$ and $N \equiv N(x, y, t)$. Therefore, Eqs. (1)-(4) can be written by

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} + \mu \frac{\partial^2 u}{\partial y^2} - \gamma_n \frac{\partial N}{\partial x} - \gamma_t \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{6}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} + \mu \frac{\partial^2 v}{\partial x^2} - \gamma_n \frac{\partial N}{\partial y} - \gamma_t \left(1 + \tau_1 \frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{7}$$

$$D_e\left(\frac{\partial^2 N}{\partial X^2} + \frac{\partial^2 N}{\partial y^2}\right) = \frac{\partial N}{\partial t} + \frac{N}{\tau} - \vartheta \frac{\theta}{\tau} + Q, \tag{8}$$

$$K\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) = \frac{-E_g}{\tau} N + \rho c_e \left(1 + \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial \theta}{\partial t} + \gamma_t T_0 \left(1 + m\tau_0 \frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 v}{\partial y \partial t}\right) + \delta_E Q. \tag{9}$$

At $y = 0$, the boundary conditions (Hobiny and Abbas, 2018) are considered by

$$\sigma_{yy} = \sigma_{yx} = 0, \qquad D_e \frac{\partial N}{\partial y} - s_0 N = 0, \qquad -K\frac{\partial \theta}{\partial y} = \frac{q_0 t^2 e^{-t/t_p}}{16 t_p^2}, \tag{10}$$

where s_0 represents the speed of surface recombination, q_0 denotes a constant and t_p denotes the characteristic time of the pulse heat flux.

It is convenient to transform the above equations into dimensionless forms. To do this, the dimensionless quantities can be introduced as

$$\begin{gathered}(t^*, \tau^*, \tau_0{}^*, \tau_1{}^*) = \eta c^2 (t, \tau, \tau_0, \tau_1), \qquad (x^*, y^*, u^*, v^*) = \eta c(x, y, u, v), \\ (\sigma_{xx}{}^*, \sigma_{yy}{}^*, \sigma_{xy}{}^*) = \frac{1}{\mu}(\sigma_{xx}, \sigma_{yy}, \sigma_{xy}), \qquad N^* = \frac{N}{n_0}, \qquad \theta^* = \frac{\theta}{T_0}, \qquad Q^* = \frac{Q}{n_0 \eta^2 c^2 D_e},\end{gathered} \tag{11}$$

where $\eta = \frac{\rho c_e}{K}$ and $c = \sqrt{\frac{\lambda + 2\mu}{\rho}}$.

By neglecting the asterisk and rewriting Eqs. (5)-(9), we obtain,

$$\frac{\partial^2 u}{\partial x^2}+\alpha_1\frac{\partial^2 v}{\partial x\partial y}+\alpha_2\frac{\partial^2 u}{\partial y^2}-\beta_n\frac{\partial N}{\partial x}-\beta_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial x}=\frac{\partial^2 u}{\partial t^2}, \tag{12}$$

$$\frac{\partial^2 v}{\partial y^2}+\alpha_1\frac{\partial^2 u}{\partial x\partial y}+\alpha_2\frac{\partial^2 v}{\partial x^2}-\beta_n\frac{\partial N}{\partial y}-\beta_t\left(1+\tau_1\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial y}=\frac{\partial^2 v}{\partial t^2}, \tag{13}$$

$$\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}=\varpi\frac{\partial N}{\partial t}+\varpi\frac{N}{\tau}-\beta\frac{\theta}{\tau}+Q_0\frac{t}{{t_0}^2}\exp\left(\frac{-x^2}{r^2}-\frac{t}{t_0}\right)\exp(-\gamma^* y), \tag{14}$$

$$\begin{gathered}\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}=\frac{-\epsilon_2}{\tau}N+\left(1+\tau_0\frac{\partial}{\partial t}\right)\frac{\partial\theta}{\partial t}+\epsilon_1\left(1+m\tau_0\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u}{\partial x\partial t}+\frac{\partial^2 v}{\partial y\partial t}\right)\\ +Q_1\frac{t}{t_0^2}\exp\left(\frac{-x^2}{r^2}-\frac{t}{t_0}\right)\exp(-\gamma^* y),\end{gathered} \tag{15}$$

