Smart Structures and Systems, Vol. 3, No. 3 (2007) 245-261
DOI: http://dx.doi.org/10.12989/sss.2007.3.3.245

Damage assessment of shear buildings by synchronous estimation of stiffness and damping using measured acceleration

Soobong Shin†

Department of Civil Engineering, Inha University, Incheon, Korea

Seong Ho Oh‡

Department of Civil Engineering, Dong-A University, Busan, Korea

(Received August 1, 2006, Accepted December 26, 2006)

Abstract. Nonlinear time-domain system identification (SI) algorithm is proposed to assess damage in a shear building by synchronously estimating time-varying stiffness and damping parameters using measured acceleration data. Mass properties have been assumed as the *a priori* known information. Viscous damping was utilized for the current research. To chase possible nonlinear dynamic behavior under severe vibration, an incremental governing equation of vibrational motion has been utilized. Stiffness and damping parameters are estimated at each time step by minimizing the response error between measured and computed acceleration increments at the measured degrees-of-freedom. To solve a nonlinear constrained optimization problem for optimal structural parameters, sensitivities of acceleration increment were formulated with respect to stiffness and damping parameters, respectively. Incremental state vectors of vibrational motion were computed numerically by Newmark-β method. No model is pre-defined in the proposed algorithm for recovering the nonlinear response. A time-window scheme together with Monte Carlo iterations was utilized to estimate parameters with noise polluted sparse measured acceleration. A moving average scheme was applied to estimate the time-varying trend of structural parameters in all the examples. To examine the proposed SI algorithm, simulation studies were carried out intensively with sample shear buildings under earthquake excitations. In addition, the algorithm was applied to assess damage with laboratory test data obtained from free vibration on a three-story shear building model.

Keywords: nonlinear time-domain SI; incomplete measurement; noise; time-window; moving average.

1. Introduction

Various system identification (SI) schemes have been developed to define an analytical model for a structure or to detect and assess structural damage using measured dynamic responses (Hjelmstad, *et al.* 1995, Kang, *et al.* 2005, Shin 1994). Depending on the final form of vibration information, SI algorithms can be classified into frequency-domain SI and time-domain SI. Even though two SI schemes use the same source of vibration response, frequency-domain SI schemes seem to have been more widely adopted

†Associate Professor, Corresponding Author, E-mail: sbshin@inha.ac.kr
‡Ph.D. Candidate

than time-domain ones partly because modal data transformed from time histories of vibration response are easier to handle with. Another desirable aspect of using a frequency-domain SI algorithm is that modal data can be obtained without any information on the applied forces. Ambient vibration tests have been widely applied to obtain modal data from field tests on bridges (Kim, *et al*. 2003). To obtain modal data from an ambient vibration test, locations of sensors except a reference sensor can be moved to other locations for the next set of ambient vibration so that more vibration information can be gathered. However, each ambient vibration test takes a long time for collecting data to produce reliable modal information so that modal information may be seriously affected by temperature variation when series of ambient vibration tests are carried out (Kim, *et al*. 2004). In addition, modal data of natural frequencies and mode shapes of measured degrees-of-freedom (DOF) are usually gathered at some limited low modes from field tests while local damage may influence more on the information of high modes (Raghavendrachar and Aktan 1992). Comparatively, a time-domain SI scheme usually utilizes every part of measured raw data so that chance of sensing local damage might be higher. One trouble in using a time-domain SI algorithm may be a large size of measured data which should be handled in estimating structural parameters. Another difficulty in applying a time-domain SI algorithm is to get the information of applied forces from field tests. Forced vibration tests with controlled force information have been carried out on bridges but are not so popular yet due to the high cost and inaccuracy in the force information (Salawu and Williams 1995). Instead of forced vibration, free vibration tests with initially imposed displacements also can be carried out on bridges (Douglas and Reid 1982). Under earthquake excitation, force information can be obtained by measuring base vibration so that the use of a time-domain SI algorithm may be more feasible.

