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Cylindrical bending of multilayered composite laminates and sandwiches

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Abstract. In a whole variety of higher order plate theories existing in the literature no consideration is given to the transverse normal strain / deformation effects on flexural response when these higher order theories are applied to shear flexible composite plates in view of minimizing the number of unknown variables. The objective of this study is to carry out cylindrical bending of simply supported laminated composite and sandwich plates using sinusoidal shear and normal deformation plate theory. The most important feature of the present theory is that it includes the effects of transverse normal strain/deformation. The displacement field of the presented theory is built upon classical plate theory and uses sine and cosine functions in terms of thickness coordinate to include the effects of shear deformation and transverse normal strain. The theory accounts for realistic variation of the transverse shear stress through the thickness and satisfies the shear stress free conditions at the top and bottom surfaces of the plate without using the problem dependent shear correction factor. Governing equations and boundary conditions of the theory are obtained using the principle of minimum potential energy. The accuracy of the proposed theory is examined for several configurations of laminates under various static loadings. Some problems are presented for the first time in this paper which can become the base for future research. For the comparison purpose, the numerical results are also generated by using higher order shear deformation theory of Reddy, first-order shear deformation plate theory of Mindlin and classical plate theory. The numerical results show that the present theory provides displacements and stresses very accurately as compared to those obtained by using other theories.

Keywords: cylindrical bending; shear deformation; normal deformation; laminated plate; sandwich plate; antisymmetric; symmetric; arbitrary laminates

1. Introduction

Structural components made with composite materials are increasingly being used in various engineering applications such as aviation and aerospace, navigation, automotive, civil, mechanical, marine etc. due to their attractive properties such as high specific strength, high specific stiffness, and high thermal resistance. Understanding their bending, buckling and vibration behaviour is of

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increasing importance.

Well known classical plate theory underestimates the deflection and overestimates natural frequencies and buckling loads due to the neglect of transverse shear and transverse normal deformation effects. The errors in deflection, stresses, natural frequencies and buckling loads are quite significant for plate made out of advanced composites. The first order shear deformation theory accounts for the transverse shear deformation effect, but require a problem dependent shear correction factor to appropriately take into account the strain energy of shear deformation of the plate (Mindlin 1951).

To avoid the use of problem dependent shear correction factor, higher order shear deformation theories were developed based on the assumption of quadratic, cubic or higher-order variations of in-plane displacements through the plate thickness. Higher order shear deformation theories can be developed by expanding the displacements in power series of the coordinate normal to the middle plane. In principle, theories developed by this means can be made as accurate as desired simply by including a sufficient number of terms in the series. These higher-order theories are cumbersome and computationally more demanding, because, with each additional power of the thickness coordinates, an additional dependent variable is introduced into the theory. It has been noted by Lo *et al*. (1977a, 1977b) that due to the higher order of terms included in their theory, the theory is not convenient to use. This observation is more or less true for many other higher order theories as well. And, thus there is a scope to develop simple to use higher order plate theory. Many simple higher order shear deformation theories are available in the literature for the bidirectional bending analysis of laminated composite plates such as higher order theory of Ambartsumian (1958), higher order theory of Kruszewski (1949), a simple higher order shear deformation theory of Reddy (1984), hyperbolic shear deformation theory of Soldatos (1992), an exponential shear deformation theory of Karama *et al*. (2003), hyperbolic shear deformation theory of Akavci (2007), two variable plate theory of Shimpi and Patel (2006), higher order shear deformation theory of Aydogdu (2009) and many more.

Pagano (1969) has obtained exact 3D solutions for cross-ply laminates under cylindrical bending and investigated limitations of classical plate theory comparing with the solutions of several specific boundary value problems to the corresponding 3D elasticity solutions. Pagano (1971) also studied cylindrical bending of angle-ply laminates considering the influence of shear coupling. These solutions are obtained for simply supported boundary conditions only. Pagano and Wang (1971) extended this solution for cross-ply laminates under uniformly distributed load and patch load.

Wan (1992) has developed sixth and twelfth order plate theories for cylindrical bending problems and demonstrated the efficiency of these theories for thick plates. Based on the assumption of the through-the-thickness inextensibility of laminates, Jalali and Taheri (1998) have developed a new semi-analytical method to study the response of cross-ply laminated plates under cylindrical and planar bending. Soldatos and Watson (1997) have proposed a new method for the stress analysis of two layered simply supported and clamped cross-ply laminated composite plates which is further extended by Shu and Soldatos (2000) for the stress analysis of angle-ply laminates subjected to different sets of edge boundary conditions.

Perel and Palazotto (2001) presented a discrete layer theory for the cylindrical bending of sandwich plates based on assumed transverse strains. It is assumed that transverse strains do not vary in the thickness direction within the face sheet and the core. A problem of cylindrical bending of a simply supported plate under uniform load is considered and results are obtained using finite element method. A new higher order shear and normal deformation theory for the bending and free vibration analysis of sandwich plates with functionally graded isotropic face sheets is developed by Bessaim *et al*. (2013). Zenkour (2007) has developed three dimensional elasticity solutions for uniformly loaded cross-ply laminates and sandwich plates. Khdeir (2001) has studied dynamic behaviour of anti-symmetric angle-ply laminated plates in cylindrical bending using classical plate theory and first order shear deformation theory. The natural frequencies are determined for arbitrary boundary conditions and loading conditions using a generalized modal approach. Vel and Batra (2000, 2001) obtained exact three-dimensional state space solution for the static cylindrical bending of simply supported laminated plates with embedded shear mode piezoelectric actuators, and subjected to mechanical and electric loading on the upper and lower surfaces. Park and Lee (2003) have presented a new shear deformation theory for cylindrical bending of laminated plates in which inplane displacements vary exponentially through plate thickness and applied the theory for cross-ply and angle-ply anti-symmetric laminates under cylindrical bending. Lu *et al*. (2007) have obtained an exact solution for free vibration of angle-ply laminates subjected to cylindrical bending using a semi-analytical approach in which thickness domain is solved analytically using the transfer matrix method based on the state space concept, while the in-plane domain is solved approximately via the technique of differential quadrature. Starovoytov *et al*. (2010) carried out cylindrical bending analysis of laminated plate resting on an elastic foundation. Kapuria and Kumari (2011) obtained a three dimensional elasticity solution of symmetric and anti-symmetric cross-ply and angle-ply laminated composite plates in cylindrical bending using extended Kantorovich method. Kant and Shiyekar (2008) have presented analytical solution for the cylindrical bending of piezoelectric laminates using higher order shear and normal deformation theory. Sayyad *et al.* (2014) and Sayyad and Ghugal (2015a) have applied *n*th order plate theory for the cylindrical bending analysis of specially orthotropic and cross-ply laminated plates and obtained the numerical results for simply supported plate subjected to sinusoidally distributed load. Saeedi *et al*. (2013) proposed two-dimensional layerwise model for the cylindrical bending of multilayered plates with multi-delamination. Lebee and Sab (2011) obtained a closed form solution for cylindrical bending of laminated plates using a bending-gradient plate theory which is an extension of the Reissner-Mindlin plate theory to arbitrarily layered plates. Afshin *et al*. (2010) studied static response of cylindrical sandwich panels with flexible core using the high order plate theory based on layerwise formulation. Zhou *et al.* (2009) carried out free vibration of cross-ply piezoelectric laminates in cylindrical bending with arbitrary edges using the state-space method and the differential quadrature method. Within a framework of the three-dimensional piezoelectricity, asymptotic formulations of functionally graded piezoelectric cylindrical shells under cylindrical bending type of electromechanical loads using the method of perturbation have been presented by Wu and Syu (2007). Chen and Lee (2005) developed an elasticity method to study the bending and free vibration of simply-supported angle-ply laminated cylindrical panels in cylindrical bending based on the state space formulations. Vel *et al*. (2004) developed an analytical solution for the cylindrical bending vibrations of linear piezoelectric laminated plates. Auricchio and Sacco (2003) developed refined first-order shear deformation theory models for the cylindrical bending of composite laminates. Alibeigloo and Shakeri (2009) presented the three-dimensional solution for static analysis of cross-ply cylindrical panel using differential quadrature method and Fourier series approach. The cylindrical bending vibration of a laminated elastic plate due to piezoelectric actuators has been presented by Yang *et al*. (1994). Toledano and Murakami (1987a, 1987b) have developed a new laminated plate theory for arbitrary laminate configurations based on Reissner's mixed variational principle. The accuracy of theory was examined by applying it to the cylindrical bending problem of symmetric, antisymmetric, arbitrary laminated plates. He (1992) presented a refined shear deformation theory for cylindrical bending of arbitrary cross-ply laminated plates.

The higher order shear deformation theories with trigonometric functions in terms of thickness coordinates, to include thickness effects, are designated as trigonometric/sinusoidal shear deformation theories. Several trigonometric shear deformation theories (TSDTs) have been developed by various researchers for the bending, buckling and free vibration analysis of laminated composite beams and plates. Stein (1986) presented a quasi-3D nonlinear theory for stability analysis of laminated and thick plates and shells including the effects of transverse shear and transverse normal strain. The displacement field of the theory consists of trigonometric terms in addition to the initial terms of a power series through-the-thickness. Equations of equilibrium and boundary conditions are obtained using principle of virtual work. Touratier (1991) has developed a trigonometric shear deformation theory using principle of virtual work. The several problems of plates investigated include bi-directional bending, buckling and free vibration analysis of laminated and sandwich plates. Wave propagation, torsion of a rectangular plate and edge effect on the stress distribution at the edge of a circular hole in a large rectangular bent plate are also addressed. Shimpi and Ghugal (2001) have developed a layerwise trigonometric shear deformation theory for the bending analysis of two layered anti-symmetric laminated beams which is further extended by Ghugal and Shinde (2013, 2014) for the flexural analysis of laminated beams of various boundary conditions using general solution technique. Shimpi *et al.* (2003) have developed trigonometric shear deformation theory for the bending and free vibration analysis of isotropic, orthotropic and laminated plates which is extended by Zenkour (2005) for the free vibration analysis of functionally graded plates by normalizing trigonometric function. Mantari *et al.* (2012) have developed a new trigonometric shear deformation theory to analyze the static behaviour of isotropic, laminated and sandwich plates. Neves *et al.* (2011, 2012) have developed a hybrid quasi-3D trigonometric shear deformation theory for the bi-directional bending and free vibration analysis of functionally graded plates using radial basis functions. The effect of transverse normal strain is taken into account. The boundary value problem is deduced from the Carrera's unified formulation and the principle of virtual displacement. Recently, Sayyad *et al*. (2015) applied trigonometric shear deformation theory for the bending analysis of laminated composite and sandwich beams.

However, it is observed from the literature reviewed that all the trigonometric shear deformation theories are not explored thoroughly for the one dimensional analysis of laminated composite and sandwich plates with transverse normal deformation effects on their static and dynamic responses.

In a whole variety of higher order plate theories existing in the literature no consideration is given to the transverse normal strain / deformation effect on flexural response when these higher order theories are applied to shear flexible composite plates in view of minimizing the number of unknown variables. The effect of transverse normal strain on static and dynamic responses of multilayered plates is highly recommended by Carrera (1999a, 1999b, 2005). Carrera recommended that any refinements of classical models are meaningless, in general, unless the effects of interlaminar continuous transverse shear and normal stresses are both taken into account in a multilayered plate/shell theory. In a whole lot of literature on this subject many researchers neglected this effect including Reddy (1984). Carrera Unified Formulation (CUF) is developed by Carrera and his co-workers in the last decade for beams, plates and shells theories (Carrera 2002, Carrera 2003, Carrera *et al*. 2008, Carrera *et al*. 2011). CUF based theories are based on the Legendre polynomials or Taylor series expansion in thickness coordinate z. The soft core effect on sandwich plates has been recently studied by Tornabene *et al.* (2014, 2015) using CUF approach with differential quadrature method and the higher order equivalent single layer theory without transverse normal strain effects. The theories reported in references Mindlin (1951), Lo *et al.* (1977a, 1977b), Ambartsumian (1958), Kruszewski (1949), Reddy (1984), Soldatos (1992), Karama *et al.* (2003), Akavci (2007), Shimpi and Patel (2006), Aydogdu (2009), Wan (1992), Jalali and Taheri (1998), Soldatos and Watson (1997), Shu and Soldatos (2000), Park and Lee (2003), Lu *et al*. (2007), Sayyad *et al*. (2014), Sayyad and Ghugal (2015), Lebee and Sab (2011), Afshin *et al*. (2010), Auricchio and Sacco (2003), Toledano and Murakami (1987a, 1987b), He (1992), Touratier (1991), Shimpi and Ghugal (2001), Ghugal and Shinde (2013, 2014), Mantari *et al*. (2012) do not consider the effect of transverse normal strain whereas the theories reported in the references Perel and Palazotto (2001), Bessaim *et al*. (2013), Kant and Shiyekar (2008), Zhou *et al.* (2009), Stein (1986), Shimpi *et al.* (2003), Zenkour (2005), Neves *et al.* (2011, 2012) considered the effect of transverse normal strain.

Ghugal and Sayyad (2011, 2013) and Sayyad and Ghugal (2013, 2014a, 2014b, 2015b) also developed a trigonometric / sinusoidal shear and normal deformation theory for the laminated composite and sandwich plates taking into account both the effects of transverse shear and normal deformations. However, in these works by authors, the theory is mostly applied for bi-directional bending of cross-ply laminated plates but not exclusively developed for the cylindrical bending or one dimensional analysis of laminated and sandwich plates taking into account the effect of transverse normal strain. The main objective of the present study is to examine the efficiency of the sinusoidal shear and normal deformation plate theory for the cylindrical bending of multilayered laminated composite and sandwich plates. The theory is built upon classical plate theory. The governing equations and boundary conditions are obtained by using the principle of minimum potential energy. Closed form solutions are obtained for the cylindrical bending analysis of single layer and multilayered laminated composite and sandwich plates and compared them to solutions of classical plate theory (CPT), first order shear deformation theory (FSDT) of Mindlin (1951), higher order shear deformation theory (HSDT) of Reddy (1984) and the exact solutions given by Pagano (1969). The results presented in this paper are specially generated according to various theories being not available in the open literature. In the present study, results for laminated and sandwich plates subjected to uniformly distributed load and patch load are presented for the first time which can be served as benchmark solutions for the future researchers in this field.