$$\frac{\partial^2 N}{\partial x^2}+\frac{\partial^2 N}{\partial y^2}=\varpi\frac{\partial N}{\partial t}+\varpi\frac{N}{\tau}-\beta\frac{\theta}{\tau}+Q_0\frac{t}{{t_0}^2}\exp\left(\frac{-x^2}{r^2}-\frac{t}{t_0}\right)\exp(-\gamma^* y), \tag{16}$$

where

$$\alpha_1=\frac{\lambda+\mu}{\lambda+2\mu},\quad \alpha_2=\frac{\mu}{\lambda+2\mu},\quad \alpha_3=\frac{\lambda}{\lambda+2\mu},\quad \beta_n=\frac{n_0\gamma_n}{\lambda+2\mu},\quad \beta_t=\frac{T_0\gamma_t}{\lambda+2\mu},$$
$$\varpi=\frac{1}{\eta D_e},\quad \beta=\frac{\vartheta T_0}{n_0\eta D_e},\quad \epsilon_1=\frac{\gamma_t}{\rho c_e},\quad \epsilon_2=\frac{I_0\gamma^*}{2\pi r},\quad Q_0=\frac{E_g n_0}{\eta K T_0},\quad Q_1=\frac{\delta_E n_0 D_e}{K T_0}\left(\frac{I_0\gamma^*}{2\pi r}\right).$$

3. Harmonic solution

The solution of the considered physical quantities can be decomposed in terms of normal mode analysis as

$$[u,v,N,\theta,\sigma_{ij}]=[u^*,v^*,N^*,\theta^*,\sigma_{ij}^*](y)\mathrm{e}^{i(ax-\omega t)}, \tag{17}$$

where u^*, v^*, N^*, θ^* and σ_{ij}^* are the amplitudes of the physical quantities u, v, N, θ and σ_{ij}. $'a'$ is the wave number in the x-direction, $i=\sqrt{-1}$ and $'\omega'$ is the frequency, respectively.

$$(\alpha_2 D^2+d_1)u^*(y)+d_2 Dv^*(y)-d_3 N^*(y)-d_4\theta^*(y)=0, \tag{18}$$

$$d_2 Du^*(y)+(D^2-d_5)v^*(y)-d_7 DN^*(y)-d_8 D\theta^*(y)=0, \tag{19}$$

$$(D^2+d_9)N^*(y)+d_{14}\theta^*(y)=f_1(x,t)\mathrm{e}^{-\gamma^*(y)}, \tag{20}$$

$$(D^2+d_{10})\theta^*(y)+d_{11}N^*(y)-d_{12}u^*(y)+d_{13}v^*(y)=f_2(x,t)\mathrm{e}^{-\gamma^*(y)}, \tag{21}$$

where

$$D=\frac{\mathrm{d}}{\mathrm{d}y},\quad d_1=\omega^2-a^2,\quad d_2=\alpha_1 ia,\quad d_3=\beta_n ia,\quad d_4=\beta_t(1-\tau_1 i\omega)ia,$$
$$d_5=\alpha_2 a^2+\omega^2,\quad d_7=\beta_n,\quad d_8=\beta_t(1-\tau_1 i\omega),\quad d_9=\varpi i\omega-a^2-\frac{\varpi}{\tau},\quad d_{11}=\frac{\epsilon_2}{\tau},$$
$$d_{10}=-a^2+i\omega(1-\tau_0 i\omega),\quad d_{12}=a\omega\epsilon_1(1-m\tau_0 i\omega),\quad d_{13}=i\omega\epsilon_1(1-m\tau_0 i\omega),$$
$$d_{14}=\frac{\beta}{\tau},\quad f_1(x,t)=Q_0\frac{t}{{t_0}^2}\left(\frac{-x^2}{r^2}-\frac{t}{t_0}\right),\quad f_2(x,t)=Q_1\frac{t}{{t_0}^2}\left(\frac{-x^2}{r^2}-\frac{t}{t_0}\right).$$

