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A two-stage damage detection approach based on subset selection and genetic algorithms

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Abstract. A two-stage damage detection method is proposed and demonstrated for structural health monitoring. In the first stage, the subset selection method is applied for the identification of the multiple damage locations. In the second stage, the damage severities of the identified damaged elements are determined applying SSGA to solve the optimization problem. In this method, the sensitivities of residual force vectors with respect to damage parameters are employed for the subset selection process. This approach is particularly efficient in detecting multiple damage locations. The SEREP is applied as needed to expand the identified mode shapes while using a limited number of sensors. Uncertainties in the stiffness of the elements are also considered as a source of modeling errors to investigate their effects on the performance of the proposed method in detecting damage in real-life structures. Through a series of illustrative examples, the proposed two-stage damage detection method is demonstrated to be a reliable tool for identifying and quantifying multiple damage locations within diverse structural systems.

Keywords: damage detection; structural health monitoring; genetic algorithms; subset selection and model updating.

1. Introduction

Structural health monitoring (SHM) has emerged in recent years as an attractive alternative to traditional visual inspection techniques. SHM is often based on the modal quantities identified from vibration response data, which are then correlated to physical changes in the structure using model updating approaches (Hu, *et al.* 2001, Doebling, *et al.* 1998). Structural damage detection problems are recast into optimization problems in which a set of damage parameters in an identification model are

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adjusted to achieve the maximum correlation with the experimental observations (Friswell and Mottershead 1995, Mottershead and Friswell 1993). As sensor technologies advance, and as sensors become more ubiquitous and inexpensive, the potential of these SHM methods for standardized use in prioritization of repair and replacement of our civil infrastructure greatly increases.

The finite element method is most widely employed to develop the identification model for large-scale civil engineering structural systems. The challenges in the application of these techniques to update finite element models include: having a large parameter space to be updated; the presence of modeling errors in the identification model due to idealization of material constitutive model, errors in manufacturing and fabrication of members, inaccurate knowledge of material properties, etc; limitations in the modal information available due to restrictions in the number of sensors available; and, measurement errors due to noise and bias errors in the modal identification of the structure. The combination of a large parameter space with limited modal information for the updating itself poses a particular challenge to the optimization problem. Thus, the damage detection and quantification problem is frequently underdetermined resulting in a non-unique solution.

One approach to accomplish damage detection is to directly solve the optimization problem to yield a set of damage parameters, including location and quantification of the damage. Alternatively, one can solve this problem in two stages by first identifying the location of damage in the structure (stage 1) and then quantifying the damage extent in those elements (stage 2). Moslem, *et al*. was one of the first to suggest a two-stage procedure for structural damage detection (Moslem and Nafaspour 2002). Conducting such a pre-screening of the damaged structure's characteristics to identify the most likely damaged elements, and appropriately reduce the search space, has been found to be advantageous in solving the optimization problem. In the pre-screening portion of this class of techniques, it is acceptable to identify too many elements with potential damage; however, it is not acceptable to discount elements that are actually damaged. Of course, it is most desirable to identify exactly the subset of elements that are truly damaged to minimize the size of the optimization problem. To quantify the advantages of using subset selection, if we assume 10 levels of damage state for 15 potentially damaged elements, then the possible number of combinations of damage state is 10^{15} in the direct approach. However, in the two-stage approach, the first damage detection problem reduces the possible number of combinations of damage states to 2^{15}. If it is assumed that 5 elements are identified as damage elements, then a total possible number of combinations in the quantification become 10^5. Finally, the total combinations under investigation can be greatly reduced to 132,768 from 10^{15}. A multivariate regression method has been successfully used for subset selection (Lallement and Piranda 1990, Friswell, *et al.* 1997, Titurus, *et al.* 2003), reducing the size of the model updating problem. Song, *et al*. (2007a, 2007b) suggested an approach to integrate this method based on the sensitivities of modal parameters in conjunction with damage functions (Teughels, *et al.* 2002) to apply SHM techniques to damage detection in complex structures. Although the subset selection method utilizing fundamental modal parameters was successful in detecting a single damaged location, the method requires iterations when attempting to detect multiple damages and typically requires weighting because of the difference in units (Friswell 2007).

The second stage focuses on the solution of the optimization problem yielding the values of the damage parameters needed to update the model, and through comparisons with the baseline, identify damage levels. Although the size of the optimization problem is reduced considerably after subset selection, there are still significant challenges in the implementation of this step. The selection of the objective function plays a significant role in the ability of the algorithm to detect damage in real world applications (Giraldo 2006). In recent years, an objective function based on the flexibility matrix has drawn considerable attention in the field of structural health monitoring. Toksoy and Aktan used a modal flexibility matrix directly obtained from modal test data for bridge-condition assessment (Toksoy and Aktan 1994). Also, Bernal suggested a flexibility matrix-based damage detection method called a Damage Locating Vector (DLV) method (Bernal 2002). The second stage in the damage detection problem is to determine the extent of damage in each of the elements. To accomplish this goal, an optimization problem must be solved. For the past couple of decades, genetic algorithms (GAs) have been exploited as a primary search tool for optimization problems. In some problems mathematically based search methods have difficulties in converging to an absolute optimal solution and get trapped at local minima or maxima. The stochastic nature and parallel computational framework of GAs are defining characteristics that avoid premature convergence. With improved computer technology, GAs are drawing much attention with their abstract ideas of solution exploration (Moslem and Nafaspour 2002, Rao, *et al.* 2004, Friswell, *et al.* 1998, Kim, *et al.* 2007). In this paper, Steady State Genetic Algorithm (SSGA) is employed for quantification of damage level of identified elements. Although there are numerous GA operators, the SSGA is chosen because the SSGA can give equivalent accuracy with much less computational time (Moslem and Nafaspour 2002).

In this paper a two-stage damage detection method for structural health monitoring is proposed. The first stage consists of identification of the subset of parameters that are potentially damaged. An improved subset selection method proposed by Yun, *et al*. (2008) based on the dynamic residual force vector is employed in this step to accurately locate damaged elements. This regularization technique is used to avoid the non-uniqueness of the finite element model-based damage detection problem (Yun, *et al.* 2008). This approach was demonstrated to accurately detect the damaged element subset with single run of the subset selection process. The proposed approach deals with the limited amount of information available in the sensors using the SEREP technique for modal expansion. Damage is then quantified in the second stage using the SSGA optimization process proposed herein with an objective function based on the modal flexibility. Performance of the proposed method is assessed through several numerical examples. The effects of limited sensor information are considered in the examples. In addition, the effects of modeling error on the performance are investigated in a portal frame structure. Finally, the efficacy of this two-stage approach for structural health monitoring is demonstrated for various types of structures.

2. Problem formulation

In this section, the proposed two-stage damage detection method is described in detail. The SEREP method, adopted herein for modal expansion of the measured mode shapes, is first discussed. It is assumed herein that partial mode shape information is obtained through ambient or forced vibration tests. An improved subset selection method based on the sensitivities of residual force vectors is employed for damage localization. Finally, the SSGA is used for quantification of the extent of damage.