2. Mathematical formulation

The mathematical formulation of present sinusoidal shear and normal deformation plate theory (SSNPT) for laminated composite and sandwich plates is based on certain kinematical and physical assumptions. The principle of minimum potential energy will be used to obtain the governing equations of equilibrium and the associated boundary conditions. The Navier solution technique will be employed to develop analytical solution for the simply supported boundary conditions. The program has been developed in FORTRAN to determine numerical results.

2.1 The plate under consideration

A general laminated composite plate with length of 'a' and width of 'b' respectively in the x

and y directions is considered (Fig. 1). The z-direction is assumed positive in downward direction. The thickness of the laminate following the z-direction and the middle plane of the plate is located at z=0. The plate is made up of N number of layers and the behaviour of all layers is in general considered orthotropic. The laminate is subjected to cylindrical bending *via* an out-of-plane loading $q(x)$ applied on its top surface (z=-h/2). It is assumed that the plate is long in the y direction (b >>a, h) so that the strain components are independent of the y coordinate. The plate is in state of plane strain condition with respect to xz plane.

2.2 Assumption made in mathematical formulation

Assumptions of present sinusoidal shear and normal deformation plate theory (SSNPT) are as follows:

1. It is assumed that the plate is of an infinite extent at the y-direction (b>>a, h) while it is simply supported at its edges x=0 and x=a.
2. The displacement component u is the inplane displacement in x- direction and w is the transverse displacement in z-direction. These displacements are small in comparison with the plate thickness.
3. The in-plane displacement u in x-direction consists of three components (extension, bending and shear)

$$u = u_0 + u_b + u_s \tag{1}$$

a) u_0 is the middle surface displacement in x- direction.
b) The bending component is assumed to be analogous to the displacement given by classical plate theory.

$$u_b = -z\frac{\partial w_0(x)}{\partial x} \tag{2}$$

c) Shear component is assumed to be sinusoidal in nature with respect to thickness coordinate, such that maximum shear stress occurs at neutral plane and zero at top and bottom surfaces of the plate.

$$u_s = \frac{h}{\pi}\sin\frac{\pi z}{h}\phi(x) \tag{3}$$

4. The transverse displacement w in z- direction is assumed to be a function of x and z coordinates.

$$w = w_0(x) + \frac{h}{\pi}\cos\frac{\pi z}{h}\xi(x) \tag{4}$$

5. The plate is subjected to transverse load only.
6. The body forces are ignored in the analysis.

3. Sinusoidal shear and normal deformation plate theory (SSNPT)

SSNPT is developed based on the before mentioned assumptions. By using Eqs. (1)-(4), the

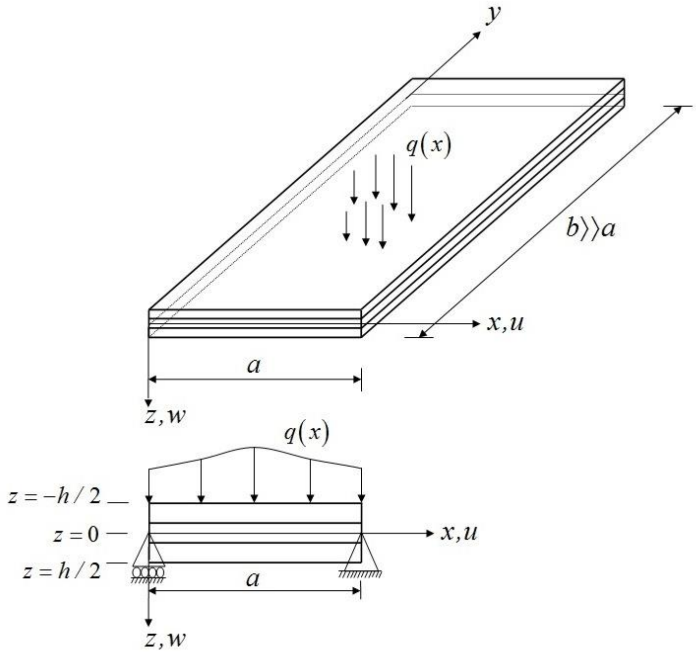


Fig. 1 Geometry of a plate strip in cylindrical bending

displacement field of the present SSNPT can be expressed as follows

$$u(x,z) = u_0(x) - z\frac{\partial w_0}{\partial x} + \frac{h}{\pi}\sin\frac{\pi z}{h}\,\phi(x), \quad v(x,z) = 0$$
$$w(x,z) = w_0(x) + \frac{h}{\pi}\cos\frac{\pi z}{h}\,\xi(x) \tag{5}$$

Here, $u(x, z)$ is the in-plane displacement in the x- direction and $w(x, z)$ is the transverse displacement in the z- direction. There is no relative motion in the y- direction at any points in the cross section of the plate, therefore v=0. $u_0(x)$ is the center line displacement and is a function of x only. The shear slopes ϕ and ξ are functions of x only. The following strain vector $\{\varepsilon\}$ is given by strain displacement relationship as per linear theory of elasticity.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial w}{\partial z} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix}^T \tag{6}$$

Considering the problem as a plane strain problem, strain quantities in y direction are zero (ε_y=γ_{xy}=γ_{yz}=0). The strain displacement relationship of Eq. (6) becomes

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{zx} \end{Bmatrix} = \left\{ \frac{\partial u}{\partial x} \quad \frac{\partial w}{\partial z} \quad \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right\}^T \tag{7}$$

$$\begin{gathered} \varepsilon_x = \varepsilon_x^0 + z k_x^b + f(z) k_x^s, \quad \varepsilon_z = g'(z)\xi, \\ \gamma_{zx} = f'(z)\gamma_{zx}^0 + g(z) k_z^s \end{gathered} \tag{8}$$

where

$$\begin{gathered} \varepsilon_x^0 = \frac{\partial u_0}{\partial x}; \quad k_x^b = -\frac{\partial^2 w_0}{\partial x^2}; \quad k_x^s = \frac{\partial \phi}{\partial x}; \quad \gamma_{zx}^0 = \phi; \quad k_z^s = \frac{\partial \xi}{\partial x}; \\ f(z) = \frac{h}{\pi}\sin\frac{\pi z}{h}; \quad g(z) = \frac{h}{\pi}\cos\frac{\pi z}{h}; \quad f'(z) = \cos\frac{\pi z}{h}; \quad g'(z) = -\sin\frac{\pi z}{h} \end{gathered} \tag{9}$$

where the prime ( )′ indicates the differentiation of function with respect to *z*.

3.1 Constitutive relations

The constitutive equations for the k^{th} lamina of laminated composite plate under cylindrical bending (plane strain problem) can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{13} & 0 \\ \bar{Q}_{13} & \bar{Q}_{33} & 0 \\ 0 & 0 & \bar{Q}_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix}^k \tag{10}$$

where $\bar{Q}_{ij}$ are the reduced elastic constants corresponding to plane strain state in *x*-*z* plane. The following relations hold between these and the engineering elastic constants:

$$\bar{Q}_{11} = \frac{E_1(1-\mu_{23}\mu_{32})}{\Delta}; \quad \bar{Q}_{13} = \frac{E_1(\mu_{31}+\mu_{21}\mu_{32})}{\Delta}; \quad \bar{Q}_{33} = \frac{E_3(1-\mu_{12}\mu_{21})}{\Delta}; \quad \bar{Q}_{55} = G_{13}; \tag{11}$$

where

$$\Delta = (1-\mu_{12}\mu_{21}-\mu_{23}\mu_{32}-\mu_{31}\mu_{13}-2\mu_{12}\mu_{23}\mu_{31}) \tag{12}$$

in which E_1, E_3 are Young's moduli, G_{13} is the shear modulus and μ_{12}, μ_{21}, μ_{13}, μ_{31}, μ_{23}, μ_{32}, are Poisson's ratios. The subscripts 1, 2, 3 correspond to *x*, *y*, *z* directions of Cartesian coordinate system, respectively.

3.2 Governing equations and boundary conditions for SSNPT

The governing equations and boundary conditions are derived by using the principle of minimum potential energy. The principle of minimum potential states that of all possible displacements that satisfy the given conditions of constraint, that system which is associated with equilibrium makes the value of the sum of the potential energy of the prescribed external forces and the potential strain energy of the internal stresses maximum and in the case of stable equilibrium a minimum. In analytical form it can be written as

$$\delta\left(U+V\right)=0 \tag{13}$$

where U is the total strain energy due to deformation, V is the potential of the external loads and δ is the variational symbol. The strain energy of the plate is given by

$$U=\frac{1}{2}\int_{x=0}^{x=a}\int_{z=-h/2}^{z=+h/2}\left(\sigma_x\,\varepsilon_x+\sigma_z\,\varepsilon_z+\tau_{zx}\,\gamma_{zx}\right)dz\,dx \tag{14}$$

and the potential energy of the external laterally distributed load $q(x,y)$ on the plate is given by

$$V=-\int_{x=0}^{x=a}q\left(x\right)w_0\,dx \tag{15}$$

Substituting the energy expressions (14) and (15) into Eq. (13), the final expression can be written as

$$\int_{x=0}^{x=a}\int_{z=-h/2}^{z=+h/2}\left(\sigma_x\,\delta\varepsilon_x+\sigma_z\,\delta\varepsilon_z+\tau_{zx}\,\delta\gamma_{zx}\right)dz\,dx-\int_{x=0}^{x=a}q\left(x\right)\delta w_0\,dx=0 \tag{16}$$

Substituting the strain components from Eq. (8) into the Eq. (16), one can obtain

$$\begin{aligned}&\int_{x=0}^{x=a}\int_{z=-h/2}^{z=+h/2}\sigma_x\left(\frac{\partial\delta u_0}{\partial x}-z\frac{\partial^2\delta w_0}{\partial x^2}+f\left(z\right)\frac{\partial\delta\phi}{\partial x}\right)dz\,dx+\int_{x=0}^{x=a}\int_{z=-h/2}^{z=+h/2}\sigma_z\,g'\left(z\right)\delta\xi\,dz\,dx\\&+\int_{x=0}^{x=a}\int_{z=-h/2}^{z=+h/2}\tau_{zx}\left(f'\left(z\right)\delta\phi+g\left(z\right)\frac{\partial\delta\xi}{\partial x}\right)dz\,dx-\int_{x=0}^{x=a}q\left(x\right)\delta w_0\,dx=0\end{aligned} \tag{17}$$

By integrating the Eq. (17) with respect to z, we get the following equation:

$$\begin{aligned}&\int_{x=0}^{x=a}\left(N_x\frac{\partial\delta u_0}{\partial x}-M_x\frac{\partial^2\delta w_0}{\partial x^2}+M_x^s\frac{\partial\delta\phi}{\partial x}\right)dx+\int_{x=0}^{x=a}Q_z\,\delta\xi\,dx\\&+\int_{x=0}^{x=a}\left(Q_x\,\delta\phi+\frac{h}{\pi}Q_x\frac{\partial\delta\xi}{\partial x}\right)dz\,dx-\int_{x=0}^{x=a}q\left(x\right)\delta w_0\,dx=0\end{aligned} \tag{18}$$

Integration of Eq. (18) by parts and collecting the coefficients of δu_0, δw_0, $\delta\phi$ and $\delta\xi$ leads to the following equation

$$\begin{aligned}&\int_{x=0}^{x=a}\frac{\partial N_x}{\partial x}\delta u_0\,dx+\int_{x=0}^{x=a}\left(\frac{\partial^2 M_x}{\partial x^2}+q\left(x\right)\right)\delta w_0\,dx+\int_{x=0}^{x=a}\left(\frac{\partial M_x^s}{\partial x}-Q_x\right)\delta\phi\,dx\\&+\int_{x=0}^{x=a}\left(\frac{h}{\pi}\frac{\partial Q_x}{\partial x}-Q_z\right)\delta\xi\,dx-N_x\,\delta u_0\Big|_{x=0}^{x=a}+M_x\frac{\partial\delta w_0}{\partial x}\Bigg|_{x=0}^{x=a}-\frac{\partial M_x}{\partial x}\delta w_0\Bigg|_{x=0}^{x=a}\\&-M_x^s\,\delta\phi\Big|_{x=0}^{x=a}-\frac{h}{\pi}Q_x\,\delta\xi\Big|_{x=0}^{x=a}=0\end{aligned} \tag{19}$$

Invoking the fundamental lemma of calculus of variations in Eq. (19), the following governing equations (Euler-Lagrange equations) are obtained

$$\delta u_0: \quad \frac{\partial N_x}{\partial x} = 0 \tag{20}$$

$$\delta w_0: \quad \frac{\partial^2 M_x}{\partial x^2} + q = 0 \tag{21}$$

$$\delta \phi: \quad \frac{\partial M_x^s}{\partial x} - Q_x = 0 \tag{22}$$

$$\delta \xi: \quad \frac{h}{\pi}\frac{\partial Q_x}{\partial x} - Q_z = 0 \tag{23}$$

And a proper set of boundary conditions at each edge of the plate is obtained as a result of the application of the principle of minimum potential energy and the calculus of variations

$$\text{Either} \quad N_x = 0 \quad \text{or} \quad u_0 \quad \text{is prescribed} \tag{24}$$

$$\text{Either} \quad M_x = 0 \quad \text{or} \quad \frac{\partial w_0}{\partial x} \text{ is prescribed} \tag{25}$$

$$\text{Either} \quad M_x^s = 0 \quad \text{or} \quad \phi \quad \text{is prescribed} \tag{26}$$

$$\text{Either} \quad \frac{\partial M_x}{\partial x} = 0 \quad \text{or} \quad w_0 \quad \text{is prescribed} \tag{27}$$

$$\text{Either} \quad Q_x = 0 \quad \text{or} \quad \xi \quad \text{is prescribed.} \tag{28}$$