Now, let us proceed to solve the nonhomogeneous coupled differential equations by an eigen-value approach. Eqs. (18)-(21) can be written in a vector-matrix differential equation as follows

$$\frac{\mathrm{d}\phi}{\mathrm{d}y} = \lambda\phi - g\ \mathrm{e}^{-\gamma^* y}, \tag{22}$$

where

$$\phi = \begin{bmatrix} u & v & N & \theta & \frac{\mathrm{d}u}{\mathrm{d}y} & \frac{\mathrm{d}v}{\mathrm{d}y} & \frac{\mathrm{d}N}{\mathrm{d}y} & \frac{\mathrm{d}\theta}{\mathrm{d}y} \end{bmatrix},$$

$$\lambda = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ b_{11} & 0 & b_{12} & b_{13} & 0 & b_{14} & 0 & 0 \\ 0 & b_{15} & 0 & 0 & b_{17} & 0 & b_{18} & b_{16} \\ 0 & 0 & b_{19} & b_{24} & 0 & 0 & 0 & 0 \\ b_{20} & 0 & b_{21} & b_{22} & 0 & b_{23} & 0 & 0 \end{bmatrix}, \qquad g = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ f_1(x,t) \\ f_2(x,t) \end{bmatrix}.$$

with

$$b_{11} = \frac{-d_1}{\alpha_2}, \quad b_{12} = \frac{d_3}{\alpha_2}, \quad b_{13} = \frac{d_4}{\alpha_2}, \quad b_{14} = \frac{d_2}{\alpha_2}, \quad b_{15} = d_5, \quad b_{16} = d_8, \quad b_{17} = d_2, \quad b_{18} = d_7,$$
$$b_{19} = -d_9, \quad b_{20} = d_{12}, \quad b_{21} = -d_{11}, \quad b_{22} = -d_{10}, \quad b_{23} = -d_{13}, \quad b_{24} = -d_{14}.$$

Then, the characteristic equation of the matrix λ is expressed as

$$\xi^8 + A\xi^6 + B\xi^4 + C\xi^2 + D = 0, \tag{23}$$

where

$$A = -b_{11} - b_{15} - b_{14}b_{17} - b_{19} - b_{22} - b_{16}b_{23},$$

$$B = b_{11}b_{15} + b_{11}b_{19} + b_{15}b_{10} + b_{14}b_{17}b_{19} - b_{13}b_{20} - b_{14}b_{16}b_{20} + b_{11}b_{22} + b_{15}b_{22} + b_{14}b_{17}b_{22} + b_{19}b_{22} + b_{11}b_{16}b_{23} + b_{13}b_{17}b_{23} + b_{16}b_{19}b_{23} - b_{21}b_{24} - b_{18}b_{23}b_{24},$$

$$C = -b_{11}b_{15}b_{19} + b_{13}b_{15}b_{20} + b_{13}b_{19}b_{20} + b_{14}b_{16}b_{19}b_{20} - b_{11}b_{15}b_{22} - b_{11}b_{19}b_{22} - b_{15}b_{19}b_{22} - b_{14}b_{17}b_{19}b_{22} - b_{11}b_{16}b_{19}b_{23} + b_{13}b_{17}b_{19}b_{23} - b_{12}b_{20}b_{24} - b_{14}b_{18}b_{20}b_{24} + b_{11}b_{21}b_{24} + b_{14}b_{17}b_{21}b_{24} - b_{12}b_{17}b_{23}b_{24} + b_{11}b_{18}b_{23}b_{24},$$

$$D = -b_{13}b_{15}b_{19}b_{20} + b_{11}b_{15}b_{19}b_{22} + b_{12}b_{15}b_{20}b_{24} - b_{11}b_{15}b_{21}b_{24}.$$

The roots of the characteristic Eq. (21) which are also the eigen values of matrix λ are of the form $\pm\xi_1$, $\pm\xi_2$, $\pm\xi_3$ and $\pm\xi_4$. The eigenvector $Y = [Y_1 \quad Y_2 \quad Y_3 \quad Y_4 \quad Y_5 \quad Y_6 \quad Y_7 \quad Y_8]$ corresponding to eigen value ξ can be calculated as

$$Y_1 = -\left(\xi b_{12}b_{17}b_{24} - \xi b_{18}b_{24}(b_{11} - \xi^2)\right) + \left(\xi b_{13} - \xi b_{16}(b_{11} - \xi^2)\right)(b_{19} - \xi^2),$$