2.1. Component 1: modal expansion

In real-world modal tests, it is challenging to measure complete mode shapes if there are a large number of DOFs in the associated mathematical model. Measuring rotational components of the mode shapes also pose a challenge in physical experiments. Incomplete measured modes may be expanded to the size of the finite element model for damage detection. Herein System Equivalent Reduction and

Expansion Process (SEREP) is used for modal expansion of the measured information. The background of SEREP is presented, which uses analytical eigenvectors to define a transformation matrix between the DOFs in the measured mode shapes and the DOFs in the analytical model to be updated. The equation for an undamped analytical model can be expressed as

$$\begin{bmatrix} \mathbf{K}_{rr} & \mathbf{K}_{rt} \\ \mathbf{K}_{tr} & \mathbf{K}_{tt} \end{bmatrix} \begin{Bmatrix} \varphi_{rj} \\ \varphi_{tj} \end{Bmatrix} - \lambda_j \begin{bmatrix} \mathbf{K}_{rr} & \mathbf{K}_{rt} \\ \mathbf{K}_{tr} & \mathbf{K}_{tt} \end{bmatrix} \begin{Bmatrix} \varphi_{rj} \\ \varphi_{tj} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{1}$$

where '*r*' indicates the retained DOFs; '*t*' denotes the truncated DOFs; **K** and **M** are stiffness and mass matrices, respectively; $\varphi_\mathbf{j}$ indicates the *j*th mode shape vector; and λ_i indicates the *j*th eigenvalue. The analytical eigenvectors are constructed from the analytical model in Eq. (1). Therefore, the transformation matrix is given by

$$T = \begin{bmatrix} \mathbf{\Phi}_{rm} \\ \mathbf{\Phi}_{tm} \end{bmatrix} \mathbf{\Phi}_{rm}^{*} \tag{2}$$

where '*r*' corresponds to the number of sensors on tested models; '*m*' indicates the number of measured modes from the testing; $\mathbf{\Phi}_{rm}$ is the *r*-by-*m* mode shape matrix corresponding to the retained DOFs in the analytical model; and $\mathbf{\Phi}_{tm}$ is the *t*-by-*m* mode shape matrix corresponding to the truncated DOFs in the analytical model. $\mathbf{\Phi}_{rm}^{*}$ is displayed in Eq. (3) and represents the Moore-Penrose pseudo-inverse of this matrix.

$$\mathbf{\Phi}_{rm}^{*} = (\mathbf{\Phi}_{rm}^{T}\mathbf{\Phi}_{rm})^{-1}\mathbf{\Phi}_{rm}^{T} \tag{3}$$

It is noteworthy to mention that the number of measured modes (*m*) is usually smaller than the number of retained DOFs (*r*) in the analytical model. For this case, sources of error when measuring modes are developed within the expansion process due to measurement error and noise as well as inherent error, which uses the pseudo-inverse from the least square minimization procedure. However, when m is equal to *r*, the SEREP error is only caused by measurement error as well as disruptive noise while modal testing. The measured mode shape matrix can then be expanded to

$$\mathbf{\Phi}_{expanded\_(neq \times m)} = \mathbf{T}_{neq \times r}\mathbf{\Phi}_{measured\_(r \times m)} \tag{4}$$

where '*neq*' indicates the total number of DOFs in the analytical model. The expanded eigenvectors resulting from this process are used for subset selection, which is explained in further detail.

2.2. Component 2: damage localization using parameter subset selection

Parameter subset selection is adopted for damage localization herein. An improved approach based on using the residual force vector is applied to enhance localization performance and facilitate the localization of multiple damage locations (Yun, *et al.* 2008). First, the residual function is expressed as

$$J = \|\mathbf{z}_m - \mathbf{z}(\theta)\|^2 \tag{5}$$

where $\mathbf{z}_m$ is a vector consisting of measured information and $\mathbf{z}(\theta)$ is the corresponding analytical values as a function of the damage parameters, θ. Because the residual function is generally a nonlinear function of the damage parameters, this results in a combinatorial optimization problem in which θ is sought by finding the minimum value of the function. The Taylor series of $\mathbf{z}_m$ is given by

$$z_{im} = z_i(0) + \theta \cdot \nabla_{\theta'} z_i(\theta')|_{\theta'=0} + \frac{1}{2!}\theta \cdot \{\theta \cdot \nabla_{\theta'}(\nabla_{\theta'} z_i(\theta'))\}|_{\theta'=0} + \cdots \tag{6}$$

where z_{im} is the ith measured data; $z_i(0)$ is the ith measured data from the healthy structure and θ' is a dummy vector representing damage parameters. When neglecting the higher order terms in Eq. (6), the linearized equation can be rewritten as

$$\mathbf{S}\theta = \mathbf{b} \tag{7}$$

where $\mathbf{b}=\mathbf{z}_m-\mathbf{z(0)}$ is the difference in the measured values between the damaged and undamaged states and $\mathbf{S} = [\nabla_{\theta'} z_1(\theta')|_{\theta'=0}; \nabla_{\theta'} z_2(\theta')|_{\theta'=0}; \ldots; \nabla_{\theta'} z_p(\theta')|_{\theta'=0}]$ **S** and **b** can be calculated in terms of either the residual force vector or the eigenvalues and eigenmodes. The ith residual force vector for the undamaged structure is given by

$$\mathbf{R}_i = \mathbf{K}\varphi_i - \lambda_i \mathbf{M}\varphi_i; \quad i = 1, 2, \ldots, m \tag{8}$$

where **K** and **M** are the stiffness and mass matrices of the structure, respectively and m is the number of measured modes. It should be noted that the mass matrix is assumed to be constant in the proposed method. Taking its derivative with respect to jth damage parameter, sensitivities of the residual force vector can be expressed as

$$\frac{\partial \mathbf{R}_i}{\partial \theta_j} = \frac{\partial \mathbf{K}}{\partial \theta_j}\varphi_i + \mathbf{K}\frac{\partial \varphi_i}{\partial \theta_j} - \frac{\partial \lambda_i}{\partial \theta_j}\mathbf{M}\varphi_i - \lambda_i \mathbf{M}\frac{\partial \varphi_i}{\partial \theta_j}; \quad i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, p \tag{9}$$

where 'p' indicated the number of damage parameters. The sensitivity of eigenvalues and mode shapes are calculated as follows (Fox and Kapoop 1968).

$$\frac{\partial \lambda_i}{\partial \theta_j} = \varphi_i^T\left(\frac{\partial \mathbf{K}}{\partial \theta_j} - \lambda_i \frac{\partial \mathbf{M}}{\partial \theta_j}\right)\varphi_i$$

$$\frac{\partial \varphi_i}{\partial \theta_j} = \sum_{k=1, k\neq i}^{n}\left[\frac{\varphi_i^T(\partial \mathbf{K}/\partial \theta_j - \lambda_i \partial \mathbf{M}/\partial \theta_j)\varphi_k}{(\lambda_k - \lambda_i)}\right]\varphi_k - \frac{1}{2}\varphi_i^T\frac{\partial \mathbf{M}}{\partial \theta_j}\varphi_i$$

$$i = 1,2,\ldots, m \text{ and } j = 1,2,\ldots, p \tag{10}$$

However, because the mass matrix does not change, the second term for the sensitivity of the mode shapes will vanish. Thus, the sensitivity matrix **S** in terms of the residual force vectors is

$$\mathbf{S} = \begin{bmatrix} \frac{\partial \mathbf{R}_1}{\partial \theta_1} & \frac{\partial \mathbf{R}_1}{\partial \theta_2} & \cdot & \frac{\partial \mathbf{R}_1}{\partial \theta_p} \\ \frac{\partial \mathbf{R}_2}{\partial \theta_1} & \frac{\partial \mathbf{R}_2}{\partial \theta_2} & \cdot & \frac{\partial \mathbf{R}_2}{\partial \theta_p} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial \mathbf{R}_m}{\partial \theta_1} & \frac{\partial \mathbf{R}_m}{\partial \theta_2} & \cdot & \frac{\partial \mathbf{R}_m}{\partial \theta_p} \end{bmatrix} \in \mathbf{R}^{(neq \times m) \times (p)} \tag{11}$$

where '*neq*', '*m*' and '*p*' are the number of DOFs, the number of measured modes, and the number of damage parameters, respectively. The **b** vector is also calculated as follows

$$\mathbf{b}_i = \mathbf{R}_{di} - \mathbf{R}_{ui} \tag{12}$$

where
$$\mathbf{R}_{di} = \mathbf{K}\boldsymbol{\varphi}_{di} - \lambda_{di}\mathbf{M}\boldsymbol{\varphi}_{di}$$