The stress resultants appeared in governing equations and associated boundary conditions are defined as

$$N_x = \sum_{k=1}^{N} \int_{-h/2}^{h/2} \sigma_x \, dz = A_{11}\varepsilon_x^0 + B_{11}k_x^b - \frac{\pi}{h} As_{13}\,\xi \tag{29}$$

$$M_x = \sum_{k=1}^{N} \int_{-h/2}^{h/2} \sigma_x \, z \, dz = B_{11}\varepsilon_x^0 + D_{11}k_x^b - \frac{\pi}{h} Bs_{13}\,\xi \tag{30}$$

$$M_x^s = \sum_{k=1}^{N} \int_{-h/2}^{h/2} \sigma_x \, f(z) dz = As_{11}\varepsilon_x^0 + Bs_{11}k_x^b - \frac{\pi}{h} Ass_{13}\,\xi \tag{31}$$

$$Q_x = \sum_{k=1}^{N} \int_{-h/2}^{h/2} \tau_{xz} f'(z) dz = Acc_{55}\,\phi + \frac{h}{\pi} Acc_{55}\,k_z^s \tag{32}$$

$$Q_z = \sum_{k=1}^{N} \int_{-h/2}^{h/2} \sigma_{zz} g'(z) dz = -\frac{\pi}{h} As_{13}\,\varepsilon_x^0 - \frac{\pi}{h} Bs_{13}\,k_x^b - \frac{\pi}{h} Ass_{13}\,k_x^s + \frac{\pi^2}{h^2} Ass_{33}\,\xi \tag{33}$$

where N are number of layers, N_x is the axial force resultant analogous to classical plate theory, M_x is the moment resultant or the stress couple analogous to classical plate theory, M_x^s is the refined moment or stress couple due to transverse shear deformation effect and Q_x, Q_z are the transverse shear and transverse normal stress resultants, respectively. The governing Eqs. (20)-(23) can be expressed in-terms of the unknown generalized displacement variables as follows

$$\delta u_0:\ A_{11}\frac{\partial^2 u_0}{\partial x^2}-B_{11}\frac{\partial^3 w_0}{\partial x^3}+As_{11}\frac{\partial^2 \phi}{\partial x^2}-\frac{\pi}{h}As_{13}\frac{\partial \xi}{\partial x}=0 \tag{34}$$

$$\delta w_0:\ B_{11}\frac{\partial^3 u_0}{\partial x^3}-D_{11}\frac{\partial^4 w_0}{\partial x^4}+Bs_{11}\frac{\partial^3 \phi}{\partial x^3}-\frac{\pi}{h}Bs_{13}\frac{\partial^2 \xi}{\partial x^2}+q=0 \tag{35}$$

$$\delta\phi:\ As_{11}\frac{\partial^2 u_0}{\partial x^2}-Bs_{11}\frac{\partial^3 w_0}{\partial x^3}+Ass_{11}\frac{\partial^2 \phi}{\partial x^2}-Acc_{55}\,\phi-\left(\frac{\pi}{h}Ass_{13}+\frac{h}{\pi}Acc_{55}\right)\frac{\partial \xi}{\partial x}=0 \tag{36}$$

$$\delta\xi:\ \frac{\pi}{h}As_{13}\frac{\partial u_0}{\partial x}-\frac{\pi}{h}Bs_{13}\frac{\partial^2 w_0}{\partial x^2}+\left(\frac{\pi}{h}Ass_{13}+\frac{h}{\pi}Acc_{55}\right)\frac{\partial \phi}{\partial x}-\frac{\pi^2}{h^2}Ass_{33}\,\xi+\frac{h^2}{\pi^2}Acc_{55}\frac{\partial^2 \xi}{\partial x^2}=0 \tag{37}$$

where the stiffness components (A_{ij}, B_{ij}… etc.) appeared in Eqs. (34) - (37) can be computed as follows.

$$\begin{gathered}
A_{ij}=\sum_{k=1}^{N}\bar{Q}_{ij}^{k}\int_{h_k}^{h_{k+1}}dz,\quad B_{ij}=\sum_{k=1}^{N}\bar{Q}_{ij}^{k}\int_{h_k}^{h_{k+1}}z\,dz,\quad D_{ij}=\sum_{k=1}^{N}\bar{Q}_{ij}^{k}\int_{h_k}^{h_{k+1}}z^2\,dz,\\
As_{ij}=\sum_{k=1}^{N}\bar{Q}_{ij}^{k}\int_{h_k}^{h_{k+1}}\frac{h}{\pi}\sin\frac{\pi z}{h}\,dz,\quad Bs_{ij}=\sum_{k=1}^{N}\bar{Q}_{ij}^{k}\int_{h_k}^{h_{k+1}}\left(z\,\frac{h}{\pi}\sin\frac{\pi z}{h}\right)dz,\\
Ass_{ij}=\sum_{k=1}^{N}\bar{Q}_{ij}^{k}\int_{h_k}^{h_{k+1}}\frac{h^2}{\pi^2}\sin^2\frac{\pi z}{h}\,dz,\quad Acc_{ij}=\sum_{k=1}^{N}\bar{Q}_{ij}^{k}\int_{h_k}^{h_{k+1}}\cos^2\frac{\pi z}{h}\,dz
\end{gathered} \tag{38}$$

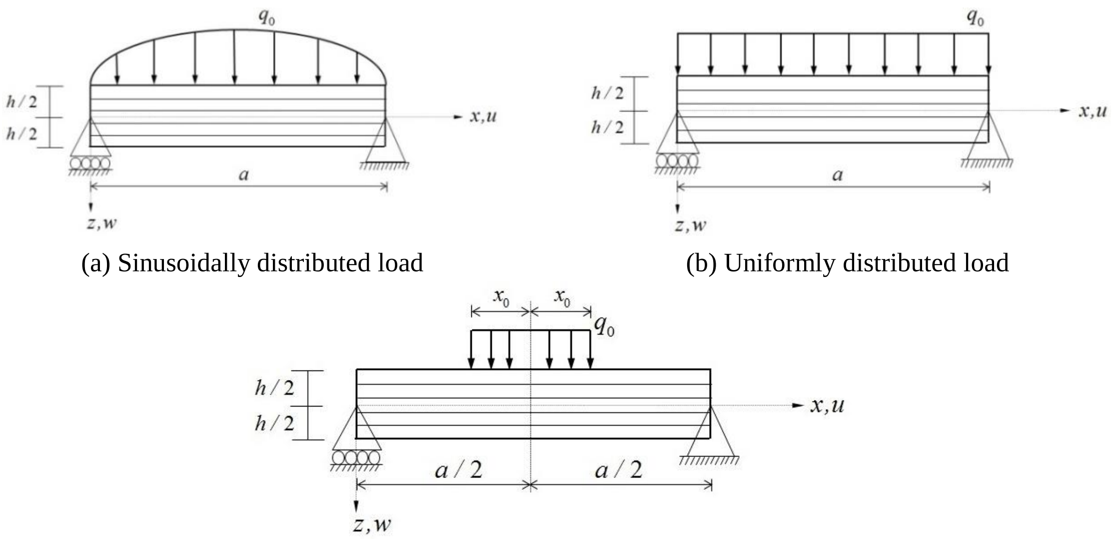
4. Navier solution

Following are the boundary conditions used for simply supported laminated composite plates along the edges x=0 and x=a

$$w_0=0,\quad N_x=0,\quad M_x=0,\quad M_x^s=0,\quad \xi=0 \tag{39}$$

Navier's solution procedure is adopted to compute displacement variables. The displacements and rotations, that satisfy the above boundary conditions exactly, can be assumed as follows

$$\begin{gathered}
u_0(x)=\sum_{m=1,3,5}^{\infty}u_m\cos\left(\frac{m\pi x}{a}\right),\quad w_0(x)=\sum_{m=1,3,5}^{\infty}w_m\sin\left(\frac{m\pi x}{a}\right),\\
\phi(x)=\sum_{m=1,3,5}^{\infty}\phi_m\cos\left(\frac{m\pi x}{a}\right),\quad \xi(x)=\sum_{m=1,3,5}^{\infty}\xi_m\sin\left(\frac{m\pi x}{a}\right)
\end{gathered} \tag{40}$$



(a) Sinusoidally distributed load (b) Uniformly distributed load

(c) Patch load

Fig. 2 Laminated plates subjected various static loadings

where u_m, w_m, ϕ_m and ξ_m are the unknowns coefficients to be determine. The various static loadings $q(x)$, on the top surface of plate (i.e., z=-h/2) as shown in Fig. 2, are presented in single trigonometric series as follows

$$q(x)=\sum_{m=1}^{\infty} q_m \sin\left(\frac{m\pi x}{a}\right) \tag{41}$$

where m is the positive integer and q_m is the coefficient of Fourier series expansion as given below for various static loadings.

$$q_m = q_0 \qquad \text{Sinusoidally distributed load (SDL)}$$

$$q_m = \frac{4q_0}{m\pi} \qquad \text{Uniformly distributed load (UDL)} \tag{42}$$

$$q_m = \frac{4q_0}{m\pi}\sin\frac{m\pi x_0}{a} \qquad \text{Patch load (PL)} \quad \text{where}\,(x_0 = 0.02\,a)$$

The value of positive integer m=1 for sinusoidally distributed load and m=25 for the uniformly distributed and patch loads.

Substitution of Eqs. (40) and (41) into governing Eqs. (34)-(37) leads to the set of algebraic equations which can be written in matrix form as follows

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{21} & K_{22} & K_{23} & K_{24} \\ K_{31} & K_{32} & K_{33} & K_{34} \\ K_{41} & K_{42} & K_{43} & K_{44} \end{bmatrix} \begin{Bmatrix} u_m \\ w_m \\ \phi_m \\ \xi_m \end{Bmatrix} = \begin{Bmatrix} 0 \\ q_m \\ 0 \\ 0 \end{Bmatrix} \tag{43}$$

Here the stiffness coefficients of matrix [K] in Eq. (43) are defined as follows.

$$K_{11}=A_{11}\left(\frac{m^2\pi^2}{a^2}\right),\quad K_{12}=-B_{11}\left(\frac{m^3\pi^3}{a^3}\right),\quad K_{13}=As_{11}\left(\frac{m^2\pi^2}{a^2}\right),\quad K_{14}=\frac{\pi}{h}As_{13}\left(\frac{m\pi}{a}\right),$$

$$K_{21}=K_{12},\quad K_{22}=D_{11}\left(\frac{m^4\pi^4}{a^4}\right),\quad K_{23}=-Bs_{11}\left(\frac{m^3\pi^3}{a^3}\right),\quad K_{24}=-\frac{\pi}{h}Bs_{13}\left(\frac{m^2\pi^2}{a^2}\right),$$

$$K_{31}=K_{13},\quad K_{32}=K_{23},\quad K_{33}=\left[Ass_{11}\left(\frac{m^2\pi^2}{a^2}\right)+Acc_{55}\right],\quad K_{34}=\left[\frac{\pi}{h}Ass_{13}+\frac{h}{\pi}Acc_{55}\right]\left(\frac{m\pi}{a}\right), \tag{44}$$

$$K_{41}=K_{14},\quad K_{42}=K_{24},\quad K_{43}=K_{34},\quad K_{44}=\left[\frac{\pi^2}{h^2}Ass_{33}+\frac{h^2}{\pi^2}Acc_{55}\left(\frac{m^2\pi^2}{a^2}\right)\right]$$

Having obtained the values of u_m, w_m, ϕ_m and ξ_m one can then calculate all the displacements and stress components within the plate using Eqs. (5)-(10). Transverse shear stress (τ_{xz}) can be obtained either by constitutive relation (τ_{xz}^{CR}) or by integrating equilibrium equation (τ_{xz}^{EE}) of theory of elasticity with respect to the thickness coordinate, satisfying shear stress free conditions on the top and bottom surfaces of the plate. When constitutive relation is used to obtain transverse shear stress, it gives a discontinuity of stress at layer interface. Therefore, transverse shear stress is obtained by integrating the equilibrium equation with respect to thickness direction which ascertains the continuity of the transverse shear stress at layer interface. This relation can be expressed as

$$\tau_{xz}=\int_{-h/2}^{z_k}\frac{\partial\sigma_x}{\partial x}\,dz+C_1 \tag{45}$$

In above expression the relation for inplane normal stress is used for individual layer. The constant of integration is obtained from boundary conditions. Since, higher order theories predicts inplane normal stress accurately, it is expected that these relations will produce accurate transverse shear stress.

4.1 Numerical results

Different configurations of the laminates are used for the cylindrical bending analysis of plate as shown in Fig. 3. (*a*) Single layered (0^0) orthotropic plates (*b*) Two layered ($0^0/90^0$) anti-symmetric cross-ply laminated plates (c) Four layered ($0^0/90^0/0^0/90^0$) anti-symmetric cross-ply laminated plates (*d*) Three layered ($0^0/90^0/0^0$) symmetric cross-ply laminated plates (*e*) Four layered ($0^0/90^0/90^0/0^0$) symmetric cross-ply laminated plates (*f*) Five layered ($0^0/90^0/0^0/90^0/0^0$) symmetric cross-ply laminated plates (g) Five layered ($90^0/0^0/0^0/90^0/0^0$) arbitrary cross-ply laminated plates (*p*) Three layered (0^0/*core*/0^0) symmetric sandwich plate (*r*) Five layered ($0^0/90^0$/*core*/$90^0/0^0$) symmetric sandwich plate (*s*) Five layered ($0^0/90^0$/*core*/$0^0/90^0$) anti-symmetric sandwich plate. Following examples are solved for the numerical study.

Example 1: Cylindrical bending of single layer orthotropic plate

Example 2: Cylindrical bending of anti-symmetric cross-ply laminated composite plates

Example 3: Cylindrical bending of symmetric cross-ply laminated composite plates

Example 4: Cylindrical bending of arbitrary cross-ply laminated composite plates

Example 5: Cylindrical bending of symmetric sandwich plates

Example 6: Cylindrical bending of anti-symmetric sandwich plates

The material properties used in various problems to obtain the numerical results are presented in Table 1.