Under severe excitation such as earthquake, a structure may easily move in and out of nonlinear regions accompanying with structural damage (Loh and Lin 2000). Therefore, it is necessary to develop a SI algorithm taking into account the nonlinearity in structural behavior. Due to the difficulties in considering nonlinearity in frequency domain, most SI algorithms integrating nonlinearity have been developed in time domain (Li 2000, Lin, *et al.* 1989, Loh and Lin 2000, Tiwari 2000, Yoshida 20001, Yoshida and Sato 2004, Yun and Shinozuka 1980). If measuring instruments are sufficiently equipped in a structure to measure dynamic responses under an excitation, a nonlinear time-domain SI algorithm can be applied directly to assess damage by estimating time-varying structural parameters during the vibration. However, if dynamic responses could not be sufficiently measured during the vibration and instead experiments could be carried before and after a damaging event, the SI scheme should be separately applied to each case and damage can be assessed by comparing the identified structural parameters. In the latter case, nonlinearity may not have a space to be involved due to mild excitation during such an experiment but the proposed SI algorithm can be applied to chase time-varying structural parameters and thus to assess damage in more diverse aspects. In recovering the nonlinear behavior from the identified results, most time-domain SI algorithms utilized pre-defined models for nonlinear behaviors and estimated parameters in the nonlinear model rather than structural parameters (Yoshida and Sato 2004). However, the nonlinear behavior may be chased directly with the time-varying structural parameters if they are identified in time (Lin, *et al*. 1989).

Damage has been defined as a reduction of stiffness in most SI applications (Hjelmstad and Shin 1996, Jang, *et al*. 20002). Many time-domain SI algorithms have estimated stiffness parameters only by assuming damping and mass properties as the *a priori* known information (Banan, *et al.* 1995, Hjelmstad, *et al*. 1995). However, it has been verified that the estimation of damping in addition to stiffness is highly required (Kang, *et al*. 2005). The study showed that the estimation of stiffness could not be improved by any means if damping was erroneously assumed and not allowed to vary from the beginning. Therefore, it is necessary to develop a SI algorithm which can estimate both stiffness and damping synchronously. By evaluating time-varying stiffness and damping together, damage may be assessed in a more qualitative manner. In general, two types of viscous damping and Rayleigh damping have been widely used in SI algorithms. Rayleigh damping may be more easily applied due to the less number of unknowns to be estimated (Kang, *et al.* 2005). However, in dealing with shear building problems, the number of unknowns by viscous damping may not increase so large compared with that by Rayleigh damping (Lin, *et al.* 1989, Yun and Shinozuka 1980). When time-varying damping need be identified separately at each floor, viscous damping is still a useful choice.

Most nonlinear time-domain SI applications were limited to the cases of complete measurements in space and state (Tiwari 2000, Yoshida 2001, Yoshida and Sato 2004, Yun and Shinozuka 1980). However, the inherent incompleteness in space and state of measured time-domain response should be cautiously considered in developing a time-domain SI algorithm (Banan, *et al.* 1995, Hjelmstad, *et al.* 1995) Incompleteness in space is related to the sparseness in measurement while incompleteness in state may occur due to the fact that only one state vector among the acceleration, velocity, and displacement can be usually measured in the field. Therefore, it is necessary to develop a SI algorithm which can provide reliable identification results regardless of the incompleteness. A time-window scheme was proposed as a way of overcoming the over-determined system of equations due to the incompleteness (Hjelmstad, *et al.* 1995). By applying a time-window scheme structural parameters can be estimated averaged around the corresponding time step. In addition to the incompleteness, noise in measured data is another factor of making it difficult to identify structural parameters correctly (Hjelmstad, *et al.* 1995, Hjelmstad and Shin 1996, Jang, *et al.* 2002, Kang, *et al.* 2005). Noise in measured data is unavoidable and random in nature so that structural parameters should be estimated by a statistical method such as the data perturbation scheme (Shin 1994).