where λ_{di} and $\boldsymbol{\varphi}_{di}$ are the ith measured eigenvalues and mode shapes of the damaged structure. Because the residual force vector for an undamaged structure, $\mathbf{R}_{ui}$ is equal to the zero vector as shown in Eq. (8), residual values of the **b** vector originate from $\mathbf{R}_{di}$. **S** and **b** corresponding to natural frequency and mode shape can be referred to reference (Titurus, *et al.* 2003). For the selection of damage parameters, sub-optimal problems are sequentially formulated using the forward selection approach (Lallement and Piranda 1990). In each sub-optimal problem, one damage parameter is selected out of the remaining damage parameters. The difference of measured data, **b**, in Eq. (12) can be viewed as a linear combination of a set of column vectors within the $\mathbf{S} = [\mathbf{a}_1\ \mathbf{a}_2\ \ldots\ \mathbf{a}_p]$ matrix as

$$\mathbf{b} = \sum_{j=1}^{p} \theta_j \mathbf{a}_j \tag{13}$$

where '*p*' indicates the number of damage parameters again. Overall, the main task of forward selection is to select a column vector in the **S** matrix that best represents the residual vector **b**, that is, yields the minimum value of the resulting residual function

$$J_j = \left\| \mathbf{b} - \mathbf{a}_j{}^e\theta_j \right\|^2 \tag{14}$$

where the summation rule with respect to j is not applied here and the least square estimate ${}^e\theta_j$ of the jth parameter can be obtained by taking a derivative of $J=\|\mathbf{b}-\mathbf{a}_j\theta_j\|^2$ with respect to θ_j as

$${}^e\theta_j = \frac{\mathbf{a}_j^T\mathbf{b}}{\mathbf{a}_j^T\mathbf{a}_j} \tag{15}$$

Finding the minimum value of the residual function $J=\|\mathbf{b}-\mathbf{a}_j\theta_j\|^2$ is equivalent to finding a vector $\mathbf{a}_j$ that forms the minimum angle with the vector **b**. This procedure seeks the best basis vector $\mathbf{a}_j$ that is closest to the damaged residual vector **b.** If the first basis vector $\mathbf{a}_{j1}$ and its corresponding damage parameter θ_{j1} are found, Gram-Schmidt orthogonalization is performed onto the remaining column vectors to ensure a well-conditioned sub-matrix of **S**. A vector orthogonal to the first vector is produced by taking the original second vector and projecting out the component of the vector that lies along the first vector. This process is accomplished through the following equations

$$\mathbf{a}_j \leftarrow \mathbf{a}_j - \alpha_j\mathbf{a}_{j1} \quad \text{and} \quad \mathbf{b} \leftarrow \mathbf{b} - \mathbf{a}_{j1}{}^e\theta_{j1}$$

$$\alpha_j = \mathbf{a}_{j1}^T\mathbf{a}_j / \mathbf{a}_{j1}^T\mathbf{a}_{j1} \tag{16}$$

where α_j is the component of the vector a_j that lies along the first vector; ${}^e\theta_j$ can be calculated by Eq. (15); and $\leftarrow$ indicates the substitution of new values into the variable. After orthogonalization, the residual function in Eq. (14) for each parameter j is calculated and the minimum value is chosen as follows

$$\min(\{J_2, J_3, \ldots, J_p\}) \rightarrow \mathbf{a}_{j2} \quad \text{and} \quad \theta_{j2} \tag{17}$$

The iterative procedure is continued to identify a number of damage parameters while retaining the parameters chosen in the previous steps. When a total of damage parameters (m) are selected in the subset, the residual sum of squares is defined by

$$RSS_m = \left\| b - \sum_{i=1}^{m} \mathbf{a}_{j_i}{}^{e}\theta_{j_i} \right\|^2 \tag{18}$$

where ${}^{e}\theta_j$ is the least squares estimator for jth parameter, Eq. (15). Efroymson suggested a stepwise regression algorithm which assists to decide whether a new parameter should be included in the damaged subset or not (Efroymson 1960). If $[\theta_1, \theta_2, \ldots, \theta_m]$ are already selected as damage parameters and a new parameter θ_{m+1} is chosen for evaluation, then F-to-enter statistic can be expressed as

$$F_a = \frac{RSS_m - RSS_{m+1}}{RSS_{m+1}/(n-m-1)} \tag{19}$$

where RSS indicates residual sum of squares and n is the number of total parameters. If the F_a value is greater than a predetermined value (F_{in}), the parameter will be included in the subset. If the criterion is not met, the parameter is excluded. However, this test can also be performed to remove a selected parameter from the subset by utilizing the F-to-remove statistic which is expressed as

$$F_r = \frac{RSS_{m-1} - RSS_m}{RSS_m/(n-m)} \tag{20}$$

If the F_r value is less than a predetermined value (F_{out}), the parameter is removed from the subset. In 1996, Miller mathematically proved that this addition and removal process terminate with a finite number of different subsets (Miller 1996). Miller introduced an objective function as seen by

$$L(s) = RSS_m \prod_{i=1}^{m} \left(1 + \frac{F}{n-i}\right) \tag{21}$$

where F is any value such that $F_{out} \leq F \leq F_{in}$ and s represents a subset of variables. Efroymson's algorithm is viewed as a heuristic algorithm that minimizes the objective function. Thus, a finite sequence of values for the objective function will decrease at each step and ultimately terminate. In this study, the F-to-enter statistic is only used for convenience.

2.3. Component 3: optimization with genetic algorithms

Within this study GAs are applied to update the finite element model. System variables are encoded into binary strings, resembling the genotype representation of solution parameters. One set of encoded system variables is defined as a chromosome. When formulated in a group, the chromosomes are referenced as a population. At each instant in time, a particular population is called a generation. The GA procedure first begins with the random selection of an initial population in a given solution space. The population evolves as generations are repeated through the GA operators, such as competition, fitness based selection, recombination, and mutation. Even though the whole process is highly stochastic, the evolutionary pressures eliminate the weak chromosomes while advancing the fitter chromosomes closer to satisfying the objective function in the optimization problem. The theory that describes the main operators in GAs is certainly robust in schema theorem as well as in the principle of minimal building blocks (Holland 1975, Goldberg 1989). In this study, the Steady State Genetic Algorithm (SSGA) is selected based on its demonstrated performance in this application.

2.3.1. Steady state genetic algorithm

In conventional GAs, such as the Simple Genetic Algorithm (SGA), the entire population is replaced in each generation through the GA operators mentioned previously. Therefore, the reproduction process for the SGA requires an enormous amount of computational effort because it evaluates the entire population. However, the SSGA can achieve equivalent performance as the SGA with elitism and scaling of fitness values, and it saves a tremendous amount of computational time by evaluating only a small percentage of the entire population during each generation. Whitley first introduced the non-generational GA into the literature (Whitley 1989) while the SSGA was first coined by Syswerda in 1989 (Syswerda 1991). Because each chromosome is not replaced in the SSGA, the percentage of those replaced is referred to as the generation gap (Goldberg 1989).

$$G = \frac{n_r}{n_{tp}} \tag{22}$$

where n_r is the number of replaced chromosomes and n_{tp} is the total population size. In this study, n_r is determined to be 2, meaning that the reduction in the number of evaluations is $G=(n_p-2)/n_p$.

Although the crossover and mutation operators for the SSGA and the SGA follow the same guidelines, reproduction is slightly altered in the SSGA process. After initialization of the parent population, all chromosomes are ranked according to their fitness values. The two chromosomes displaying the highest fitness values are chosen for crossover and therefore produce new offspring. To ensure diversity, mutation is applied to the new offspring after they have replaced the two worst chromosomes in the population. Because the procedure essentially only assesses the two worst chromosomes, the SSGA does not require evaluation of the entire population, rather only two chromosomes. This characteristic of the SSGA is very beneficial as it significantly reduces the computational effort. Moreover, the population evolves steadily toward optimal solutions without endangering the highest fit schemata that already exists in the population. As illustrated in Fig. 1, the computational procedures adopted in this paper can be described as follows.