$$Y_2 = -(b_{19} - \xi^2)(\xi^2 b_{14}b_{17} - \xi b_{18}b_{11} + \xi^3 b_{18} - b_{11}b_{15} - \xi^2 b_{15} - \xi^2 b_{11} + \xi^4),$$

$$Y_3 = b_{24}\left(\xi^2 b_{17}b_{14} - (b_{11} - \xi^2)(b_{15} - \xi^2)\right),$$

$$Y_4 = \frac{b_{14}\xi}{(b_{11}-\xi^2)}(\xi b_{12}b_{17}b_{24} - \xi b_{11}b_{18}b_{24} + \xi^3 b_{18}b_{24} + \xi b_{13}b_{19} - \xi^3 b_{13} - \xi b_{11}b_{16}b_{19} + \xi^3 b_{16}b_{11} + \xi^3 b_{16}b_{19} - \xi^5 b_{16}) - \frac{b_{12}b_{24}}{(b_{11}-\xi^2)}\left(\xi^2 b_{17}b_{14} - (b_{11} - \xi^2)(b_{15} - \xi^2)\right) + b_{13}(\xi^2 b_{14}b_{17} - \xi b_{18}b_{11} + \xi^3 b_{18} - b_{11}b_{15} - \xi^2 b_{15} - \xi^2 b_{11} + \xi^4),$$

$$Y_5 = \xi Y_1, \qquad Y_6 = \xi Y_2, \qquad Y_7 = \xi Y_3, \qquad Y_8 = \xi Y_4.$$

The solution of Eq. (22) has the following form

$$\phi = \sum_{i=1}^{4} B_i Y_i \mathrm{e}^{-\xi_i y} + \sum_{i=1}^{4} B_{i+4} Y_{i+4} \mathrm{e}^{\xi_i y} + g_2 \mathrm{e}^{-\gamma^* y}, \tag{24}$$

where B_i, $(i = 1,2,\dots,8)$ are constants to be determined by the boundary conditions of the problem and

$$g^* = [g_1 \quad g_2 \quad g_3 \quad g_4 \quad g_5 \quad g_6 \quad g_7 \quad g_8]^T,$$

$$g = -\left(b_{20}(\gamma^{*^2} - b_{15}) + \gamma^{*^2} b_{23} b_{17}\right)\begin{pmatrix} \gamma^* b_{17}\left(b_{12}b_{24} + b_{13}(\gamma^{*^2} - b_{19})\right) + \\ (\gamma^{*^2} - b_{11})\left(\gamma^* b_{18} b_{24} + \gamma^* b_{16}(\gamma^{*^2} - b_{19})\right)\end{pmatrix} - \left(\gamma^* b_{17} b_{14} - (\gamma^{*^2} - b_{15})(\gamma^{*^2} - b_{11})\right)\left(\left(-\gamma^* b_{17} b_{21} b_{24} + \gamma^* b_{17}(\gamma^{*^2} - b_{22})(\gamma^{*^2} - b_{19})\right) + \left(\gamma^* b_{18} b_{20} + \gamma^* b_{16} b_{20}(\gamma^{*^2} - b_{19})\right)\right),$$

$$g_3 = \frac{1}{g}\left(b_{20}(\gamma^{*^2} - b_{15}) + \gamma^{*^2} b_{23} b_{17}\right)\left(\gamma^* b_{13} b_{17} + \gamma^* b_{16}(\gamma^{*^2} - b_{11})\right) - f_1(x,t)\left(\gamma^* b_{17} b_{14} - (\gamma^{*^2} - b_{15})(\gamma^{*^2} - b_{11})\right)\left(\gamma^* b_{16} b_{20} + \gamma^* b_{17}(\gamma^{*^2} - b_{24})\right) + f_2(x,t)\left(\gamma^* b_{17} b_{24}\left(\gamma^* b_{17} b_{14} - (\gamma^{*^2} - b_{15})(\gamma^{*^2} - b_{11})\right)\right),$$

$$g_4 = \frac{\left(\gamma^{*^2} - b_{19}\right)}{b_{24}} g_3 + f_1(x,t)\frac{1}{b_{24}},$$