The current study proposed a nonlinear time-domain SI algorithm to estimate stiffness and damping synchronously at each time step. The proposed algorithm can be mainly applied to detect and assess damage in a structure through the SI application. For the current research, viscous damping was applied. The SI algorithm is based on the minimization of a response error between the measured and computed acceleration increments at limited DOF. Incompleteness in state and also that in space were considered in developing the algorithm. To overcome the under-determined condition due to the incompleteness, a time-window scheme was incorporated in the formulation of the objective function to be minimized. The proposed SI algorithm was examined through simulation studies on sample shear buildings and laboratory experiments on a model shear building. In the simulation studies, the effects of time-window size and measurement noise on the identification were investigated. Monte Carlo iterations with simulated white noise were applied to estimate structural parameters with sparse and noisy measured data. Identifications of damaging event and damage severity under earthquake excitation were examined by moving average of estimated parameters in time. Recovery of nonlinear behavior was examined through a simulation study under earthquake excitation. In addition to the simulation studies, the SI algorithm was applied to laboratory experimental data in which artificial damage was simulated on the model building. Damage was assessed by comparing the identification outcomes separately obtained before and after the damage.

2. Nonlinear time-domain SI algorithm

2.1. Incremental governing equation

To describe the nonlinear dynamic motion of a structure, the incremental equation of Eq. (1) can be applied.

$$\mathbf{M}\Delta\ddot{\mathbf{u}}^i + \mathbf{C}(\mathbf{x}_C^i)\Delta\dot{\mathbf{u}}^i + \mathbf{K}(\mathbf{x}_K^i)\Delta\mathbf{u}^i = \Delta\mathbf{f}^i \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}(N \times N)$, = mass, damping, and stiffness matrix, $\Delta\ddot{\mathbf{u}}^i$, $\Delta\dot{\mathbf{u}}^i$, $\Delta\mathbf{u}^i$, $\Delta\mathbf{f}^i$ $(N\times 1)$ = vector increments of acceleration, velocity, displacement, and force, $\mathbf{x}^i = \{\mathbf{x}_K^{iT}, \mathbf{x}_C^{iT}\}^T((N_K + N_C)\times 1)$ = vector of stiffness and damping parameters at time step i, and N, N_K, N_C = number of degrees-of-freedom (DOF), stiffness and damping parameters, respectively.

In Eq. (1), stiffness and damping are two types of time-varying parameters estimated and mass property has been assumed as known and not to change in time. Stiffness and damping matrices can be decomposed by a linear combination of unknown parameters and kernel matrices as Eq. (2).

$$\mathbf{K}(\mathbf{x}) = \mathbf{K}_o + \sum_{p=1}^{N_K} x_{Kp} G_{Kp}, \quad \mathbf{C}(\mathbf{y}) = \mathbf{C}_o + \sum_{q=1}^{N_C} x_{Cp} \mathbf{G}_{Cq} \tag{2}$$

where $\mathbf{K}_o$, $\mathbf{C}_o$ = known part of stiffness and damping matrices if any and $\mathbf{G}_{Kp}$, $\mathbf{G}_{Cq}$ = constant kernel matrices for the p-th stiffness and the q-th damping parameters, respectively (Hjelmstad and Shin 1996). As shown in Eq. (2), viscous damping were used in the current research because viscous damping may be more efficient to identify time-varying damping property separately at each floor (Yun and Shinozuka 1980). The number of unknowns by viscous damping may be controllable in a shear building even if it is larger than that by Rayleigh damping. The current algorithm was devised to estimate time-varying properties of damping and stiffness synchronously at each time step, which is different from most available algorithms estimating stiffness parameters only or estimating damping parameters dependent on stiffness and mass by Rayleigh damping (Kang, *et al*. 2005, Tiwari 2000).