Step 1 Initialize the Population
Step 2 Evaluate Objective Function for Every Chromosome
Step 3 Start Generation
Step 4 Rank the Chromosomes according to Fitness Values
Step 5 Select Two Best Chromosomes
Step 6 Apply Crossover to Obtain Offspring
Step 7 Replace Two Worst Chromosomes with Offspring
Step 8 Apply Mutation on the Offspring
Step 9 Evaluate Objective Function for Two Offspring
Step 10 Linear Scale the Fitness Values for Every Chromosome
Step 11 Check if Solution is Obtained or Maximum Generation has reached the Solution; If Yes, evolution terminated. If No, proceed to Step 4

2.3.2. Objective function: modal flexibility matrix approach

Selecting an objective function that is sensitive to damage is not only of paramount importance to the success of the damage detection, but it also affects the performance of the numerical optimization procedures. In this study, the modal flexibility matrix is employed as the objective function. A finite element model containing damage parameters is used for the analytical model. These damage parameters are related to changes in the stiffness matrix and/or mass matrix. It is assumed that structural damage is described as a reduction in Young's modulus (E) for each finite element. Thus, the global stiffness matrix can be expressed as the summation of damaged and undamaged element stiffness matrices, where the local element stiffness is multiplied by a reduction factor as

$$\mathbf{K} = \sum_{i=1}^{nelem} (1-\theta_i)[\mathbf{k}]_i \tag{23}$$

where 'nelem' is the total number of elements in the analytical model; θ_i is the damage parameter associated with the ith element, which equals zero for healthy state and unity for complete damage state; and [$\mathbf{K}$] is the structural stiffness matrix, where [$\mathbf{K}$] is assembled from the elemental stiffness matrices $[\mathbf{k}]_i$ as expressed in Eq. (23).

With its particular sensitivity to structural damage, the modal flexibility methodology is numerically advantageous to use over the flexibility matrix because it does not require the inverse of the modal matrix. Therefore, the computation time is significantly less demanding.

To formulate the modal flexibility matrix, consider the characteristic equation for a linear system

$$\mathbf{K\Phi} = \mathbf{M\Phi\Lambda} \tag{24}$$

where $\mathbf{\Phi}$ is a mode shape matrix and Λ represents a diagonal matrix containing the square of the modal frequencies. Due to orthogonality properties of the mode shape with respect to stiffness and mass matrices, the following relationships are established

$$\mathbf{\Phi}^T\mathbf{K\Phi} = \mathbf{\Lambda} \quad \text{and} \quad \mathbf{\Phi}^T\mathbf{M\Phi} = \mathbf{I} \tag{25}$$

where $\mathbf{I}$ is the unity matrix. From Eq. (25), the stiffness matrix can be rewritten as

$$\mathbf{K} = \mathbf{\Phi}^{-T}\mathbf{\Lambda\Phi}^{-1} = (\mathbf{\Phi\Lambda}^{-1}\mathbf{\Phi}^T)^{-1} \tag{26}$$

Proceeding with the relationship between the stiffness and the flexibility matrix, $\mathbf{K}=\mathbf{F}^{-1}$, the modal flexibility matrix can be derived as

$$\mathbf{F} = \mathbf{\Phi\Lambda}^{-1}\mathbf{\Phi}^T \tag{27}$$

The difference between the tested model and the analytical model can then be utilized as an objective function for damage quantification

$$\Pi(\theta) = \|\mathbf{F}_{measured} - \mathbf{F}_{updated}(\theta)\|_{Fro} \tag{28}$$

where θ is a vector depicting the damage parameters and $\| \|_{Fro}$ represents the Frobenius norm for the residual matrix. From the expanded modal matrix in Eq. (4), $\mathbf{F}_{measured}$ is expressed as

$$\mathbf{F}_{measured} = \mathbf{\Phi}_{\text{exp}anded}\mathbf{\Lambda}_{measured}^{-1}\mathbf{\Phi}_{\text{exp}anded}^{T} \tag{29}$$

Similarly, for a particular damage scenario, $\mathbf{F}_{updated}$ can be rewritten as

$$\mathbf{F}_{updated} = \mathbf{\Phi}_{updated}\mathbf{\Lambda}_{updated}^{-1}\mathbf{\Phi}_{updated}^{T} \tag{30}$$

where the modal matrix and eigenvalue matrix are calculated from the analytical model with a vector of damage parameters. One of the unique advantages of using the modal flexibility matrix is that the modal flexibility matrix is insensitive to high frequency modes because there is an inverse relationship between the flexibility matrix and the eigenvalues of the dynamic system. Moreover, unlike the flexibility matrix, calculation of the modal flexibility matrix is numerically advantageous over that of flexibility matrix as it does not require the inverse of the modal matrix, reducing the computation time.

3. Illustrative examples

In this section, the proposed two-stage damage detection method is demonstrated and validated for various types of structures. A relatively simple portal frame and two truss structures are considered in these examples. The first example considered the case in which both incomplete measurements and modeling errors are introduced, to validate the fact that the proposed method can be applied to real-world structures. In the subsequent examples, only effects of limited sensor information are considered because modeling of steel truss type structure is relatively straightforward. The SEREP technique is applied to deal with incomplete sensor measurements when necessary.

3.1. Two-dimensional steel moment resisting frame

In verification of the proposed damage detection method, a one-story steel portal frame is utilized as the first example. A finite element model is constructed using 30 evenly spaced elements, with each element being 100 mm in length. Thus, the total length for each beam and column is 1000 mm. Fig. 3 illustrates the layout for the portal frame. Four damage locations are induced in the structure at elements 4, 11, 15 and 28. Young's modulus is reduced in these elements by 40%. The nominal specifications of the frame are as follows: Young's modulus = 200,000 N/mm^2, mass density = 7670 × 10^{-9} kg/mm^3. The dimensions for the columns and beams are 50.50 × 6.0 mm^2 and 40.50 × 6.0 mm^2, respectively. In this example, to consider the ability of the method to deal with modeling errors, Young's modulus of each element is sampled from a random process with a mean value of 200,000 N/mm^2 and coefficient of variation (COV) of 3.3% as shown in Fig. 2. Gaussian distribution is assumed for the random variable. Therefore, spatial variations of the structural stiffness are considered. In addition, to consider incomplete measurement of mode shapes, a limited number of DOFs as shown in Fig. 3(b) is used and the results are expanded to the size of the identification model.