Displacements and stresses of various laminate configurations obtained by using present theory (SSNPT) are compared with those obtained by classical plate theory (CPT), FSDT of Mindlin (1951), HSDT of Reddy (1984), exact 3D solution given by Pagano (1969). When FSDT is used, a shear correction factor K has been introduced equal to 5/6.The displacements and stresses are obtained at typical important locations and presented in the following non-dimensional form.

$$\bar{u}\left(0,-\frac{h}{2}\right)=\frac{b\,E_3\,u}{q_0\,h},\quad \bar{w}\left(\frac{a}{2},0\right)=\frac{100E_3\,w\,h^3}{q_0\,a^4},\quad \bar{\sigma}_x\left(\frac{a}{2},-\frac{h}{2}\right)=\frac{b\,\sigma_x}{q_0},\quad \bar{\tau}_{zx}\left(0,0\right)=\frac{b\,\tau_{zx}}{q_0} \tag{46}$$

The through-the-thickness profiles for in-plane displacement ($\bar{u}$), in-plane normal stress ($\bar{\sigma}_x$) and transverse shear stress ($\bar{\tau}_{zx}$) for laminated and sandwich plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL) are obtained at the following locations (shown in brackets)

$$\bar{u}\left(x=0,z\right),\qquad \bar{\sigma}_x\left(x=a/2,z\right)\qquad \text{and}\qquad \bar{\tau}_{zx}\left(x=0,z\right) \tag{47}$$

Table 1 Elastic properties of materials

Material	1	2	3	4	5	6
E_1 (GPa)	172.5	181	224.25	131.1	0.002208	0.04
E_2 (GPa)	6.9	10.3	6.9	6.9	0.002001	0.04
E_3 (GPa)	6.9	10.3	6.9	6.9	27.60	0.5
G_{12} (GPa)	3.45	7.17	56.58	3.588	0.1656	0.016
G_{13} (GPa)	3.45	7.17	56.58	3.588	5.451	0.06
G_{23} (GPa)	1.38	2.87	1.38	3.088	4.554	0.06
μ_{12}	0.25	0.25	0.25	0.32	0.99	0.25
μ_{13}	0.25	0.25	0.25	0.32	0.00003	0.25
μ_{23}	0.25	0.33	0.25	0.49	0.00003	0.25

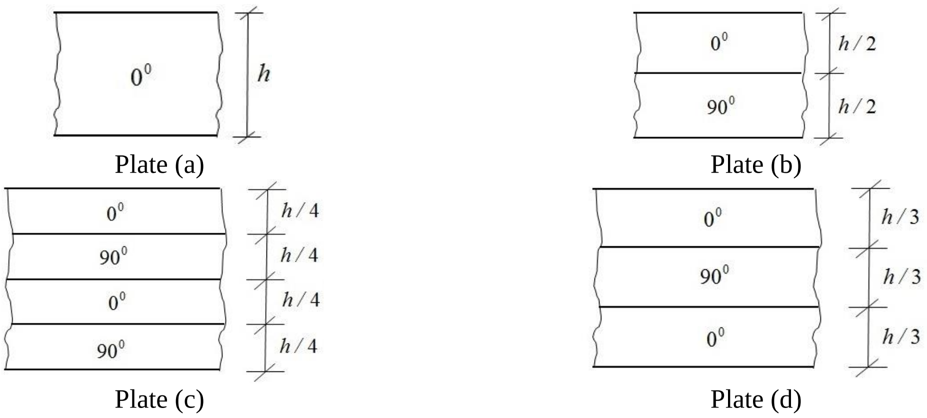


Fig. 3 Laminate configurations

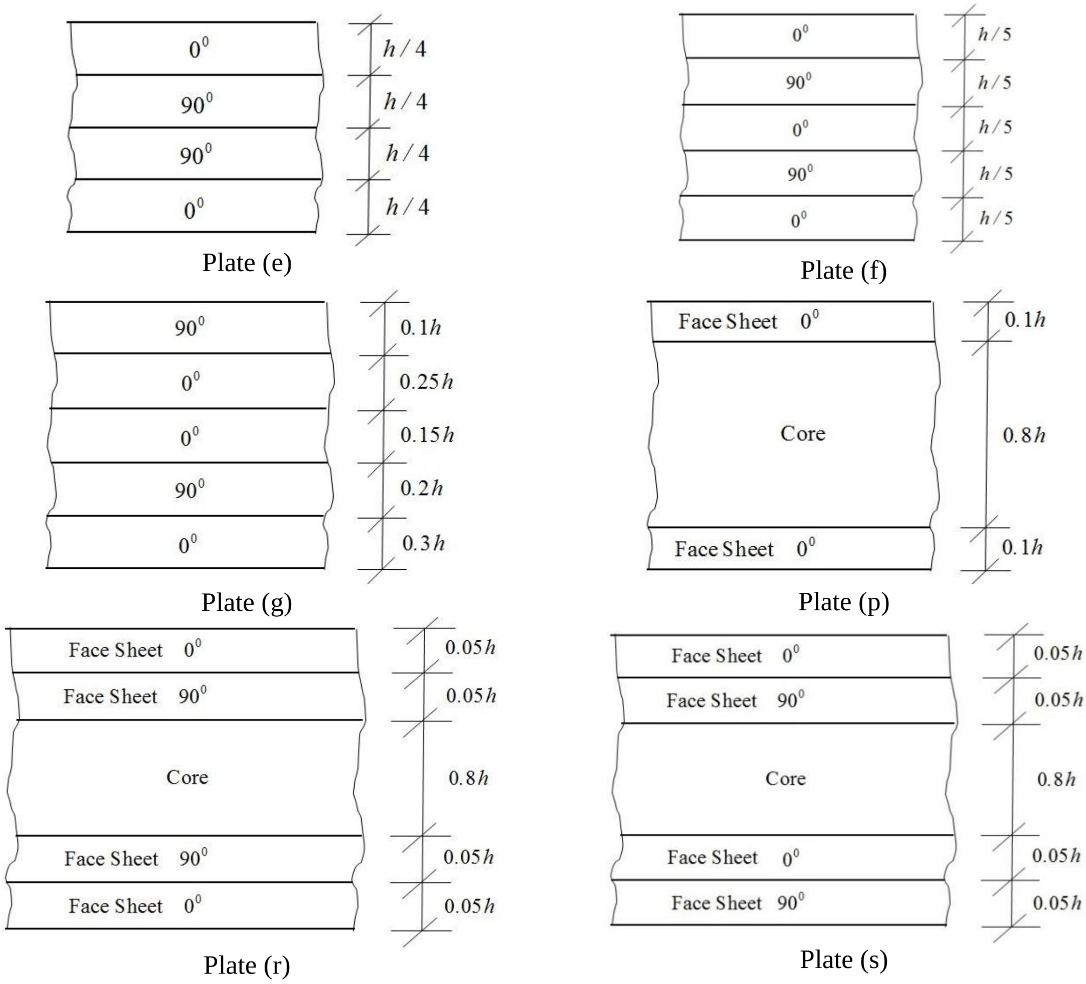


Fig. 3 Continued

4.2 Discussion of numerical results

Example 1: In this example cylindrical bending analysis of single layer orthotropic plate shown in Fig. 3(a) is carried out. The plate has overall thickness ‘*h*’ and made up of material 1. The plate is analyzed for different static loadings (SDL, UDL and PL) as shown in Fig. 2. The non-dimensional displacements and stresses are reported in Table 2. It is pointed out that the present theory predicts excellent values of displacements and stresses compared to those obtained by 3D exact solution given by Pagano (1969) when plate is subjected SDL. In case of other static loadings (UDL and PL) results obtained by present theory are in good agreement with those given by HSDT of Reddy (1984), however, numerical results from Pagano and Wang (1971) are not available for the purpose of comparison.

Example 2: In this example an efficiency of present theory is demonstrated for cylindrical bending analysis of two layered ($0^0/90^0$) and four layered ($0^0/90^0/0^0/90^0$) anti-symmetric laminated composite plates. Figs. 3 (b) and (c) show laminate configurations and distribution of overall

Table 2 Comparison of displacements and stresses of single layered (0^0) orthotropic plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, 0)
			Sinusoidally distributed load			
4	Present	SSNPT	0.7523	1.9233	14.9471	1.6374
	Reddy (1984)	HSDT	0.7397	1.9574	14.5613	1.6725
	Mindlin (1951)	FSDT	0.4941	1.7580	9.72682	1.9099
	Kirchhoff	CPT	0.4941	0.4915	9.72682	1.9099
	Pagano (1969)	Elasticity	0.7215	1.9420	14.4091	1.7314
10	Present	SSNPT	8.3737	0.7314	66.0994	4.6638
	Reddy (1984)	HSDT	8.3539	0.7333	65.7759	4.6768
	Mindlin (1951)	FSDT	7.7210	0.6942	60.7927	4.7746
	Kirchhoff	CPT	7.7210	0.4915	60.7927	4.7746
	Pagano (1969)	Elasticity	8.3140	0.7317	65.7120	4.6826
100	Present	SSNPT	175522.0	0.4939	6087.28	47.729
	Reddy (1984)	HSDT	176115.5	0.4940	6084.32	47.737
	Mindlin (1951)	FSDT	176129.5	0.4936	6079.34	47.747
	Kirchhoff	CPT	176129.5	0.4915	6079.34	47.747
			Uniformly distributed load			
4	Present	SSNPT	1.0114	2.4069	17.3065	2.5711
	Reddy (1984)	HSDT	0.9894	2.4425	17.0148	2.4641
	Mindlin (1951)	FSDT	0.6384	2.1860	12.0003	2.9533
	Kirchhoff	CPT	0.6384	0.6234	12.0003	2.9533
10	Present	SSNPT	10.9632	0.9198	80.3520	6.2912
	Reddy (1984)	HSDT	10.9249	0.9220	80.0392	6.4799
	Mindlin (1951)	FSDT	9.9749	0.8734	75.0022	7.3831
	Kirchhoff	CPT	9.9749	0.6234	75.0022	7.3831
100	Present	SSNPT	9984.65	0.6264	7509.00	73.6407
	Reddy (1984)	HSDT	9984.81	0.6264	7505.40	73.6711
	Mindlin (1951)	FSDT	9975.03	0.6259	7500.32	73.8323
	Kirchhoff	CPT	9974.90	0.6234	7500.22	73.8310
			Patch load			
4	Present	SSNPT	0.0548	0.1638	2.4563	0.1167
	Reddy (1984)	HSDT	0.0543	0.1712	2.3096	0.1155
	Mindlin (1951)	FSDT	0.0383	0.1621	0.9387	0.1218
	Kirchhoff	CPT	0.0383	0.0399	0.9387	0.1218
10	Present	SSNPT	0.6394	0.0621	8.8783	0.2907
	Reddy (1984)	HSDT	0.6383	0.0626	8.5141	0.2919
	Mindlin (1951)	FSDT	0.5982	0.0594	5.8667	0.3045
	Kirchhoff	CPT	0.5982	0.0399	5.8667	0.3045
100	Present	SSNPT	598.5865	0.0401	591.6621	3.0375
	Reddy (1984)	HSDT	598.6016	0.0401	591.1219	3.0388
	Mindlin (1951)	FSDT	598.1996	0.0401	586.6761	3.0448
	Kirchhoff	CPT	598.1917	0.0399	586.6696	3.0448

Table 3 Comparison of displacements and stresses of two layered ($0^0/90^0$) anti-symmetric laminated composite plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, -0.24*h*)
Sinusoidally distributed load						
4	Present	SSNPT	1.7155	4.3904	33.855	2.9900
	Reddy (1984)	HSDT	1.7071	4.4444	33.606	2.9770
	Mindlin (1951)	FSDT	1.4176	4.7900	27.905	2.9468
	Kirchhoff	CPT	1.4176	2.6188	27.905	2.9468
	Pagano (1969)	Elasticity	1.5500	4.3276	30.029	2.7000
10	Present	SSNPT	22.892	2.9066	180.66	7.3879
	Reddy (1984)	HSDT	22.886	2.9159	180.20	7.3780
	Mindlin (1951)	FSDT	22.150	2.9662	174.40	7.3670
	Kirchhoff	CPT	22.150	2.6188	174.40	7.3670
	Pagano (1969)	Elasticity	23.423	2.9569	175.00	7.3000
100	Present	SSNPT	22137.74	2.6198	17470.0	73.743
	Reddy (1984)	HSDT	22158.77	2.6219	17447.0	73.674
	Mindlin (1951)	FSDT	22151.56	2.6224	17441.4	73.674
	Kirchhoff	CPT	22205.98	2.6188	17441.4	73.674
Uniformly distributed load						
4	Present	SSNPT	2.2640	5.5166	40.5293	5.0804
	Reddy (1984)	HSDT	2.2534	5.5817	40.2535	5.0008
	Mindlin (1951)	FSDT	1.8315	6.0002	34.4272	4.5567
	Kirchhoff	CPT	1.8315	3.3216	34.4272	4.5567
10	Present	SSNPT	29.7435	3.6769	221.5495	11.5918
	Reddy (1984)	HSDT	29.7327	3.6883	221.0314	11.5446
	Mindlin (1951)	FSDT	28.6165	3.7502	215.1701	11.3918
	Kirchhoff	CPT	28.6165	3.3216	215.1701	11.3918
100	Present	SSNPT	28602.04	3.3227	21552.17	114.0521
	Reddy (1984)	HSDT	28629.14	3.3254	21523.89	113.9404
	Mindlin (1951)	FSDT	28617.95	3.3260	21518.13	113.9230
	Kirchhoff	CPT	28688.27	3.3216	21517.03	113.9184
Patch load						
4	Present	SSNPT	0.1290	0.3726	4.6914	0.1894
	Reddy (1984)	HSDT	0.1284	0.3791	4.7085	0.1891
	Mindlin (1951)	FSDT	0.1098	0.4219	2.6929	0.1879
	Kirchhoff	CPT	0.1101	0.2124	2.6929	0.1879
10	Present	SSNPT	1.7628	0.2396	20.4896	0.4728
	Reddy (1984)	HSDT	1.7628	0.2406	20.4318	0.4720
	Mindlin (1951)	FSDT	1.7161	0.2459	16.8307	0.4698
	Kirchhoff	CPT	1.7204	0.2124	16.8307	0.4698
100	Present	SSNPT	1715.033	0.2125	1690.7250	4.7033
	Reddy (1984)	HSDT	1716.668	0.2127	1688.3180	4.6988
	Mindlin (1951)	FSDT	1716.212	0.2128	1683.1380	4.6982
	Kirchhoff	CPT	1720.425	0.2124	1683.0700	4.6979

