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Hygrothermal effect on the moisture absorption in composite laminates with transverse cracks and delamination

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Abstract. The stiffness degradation of the cross-ply composite laminates containing a transverse cracking and delamination in 90° layer is predicted by using a modified shear-lag model by introducing the stress perturbation function. The prediction shows better agreement with the experimental results published by Ogihara and Takeda 1995, especially for laminates with thicker 90°plies in which extensive delamination occurs. A homogenised analytic model for average transient moisture uptake in composite laminates containing periodically distributed matrix cracks and delamination is presented. It is shown that the model well describes the moisture absorption in a cross-ply composite laminate containing periodically distributed transverse matrix cracks in the 90° plies. The obtained results represent well the dependence of the stiffness degradation on the crack density, thickness ratio and moisture absorption. The present study has proved to be important to the understanding of the degradation of the material properties in the failure process when the laminates in which the delamination grows extensively.

Keywords: delamination; stiffness; hygrothermal effect; absorption; matrix cracks

1. Introduction

When the composite laminate is subjected to hygrothermal and mechanical loads reaches final failure, different kinds of damage modes are expected such as transverse matrix cracking, delamination, fibre-matrix debonding and fibre fractures (Adolfsson and Gudmundson 1997). The damage mode that often first is observed during increased loading is transverse matrix cracking. These cracks are generally not critical for the final failure of the laminate, but they have an influence in the material properties such as elastic modulus, hygrothermal expansion coefficient and diffusion coefficients.

Previously, Adda bedia et *al.* (2008) and Amara et *al.* (2014) studied the stiffness degradation of symmetric hygrothermal aged composite laminates containing a cracked mid-layer. For that, the material properties of the composite are affected by the variation of temperature and moisture, and are based on a micro-mechanical model of laminates. This hygrothermal effect is taken into account to assess the changes in the longitudinal modulus due to transverse cracking.

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Generally, the previous analyses consider only transverse cracking. In fact, a comparison (Berthelot *et al.* 1996) of experimental results and results derived from analytical models shows that transverse cracking is not the only process of damage which occurs for high crack densities. Transverse cracks induce local stress concentrations at crack tips and involve inter-laminar delamination between 0° and 90° layers.

Delamination develops from the crack tips and releases the local stress concentrations (Zhang and Minnetyan 2006). For most composite structures, onset of delamination emanating from transverse cracks is considered sufficient cause to withdraw the structure from service (Hallett *et al*. 2008). Thus, it is necessary to investigate the effect of local delamination that initiate and grow from transverse crack tips. Only a few studies have been conducted on the analysis of delamination induced by transverse cracks. An experimental campaign is performed by Zubillaga *et al*. (2015) to better understand matrix crack induced delamination. Its objective is to analyse delamination induced by matrix cracks, i.e., to observe the creation of transversal cracks in the matrix and then delamination onset from the crack tips. The comparison of the model with the experimental data has shown the suitability of the failure criterion developed to anticipate the apparition of delamination cracks within the different plies of the laminate.

If the cracked laminate is exposed to moisture environment, the moisture may diffuse through the matrix cracks and the local delamination at the crossing points. The moisture environments can the reach matrix crack surfaces in the interior and from there diffuse into the composite material. As a consequence, a cracked laminate is exposing a larger surface to the moisture environment and, since the diffusion coefficient for air is much larger than for the constituents in the composite (Landolt 1969, Loos and Springer 1981), the moisture absorption in a cracked material compared with that in an un-cracked material will generally be faster (Lundgren and Gudmundson 1998, Lundgren and Gudmundson 1999).

Recently, Khodjet Kesba et *al.* (2016a, b) started the prediction of the mechanical properties of the cracked composite laminate with the transverse cracks only and under the transient and non-uniform moisture concentration distribution. For that, the adopted model which will enable to introduce ageing and to see its development on the fibre and matrix scales is the transient Tsai model. This model allows predicting the most representation of hygrothermal effects on cracked composite laminate, compared to other works already done (Amara *et al*. 2014, Bouazza *et al*. 2007, Rezoug *et al.* 2011).

In this paper, in comparison with the few studies done on the evolution of delamination from transverse cracking and the good results given by the transient hygrothermal model (Khodjet kesba et *al.* 2016 a,b), it has been found that it's necessary to predict the effect of transverse cracks and delamination on the stiffness degradation under the moisture absorption. A comparison is done between the glass-fibre/epoxy laminates with transverse cracks only and the same one with the transverse crack and delamination together under the variation of the temperature and moisture concentration. The obtained results show that the stiffness degradation for the cracked composite laminate with transverse cracks and delamination is significantly influencing by the temperature and the moisture absorption compared with the one with transverse crack only.

2. Theoretical analysis

In the absence of a unified theory for the mechanical characterization of the composite materials with long fibres, many formulations were proposed. In the bibliography, we can quote the rule of mixtures method, the contiguity method that is based on the fibre arrangement (Staab

1999, Maurice 2001), the semi-empirical method based of Halpin-Tsai (Haplin and Tsai 1968). In this paper, we used the rule of mixtures method applied to the composites with anisotropic fibres, which it's based on the fibres emplacement (Chamis 1983). Therefore, the elastic constants are obtained from the following equations.

$$E_x = F_m E_m \cdot V_m + E_{fx} \cdot V_f \tag{1}$$

$$E_y = \left(1-\sqrt{V_f}\right) F_m E_m \frac{F_m E_m \sqrt{V_f}}{1-\sqrt{V_f}\left(1-\frac{F_m E_m}{E_{fy}}\right)} \tag{2}$$

$$G_{xy} = \left(1-\sqrt{V_f}\right) F_m G_m + \frac{F_m G_m \sqrt{V_f}}{1-\sqrt{V_f}\left(1-\frac{F_m G_m}{G_f}\right)} \tag{3}$$

$$\nu_{xy} = V_f \cdot \nu_f + V_m \cdot \nu_m \tag{4}$$

In the above equations, E_x , G_{xy} , ν_{xy} are the longitudinal Young modulus, shear modulus and Poisson's ratio respectively. E_y is the transversal Young modulus.