After damage, the Modal Assurance Criteria (MAC) values are calculated to ensure correlation between the damaged and undamaged mode shapes. These values are illustrated in Fig. 4. In the first stage of the damage detection approach, localization of the damage is obtained through the proposed subset selection. Fig. 5 shows the sub-space angles between the residual vector and the orthogonalized basis vector for each element. The numbers with arrows indicate the order of selections. 10 modes are used to obtain these results. Clearly, the subset of damaged elements is identified; the smaller sub-space

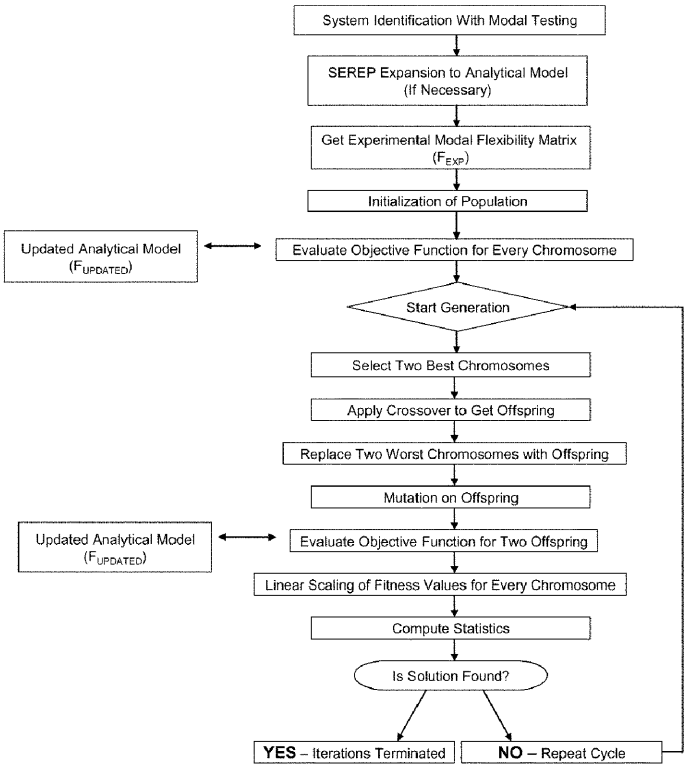


Fig. 1 Flow chart of the steady-state computational procedure for damage detection

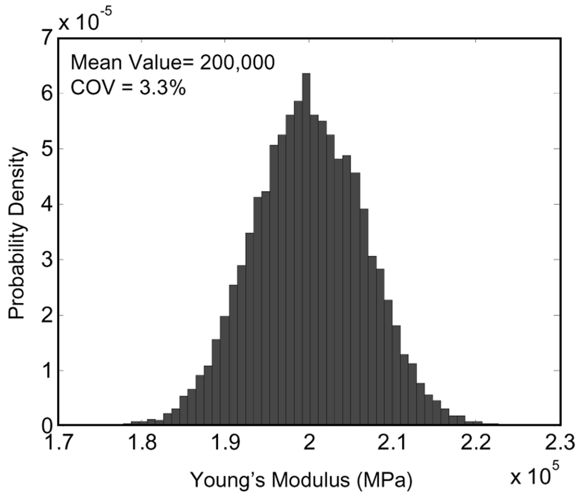
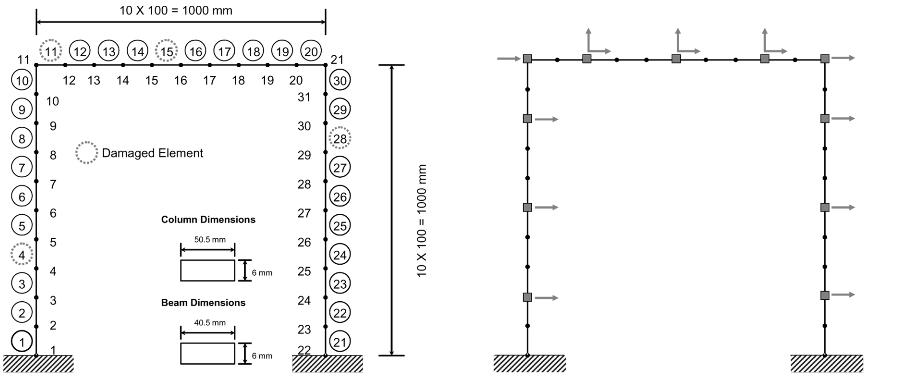


Fig. 2 Probability density function of Young's modulus for considering modeling error (COV=3.3%)

angles appear in elements 4, 11, 15, and 28.

Once the subset of damaged elements has been identified, the second stage of damage detection is applied to quantify the damage through the SSGA procedure. Here, the SSGA search utilizes one point crossover with a mutation probability of 12%. The number of bits per each damage parameter string is



(a) Finite Element Model (b) Sensor Location with Measured DOFs

Fig. 3 Finite element model for steel portal frame with four damaged elements and sensor locations

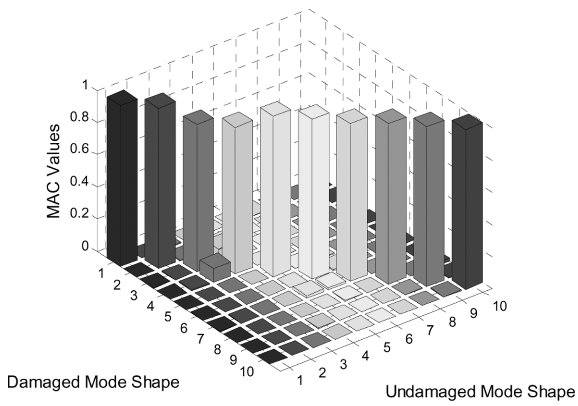


Fig. 4 MAC (modal assurance criteria) values for damaged and undamaged structures with modeling error (COV=3.3%)

55. Thus, with 4 damage parameters, the total bit length for each chromosome is 220. The population size is comprised of 55 chromosomes that adhere to the maximum number of generations, 10,000. Although both modeling errors and incomplete sensor information are considered, the results of the damage quantification stage using the SSGA shown in Fig. 6 demonstrate that the proposed method can quantify damage level reasonably.

3.2. Fourteen-bay planar truss structure

In this example, a 14 bay planar truss (Duan, *et al.* 2007) is selected to demonstrate the performance of the two-stage damage detection method. The truss is modeled using 53 truss elements with 28 nodes as shown in Fig. 7. The total length of the structure is 5.56 m with 0.40 m in each bay, and the height of structure is 0.40 m. The members are steel bars with a tube cross section having an inner diameter of 3.1 mm and an outer diameter of 17.0 mm. The physical properties are: the elastic modulus of the material=1.999×10^5 N/mm^2; and the mass density=$7,827\times10^{-9}$ kg/mm^3: The members are connected using pinned joints. There are two supports at each end of the structure: a pin support at the left end and

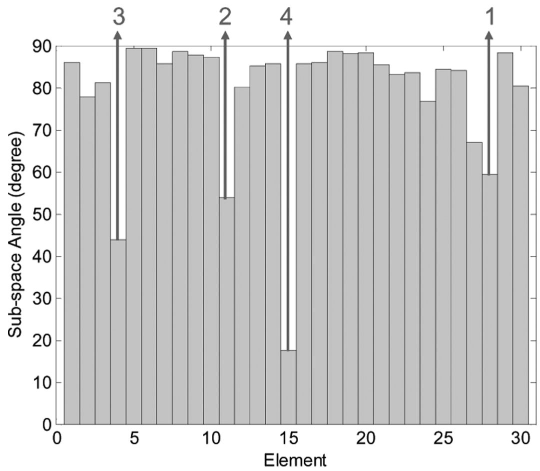


Fig. 5 Sub-space angles of damaged elements in portal frame with first 10 measured modes and modeling error (COV=3.3%); the umbers with arrows indicates the order of selections

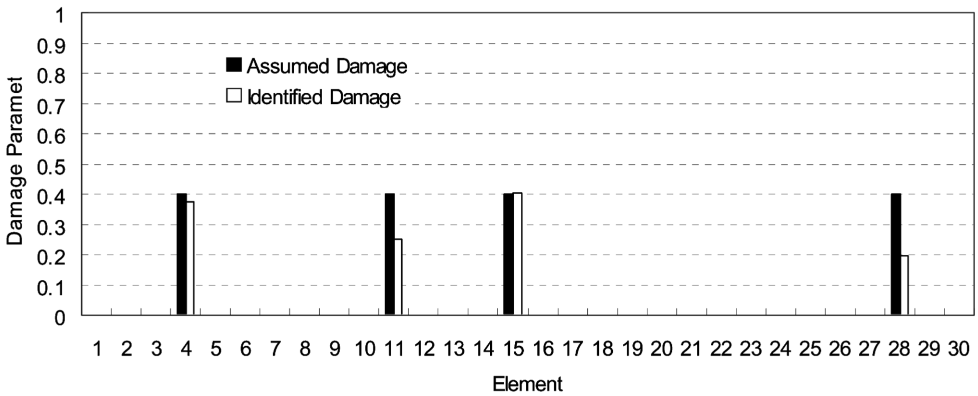


Fig. 6 Damage quantification for multi-damaged portal frame with first 10 measured modes and modeling error (COV=3.3%)

a pinned roller support at the right end. The resulting model has 53 DOFs. Two cases are considered herein: in the first case all DOFs are measured and in the second case only a subset of the DOFs are measured and SEREP is used to expand the measured mode shapes. In the second case sensors are simulated at 26 DOF, i.e., vertical DOFs at nodes 2~14 and 16~28. Damage is induced in elements 2, 18, 32, 41 and 47, with elements 32 and 47 having a 70% loss in the area of the section, and the rest having a 90% loss.