$$g_2 = \frac{\gamma^* b_{17}\left(b_{12}b_{24} + b_{13}(\gamma^{*^2} - b_{19})\right) + (\gamma^{*^2} - b_{11})\left(\gamma^* b_{18} b_{24} + \gamma^* b_{16}(\gamma^{*^2} - b_{19})\right)}{b_{24}\left(\gamma^* b_{17} b_{14} - (\gamma^{*^2} - b_{15})(\gamma^{*^2} - b_{11})\right)} g_3 + \frac{\gamma^* b_{17} b_{13} + \gamma^* b_{16}(\gamma^{*^2} - b_{11})}{b_{24}\left(\gamma^* b_{17} b_{14} - (\gamma^{*^2} - b_{15})(\gamma^{*^2} - b_{11})\right)} f_1(x,t),$$

$$g_1 = -\frac{\gamma^* b_{14}}{(\gamma^{*^2} - b_{11})} g_2 + \frac{b_{12}b_{24} + b_{13}(\gamma^{*^2} - b_{19})}{b_{24}(\gamma^{*^2} - b_{11})} g_3 + \frac{b_{13}}{b_{24}(\gamma^{*^2} - b_{11})} f_1(x,t),$$

$$g_5 = -\gamma^* g_1, \qquad g_6 = -\gamma^* g_2, \qquad g_7 = -\gamma^* g_3, \qquad g_8 = -\gamma^* g_4.$$

The general solution of the field variables can be written as

$$\bar{u} = \sum_{i=1}^{4} B_i U_i e^{-\xi_i y} + \sum_{i=1}^{4} B_{i+4} U_{i+4} e^{\xi_i y} + g_1 e^{-\gamma^* y}, \tag{25}$$

$$\bar{v} = \sum_{i=1}^{4} B_i V_i e^{-\xi_i y} + \sum_{i=1}^{4} B_{i+4} V_{i+4} e^{\xi_i y} + g_2 e^{-\gamma^* y}, \tag{26}$$

$$\bar{N} = \sum_{i=1}^{4} B_i N_i e^{-\xi_i y} + \sum_{i=1}^{4} B_{i+4} N_{i+4} e^{\xi_i y} + g_3 e^{-\gamma^* y}, \tag{27}$$

$$\bar{\theta} = \sum_{i=1}^{4} B_i \theta_i e^{-\xi_i y} + \sum_{i=1}^{4} B_{i+4} \theta_{i+4} e^{\xi_i y} + g_4 e^{-\gamma^* y}, \tag{28}$$

and

$$\bar{\sigma}_{yy} = \sum_{i=1}^{4} B_i(-\xi_i V_i + \alpha_3 ia U_i - \beta_n N_i - \beta_t(1 - \tau_1 i\omega)\theta_i)e^{-\xi_i y} + \sum_{i=1}^{4} B_{i+4}(-\xi_{i+4} V_{i+4} + \alpha_3 ia U_{i+4} - \beta_n N_{i+4} - \beta_t(1 - \tau_1 i\omega)\theta_{i+4})e^{\xi_i y} - (\gamma^* g_2 - \alpha_3 ia g_1 + \beta_n g_3 + \beta_t(1 - \tau_1 i\omega) g_4)e^{-\gamma^* y},$$

$$\bar{\sigma}_{xy} = \sum_{i=1}^{4} B_i(-\alpha_2 \xi_i U_i + \alpha_2 ia V_i)e^{-\xi_i y} + \sum_{i=1}^{4} B_{i+4}(-\alpha_2 \xi_i U_{i+4} + \alpha_2 ia V_{i+4})e^{\xi_i y} + (-\alpha_2 \gamma^* g_1 + \alpha_2 ia g_2)e^{-\gamma^* y}.$$