V_f and V_m are the fibre and matrix volume fractions and are related by

$$V_f + V_m = 1 \tag{5}$$

where F_m is the matrix mechanical property retention ratio.

E_f, G_f and ν_f are the Young's modulus, shear modulus and Poisson's ratio, respectively, of the fibre and E_m, G_m and ν_m are the corresponding properties of the matrix.

2.1 Stiffness reduction model

We consider a symmetric $[0/90]_s$ laminate subjected to uniaxial loads. It is assumed that the 90° ply has developed continuous intra-laminar cracks and the state that a delamination with a length of *2a* has developed at each tip of the transverse cracks (the crack spacing is *2l₀*) as shown in (Fig. 1). t_0 and t_{90} are the 0°, 90° layers thickness respectively.

Loading is applied only in x-direction and the far field applied stress is defined by $\sigma_c = (1/2h)N_x$, where N_x is applied load and h is the thickness plate.

By using the strains in the 0° layer (which is not damaged and, hence, strains are equal to laminate strains, $\varepsilon_x = \bar{\varepsilon}_x^0$, etc.) and assuming that the residual stresses are zero, the Young's modulus of the laminate with cracks may be defined by the following expression

$$E_x = \frac{\sigma_c}{\bar{\varepsilon}_x^0} \tag{6}$$

Note that the initial laminate modulus measured at the same load is $E_{x0} = \frac{\sigma_c}{\varepsilon_{x0}}$ and, hence

$$\frac{E_x}{E_{x0}} = \frac{\varepsilon_{x0}}{\bar{\varepsilon}_x^0} \tag{7}$$

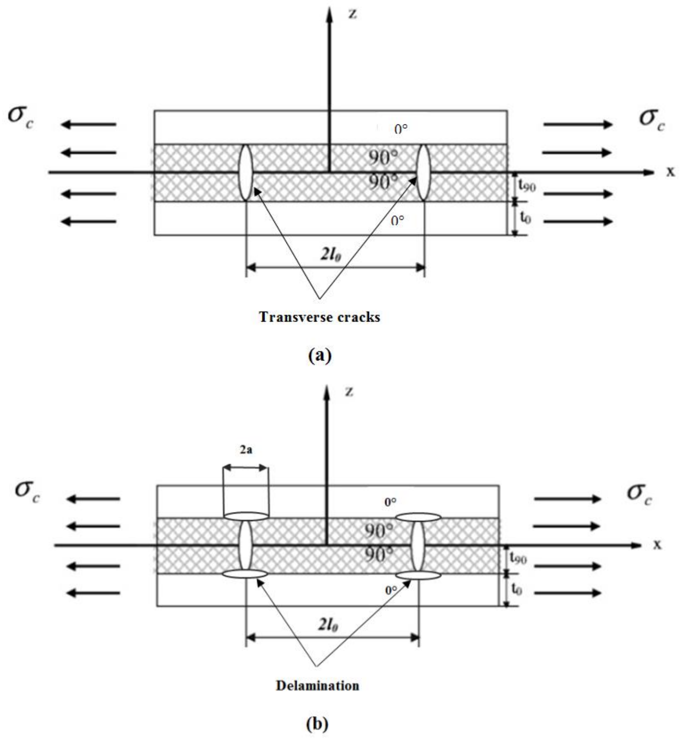

Fig. 1 Cross-ply composite laminates, (a) with transverse cracks and (b) with transverse cracks and delamination

Constitutive equations that give the relationship between strain and stresses are:

- In the 90° layer.

$$\begin{Bmatrix} \varepsilon_x^{90} \\ \varepsilon_y^{90} \\ \varepsilon_z^{90} \end{Bmatrix} = \begin{bmatrix} S_{22} & S_{12} & S_{23} \\ S_{12} & S_{11} & S_{12} \\ S_{32} & S_{12} & S_{22} \end{bmatrix} \begin{Bmatrix} \sigma_x^{90} \\ \sigma_y^{90} \\ \sigma_z^{90} \end{Bmatrix} \tag{8}$$

- In the 0° layer.

$$\begin{Bmatrix} \varepsilon_x^{0} \\ \varepsilon_y^{0} \\ \varepsilon_z^{0} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \begin{Bmatrix} \sigma_x^{0} \\ \sigma_y^{0} \\ \sigma_z^{0} \end{Bmatrix} \tag{9}$$

where S_{ij} is the compliance matrix for unidirectional composite with 0° fibre orientation.

In order to obtain an expression for average stress $\bar{\sigma}_x^{90}$ in the repeatable unit, we consider the axial stress perturbation caused by the presence of two cracks. Without losing generality the axial stress distribution may be written in the following form

$$\sigma_x^{90} = \sigma_{x0}^{90} - \sigma_{x0}^{90} f_1(\bar{x}, \bar{z}) \tag{10}$$

$$\sigma_x^{0} = \sigma_{x0}^{0} + \sigma_{x0}^{90} f_2(\bar{x}, \bar{z}) \tag{11}$$

where σ_{x0}^{90} is the CLT stress in 90° layer and σ_{x0}^{0} is CLT stress in the 0° layer (laminate theory routine), $-\sigma_{x0}^{90} f_1(\bar{x},\bar{z})$ and $\sigma_{x0}^{90} f_2(\bar{x},\bar{z})$ are stress perturbation caused by the presence of crack. Normalising factors in form of far field stresses in perturbation functions are used for convenience. Averaging Eqs. (10) and (11) using the integral force equilibrium in the x-direction, we obtain

$$\bar{\sigma}_x^{90} = \sigma_{x0}^{90} - \sigma_{x0}^{90} \frac{1}{2\bar{l_0}} R(\bar{l_0}) \tag{12}$$

$$\bar{\sigma}_x^{0} = \sigma_{x0}^{0} + \sigma_{x0}^{90} \frac{1}{2\alpha\bar{l_0}} R(\bar{l_0}) \tag{13}$$

In the following function

$$R(\bar{l_0}) = \int_{-\bar{l_0}}^{+\bar{l_0}} \int_0^1 f_1(\bar{x},\bar{z})\, d\bar{z}\, d\bar{x} \tag{14}$$

$\bar{l}_0$ is the half-length between two consecutive cracks standardized and α is the stacking parameter of layer 0° and 90°.