Fig. 8 graphically provides the change in the mode shapes between the damaged and undamaged structures in terms of MAC values. Higher modes are clearly more sensitive to damage than lower modes. Subset selection is first applied to identify the subset of most likely damaged elements. Two scenarios are considered in which either 8 modes or 15 modes are measured Fig. 9. shows the resulting sub-space angles for each element. All damaged elements are selected sequentially within the first five trials in both cases. The angle shown indicates the subspace angle between the orthogonalized column vector and the residual vector. Therefore, the angles of healthy elements are very close to 90 degrees.

After the subset of damaged elements has been identified, SSGA is applied to determine the extent of

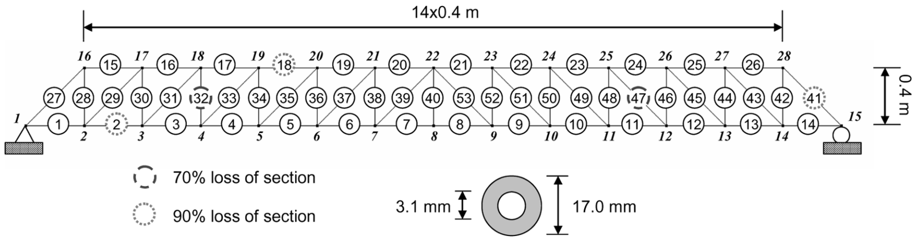
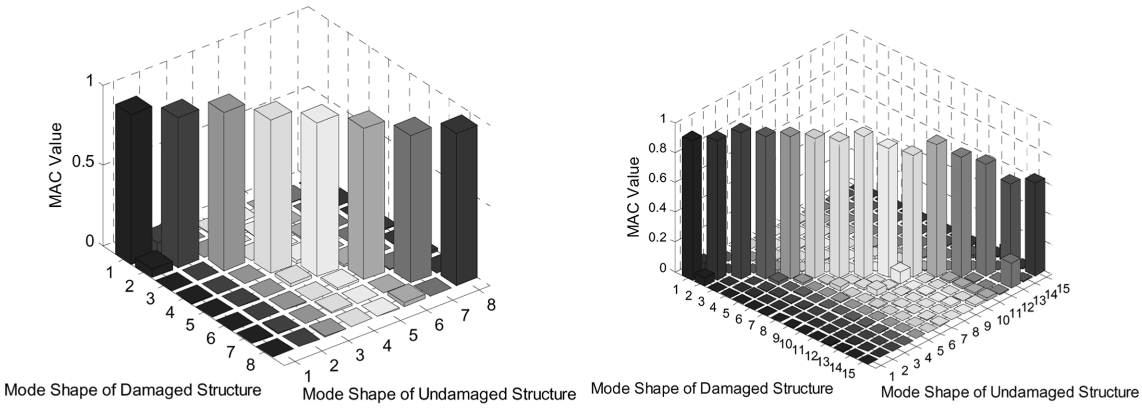


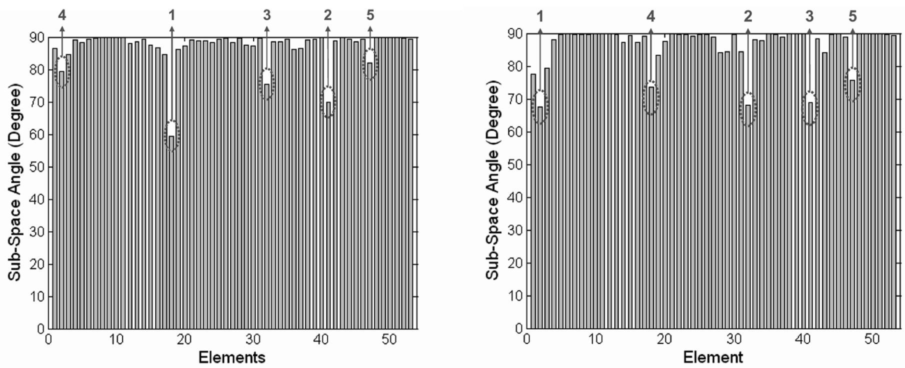
Fig. 7 Fourteen-bay planar truss structures and damage scenario



(a) 8 Measured Modes (b) 15 Measured Modes

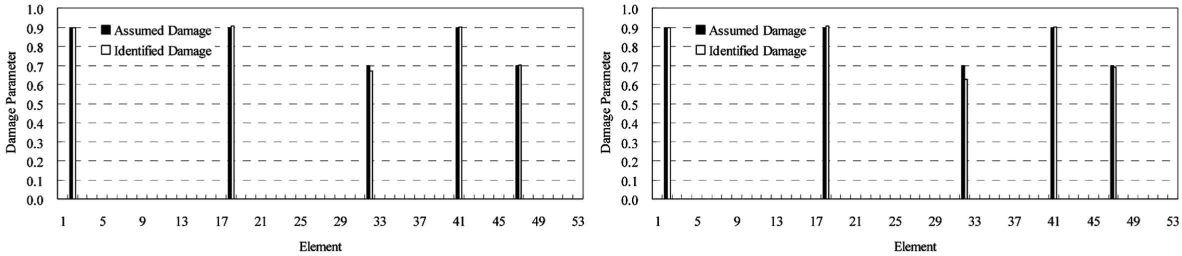
Fig. 8 MAC value between damaged and undamaged structure

damage. Here, the SSGA search utilizes uniform crossover with a mutation probability of 3%. The number of bits per each damage parameter string is 32. Thus, with 5 damage parameters, the total bit length for each chromosome is 160. The population size is comprised of 40 chromosomes that adhere to the maximum number of generations, 20,000. In both cases the results indicate very accurate identification of the damage as shown in Fig. 10. Fig. 11 also depicts the variation of the fitness values up to the maximum number of generations, 20,000.



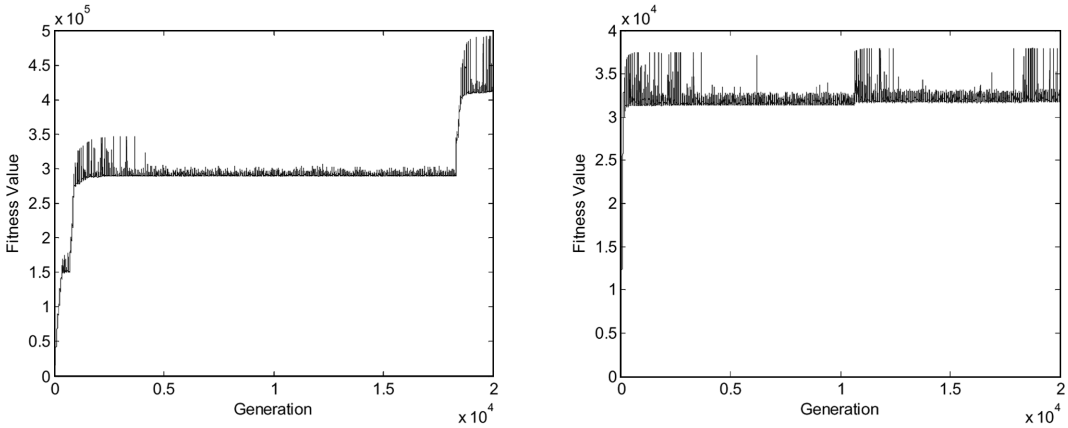
(a) 8 Measured Modes without SEREP (b) 15 Measured Modes with SEREP