After applying the boundary conditions of the problem as mentioned above, we have the following set of equations:

$$\sum_{i=1}^{4} B_i(-\xi_i V_i + \alpha_3 ia U_i - \beta_n N_i - \beta_t(1 - \tau_1 i\omega)\theta_i) = (\gamma^* g_2 - \alpha_3 ia g_1 + \beta_n g_3 + \beta_t(1 - \tau_1 i\omega) g_4),$$

$$\sum_{i=1}^{4} B_i(-\alpha_2 \xi_i U_i + \alpha_2 ia V_i) = \alpha_2(\gamma^* g_1 - ia g_2), \sum_{i=1}^{4} B_i(-D_e \xi_i N_i - s_0 N_i) = \gamma^* D_e g_3 + s_0 g_3, \sum_{i=1}^{4} B_i(-\xi_i \theta_i) = \gamma^* g_4 + \frac{q_0 t^2 e^{-t/t_p}}{16 c t_p^2},$$

where B_i, $(\mathrm{i}=1,2,3,4)$ are constants based on the boundary conditions of the problem, which can be determined by:

$$[B]^T = [M]^{-1}[H]^T. \tag{29}$$

Therefore,

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix}^{-1} \begin{bmatrix} \gamma^* g_2 - \alpha_3 iag_1 + \beta_n + \beta_t(1-\tau_1 i\omega)g_4 \\ \alpha_2\gamma^* g_1 - \alpha_2 iag_2 \\ \gamma^* D_e g_3 + s_0 g_3 \\ \gamma^* g_4 + \dfrac{q_0 t^2 e^{-\left(t/t_p\right)}}{16ct_p{}^2} \end{bmatrix},$$

in which

$$M_{1p} = \sum_{p=1}^{4}\left(-\xi_p V_p + \alpha_3 iaU_p - \beta_n N_p - \beta_t(1-\tau_1 i\omega)\theta_p\right),\ M_{2p} = \sum_{p=1}^{4}\left(-\alpha_2\xi_p U_p + \alpha_2 iaV_p\right),$$

$$M_{3p} = \sum_{p=1}^{4}\left(-D_e\xi_p N_p - s_0 N_p\right),\ M_{4p} = \sum_{p=1}^{4}\left(-\xi_p\theta_p\right).$$

Hence, we obtain the solution of each variable.

4. Numerical results and discussions

To evaluate with the numerical examples, we consider for the computational purpose the silicon (Si) material. The thermoelastic properties of such material are (Alzahrani and Abbas 2018)

$$E = 2.33\ \text{(eV)},\quad E_g = 1.11\ \text{(eV)},\quad s_0 = 2\ (\text{m}s^{-1}),\quad D_e = 2.5\times10^{-3}\ (\text{m}^2\text{s}^{-1}),$$

$$\rho = 2330\ (\text{kg m}^{-3}),\quad \lambda = 3.64\times10^{10}(\text{Nm}^{-2}),\quad \mu = 5.64\times10^{10}\ (\text{Nm}^{-2}),$$

$$n_0 = 10^{20}\ \text{m}^{-3},\quad d_n = -9\times10^{-31}\ (\text{m}^3),\quad \tau = 5\times10^{-5}\ (\text{s}),\quad \alpha_t = 3\times10^{-6}\ (\text{K}^{-1}),$$

$$c_e = 695\ (\text{Jkg}^{-1}\text{K}^{-1}),\quad T_0 = 300\ (\text{K}),\quad K = 1.7\times10^2\ (\text{Wm}^{-1}\text{K}),\quad \omega = -0.1+0.1i,$$

$$a = 0.5,\quad q_0 = 0.1.$$

Based on the data set, the following graphs represent the numerically computed physical quantities at different values of the distance y. Numerical computations are carried out for the displacement, distribution of the temperature, the density of carriers along the y-axis for the two-dimensional isotropic and homogenous medium in context of the coupled photo-thermo-elastic conditions.