$R(\bar{l_0})$ is called the stress perturbation function. It's related to axial stress perturbation in the 90° layer and depends on the crack density.

The first term according to the discussion above is equal to CLT strain but the second one is a new term related to the stress perturbation function $R(\bar{l_0})$

$$\varepsilon_y = \varepsilon_{y0} - \sigma_{x0}^{90} g_1 \frac{1}{2\bar{l_0}} R(\bar{l_0}) \tag{15}$$

$$\bar{\varepsilon}_x^{90} = \varepsilon_{x0} - \sigma_{x0}^{90} g_2 \frac{1}{2\bar{l_0}} R(\bar{l_0}) \tag{16}$$

$$\bar{\varepsilon}_x^{0} = \varepsilon_{x0} - \sigma_{x0}^{90} g_3 \frac{1}{2\bar{l_0}} R(\bar{l_0}) \tag{17}$$

The stress σ_{x0}^{90} in the 90° layer of an undamaged laminate under mechanical loading may be calculated using CLT

$$\sigma_{x0}^{90} = Q_{22}\varepsilon_{x0}(1 - \upsilon_{12}\upsilon_{xy}^{0}) \tag{18}$$

Here, υ_{xy}^{0} is the Poisson's ratio of the undamaged laminate.

In the delaminate damage range, $l - a \le x \le l$, 90°layer cannot bear any load due to the transverse cracks, we therefore have

$$\overline{\sigma_{xx}^{0}} = \frac{t_0 + t_{90}}{t_0} \bar{\sigma}_c \tag{19}$$

$$\overline{\sigma_{xx}^{90}} = 0 \tag{20}$$

$\overline{\sigma_c}$ is the average stress. However, in the non-delaminated range, $0 \le x \le l - a$, 0° layer and 90° layer bearing the load jointly. Stress of 0° layer is determined as

$$\overline{\sigma_{xx}^{0}} = \bar{\sigma}_c \frac{E_0}{E_{x0}} \left[1 + \frac{t_{90}}{t_0}\frac{E_{90}}{E_0} \frac{\cosh(\xi \bar{l_0} \frac{x}{\bar{l_0}})}{cosh(\xi \bar{l_0})} \right] \tag{21}$$

The Stress of 90° layer is

$$\overline{\sigma_{xx}^{90}} = \bar{\sigma_c}\,\frac{E_0}{E_{x0}}\left[1-\frac{\cosh(\xi\overline{l_0}\frac{x}{\overline{l_0}})}{cosh(\xi\overline{l_0})}\right] \tag{22}$$

where ξ is the shear-lag parameter.

Since the longitudinal strains in the delaminated range and non–delaminated range are different. We take the longitudinal average strain as follows

$$\varepsilon_c \approx \bar{\varepsilon}_x^0 = \frac{1}{l_0}\int_0^{l_0}\frac{\overline{\sigma_{xx}^0}}{E_0}dx$$

$$= \frac{1}{l_0}\int_{l_0-a}^{l_0}\frac{t_0+t_{90}}{E_0t_0}\sigma_c dx + \frac{1}{l_0}\int_{l_0-a}^{l_0}\frac{\sigma_c}{E_{x0}}\left(1+\frac{t_{90}E_{90}\cosh(\xi\overline{l_0}\frac{x}{\overline{l_0}})}{t_0E_0cosh(\xi\overline{l_0})}\right)dx \tag{23}$$

$$= \frac{a}{l_0}\frac{t_0+t_{90}}{E_0t_0}\sigma_c + \frac{l_0-a}{l_0}\frac{\sigma_c}{E_{x0}} + \frac{t_{90}E_{90}}{t_0E_0E_{x0}cosh(\xi\overline{l_0})}\frac{\sigma_c}{\xi\overline{l_0}}sinh\left(\xi\overline{l_0}(1-\frac{a}{l_0})\right)$$

Defining *n* as the delaminate propagation rate, a is half of the delaminated crack length.

In order to derive expression for longitudinal modulus, E_x of the damaged laminate with only transverse cracks, we use definition Eq. (7) and substitute Eq. (17) in this relationship. Finally we use Eq. (18). This procedure yields

$$\frac{E_x}{E_{x0}} = \frac{1}{1+b\bar{\rho}R(\overline{l_0})} \tag{24}$$

where $\bar{\rho} = \frac{1}{2\overline{l_0}}, \overline{l_0} = \frac{l_0}{t_{90}}$ is the noramalized crack density and a , b are known functions, dependent on elastic properties and geometry of the 0° and 90° layer

$$b = \frac{E_{90}t_{90}}{E_x^0t_0}\left(\frac{1-\upsilon_{12}\upsilon_{xy}^0}{1-\upsilon_{12}\upsilon_{21}}\right)\left(1+\upsilon_{xy}^0\frac{S_{12}t_{90}+S_{12}t_0}{S_{22}t_0+S_{11}t_0}\right) \tag{25}$$

E_x^0 and E_{90} are the Young's moduli of 0° and 90° layers respectively.