2.2. Parameter estimation with incomplete measurement

Accelerometer is one of the most frequently used sensors in the field for dynamic measurements. It is still hard to measure dynamic displacements directly at multiple locations of a civil structure. Because a limited number of accelerometers can be usually used for measuring transient dynamic vibration, a SI algorithm is developed to estimate structural parameters by minimizing a response error between measured and computed acceleration increment at the measured DOF. To chase the variations of stiffness and damping in time, the response error vector $\mathbf{e}^i(\mathbf{x}^i)$ at time step i is defined by Eq. (3).

$$\mathbf{e}^i(\mathbf{x}^i) = \Delta\tilde{\ddot{\mathbf{u}}}_a^i(\mathbf{x}^i) - \Delta\tilde{\ddot{\mathbf{u}}}_m^i (N_m\times 1) \tag{3}$$

where $\Delta\tilde{\ddot{\mathbf{u}}}_a^i(\mathbf{x}^i)$, $\Delta\tilde{\ddot{\mathbf{u}}}_m^i$ = analytically computed and measured acceleration increment vector at the measured DOF respectively and N_m = number of measured DOF.

Analytical increments of all the state vectors are computed numerically by Newmark-β method as Eq. (4) and thus the state vectors can be updated for the next time step by Eq. (5) with known initial conditions on velocity and displacement.

$$\Delta\ddot{\mathbf{u}}_a^i = S^{i^{-1}}\Delta\mathbf{r}^i, \quad \Delta\dot{\mathbf{u}}_a^i = \Delta t\ddot{\mathbf{u}}_a^i + \gamma\Delta t\Delta\ddot{\mathbf{u}}_a^i, \quad \Delta\mathbf{u}_a^i = \Delta t\dot{\mathbf{u}}_a^i + \tfrac{1}{2}\Delta t^2\Delta\ddot{\mathbf{u}}_a^i + \beta\Delta t^2\Delta\ddot{\mathbf{u}}_a^i \tag{4}$$

$$\ddot{\mathbf{u}}_a^i = \ddot{\mathbf{u}}_a^{i-1} + \Delta\ddot{\mathbf{u}}_a^{i-1},\ \dot{\mathbf{u}}_a^i = \dot{\mathbf{u}}_a^{i-1} + \Delta\dot{\mathbf{u}}_a^{i-1},\ \mathbf{u}_a^i = \mathbf{u}_a^{i-1} + \Delta\mathbf{u}_a^{i-1} \tag{5}$$

where β, γ = constants depending on the applied Newmark method and structural matrix $\mathbf{S}^i$ and redundant force increment $\Delta\mathbf{r}^i$ are defined by Eq. (6).

$$\mathbf{S}^i(\mathbf{x}^i) = \mathbf{M} + \gamma\Delta t\mathbf{C}(\mathbf{x}_C^i) + \beta\Delta t^2\mathbf{K}(\mathbf{x}_K^i)$$

$$\Delta\mathbf{r}^i(\mathbf{x}^i) = \Delta\mathbf{f}^i - \Delta t\mathbf{K}(\mathbf{x}_K^i)\dot{\mathbf{u}}_a^i - [\Delta t\mathbf{C}(\mathbf{x}_C^i) + \tfrac{1}{2}\Delta t^2\mathbf{K}(\mathbf{x}_K^i)]\ddot{\mathbf{u}}_a^i \tag{6}$$

Since Eq. (3) is defined only with the measured DOF, the analytical acceleration increment can be defined by using a Boolean matrix indicating measured DOF as Eq. (7).

$$\Delta\tilde{\ddot{\mathbf{u}}}_a^i = \mathbf{B}\Delta\ddot{\mathbf{u}}_a^i = \mathbf{B}{\mathbf{S}^i}^{-1}\Delta\mathbf{r}^i \tag{7}$$

To estimate optimal stiffness and damping parameters at time step i, the minimization problem of Eq. (8) can be defined with measured data in a time-window.

$$\text{Minimize } J^i(\mathbf{x}^i) = \frac{1}{2}\sum_{t=i-n_i}^{i+n_f} \|\Delta\tilde{\ddot{\mathbf{u}}}_a^t(\mathbf{x}^i) - \Delta\tilde{\ddot{\mathbf{u}}}_m^t\|^2\Delta t$$

$$\text{Subject to } \mathbf{x}_{lo} \le \mathbf{x}^i \le \mathbf{x}_{up} \tag{8}$$

where n_i, n_f = time steps to open a time-window around the current time step i and $\mathbf{x}_{lo}$, $\mathbf{x}_{up}$ ($N_m\times 1$) = upper and lower bounds for the structural parameters, respectively.