Fig. 9 Sub-space angle used in identification of damaged elements; the numbers with arrows indicates the order of selections



(a) 8 Measured Modes without SEREP (b) 15 Measured Modes with SEREP

Fig. 10 Final damage identification using SSGA optimization process



(a) 8 Measured Modes without SEREP (b) 15 Measured Modes with SEREP

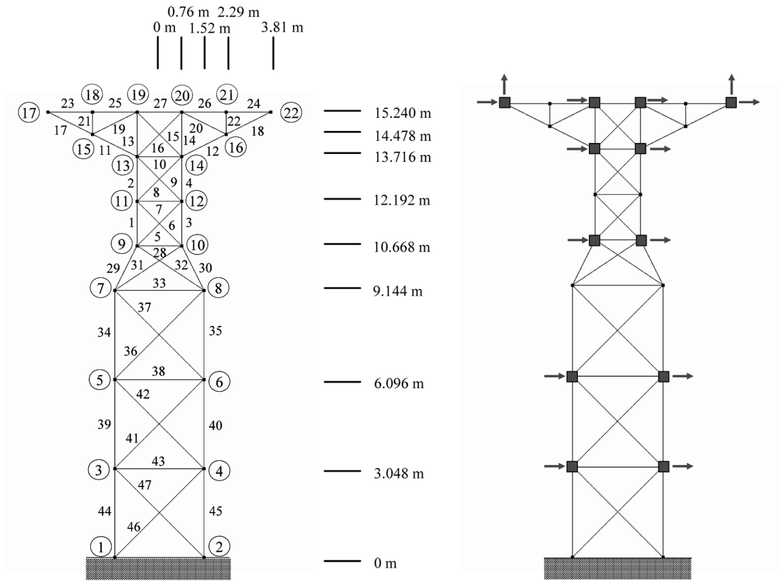
Fig. 11 Variations of fitness values in SSGA optimization process

3.3. 47-element truss-type electrical tower

The final numerical example considered herein considers a typical tower design to verify the proposed damage detection method. The tower stands 15.24 meters tall and contains a maximum span of 7.62 meters at the highest elevation (see Fig. 12). Modal expansion is used because it is assumed that there are limited amount of sensors installed to identify the full mode shapes. A finite element model with 47 truss elements and 22 nodes is used as the identification model. Damage is applied to elements 3, 4, 30, 32, 33, 35, 36, 44, and 46 using a 50% reduction in the elastic stiffness. The structural details are as follows: Young's modulus = 206,829.84 N/mm^2, mass density = 8.30396×10^{-6} kg/mm^3, and cross sectional area ($b \times h$) = 50.8 mm × 50.8 mm = 2580.64 mm.

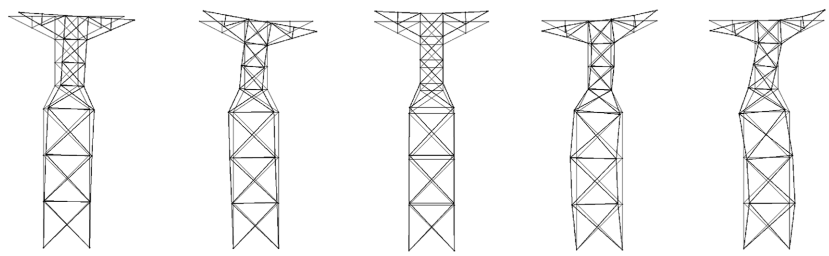
The mode shapes for the healthy and damaged structures are shown in Fig. 13 and Fig. 14. Clearly the mode shapes for these two cases are shown to be quite similar. However, it is apparent that the damaged elements reduce the natural frequencies. The MAC values between these two cases are also slightly altered by the applied damage (see Fig. 15). However, the damaged modes shapes are acceptably correlated to the undamaged modes shapes as the MAC values are close to unity.

Table 1 provides a comparison of the selected parameters and the sub-space angles for different cases in which the first 5, 8, and 12 modes used. The order elements are listed in the table signifies the order in which the corresponding damaged element is selected through subset selection. Also, the angle



(a) Finite Element Model (b) Sensor Locations with Measured DOFs

Fig. 12 47-element truss tower and sensor locations



NF: 0.2459 Hz (a) NF: 0.924 Hz (b) NF: 1.310 Hz (c) NF: 2.056 Hz (d) NF: 3.277 Hz (e)

Fig. 13 Undamaged mode shapes of truss tower a) Mode 1 b) Mode 2 c) Mode 3 d) Mode 4 e) Mode 5

indicates the subspace angle between the orthogonalized column vector in the sensitivity matrix and the residual vector (**b** vector). In the 8 mode trial, the first 9 selected parameters contained all of the damaged elements and displayed the lowest sub-space angle, 68.541°, for the whole experiment. When either increasing or decreasing the number of modes, the number of selected damage parameters needed to obtain all damaged elements, drastically increases, along with the sub-space angle. The results shown in Table 1 indicate that 8 measured modes are the most appropriate number of modes for this problem. According to test results, if the number of measured modes is too small, the subset selection does not seem to be successful due to limited information on the effect of damage on mode shapes. On the contrary, if the number of measured modes becomes large, the mode shape is more likely to be contaminated with measurement errors and/or expansion error. It is noteworthy that the number of modes giving the best selection of damaged elements depends on the properties of the problems given (e.g. geometrical dimension, material properties, etc.), according to the observations from tests with

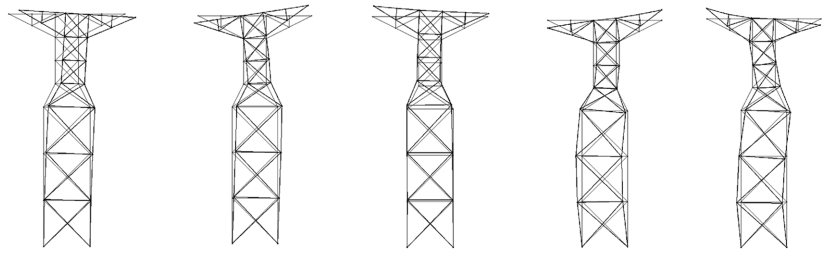


Fig. 14 Damaged mode shapes of truss tower a) Mode 1 b) Mode 2 c) Mode 3 d) Mode 4 e) Mode 5

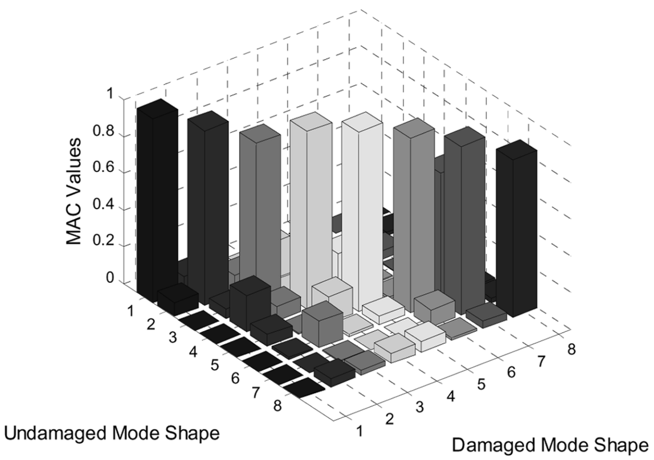


Fig. 15 MAC values between undamaged and damaged structures

Table 1 Comparison of selection order and sub-space angle for multiple damage locations

Damaged Element #	5 Modes		8 Modes		12 Modes	
	Order	Angle (Degree)	Order	Angle (Degree)	Order	Angle (Degree)
3	5	79.053	7	73.897	9	85.018
4	25	85.152	8	68.541	16	86.420
30	1	74.492	2	72.434	4	82.591
32	19	85.536	6	73.076	10	85.755
33	6	81.753	3	69.654	3	78.274
35	13	83.924	1	68.649	2	74.863
36	4	77.897	9	79.399	1	65.618
44	17	85.194	4	74.110	13	86.753
46	8	81.517	5	73.639	19	86.429

various structures. In typical structures of this type, only 5 to 10 modes are likely to be measured.