From Eqs. (21) and (23) and also $\sigma_c = E_x\varepsilon_c$, $a = nl_0$ and $l_0 = \frac{1}{2\rho}$, the longitudinal modulus, E_x of the damaged laminate with transverse cracks and delamination can be obtained

$$\frac{E_x}{E_{x0}} = \frac{1}{E_{x0}\left(n\frac{t_0+t_{90}}{E_0t_0}+\frac{1-n}{E_{x0}}+\frac{t_{90}E_{90}\left[\frac{2\rho}{n\xi}sinh\left(\frac{(1-n)\xi}{2\rho}\right)\right]}{t_0E_0E_{x0}cosh(\frac{n\xi}{2\rho})}\right)} \tag{26}$$

From Eq. (24) and (26) it's clear that the function $R(\overline{l_0})$ and ξ are only the unknown function. Hence, reduction of the Young's modulus depends on the form of this function of crack density. Solution for this function can be found by using different analytical models such as shear-lag models.

In this work, we have used two models developed by Berthelot *et al*. (1996). This latter is modified by introducing the stress perturbation function. The stress perturbation function $R(\overline{l_0})$ is found as

$$R(a) = \int_{-\bar{l_0}}^{+\bar{l_0}} \frac{\cosh(\xi\bar{x})}{\cosh(\xi\bar{l_0})} d\bar{x} = \frac{2}{\xi}\tanh(\xi\bar{l_0}) \tag{27}$$

where,

$$\xi^2 = \bar{G}\frac{t_{90}(t_{90}E_{90} + t_\theta E_\theta)}{t_\theta E_{90} E_\theta} \tag{28}$$

The coefficient $\bar{G}$ depends on used assumptions about the longitudinal displacement and shears stress distribution

• The first case assumes that the longitudinal displacement is parabolic in the thickness of 90° layer

$$u_{90}(x,z) = \overline{u_{90}}(x) + \left(z^2 - \frac{t_{90}^2}{3}\right) A_{90}(x) \tag{29}$$

The variation of the longitudinal displacement is to be determined by the thickness of θ° layers

$$u_0(x,z) = \overline{u_0}(x) + f(z)A_0(x) \tag{30}$$

• The second case assumes that the shear stresses, similar in 0° and 90° layers, can be obtained by assuming that the transverse displacement is independent of the longitudinal coordinate

$$\sigma_{xz}^i = G_{xz}^i \gamma_{xz}^i \tag{31}$$

$$\gamma_{xz}^i = \frac{\partial u_i}{\partial z} + \frac{\partial w_i}{\partial x} \approx \frac{\partial u_i}{\partial z} \qquad i = 0°, 90° \tag{32}$$

The coefficient $\bar{G}$ is done by

$$\bar{G} = \frac{3G}{t_{90}} \tag{33}$$

The shear modulus G of the elementary cell

$$G = \frac{G_{xz}^{90}}{1 - 3\frac{G_{xz}^{90}}{G_{xz}^{0} t_{90}} \frac{f(t_{90})}{f'(t_{90})}} \tag{34}$$

Two different analytical functions of the variation function have been considered (Berthelot *et al*. 1996):

• In the case of the assumption of a parabolic variation of longitudinal displacement in both 0° and 90° layers,

By replacing the function $f(z) = z^2 - 2(t_0 + t_{90})z + \frac{2}{3}t_0^2 + 2t_0 t_{90} + t_{90}^2$ in the Eq. (34), the shear modulus for parabolic assumption will be in the form

$$G = \frac{G_{xz}^{90}}{1 + \alpha\frac{G_{xz}^{90}}{G_{xz}^{0}}} \tag{35}$$

• In the case when the variation of the longitudinal displacement is supposed progressive in 0° - layer:

We use the function $f(z) = \frac{\sinh\alpha\eta_t}{\alpha\eta_t} - \cosh\eta_t(1 + \alpha - \frac{z}{t_{90}})$ in the Eq. (34), the shear modulus for progressive shear assumption will be in the form

$$G = \frac{G_{xz}^{90}}{1 + 3\alpha \dfrac{\alpha\eta_t (tanh\alpha\eta_t)^{-1} - 1}{\alpha{\eta_t}^2} \dfrac{G_{xz}^{90}}{G_{xz}^{0}}} \tag{36}$$

η_t is the shear transfer parameter. G_{xz}^{90} and G_{xz}^{0} are the longitudinal shear modulus of the 90°, 0° layers respectively.

3. Results and discussion

A computer code based on the preceding equations was written to compute the stiffness loss for cross-ply laminates due to the transverse ply cracking and delamination.

3.1 Comparison of predictions with experimental data

In this section, we will validate the results of the present programme without taking into account the hygrothermal effect on the material properties. The results are compared with experimental data for T800H/3631 laminates with elastic properties in Table 1. (Takeda and Ogihara 1994). The thickness of each ply was approximately 0.132 mm.

Table 1 Material properties of T800H/3631 laminate used in calculations (Takeda and Ogihara 1994)

T800H/3631	Room temperature (RT)	80°C
E_0 (GPa)	152.2	144.2
E_{90} (GPa)	9.57	8.09
G_0 (GPa)	4.5	4.26
G_{90} (GPa)	3.21	2.75
ν_0	0.349	0.349
ν_{90}	0.490	0.490

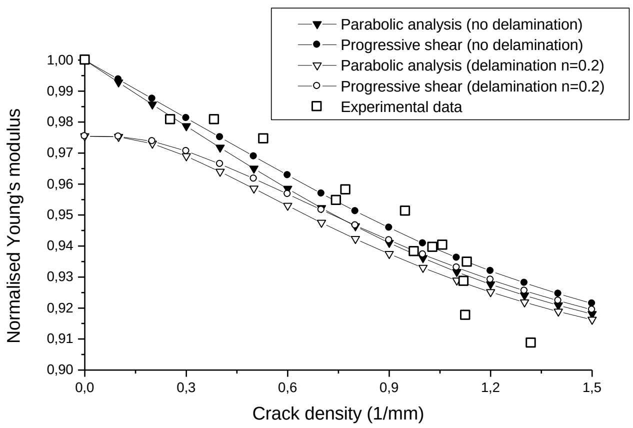

Fig. 2 Young's modulus reduction as function of crack density for T800H/3631 $[0/90_2]_s$ laminate at room temperature