Table 4 Comparison of displacements and stresses of four layered ($0^0/90^0/0^0/90^0$) anti-symmetric laminated composite plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, -0.1*h*)
			Sinusoidally distributed load			
4	Present	SSNPT	1.2298	3.3416	24.2756	2.2025
	Reddy (1984)	HSDT	1.2127	3.3438	23.8705	2.2091
	Mindlin (1951)	FSDT	0.8700	3.2962	17.1247	2.2821
	Kirchhoff	CPT	0.8722	1.1250	17.1247	2.2821
10	Present	SSNPT	14.4884	1.4857	114.357	5.6715
	Reddy (1984)	HSDT	14.4656	1.4864	113.897	5.6734
	Mindlin (1951)	FSDT	13.5933	1.4724	107.029	5.7054
	Kirchhoff	CPT	13.6274	1.1250	107.029	5.7054
100	Present	SSNPT	13575.31	1.1264	10713.94	57.0612
	Reddy (1984)	HSDT	13602.34	1.1286	10710.02	57.0514
	Mindlin (1951)	FSDT	13593.63	1.1285	10703.17	57.0548
	Kirchhoff	CPT	13627.41	1.1250	10702.94	57.0548
			Uniformly distributed load			
4	Present	SSNPT	1.6454	4.1737	28.4633	3.7133
	Reddy (1984)	HSDT	1.6222	4.1744	28.0294	3.6618
	Mindlin (1951)	FSDT	1.1239	4.1055	21.1274	3.5289
	Kirchhoff	CPT	1.1267	1.4269	21.1274	3.5289
10	Present	SSNPT	18.9205	1.8722	139.4595	8.4872
	Reddy (1984)	HSDT	18.8824	1.8731	138.9880	8.5055
	Mindlin (1951)	FSDT	17.5615	1.8555	132.0461	8.8223
	Kirchhoff	CPT	17.6055	1.4269	132.0461	8.8223
100	Present	SSNPT	17540.51	1.4286	13216.65	88.1829
	Reddy (1984)	HSDT	17575.30	1.4314	13211.91	88.1767
	Mindlin (1951)	FSDT	17561.83	1.4312	13204.90	88.2249
	Kirchhoff	CPT	17605.47	1.4269	13204.61	88.2249
			Patch load			
4	Present	SSNPT	0.0906	0.2915	4.0039	0.1426
	Reddy (1984)	HSDT	0.0894	0.2934	3.9833	0.1424
	Mindlin (1951)	FSDT	0.0674	0.3008	1.6526	0.1455
	Kirchhoff	CPT	0.0676	0.0912	1.6526	0.1455
10	Present	SSNPT	1.1095	0.1253	14.6558	0.3579
	Reddy (1984)	HSDT	1.1084	0.1255	14.5304	0.3579
	Mindlin (1951)	FSDT	1.0532	0.1248	10.3287	0.3638
	Kirchhoff	CPT	1.0558	0.0912	10.3287	0.3638
100	Present	SSNPT	1051.63	0.0914	1039.746	3.6368
	Reddy (1984)	HSDT	1053.73	0.0916	1039.031	3.6365
	Mindlin (1951)	FSDT	1053.17	0.0916	1032.888	3.6383
	Kirchhoff	CPT	1055.79	0.0912	1032.869	3.6383

Table 5 Comparison of displacements and stresses of three layered ($0^0/90^0/0^0$) symmetric laminated composite plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, 0)
			Sinusoidally distributed load			
4	Present	SSNPT	0.8885	2.7342	17.5753	1.5278
	Reddy (1984)	HSDT	0.8640	2.6985	17.0063	1.5565
	Mindlin (1951)	FSDT	0.5124	2.4094	10.0854	1.7690
	Kirchhoff	CPT	0.5124	0.5097	10.0854	1.7690
	Pagano (1969)	Elasticity	0.9500	2.8870	17.9500	1.4300
10	Present	SSNPT	8.9765	0.8802	70.8563	4.3214
	Reddy (1984)	HSDT	8.9197	0.8738	70.2307	4.3342
	Mindlin (1951)	FSDT	8.0057	0.8136	63.0339	4.4226
	Kirchhoff	CPT	8.0057	0.5097	63.0339	4.4226
	Pagano (1969)	Elasticity	9.1850	0.8900	71.5000	4.2500
100	Present	SSNPT	8003.87	0.5127	6312.71	44.2092
	Reddy (1984)	HSDT	8014.88	0.5133	6310.65	44.2172
	Mindlin (1951)	FSDT	8005.65	0.5127	6303.38	44.2260
	Kirchhoff	CPT	8005.65	0.5097	6303.38	44.2260
			Uniformly distributed load			
4	Present	SSNPT	1.1930	3.3916	20.3081	1.7602
	Reddy (1984)	HSDT	1.1600	3.3664	19.6888	1.8298
	Mindlin (1951)	FSDT	0.6619	2.9902	12.4428	2.7355
	Kirchhoff	CPT	0.6619	0.6464	12.4428	2.7355
10	Present	SSNPT	11.7920	1.1044	85.6845	6.0143
	Reddy (1984)	HSDT	11.7075	1.0963	85.0477	6.0887
	Mindlin (1951)	FSDT	10.3426	1.0214	77.7673	6.8388
	Kirchhoff	CPT	10.3426	0.6464	77.7673	6.8388
100	Present	SSNPT	10342.85	0.6501	7786.573	68.2126
	Reddy (1984)	HSDT	10356.90	0.6509	7784.159	68.2428
	Mindlin (1951)	FSDT	10342.64	0.6502	7776.720	68.3877
	Kirchhoff	CPT	10342.64	0.6464	7776.720	68.3877
			Patch load			
4	Present	SSNPT	0.0645	0.2338	2.8058	0.1082
	Reddy (1984)	HSDT	0.0628	0.2358	2.7521	0.1081
	Mindlin (1951)	FSDT	0.0397	0.2247	0.9733	0.1128
	Kirchhoff	CPT	0.0397	0.0413	0.9733	0.1128
10	Present	SSNPT	0.6817	0.0757	9.8016	0.2728
	Reddy (1984)	HSDT	0.6783	0.0754	9.6201	0.2729
	Mindlin (1951)	FSDT	0.6202	0.0707	6.0830	0.2820
	Kirchhoff	CPT	0.6202	0.0413	6.0830	0.2820
100	Present	SSNPT	619.9702	0.0400	615.3944	2.8137
	Reddy (1984)	HSDT	620.8328	0.0417	614.7140	2.8149
	Mindlin (1951)	FSDT	620.2444	0.0416	608.2980	2.8203
	Kirchhoff	CPT	620.2444	0.0413	608.2980	2.8203

Table 6 Comparison of displacements and stresses of four layered ($0^0/90^0/90^0/0^0$) symmetric laminated composite plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, 0)
			Sinusoidally distributed load			
4	Present	SSNPT	1.0957	2.7530	15.2548	1.5687
	Reddy (1984)	HSDT	1.0790	2.7293	14.9452	1.5777
	Mindlin (1951)	FSDT	0.7961	2.0912	11.0267	1.6546
	Kirchhoff	CPT	0.7961	0.7919	11.0267	1.6546
10	Present	SSNPT	13.1301	1.1086	73.2341	4.1007
	Reddy (1984)	HSDT	13.1589	1.1074	72.9052	4.1053
	Mindlin (1951)	FSDT	12.4391	0.9998	68.9171	4.1366
	Kirchhoff	CPT	12.4391	0.7919	68.9171	4.1366
100	Present	SSNPT	12414.10	0.7930	6898.142	41.3518
	Reddy (1984)	HSDT	12446.88	0.7951	6896.028	41.3643
	Mindlin (1951)	FSDT	12439.09	0.7940	6891.714	41.3656
	Kirchhoff	CPT	12439.09	0.7919	6891.714	41.3656
			Uniformly distributed load			
4	Present	SSNPT	1.4648	3.4354	17.9237	2.3734
	Reddy (1984)	HSDT	1.4401	3.4033	17.6119	2.3250
	Mindlin (1951)	FSDT	1.0285	2.6074	13.6041	2.5586
	Kirchhoff	CPT	1.0285	1.0044	13.6041	2.5586
10	Present	SSNPT	17.1903	1.3986	89.3921	6.0174
	Reddy (1984)	HSDT	17.1607	1.3939	89.0567	6.0575
	Mindlin (1951)	FSDT	16.0703	1.2609	85.0255	6.3965
	Kirchhoff	CPT	16.0703	1.0044	85.0255	6.3965
100	Present	SSNPT	16039.93	1.0057	8509.589	63.8913
	Reddy (1984)	HSDT	16082.14	1.0084	8507.056	63.9154
	Mindlin (1951)	FSDT	16070.26	1.0069	8502.562	63.9646
	Kirchhoff	CPT	16070.26	1.0044	8502.562	63.9646
			Patch load			
4	Present	SSNPT	0.0809	0.2404	2.5097	0.1023
	Reddy (1984)	HSDT	0.0798	0.2409	2.4272	0.1023
	Mindlin (1951)	FSDT	0.0617	0.1896	1.0641	0.1055
	Kirchhoff	CPT	0.0617	0.0642	1.0641	0.1055
10	Present	SSNPT	1.0097	0.0943	9.2962	0.2572
	Reddy (1984)	HSDT	1.0093	0.0941	9.1016	0.2577
	Mindlin (1951)	FSDT	0.9637	0.0843	6.6507	0.2638
	Kirchhoff	CPT	0.9637	0.0642	6.6507	0.2638
100	Present	SSNPT	961.685	0.0644	669.122	2.6350
	Reddy (1984)	HSDT	964.233	0.0645	668.667	2.6360
	Mindlin (1951)	FSDT	963.727	0.0644	665.073	2.6378
	Kirchhoff	CPT	963.727	0.0642	665.073	2.6378

Table 7 Comparison of displacements and stresses of five layered ($0^0/90^0/0^0/90^0/0^0$) symmetric laminated composite plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, 0)
Sinusoidally distributed load						
4	Present	SSNPT	0.8893	2.5673	17.5614	1.5044
	Reddy (1984)	HSDT	0.8791	2.5872	17.3034	1.5078
	Mindlin (1951)	FSDT	0.6174	2.6139	12.1537	1.6533
	Kirchhoff	CPT	0.6174	0.6142	12.1537	1.6533
	Pagano (1969)	Elasticity	0.9509	3.0440	18.4615	---
10	Present	SSNPT	10.3246	0.9324	81.4962	4.0686
	Reddy (1984)	HSDT	10.3183	0.9380	81.2431	4.0735
	Mindlin (1951)	FSDT	9.6474	0.9341	75.9605	4.1332
	Kirchhoff	CPT	9.6716	0.6142	75.9605	4.1332
100	Present	SSNPT	9633.854	0.6161	7602.827	41.3212
	Reddy (1984)	HSDT	9654.246	0.6174	7601.430	41.3263
	Mindlin (1951)	FSDT	9647.451	0.6174	7596.080	41.3305
	Kirchhoff	CPT	9671.590	0.6142	7596.080	41.3305
Uniformly distributed load						
4	Present	SSNPT	1.1907	3.1932	20.5494	2.3808
	Reddy (1984)	HSDT	1.1738	3.2255	20.3076	2.3018
	Mindlin (1951)	FSDT	0.7977	3.2462	14.9944	2.5565
	Kirchhoff	CPT	0.7997	0.7790	14.9944	2.5565
10	Present	SSNPT	13.4906	1.1719	99.3115	5.8002
	Reddy (1984)	HSDT	13.4739	1.1789	99.0537	5.8098
	Mindlin (1951)	FSDT	12.4636	1.1737	93.7152	6.3913
	Kirchhoff	CPT	12.4949	0.7790	93.7152	6.3913
100	Present	SSNPT	12447.91	0.7813	9378.722	63.8002
	Reddy (1984)	HSDT	12474.18	0.7830	9377.051	63.8148
	Mindlin (1951)	FSDT	12463.70	0.7829	9371.564	63.9107
	Kirchhoff	CPT	12494.88	0.7790	9371.564	63.9107
Patch load						
4	Present	SSNPT	0.0654	0.2268	2.9008	0.1002
	Reddy (1984)	HSDT	0.0648	0.2276	2.7327	0.1010
	Mindlin (1951)	FSDT	0.0478	0.2428	1.1729	0.1054
	Kirchhoff	CPT	0.0480	0.0498	1.1729	0.1054
10	Present	SSNPT	0.7900	0.0799	10.5489	0.2530
	Reddy (1984)	HSDT	0.7899	0.0803	10.2789	0.2548
	Mindlin (1951)	FSDT	0.7474	0.0807	7.3304	0.2636
	Kirchhoff	CPT	0.7493	0.0498	7.3304	0.2636
100	Present	SSNPT	746.2938	0.0501	738.1019	2.6314
	Reddy (1984)	HSDT	747.8774	0.0501	737.7702	2.6320
	Mindlin (1951)	FSDT	747.4434	0.0501	733.0467	2.6356
	Kirchhoff	CPT	749.3135	0.0498	733.0467	2.6356

Table 8 Comparison of displacements and stresses of five layered ($90^0/0^0/0^0/90^0/0^0$) arbitrary laminated composite plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, 0)
			Sinusoidally distributed load			
4	Present	SSNPT	0.7063	1.0028	0.5712	1.9474
	Reddy (1984)	HSDT	0.6976	0.9812	0.5492	1.9525
	Mindlin (1951)	FSDT	0.6173	0.9342	0.4861	1.9767
	Kirchhoff	CPT	0.6173	0.6088	0.4861	1.9767
10	Present	SSNPT	9.8523	0.6703	3.0993	4.9302
	Reddy (1984)	HSDT	9.8471	0.6686	3.1013	4.9319
	Mindlin (1951)	FSDT	9.6457	0.6608	3.0379	4.9416
	Kirchhoff	CPT	9.6458	0.6088	3.0379	4.9416
100	Present	SSNPT	9631.7500	0.6083	301.1299	49.4171
	Reddy (1984)	HSDT	9648.5190	0.6094	303.8768	49.4192
	Mindlin (1951)	FSDT	9645.2670	0.6093	303.7744	49.4222
	Kirchhoff	CPT	9645.7660	0.6088	303.7901	49.4222
			Uniformly distributed load			
4	Present	SSNPT	0.9368	1.2502	0.6835	2.7554
	Reddy (1984)	HSDT	0.9184	1.2321	0.6635	2.8250
	Mindlin (1951)	FSDT	0.7975	1.1736	0.5997	3.0566
	Kirchhoff	CPT	0.7975	0.7721	0.5997	3.0566
10	Present	SSNPT	12.7886	0.8480	3.8047	7.4271
	Reddy (1984)	HSDT	12.7709	0.8460	3.8125	7.5023
	Mindlin (1951)	FSDT	12.4615	0.8364	3.7480	7.6414
	Kirchhoff	CPT	12.4615	0.7721	3.7480	7.6414
100	Present	SSNPT	12443.98	0.7715	371.4964	76.3974
	Reddy (1984)	HSDT	12465.58	0.7729	374.8915	76.4026
	Mindlin (1951)	FSDT	12460.88	0.7727	374.7771	76.4212
	Kirchhoff	CPT	12460.88	0.7721	374.7771	76.4212
			Patch load			
4	Present	SSNPT	0.0534	0.0847	0.1041	0.1217
	Reddy (1984)	HSDT	0.0529	0.0845	0.0825	0.1224
	Mindlin (1951)	FSDT	0.0478	0.0808	0.0469	0.1260
	Kirchhoff	CPT	0.0478	0.0494	0.0469	0.1260
10	Present	SSNPT	0.7602	0.0552	0.3743	0.3077
	Reddy (1984)	HSDT	0.7601	0.0551	0.3434	0.3111
	Mindlin (1951)	FSDT	0.7473	0.0544	0.2932	0.3151
	Kirchhoff	CPT	0.7473	0.0494	0.2932	0.3151
100	Present	SSNPT	746.1965	0.0494	29.1314	3.1506
	Reddy (1984)	HSDT	747.5002	0.0494	29.3755	3.1509
	Mindlin (1951)	FSDT	747.2731	0.0494	29.3155	3.1517
	Kirchhoff	CPT	747.2731	0.0494	29.3155	3.1517