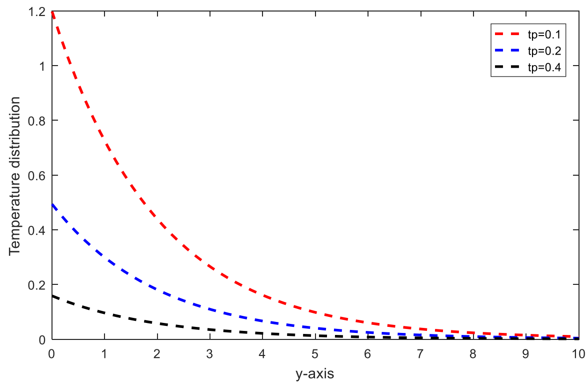

Fig. 1 Variation of temperature against y for different values of t_p

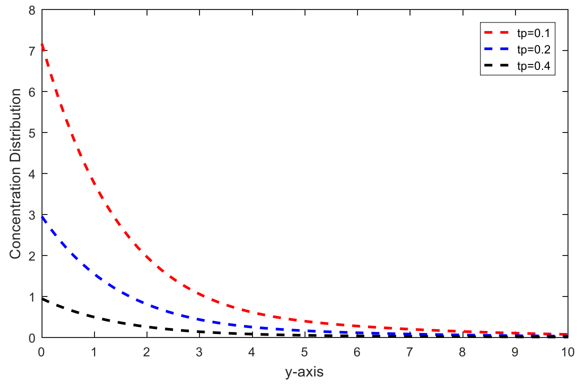

Fig. 2 Variation of concentration against y for different values of t_p

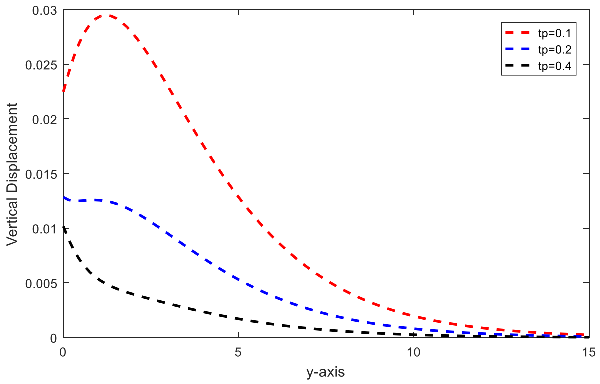

Fig. 3 Variation of vertical displacement against y for different values of t_p

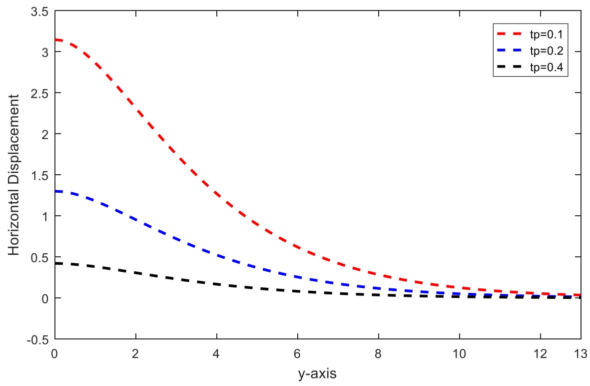

Fig. 4 Variation of horizontal displacement against y for different values of t_p

As shown graphically in Figures 1-4, the distributions with different values of characteristic time of heat pulse t_p i.e., $t_p = 0.1$, 0.2 and 0.4. The solid line refers to $t_p = 0.1$, while the dashed line shows to the time $t_p = 0.2$ and the dotted line refers to the time $t_p = 0.4$. Fig. 1 shows the variation in temperature distribution and it starts with its maximum value and gradually decreases with the distance y. Variation in carrier density is shown in the Fig. 2 and it also starts