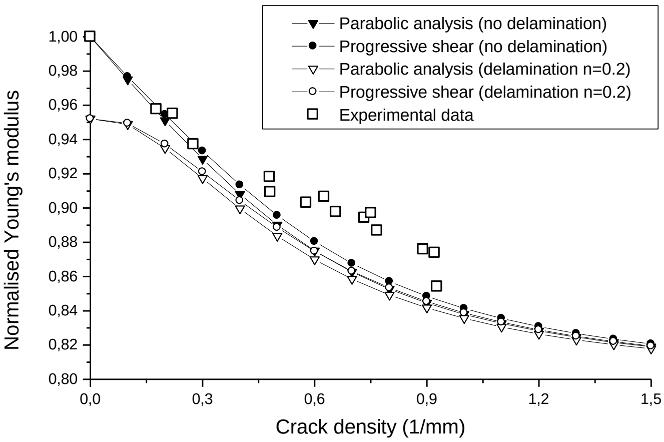

Fig. 3 Young's modulus reduction as function of crack density for T800H/3631 $[0/90_4]_s$ laminate at room temperature

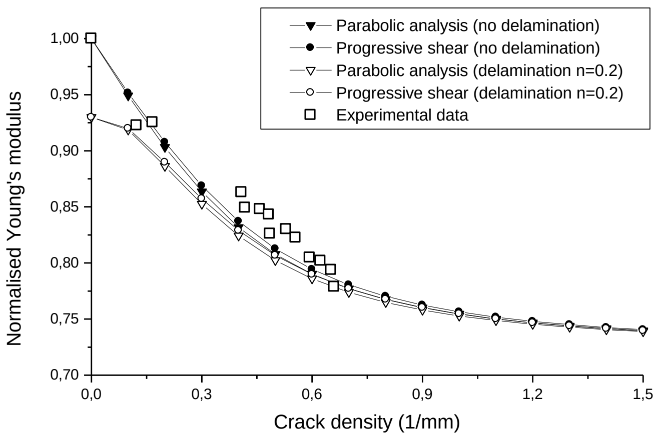

Fig. 4 Young's modulus reduction as function of crack density for T800H/3631 $[0/90_6]_s$ laminate at room temperature

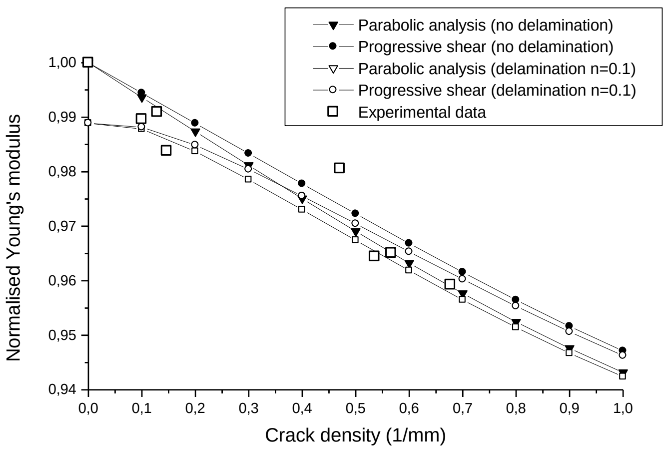

Fig. 5 Young's modulus reduction as function of crack density for T800H/3631 $[0/90_2]$s laminate at 80°C

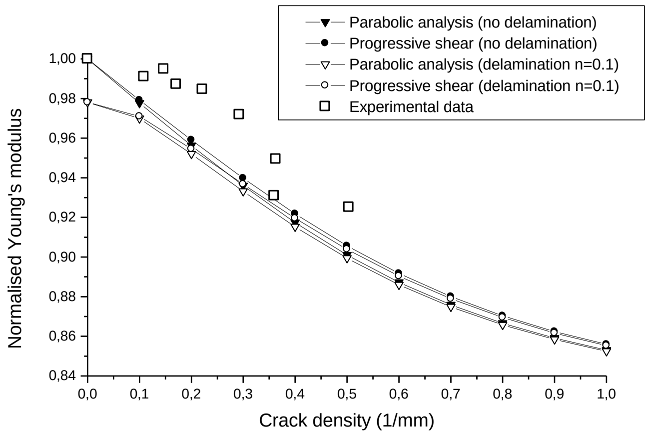

Fig. 6 Young’s modulus reduction as function of crack density for T800H/3631 $[0/90_4]s$ laminate at 80°C

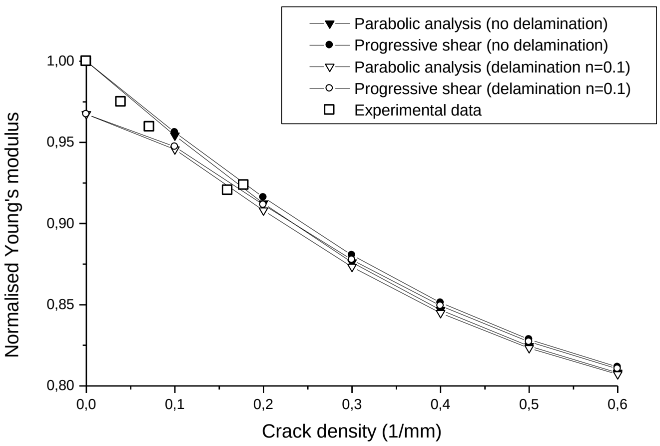

Fig. 7 Young’s modulus reduction as function of crack density for T800H/3631 $[0/90_6]s$ laminate at 80°C

Fig. 2 to 7 show the degradation in longitudinal Young’s modulus as a function of crack density for T800/3631 $[0/90_m]_s$ (m=2,4,6) laminates at room temperature and at 80°C. The stiffness degradation in $[0/90_6]_s$ laminates (Fig. 4 and 7) are more than that in $[0/90_2]_s$ and $[0/90_4]_s$ laminates. The reason is that 90° layers take more portion of the applied load than 0° layer because of a greater thickness of 90° layers in the uncracked $[0/90_6]_s$ laminates, so that local delamination leads to more loss in load carrying capacity for the $[0/90_6]_s$ laminates. We can note also that the stiffness decrease with the increase of the crack density and also with an increase in temperature.