2.3. Error sensitivity

The parameters at time i were averaged within a time-window opened around the time step. To obtain optimal parameters by solving the constrained nonlinear optimization problem of Eq. (8), gradient vector and Gauss-Newton Hessian matrix were computed by Eq. (10).

$$\nabla\mathbf{J}^i(\mathbf{x}^i) = \sum_{t=i-n_i}^{t+n_f} \nabla\mathbf{e}^t(\mathbf{x}^i)^T\mathbf{e}^t(\mathbf{x}^i) \qquad \{(N_K+N_C)\times 1\}$$

$$\mathbf{H}_{GN}^i(\mathbf{x}^i) = \sum_{t=i-n_i}^{t+n_f} \nabla\mathbf{e}^t(\mathbf{x}^i)^T\nabla\mathbf{e}^t(\mathbf{x}^i) \qquad \{(N_K+N_C)\times(N_K+N_C)\} \tag{9}$$

In Eq. (9), sensitivity of the error vector and thus sensitivity of acceleration increment are computed with respect to stiffness and damping parameters by Eq. (10).

$$\frac{\partial\mathbf{e}^t}{\partial x_l^i} = \frac{\partial\Delta\tilde{\ddot{\mathbf{u}}}_a^t}{\partial x_l^i} = -\mathbf{B}{\mathbf{S}^t}^{-1}\frac{\partial\mathbf{S}^t}{\partial x_l^i}{\mathbf{S}^t}^{-1}\Delta\mathbf{r}^t + \mathbf{B}{\mathbf{S}^t}^{-1}\frac{\partial\Delta\mathbf{r}^t}{\partial x_l^i} \tag{10}$$

where x_l^i = the l-th structural parameter at time step i. Sensitivities of structural matrix $\mathbf{S}^t$ and redundant force increment $\Delta\mathbf{r}^t$ at time step t with respect to stiffness and damping parameters are defined by Eq. (11), respectively.

$$\frac{\partial \mathbf{S}^t}{\partial x_{Kp}^i} = \beta \Delta t^2 \mathbf{G}_{Kp}, \quad \frac{\partial \Delta \mathbf{r}^t}{\partial x_{Kp}^i} = -\Delta t \mathbf{G}_{Kp} \dot{\mathbf{u}}_a^t - \Delta t \mathbf{K} \frac{\partial \dot{\mathbf{u}}_a^t}{\partial x_{Kp}^i} - \frac{1}{2} \Delta t^2 \mathbf{G}_{Kp} \ddot{\mathbf{u}}_a^t - \left[\Delta t \mathbf{C} + \frac{1}{2} \Delta t^2 \mathbf{K} \right] \frac{\partial \ddot{\mathbf{u}}_a^t}{\partial x_{Kp}^i}$$

$$\frac{\partial \mathbf{S}^t}{\partial x_{Cq}^i} = \gamma \Delta t \mathbf{G}_{Cq}, \quad \frac{\partial \Delta \mathbf{r}^t}{\partial x_{Cq}^i} = -\Delta t \mathbf{K} \frac{\partial \dot{\mathbf{u}}_a^t}{\partial x_{Cq}^i} - \Delta t \mathbf{G}_{Cq} \ddot{\mathbf{u}}_a^t - \left[\Delta t \mathbf{C} + \frac{1}{2} \Delta t^2 \mathbf{K} \right] \frac{\partial \ddot{\mathbf{u}}_a^t}{\partial x_{Cp}^i} \tag{11}$$

where x_{Kp}^i, x_{Cp}^i = the p-th stiffness and q-th damping parameter at time step i, respectively.