Table 9 Comparison of displacements and stresses of three layered (0^0/*core*/0^0) symmetric sandwich plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, 0)
Sinusoidally distributed load						
4	Present	SSNPT	1.8901	8.4532	28.9670	1.3841
	Reddy (1984)	HSDT	1.9081	8.5369	28.6061	1.3855
	Mindlin (1951)	FSDT	1.3295	5.4694	19.9320	1.4089
	Kirchhoff	CPT	1.3295	1.3225	19.9320	1.4089
10	Present	SSNPT	22.0925	2.4739	133.7540	3.5122
	Reddy (1984)	HSDT	22.2353	2.4889	133.3406	3.5128
	Mindlin (1951)	FSDT	20.7736	1.9860	124.5752	3.5223
	Kirchhoff	CPT	20.7736	1.3225	124.5752	3.5223
100	Present	SSNPT	20680.22	1.3272	12477.56	35.2203
	Reddy (1984)	HSDT	20788.46	1.3342	12466.41	35.2221
	Mindlin (1951)	FSDT	20773.27	1.3291	12457.30	35.2216
	Kirchhoff	CPT	20773.27	1.3225	12457.30	35.2216
Uniformly distributed load						
4	Present	SSNPT	2.5545	10.5016	33.7147	2.4767
	Reddy (1984)	HSDT	2.5717	10.5978	33.3963	2.3642
	Mindlin (1951)	FSDT	1.7176	6.7935	24.5909	2.1786
	Kirchhoff	CPT	1.7176	1.6774	24.5909	2.1786
10	Present	SSNPT	28.8841	3.0984	163.0148	5.3152
	Reddy (1984)	HSDT	29.0666	3.1170	162.5669	5.3305
	Mindlin (1951)	FSDT	26.8378	2.4960	153.6930	5.4466
	Kirchhoff	CPT	26.8378	1.6774	153.6930	5.4466
100	Present	SSNPT	26720.74	1.6830	15392.13	54.4470
	Reddy (1984)	HSDT	26860.67	1.6918	15378.42	54.4506
	Mindlin (1951)	FSDT	26837.32	1.6855	15369.02	54.4643
	Kirchhoff	CPT	26837.32	1.6774	15369.02	54.4643
Patch load						
4	Present	SSNPT	0.1384	0.7554	5.9734	0.0882
	Reddy (1984)	HSDT	0.1399	0.7743	5.5542	0.0883
	Mindlin (1951)	FSDT	0.1030	0.5075	1.9235	0.0898
	Kirchhoff	CPT	0.1030	0.1073	1.9235	0.0898
10	Present	SSNPT	1.6915	0.2165	18.7023	0.2215
	Reddy (1984)	HSDT	1.7021	0.2184	18.0517	0.2221
	Mindlin (1951)	FSDT	1.6095	0.1713	12.0219	0.2246
	Kirchhoff	CPT	1.6095	0.1073	12.0219	0.2246
100	Present	SSNPT	1606.02	0.1078	1211.38	2.2454
	Reddy (1984)	HSDT	1610.40	0.1084	1210.05	2.2456
	Mindlin (1951)	FSDT	1609.42	0.1079	1202.17	2.2460
	Kirchhoff	CPT	1609.42	0.1073	1202.17	2.2460

Table 10 Comparison of displacements and stresses of five layered ($0^0/90^0/core/90^0/0^0$) symmetric sandwich plates

a/h	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, 0)
Sinusoidally distributed load						
4	Present	SSNPT	2.19416	10.7889	43.3903	1.3382
	Reddy (1984)	HSDT	2.20259	10.9251	43.3562	1.3376
	Mindlin (1951)	FSDT	1.76165	7.11890	34.6766	1.3470
	Kirchhoff	CPT	1.76165	1.75230	34.6766	1.3469
10	Present	SSNPT	28.48009	3.1979	225.1449	3.3629
	Reddy (1984)	HSDT	28.63345	3.2271	225.4502	3.3623
	Mindlin (1951)	FSDT	27.52578	2.6110	216.7288	3.3674
	Kirchhoff	CPT	27.52578	1.7523	216.7288	3.3674
100	Present	SSNPT	27379.71	1.7569	21641.47	33.6837
	Reddy (1984)	HSDT	27536.97	1.7671	21681.69	33.6734
	Mindlin (1951)	FSDT	27525.88	1.7609	21672.96	33.6739
	Kirchhoff	CPT	27525.88	1.7523	21672.88	33.6739
Uniformly distributed load						
4	Present	SSNPT	2.92799	13.3846	51.5722	2.2126
	Reddy (1984)	HSDT	2.93813	13.5511	51.5548	2.1906
	Mindlin (1951)	FSDT	2.27590	7.50150	42.7818	2.0828
	Kirchhoff	CPT	2.27590	2.2226	42.7818	2.0828
10	Present	SSNPT	37.06437	4.0061	275.8507	5.1684
	Reddy (1984)	HSDT	37.26111	4.0423	276.2472	5.1653
	Mindlin (1951)	FSDT	35.56100	3.0672	267.3861	5.2071
	Kirchhoff	CPT	35.56100	2.2226	267.3861	5.2071
100	Present	SSNPT	35375.11	2.2279	26698.03	52.0776
	Reddy (1984)	HSDT	35578.27	2.2408	26747.67	52.0622
	Mindlin (1951)	FSDT	35561.13	2.2311	26739.46	52.0706
	Kirchhoff	CPT	35561.00	2.2226	26738.63	52.0710
Patch load						
4	Present	SSNPT	0.16375	0.9876	8.0693	0.0854
	Reddy (1984)	HSDT	0.16443	1.0042	7.9922	0.0853
	Mindlin (1951)	FSDT	0.13649	0.6600	3.3464	0.0859
	Kirchhoff	CPT	0.13649	0.1421	3.3464	0.0859
10	Present	SSNPT	2.19095	0.2807	27.7508	0.2133
	Reddy (1984)	HSDT	2.20288	0.2835	27.6826	0.2133
	Mindlin (1951)	FSDT	2.13258	0.2250	20.9150	0.2147
	Kirchhoff	CPT	2.13258	0.1421	20.9150	0.2147
100	Present	SSNPT	2121.11	0.1427	2095.533	2.1477
	Reddy (1984)	HSDT	2133.29	0.1436	2099.334	2.1470
	Mindlin (1951)	FSDT	2132.58	0.1430	2091.511	2.1474
	Kirchhoff	CPT	2132.58	0.1421	2091.505	2.1473

Table 11 Comparison of displacements and stresses of five layered ($0^0/90^0/core/0^0/90^0$) anti-symmetric sandwich plates

a/*h*	Theory	Model	$\bar{u}$ (0, -*h*/2)	$\bar{w}$ (*a*/2, 0)	$\bar{\sigma}_x$ (*a*/2, -*h*/2)	$\bar{\tau}_{zx}$ (0, 0)
			Sinusoidally distributed load			
4	Present	SSNPT	2.31898	11.4753	45.8549	1.4093
	Reddy (1984)	HSDT	2.32380	11.5300	45.7420	1.4091
	Mindlin (1951)	FSDT	1.85731	7.3023	36.5596	1.4157
	Kirchhoff	CPT	1.85731	1.9358	36.5596	1.4157
10	Present	SSNPT	30.03568	3.4597	237.4371	3.5346
	Reddy (1984)	HSDT	30.19151	3.4772	237.7180	3.5347
	Mindlin (1951)	FSDT	29.02044	2.7944	228.4972	3.5394
	Kirchhoff	CPT	29.02044	1.9358	228.4972	3.5394
100	Present	SSNPT	28861.13	1.9392	22812.24	35.3963
	Reddy (1984)	HSDT	29032.40	1.9512	22859.14	35.3934
	Mindlin (1951)	FSDT	29019.82	1.9443	22849.23	35.3937
	Kirchhoff	CPT	29020.44	1.9358	22849.72	35.3942
			Uniformly distributed load			
4	Present	SSNPT	3.09699	14.2369	54.4799	2.3778
	Reddy (1984)	HSDT	3.10187	14.3026	54.3845	2.3538
	Mindlin (1951)	FSDT	2.39949	9.0762	45.1049	2.1892
	Kirchhoff	CPT	2.39949	2.4552	45.1049	2.1892
10	Present	SSNPT	39.0934	4.3353	290.8927	5.4484
	Reddy (1984)	HSDT	39.2909	4.3573	291.2801	5.4461
	Mindlin (1951)	FSDT	37.4919	3.5146	281.9054	5.4730
	Kirchhoff	CPT	37.4919	2.4552	281.9054	5.4730
100	Present	SSNPT	37289.17	2.4590	28142.33	54.7261
	Reddy (1984)	HSDT	37510.41	2.4743	28200.24	54.7225
	Mindlin (1951)	FSDT	37491.18	2.4657	28189.91	54.7301
	Kirchhoff	CPT	37491.90	2.4552	28190.54	54.7301
			Patch load			
4	Present	SSNPT	0.17299	1.0516	8.7285	0.0899
	Reddy (1984)	HSDT	0.17345	1.0609	8.6450	0.0902
	Mindlin (1951)	FSDT	0.14390	0.6749	3.5281	0.0903
	Kirchhoff	CPT	0.14390	0.1570	3.5281	0.0903
10	Present	SSNPT	2.31044	0.3032	29.4328	0.2247
	Reddy (1984)	HSDT	2.32273	0.3049	29.3245	0.2246
	Mindlin (1951)	FSDT	2.24838	0.2399	22.0507	0.2257
	Kirchhoff	CPT	2.24838	0.1570	22.0507	0.2257
100	Present	SSNPT	2235.87	0.1575	2208.99	2.2569
	Reddy (1984)	HSDT	2249.14	0.1585	2213.36	2.2567
	Mindlin (1951)	FSDT	2248.33	0.1578	2205.03	2.2570
	Kirchhoff	CPT	2248.38	0.1570	2205.07	2.2570

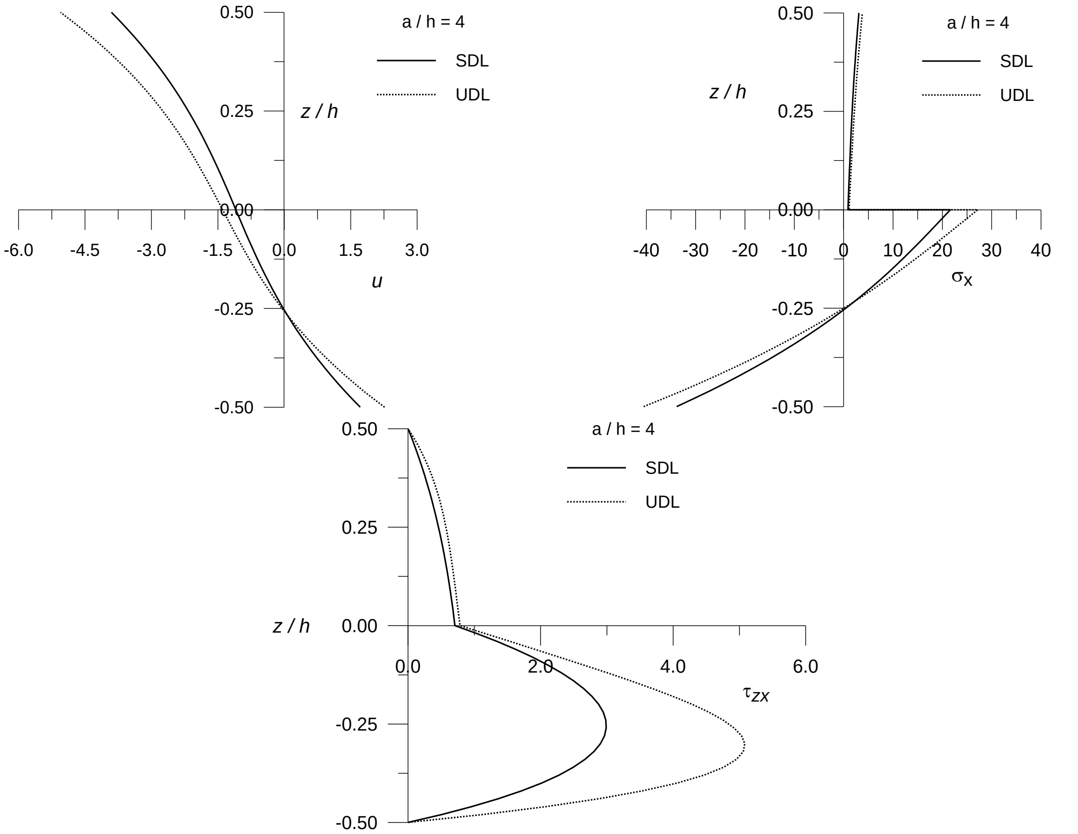


Fig. 4 Through thickness variation of in-plane displacement ($\bar{u}$) at (x=0, z), in-plane normal stress ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x=0, z) for two layered ($0^0/90^0$) antisymmetric laminated composite plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