The predictions for the laminates with only the transverse cracks seem to be accurate with experimental data (Ogihara and Takeda 1995) for $[0/90_m]_s$ laminates at lower crack densities. But at higher crack densities, the prediction for the laminates with only the transverse cracks tends to be inadequate and the consideration of the delamination proves to be necessary. For that, the laminates in which the delamination grows extensively, the modified shear-lag analysis described in the present study has proved to be important to the understanding of the degradation of the material properties in the failure process. The simultaneous observation of the transverse cracks and the delamination is necessary for better prediction of the Young’s modulus reduction.

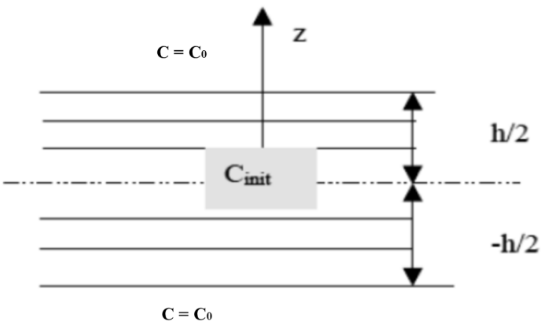

Fig. 8 The one-dimensional problem of moisture diffusion in plates

3.2 Influence of transient hygrothermal conditions on the reduced stiffness

The study here has been focused on the axial and flexural stiffness properties reduction due to transverse cracking in $[0/90]_{2s}$ composite laminate when this latter is initially exposed to different environmental conditions. The model which will enable us to introduce ageing and to see its development on the fibre and matrix scales is Chamis model (1983). Since the effect of temperature and moisture is dominant in matrix material, the hygrothermal degradation of the fibre composite material property is estimated by degrading the matrix property only. The matrix mechanical property retention ratio is expressed as

$$F_m = \left(\frac{T_{gw} - T_{opr}}{T_{go} - T_{rm}} \right)^{1/2} \tag{37}$$

where T_{gw} and T_{go} are the glass transition temperature for wet and reference dry conditions, T_{opr} is the operating temperature and T_{rm} is the room temperature. The glass transition temperature for wet material is determined as (Chamis 1983)

$$T_{gw} = \left(0.005C^2 - 0.1C + 1\right) T_{go} \tag{38}$$

where C is the weight percent of moisture in the matrix material.

Let us consider a laminated plate of thickness h made of polymer matrix composite, submitted on it two sides to the different dry environment. If the laminate edges are moisture insulated and the free surfaces x = 0 and x = h are exposed to a constant moisture environmental change C_0 for t = 0, then a one-dimensional moisture diffusion through the thickness is obtained. Provided that the laminate has a uniform moisture concentration C_I for t=0. The moisture concentration inside the plate is described by Fick equation (Shen and Springer 1981, Tounsi *et al.* 2005, Benkhedda *et al.* 2008) with diffusivity D_z.

$$\frac{\partial C}{\partial t} = D_z \frac{\partial^2 C}{\partial z^2} \tag{39}$$

with

$$C_I = C_{init} \quad \text{at} \quad t = 0 \quad \text{for} \quad 0 < z < h \tag{40}$$

$$C = C_0 \quad \text{at} \quad t = 0 \quad \text{for} \quad z = 0 \text{ and } z = h \tag{41}$$

Table 2 Fibre and matrix characteristics of glass/epoxy material (Talerja 1986)

	E (Gpa)	G_{12} (Gpa)	υ_{12}
Fibre	84	33.6	0.27
Matrix	3.2	1.26	0.27

The solution is well known (Crank 1975) and can be written as

$$C(z,\tau) = C_0 + (C_I - C_0)\sum_{n=1}^{\infty} \frac{4}{(2n+1)\pi} e^{-(2n+1)^2\pi^2\tau} \sin\left(\frac{(2n+1)\pi z}{h}\right) \quad ; \quad \begin{cases} 0 \le z \le h \\ \tau \ge 0 \end{cases} \tag{42}$$

where, the closure time

$$\tau = \frac{D_z t}{h^2} \tag{43}$$

If the composite laminate also contains matrix cracks that are in connection with the surrounding moisture environment, a faster moisture uptake is expected. This change in moisture uptake, for a cracked cross-ply laminate with matrix cracks in the 90° plies, is here estimated by the model given by Lundgren and Gudmundson (1998). For the case when the moisture environment instantly reaches the crack surfaces in the interior of the laminate the model becomes

$$C(z,\tau) = C_0 + (C_I - C_0)e^{-\mu\tau}\sum_{n=1}^{\infty} \frac{4}{(2n+1)\pi} e^{-(2n+1)^2\pi^2\tau} \sin\left(\frac{(2n+1)\pi z}{h}\right) \tag{44}$$

μ is the moisture transfer coefficient which is related to the crack density, ρ, according to Lundgren and Gudmundson (1998) as

$$\mu = N^2 \cdot a \cdot \rho^b \tag{45}$$

a and b are constants and N is the total number of plies. From Lundgren and Gudmundson (1998) the values of a=5.96 and b=1.43 are obtained.