When utilizing only the first 8 measured modes, damage levels of the previously identified elements are determined through the SSGA optimization process. The GA related parameters are set as follows; total population size is 55; probability of mutation is set to 12%, and bit length for each damage

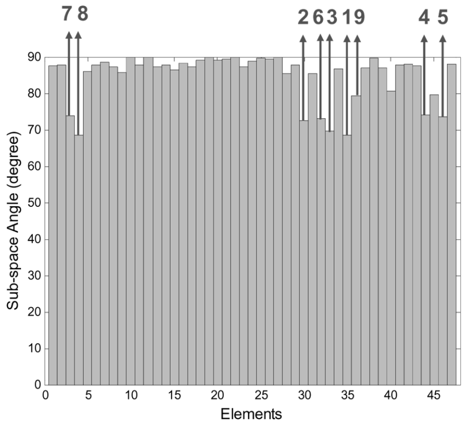


Fig. 16 Sub-space angles of all elements from subset selection process with 8 measured modes; the numbers with arrows indicates the order of selections

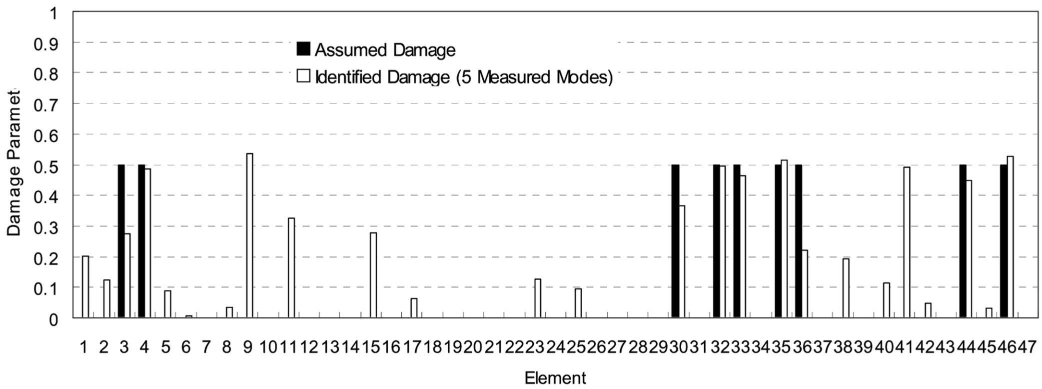


Fig. 17 Damage quantification by SSGA optimization process with 5 measured modes

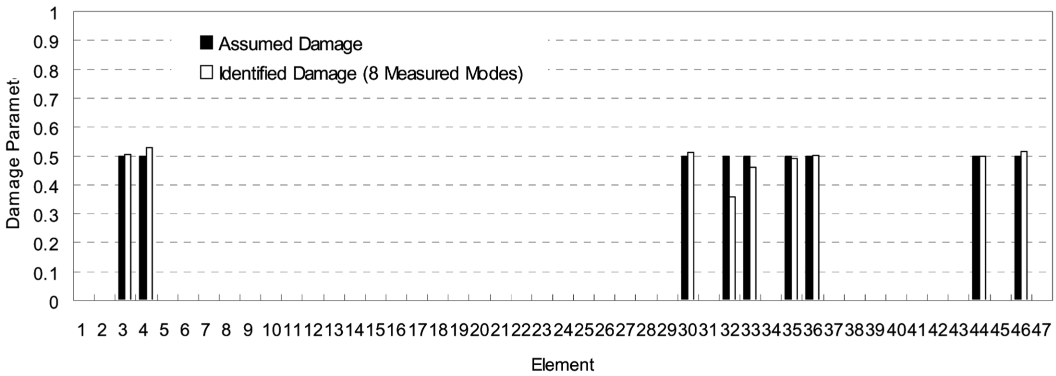


Fig. 18 Damage quantification by SSGA optimization process with 8 measured modes

parameter is 55. Although element 14 was mistakenly selected as a damaged element, elements 3, 4, 14, 30, 32, 33, 35, 36, 44, and 46 are encoded into the SSGA search. The SSGA results found the damage in element 14 to be insignificant as shown in Fig. 18. As mentioned previously, when the first 5 and 12 modes are used for the damage detection, the subset selection seems to be less successful as shown in

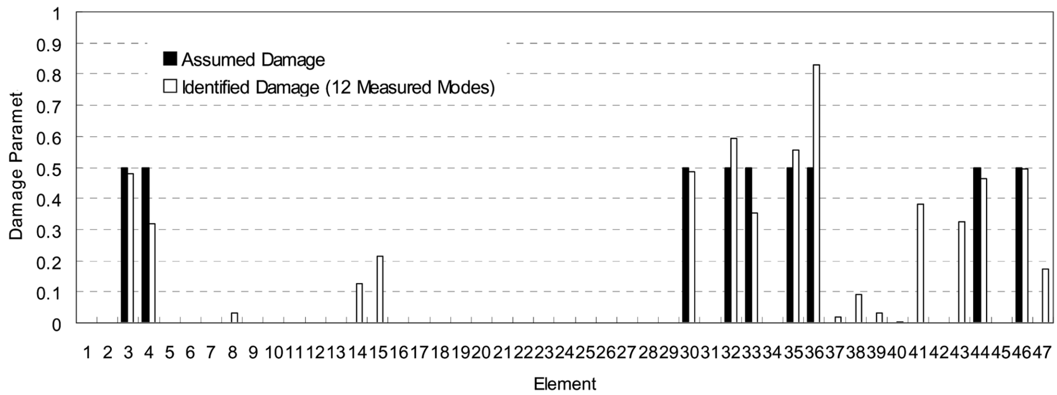


Fig. 19 Damage quantification by SSGA optimization process with 12 measured modes

Fig. 17 and Fig. 19. However, the accuracy of the identified damage level could potentially be improved with optimized GA related parameters. According to the test not demonstrated in this paper, less severe damage, for example 10% reduction of the stiffness, was also successfully detected using the proposed method.

4. Conclusions

A two-stage structural damage detection method has been proposed for locating and quantifying damage within structural systems. The two stages of the proposed method include: 1) subset selection to determine the subset of potentially damaged elements, which requires the modal characteristics of the damaged structure be obtained; and 2) SSGA for quantification of the damage in the subset of potentially damaged elements. The mode shapes are identified through ambient or forced vibration testing and, if necessary, these measured mode shapes are expanded using system equivalent reduction and expansion process (SEREP) to match the size of the analytical model to be updated. For damage localization, a parameter subset selection method using residual force vectors has been employed. The technique is advantageous because the subset selection allows one to greatly reduce the size of the optimization problem. The advantage of implementing the residual force vector in the subset selection method for damage localization is that it can more accurately identify multiple damage locations than the subset selection method using fundamental modal properties. Steady-state genetic algorithm (SSGA), with comparable searching capability and computational efficiency to the simple genetic algorithm (SGA), is adopted herein. The SSGA is shown to be a reliable optimization tool for the damage detection problem.

Three examples using different types of structures are presented in this study. Throughout the examples, the two-stage damage detection method proposed in this paper has demonstrated to be a reliable tool for identifying and quantifying multiple damages within diverse structural systems.

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