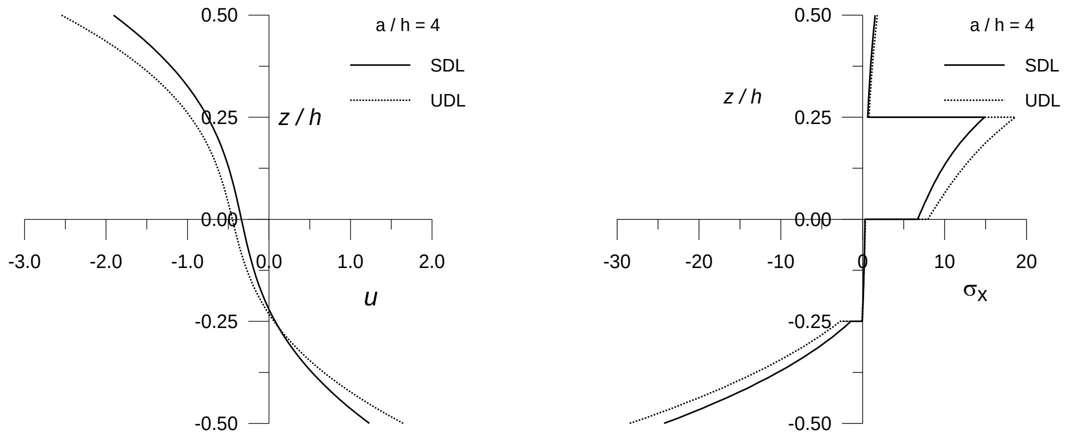


Fig. 5 Through thickness variation of in-plane displacement ($\bar{u}$) at (x = 0, z), in-plane normal stress ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x = 0, z) for four layered ($0^0/90^0/0^0/90^0$) antisymmetric laminated composite plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

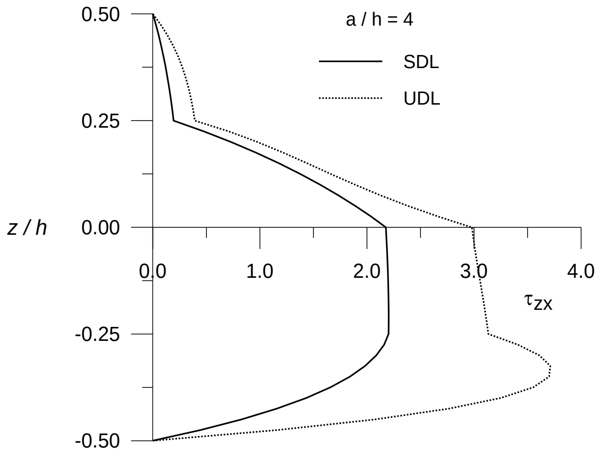


Fig. 5 Continued

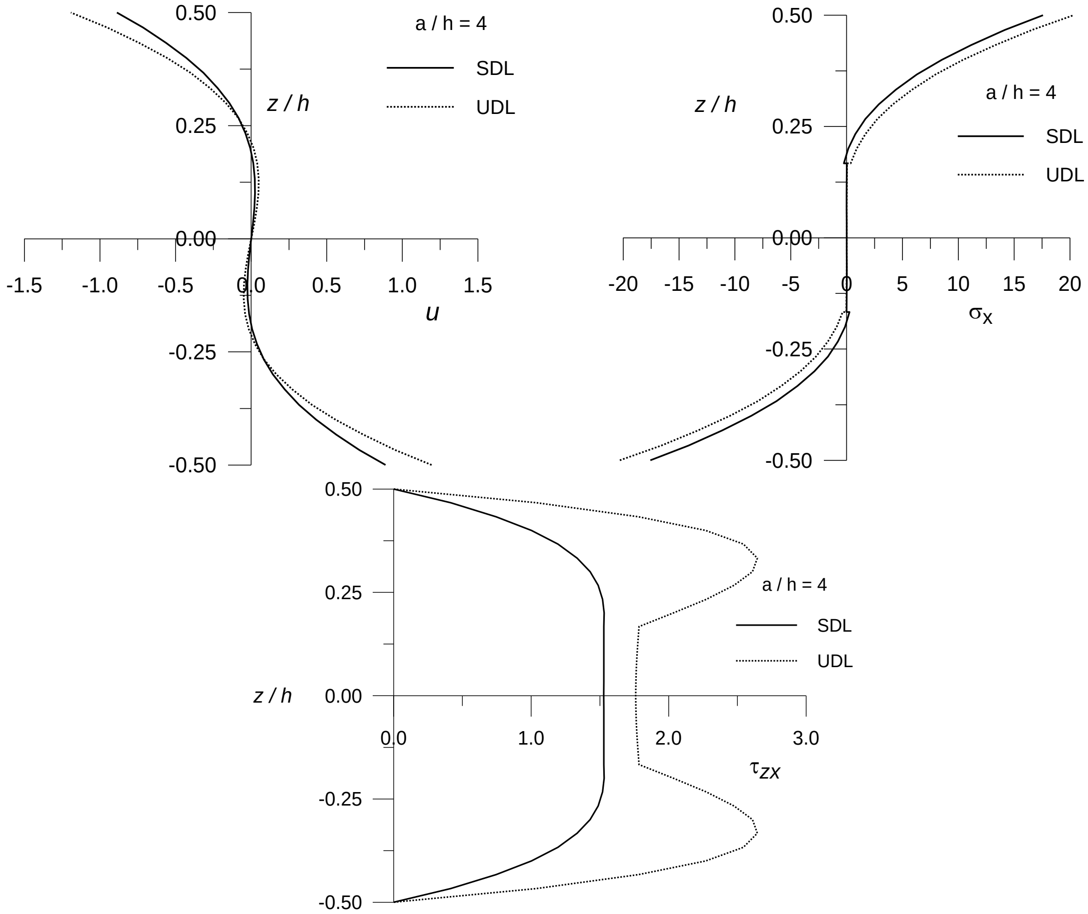


Fig. 6 Through thickness variation of in-plane displacement ($\bar{u}$) at (x = 0, z), in-plane normal stress ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x = 0, z) for three layered ($0^0/90^0/0^0$) symmetric laminated composite plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

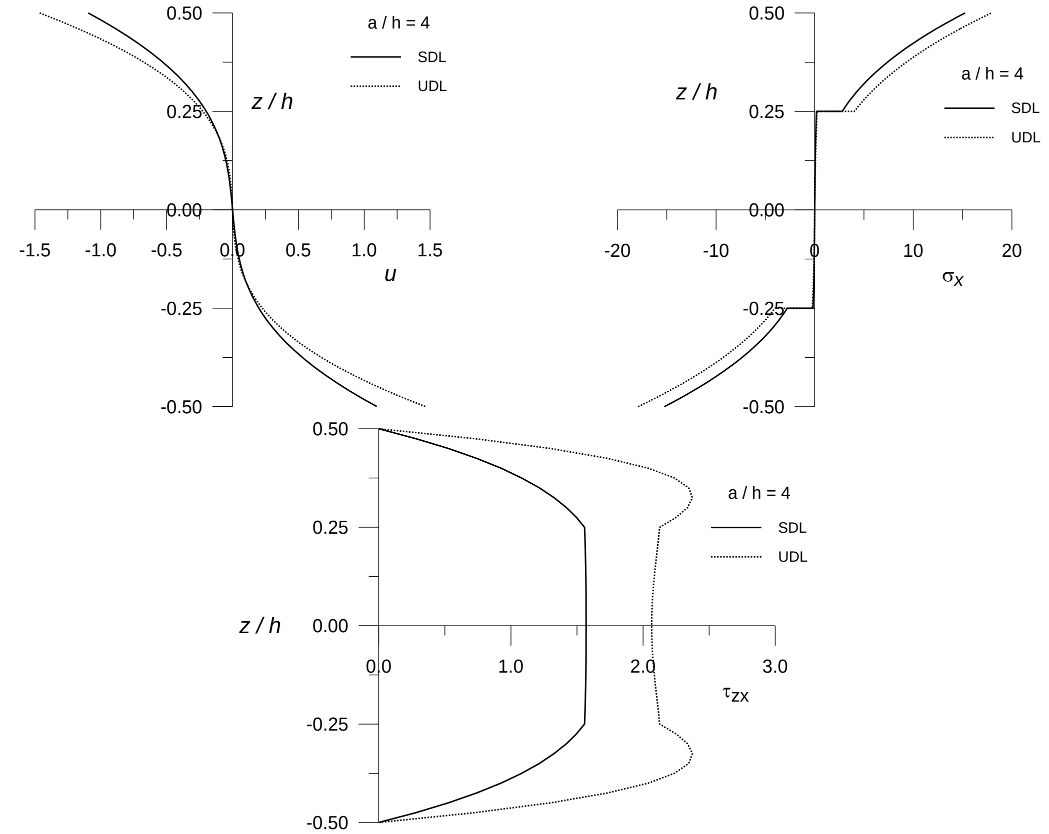


Fig. 7 Through thickness variation of in-plane displacement ($\bar{u}$) at (x = 0, z), in-plane normal stress ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x = 0, z) for four layered ($0^0/90^0/90^0/0^0$) symmetric laminated composite plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

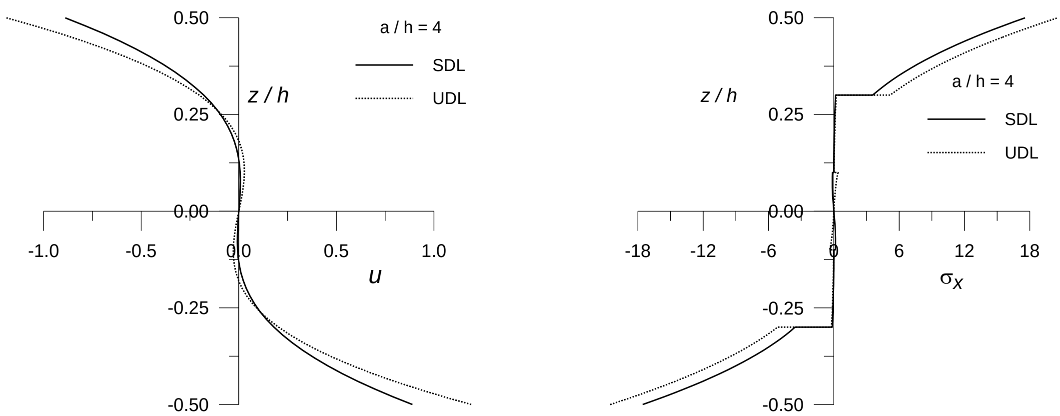


Fig. 8 Through thickness variation of in-plane displacement ($\bar{u}$) at (x=0, z), in-plane normal stress ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x=0, z) for five layered ($0^0/90^0/0^0/90^0/0^0$) sy mmetric laminated composite plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

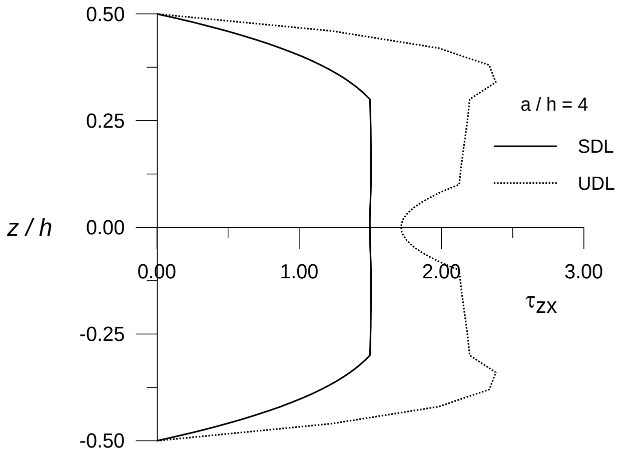


Fig. 8 Continued

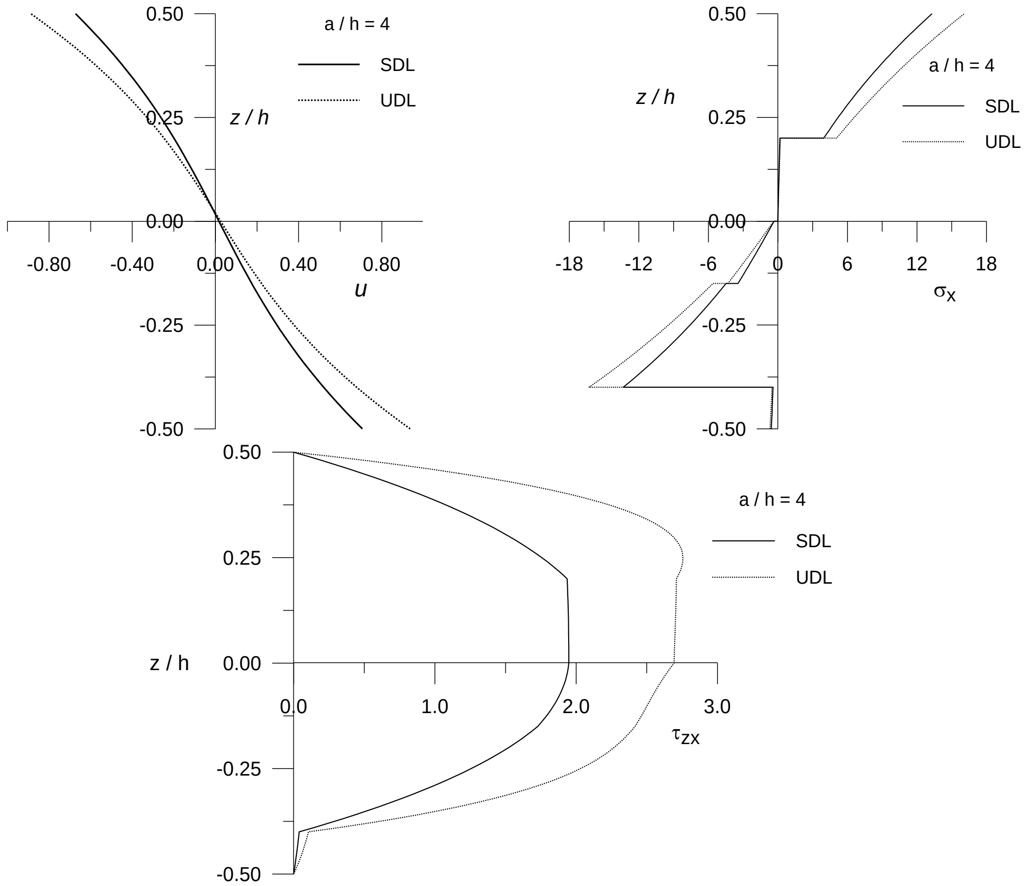


Fig. 9 Through thickness variation of in-plane displacement ($\bar{u}$) at (x = 0, z), in-plane normal stress ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x=0, z) for five layered ($90^0/0^0/0^0/90^0/0^0$) arbitrary laminated composite plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