Very often, the laminated composite plate is symmetrical about the central plane (Fig. 8), and for the formulae of concentrations are the most convenient if we take the surfaces at $z = \pm\frac{h}{2}$. A few modification are considered for Eq. (42), when z is replaced by $z \pm \frac{h}{2}$ and

$$C_I = C_{init} \quad \text{at} \quad t = 0 \quad \text{for} \quad -\frac{h}{2} < z < \frac{h}{2} \tag{46}$$

$$C = C_0 \quad \text{at} \quad t = 0 \quad \text{for} \quad z = \frac{h}{2} \text{ and } z = -\frac{h}{2} \tag{47}$$

The presented model is applied to $[0/90]_{2s}$ glass/epoxy reported by Talerja 1986, which the mechanical properties of fibre and matrix of the lamina are listed in Table 2. The Single ply thickness = 0.203 mm, $\sqrt{\tau} = 0.3$ and Fibre volume fraction V_f=0.45.

3.2.1 Analysis of relative stiffness reduction

The stiffness degradation in $[0/90_3]_s$ composite laminate as of crack density with only transverse cracks and with transverse crack and delamination are evaluated compared to the initial stiffness of the same uncracked laminate and for the same environmental case. We note that this

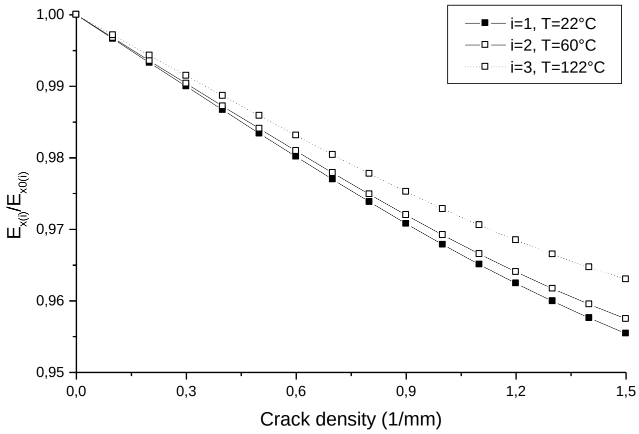

Fig. 9 Hygrothermal effect on the relative stiffness degradation due to: transverse cracks only in a $[0/90]_{2s}$ glass/epoxy laminate

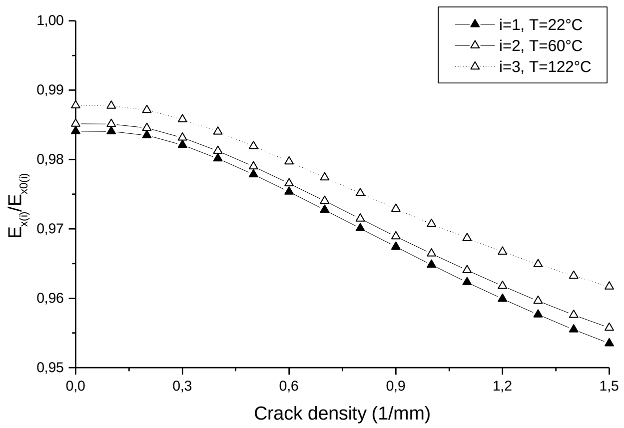

Fig. 10 Hygrothermal effect on the relative stiffness degradation due to: transverse cracks and delamination in a $[0/90]_{2s}$ glass/epoxy laminate (n=0.2)

initial stiffness of the uncracked laminate is a function of temperature and moisture distribution. Consequently, Eqs. (24) and (26) become

a) For laminate with only transverse cracks

$$\frac{E_{x(i)}}{E_{x0(i)}} = \frac{1}{1 + b_{(i)}\bar{\rho}R_{(i)}(\overline{l_0})} \tag{48}$$

b) For laminate with transverse cracks and delamination

$$\frac{E_{x(i)}}{E_{x0(i)}} = \frac{1}{E_{x0(i)}\left(n\frac{t_0+t_{90}}{E_{0(i)}t_0} + \frac{1-n}{E_{x0(i)}} + \frac{t_{90}E_{90(i)}\left[\frac{2\rho}{n\xi_{(i)}}sinh\left(\frac{(1-n)\xi_{(i)}}{2\rho}\right)\right]}{t_0E_{0(i)}E_{x0(i)}cosh(\frac{n\xi_{(i)}}{2\rho})}\right)} \tag{49}$$

The index (i) represents the considered case of the environmental conditions.

The normalised stiffness degradation is represented in cross-ply $[0/90_3]_s$ cracked laminate

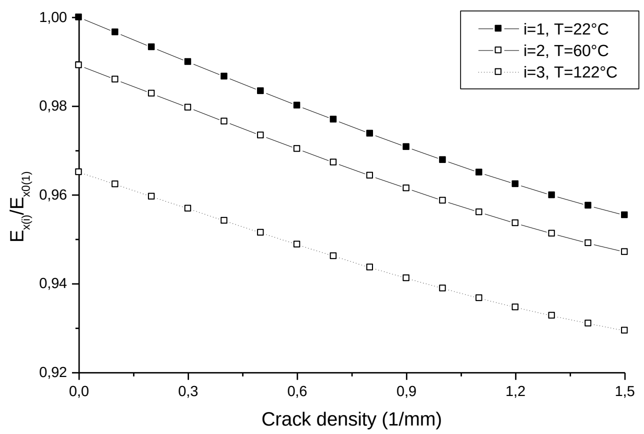

Fig. 11 Hygrothermal effect on the total stiffness degradation due to: transverse cracks only in a $[0/90]_{2s}$ glass/epoxy laminate