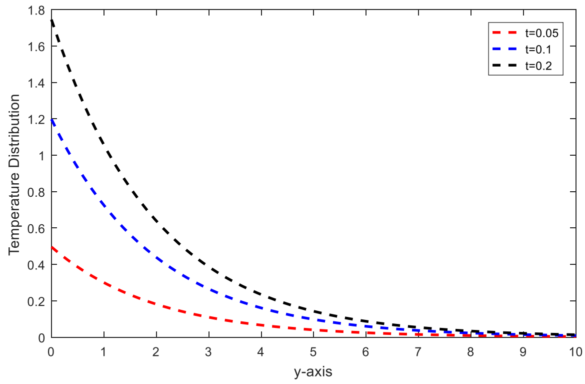

Fig. 5 Variation of temperature against y for different values of t

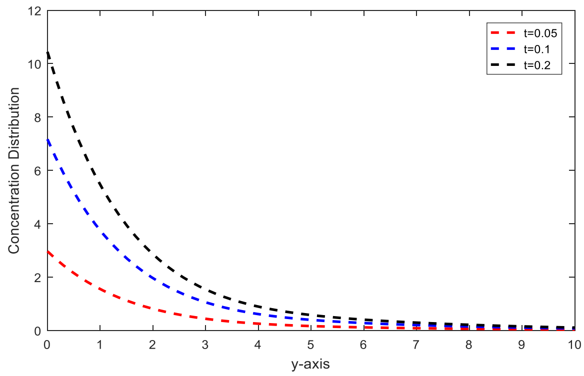

Fig. 6 Variation of concentration against y for different values of t

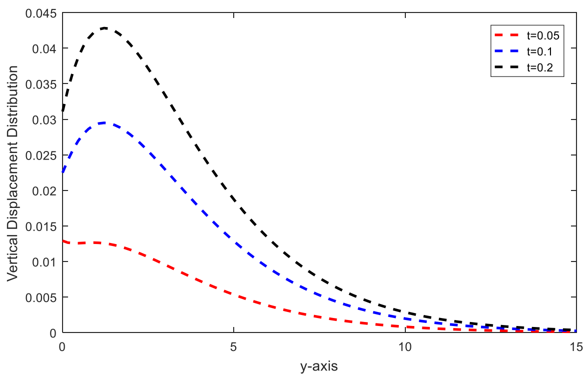

Fig. 7 Variation of vertical displacement against y for different values of t

with its peak value and the graph comes down with the increasing distance y with a range of $0 \leq y \leq 10$. In Fig. 3, the variation in vertical displacement is shown with a wide range of $0 \leq y \leq 15$, initially graph goes up to its maximum value and after reaching its peak point it decreases to its minimum value of zero. Fig. 4 displays the distribution of the horizontal component with the same range of $0 \leq y \leq 15$; it starts with its maximum value and decreases with the increasing distance.

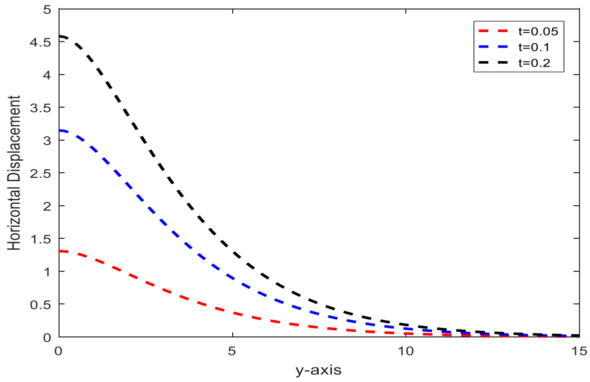

Fig. 8 Variation of horizontal displacement against y for different values of t

Graphs 5-8 show the variation with different values of thermal source for time $t = 0.05$, 0.1 and 0.2. The solid line refers to the time $t = 0.05$, while the dashed line refers to the time $t = 0.01$ and the dotted line refers to the time $t = 0.2$. Fig. 5 displays the distribution of the values of the temperature T with respect to y-axis for different values of time t with respect to wide range $0 \leq y \leq 10$. It is noted that the temperature starts by its maximum value at the first end of the strip $y = 0$ and gradually decreases with increasing distance up to zero. Figure 6 shows the distribution of the values of the carrier density with respect to y-axis for different values of time t with the same range. It also starts with its maximum value and decreases with increasing distance y. Figure 7 shows the distribution of the values of the vertical displacement component v with respect to y-axis for different times t with wide range of $0 \leq y \leq 15$. It is noted that the vertical displacement first increases and goes to its peak and then decreases with the increasing distance up to zero. Figure 8 shows the distribution of the values of the displacement component u with respect to y-axis for different times t, it starts with the maximum value and the decreases with a range of $0 \leq y \leq 15$.

5. Conclusions

According to the proceeding results, the time parameters t_p and t are having significant effects on distribution function of each variable. Based upon Eigen value approach, an analytical solution is analyzed for thermoelastic problem in semiconductor. Based on the graphical representation, it can be concluded that the characteristic time of pulse is having a decreasing effect on each variable, while the temporal variable is directly proportional to the amplitudes of each variable. Values of all physical quantities converge to zero with the increase in the distance y.

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