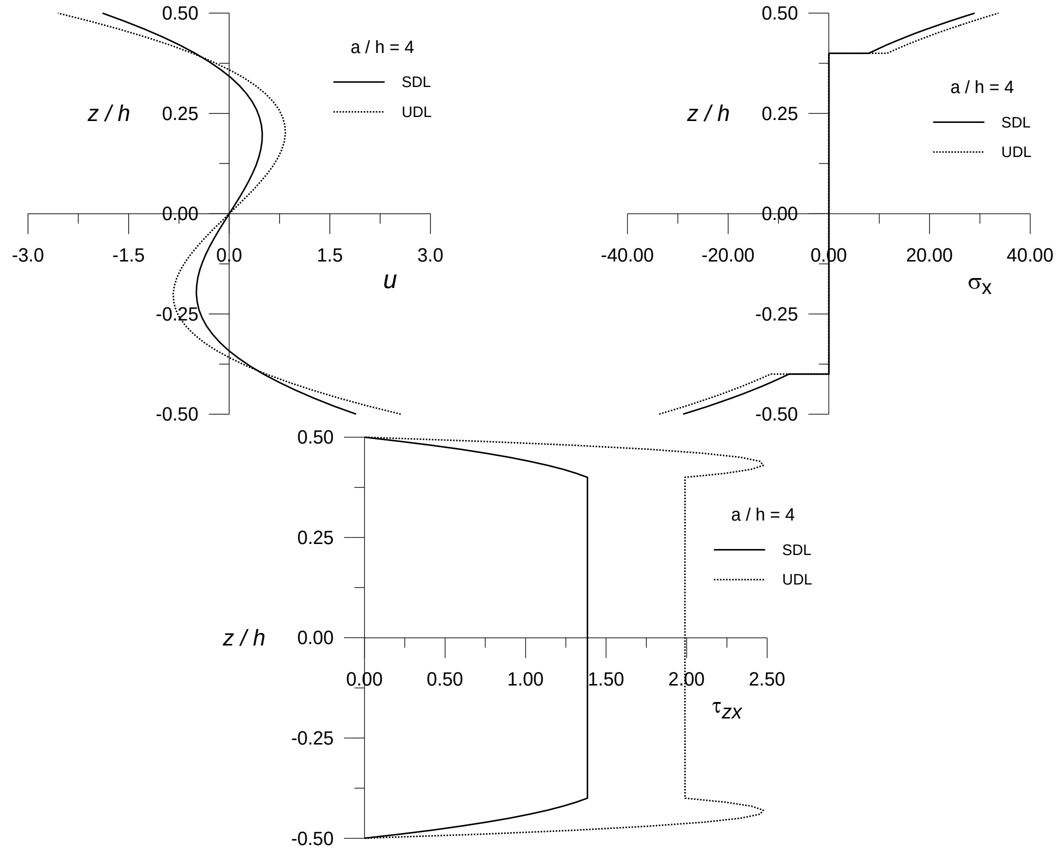


Fig. 10 Through thickness variation of in-plane displacement ($\bar{u}$) at (x=0, z), in-plane normal stress ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x=0, z) for three layered (0^0/core/0^0) symmetric sandwich plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

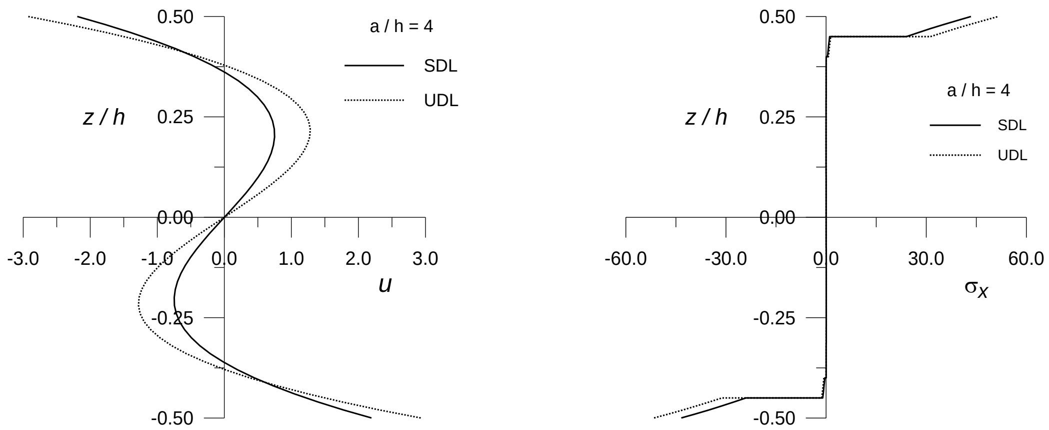


Fig. 11 Through thickness variation of in-plane displacement ($\bar{u}$) at (x=0, z), in-plane normal str ess ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x=0, z) for five layered (0^0/90^0/core /90^0/0^0) symmetric sandwich plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

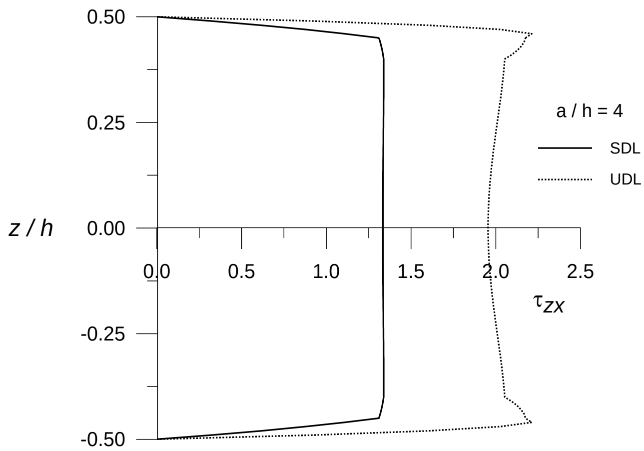


Fig. 11 Continued

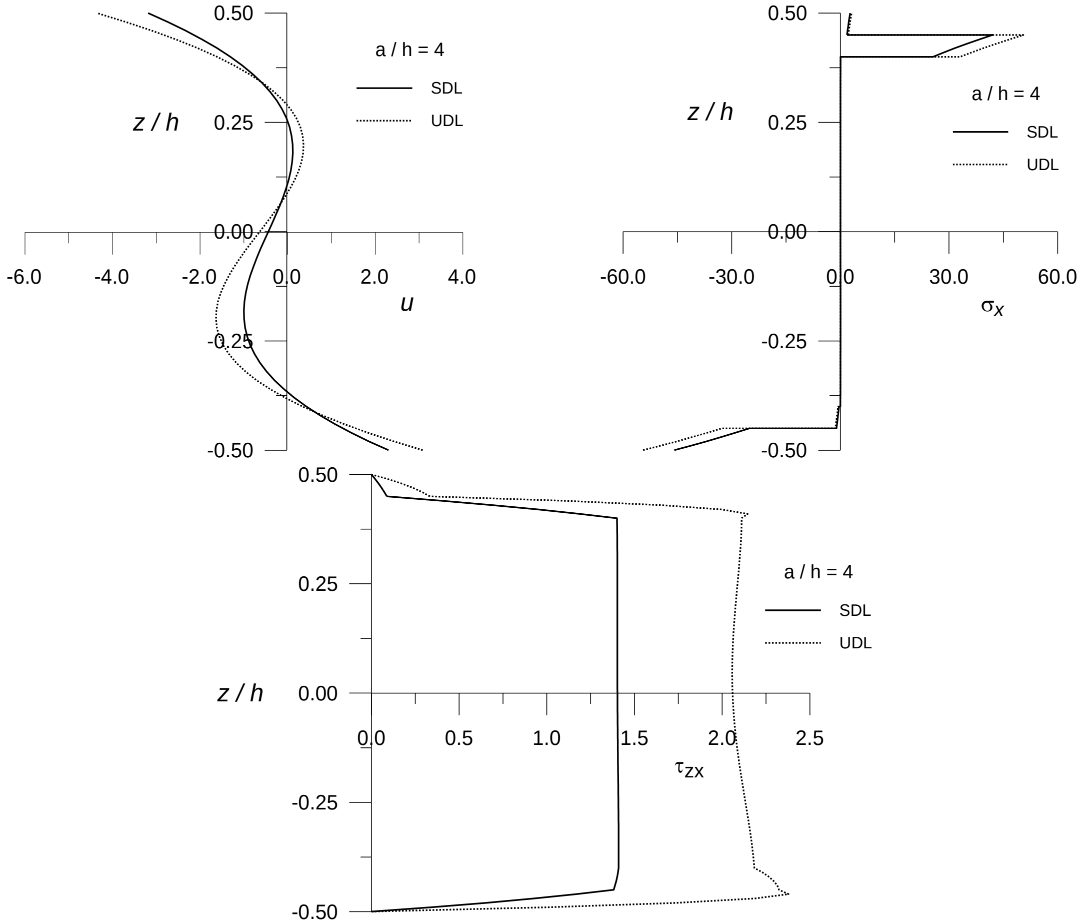


Fig. 12 Through thickness variation of in-plane displacement ($\bar{u}$) at (x=0, z), in-plane normal stress ($\bar{\sigma}_x$) at (x=a/2, z) and transverse shear stress ($\bar{\tau}_{zx}$) at (x=0, z) for five layered ($0^0/90^0$/core/$0^0/90^0$) antisymmetric sandwich plates subjected to sinusoidally distributed load (SDL) and uniformly distributed load (UDL)

thickness among the layers. Both the plates are made up of orthotropic material 1. The static loadings acting on plates are shown in Fig. 2. The comparison of non-dimensional displacements and stresses for these plates are presented in Tables 3 and 4.Through thickness distributions of in-plane displacement at support, normal stress at mid span and transverse shear stress at boundary edges under SDL and UDL are plotted in Figs. 4 and 5. It is observed from Table 3 that the results obtained using present theory for two layered anti-symmetric laminated composite plates are in excellent agreement with respect to those obtained by 3D elasticity solution (Pagano 1969). Examination of Table 4 reveals that the present results are in good agreement with those of HSDT of Reddy (1984).

Example 3: To check the efficiency of present theory for symmetric laminates, three different problems are considered. Figs. 3 (d), (e) and (f) show the laminate configurations and distributions of overall thickness among the layers. In first problem, three layered ($0^0/90^0/0^0$) symmetric laminated composite plate made up of material 1 is analyzed under different static loadings. The non-dimensional numerical results are presented in Table 5 and graphically in Fig. 6. In second problem, four layered ($0^0/90^0/90^0/0^0$) symmetric laminated composite plate made up of material 2 is analyzed under same loading conditions and obtained results are presented in Table 6 and Fig. 7. In the third and the last example of symmetric laminates, five layered ($0^0/90^0/0^0/90^0/0^0$) laminated plate made up of material 1 is considered. Results of this plate are shown in Table 7 and Fig. 8. The numerical results obtained by using present theory for these problems are compared with those generated by using CPT, FSDT of Mindlin (1951) and HSDT of Reddy (1984); and found to agree well with those of exact solution wherever applicable.

Example 4: In this section, an arbitrarily layered laminated composite plate is analyzed using present theory. The laminate configurations and distribution of overall thickness among the layers ($0.1h/0.25h/0.15h/0.2h/0.3h$) is shown in Fig. 3(g). The layers are made up of materials 1/3/1/1/1.Table 8 shows the comparison of non-dimensional displacements and stresses obtained by present theory and other theories under various static loadings (SDL, UDL and PL). Through thickness distributions of in-plane displacement, in-plane normal stress and transverse shear stress are plotted in Fig. 9. Examination of Table 8 reveals that the results obtained by present theory and theory of Reddy (1984) are in close agreement with each other. FSDT of Mindlin (1951) and CPT underestimate the values of in-plane displacement, transverse displacement and in-plane normal stress and overestimate the values of transverse shear stress.

Example 5: To study the efficiency of the present theory for symmetric sandwich plates under cylindrical bending two different problems are solved. In first problem a three layered ($0^0/core/0^0$) sandwich plate is considered. The distribution of overall thickness among all the layers as ($0.1h/0.8h/0.1h$) is shown Fig. 3(p). The plate is made up of materials 4/5/4. The non-dimensional displacements and stresses are reported in Table 9. Through thickness distributions of in-plane displacement, in-plane normal stress and transverse shear stress are plotted in Fig. 10. In second problem a five layered ($0^0/90^0/core/90^0/0^0$) symmetric sandwich plate is considered. The layers are of thickness ($0.05h/0.05h/0.8h/0.05h/0.05h$) and are made up of materials 1/1/6/1/1. The numerical results are reported in Table 10 whereas graphical results are plotted in Fig. 11. Since exact results are not available in the literature for these problems, authors have generated the results by using CPT, FSDT of Mindlin (1951) and HSDT of Reddy (1984). From the numerical results presented in Tables 9 and 10, it is observed that the present theory results are in good agreement with those obtained by using HSDT of Reddy.

Example 6: In this example, efficiency of present theory is checked for anti-symmetric sandwich plates. A five layer ($0^0/90^0/core/0^0/90^0$) anti-symmetric sandwich plate is analyzed under various static loadings as shown in Fig. 2. The laminate configuration is shown in Fig. 3(s). The distribution of overall thickness and material properties are similar to five layer symmetric sandwich plate. The numerical results of non-dimensional displacements and stresses are reported in Table 11 and through thickness distributions are plotted in Fig. 12. These results are not available in the literature and presented for the first time in this paper. To prove the validity of present theory similar results are also generated by using CPT, FSDT of Mindlin (1951) and HSDT of Reddy (1984). From Table 11 it is seen that the results obtained by present theory and theory of Reddy (1984) are in close agreement with each other for all aspect ratios (*a/h*).

5. Conclusions

In the present study, the effect of transverse normal strain on cylindrical bending of multilayered laminated composite and sandwich plates is examined. A sinusoidal shear and normal deformation plate theory (SSNPT) has been developed for the analysis. The theory is built upon the classical plate theory in which in-plane displacement includes the effects of shear deformation and transverse displacement includes the effect of transverse normal strain/stress. The theory is variationally consistent and obviates the need of shear correction factor. Several problems on laminated composite and sandwich plates are solved and obtained results are compared with the results available in the literature. For the comparison purpose, the numerical results are also generated by using higher order shear deformation theory of Reddy, first-order shear deformation plate theory of Mindlin and classical plate theory. From the examples studied it is observed that the present theory is in good agreement with the exact theory wherever applicable for predicting the static response of laminated composite and sandwich plates under cylindrical bending. The present study also successfully presents the unavailable results of sandwich plates which can be served as benchmark solutions for future research. Therefore, the present theory is recommended for the accurate structural analysis of laminated composite and sandwich plates under cylindrical bending.

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