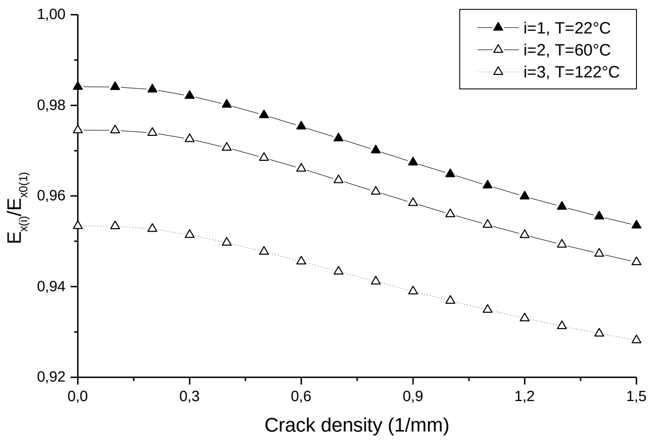

Fig. 12 Hygrothermal effect on the total stiffness degradation due to: transverse cracks and delamination in a $[0/90]_{2s}$ glass/epoxy laminate (n=0.2)

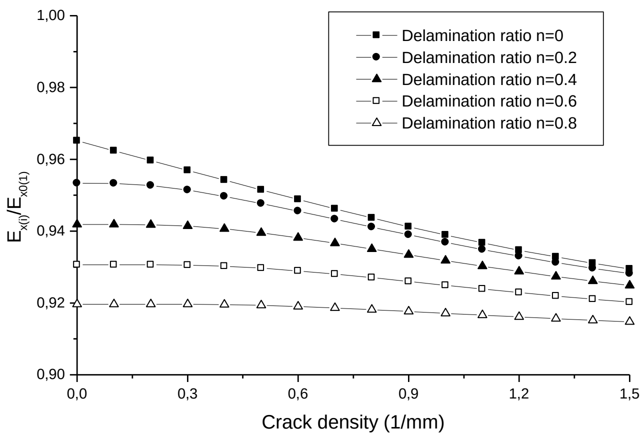

Fig. 13 The total stiffness degradation as a function of crack density for a $[0/90]_{2s}$ glass/epoxy laminate with different delamination ratio (T=122°C)

exposed to environmental conditions with a parabolic variation of longitudinal displacement in both 0° and 90° layers (transverse cracks are in 90° layers). Moisture absorption in glass-fibre/epoxy laminate with transverse cracks has been selected to represent the effect of high temperature.

In Figs. 9 and 10, the relative stiffness are plotted as a function of crack density with different values of operating temperatures. It can be observed that the axial modulus reduces monotonically with an increase of crack density and also with a decrease in temperature. We note that, when the laminate is under transverse crack and delamination, the hygrothermal effect is more significant even at small crack density. On the other hand, very small effect related to hygrothermal conditions is observed when is the laminate is under only the transverse crack.

3.2.2 Analysis of total stiffness reduction

In this section, the total reduction of stiffness modulus is determined compared to the axial modulus of the un-cracked laminate when this latter is exposed initially to the environmental condition of case 1. Consequently, this total reduction of stiffness takes into account the reduction due to the crack density and to the variation of moisture and temperature. Eq. (24) and (26) become:

a) For laminate with only transverse cracks

$$\frac{E_{x(i)}}{E_{x0(1)}} = \frac{\left(t_0 E_{0(i)} + t_{90} E_{90(i)}\right)}{\left(1 + b_{(i)} \bar{\rho} R_{(i)}(\overline{l_0})\right)\left(t_0 E_{0(1)} + t_{90} E_{90(1)}\right)} \tag{50}$$

b) For laminate with transverse cracks and delamination

$$\frac{E_{x(i)}}{E_{x0(1)}} = \frac{\left(t_0 E_{0(i)} + t_{90} E_{90(i)}\right)}{E_{x0(i)}\left(n\frac{t_0 + t_{90}}{E_{0(i)} t_0} + \frac{1-n}{E_{x0(i)}} + \frac{t_{90} E_{90(i)}\left[\frac{2\rho}{n\xi_{(i)}} sinh\left(\frac{(1-n)\xi_{(i)}}{2\rho}\right)\right]}{t_0 E_{0(i)} E_{x0(i)} cosh(\frac{n\xi_{(i)}}{2\rho})}\right)\left(t_0 E_{0(1)} + t_{90} E_{90(1)}\right)} \tag{51}$$

The total stiffness are plotted in Figs. 11 and 12, as a function of crack density with different environmental conditions for $[0/90]_{2s}$ glass/epoxy when this latter is under transverse cracks only and under transverse cracks and delamination respectively. It can be observed that the total stiffness reduces monotonically with an increase of operating temperature and moisture absorption also increase of crack density. When we analyse the hygrothemal effect of cracked laminate under hygrothermal effect compared to un-cracked laminate with the standard condition (case 1: T=22°C), we note that the big degradation of the total stiffness is at zero crack density. This leads that, the hygrothermal effect has more impact on the axial stiffness degradation for un-cracked composite laminates than one with the transverse crack and delamination.

The total stiffness degradation as a function of crack density and different delamination ratio under environmental case 3 (i=3) are shown in Fig. 13 for $[0/90]_{2s}$ glass/epoxy. It shows that the total stiffness decrease with the increase of the delamination ratio (n), until we note practically no effect of the crack density. This leads that, our cracked composite laminate are near saturation stage. We note also that, this mode is quickly reached with an increase of the temperature and moisture absorption.

4. Conclusions

The stiffness degradation was predicted using analytical models on the cross-ply composite laminate including the effect of transverse cracks and delamination under uniaxial loading. The results show good agreement between prediction models and experimental data. In the second case, the influence of matrix cracks and delamination on moisture uptake in glass-fibre/epoxy laminates have been studied. On the basis of the present results, the following conclusions can be drawn:

• The stiffness degradation of the cracked cross-ply composite laminates with transverse crack and delamination largely depend on the increase of crack density and the increase of thickness ratio.

• The relative stiffness modulus reduces monotonically with increased crack density and also with a decrease in temperature and moisture absorption.

• The total stiffness modulus reduces monotonically with an increase of temperature, moisture uptake and also increase of crack density.

• The saturation mode is quickly reached with the increase of the crack density, delamination ratio and also the increase in the operating temperature and moisture uptake.

Finally, through this theoretical study, we hope that our prediction will be a support for future experimental research.

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