Sensitivities of velocity and acceleration with respect to stiffness and damping used in Eq. (12) can be computed by Eq. (12). Since the initial conditions for velocity and displacement were assumed as known, initial sensitivities with respect to stiffness and damping can be ignored.

$$\frac{\partial \dot{\mathbf{u}}_a^t}{\partial x_l^i} = \frac{\partial \dot{\mathbf{u}}_a^{t-1}}{\partial x_l^i} + \frac{\partial \Delta \dot{\mathbf{u}}_a^{t-1}}{\partial x_l^i}, \quad \frac{\partial \mathbf{u}_a^t}{\partial x_l^i} = \frac{\partial \mathbf{u}_a^{t-1}}{\partial x_l^i} + \frac{\partial \Delta \mathbf{u}_a^{t-1}}{\partial x_l^i} \tag{12}$$

2.4. Identifiability criterion

The identifiability criterion to solve for optimal parameters using the above equations can be defined by Eq. (13).

$$N_m \times (n_f - n_i + 1) \geq N_p \tag{13}$$

Eq. (13) indicates that the number of measured time points in a time-window multiplied by the number of measured DOF should be larger than the number of unknown parameters N_p. In other words, the amount of measured information used for the identification should be larger than that of unknowns. Since the minimization using gradient vector and the Hessian matrix in Eq. (9) was basically a least squared problem, a under-determined case should be avoided by preserving the condition of Eq. (13).

3. Simulation study

3.1. Effects of time-window size and measurement noise on the identification

The basic aspects of the developed algorithm have been examined through a simulation study on a SDOF system as shown in Fig. 1. The applied earthquake excitation for the simulation study is shown in the first figure of Fig. 2. The natural period of the system is 0.314 sec and the data sampling rate was 200 Hz.

Fig. 3 shows the effect of time-window size on the identification results. Damage in Fig. 2 was not imposed in the simulation and the system behavior was limited to the elastic region. As the size of a time-window grows the estimated parameters converge to the exact ones rapidly. Each point in the figures was obtained by averaging and normalizing estimated parameters over 10 seconds. Three lines in each figure were drawn with different number of Monte Carlo iterations with maximum 5% random error in simulated acceleration. As the number of Monte Carlo iterations increases, the estimated stiffness and damping converge to the actual values with a smaller size of time-window. In overall, stiffness could be estimated reliably with a smaller size of time-window than damping.

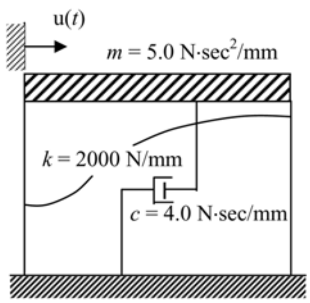

Fig. 1 SDOF system for the simulation study

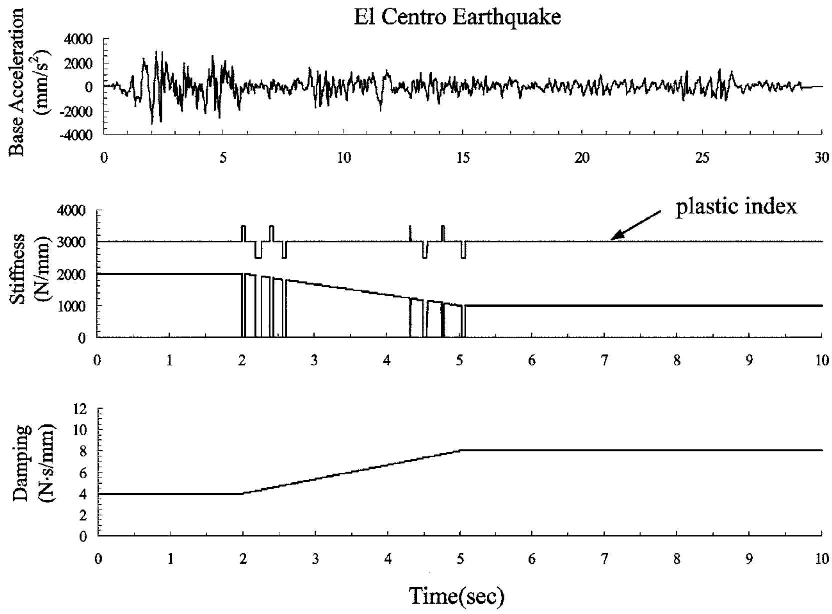

Fig. 2 Simulated earthquake excitation and damaging event with elasto-plastic behavior

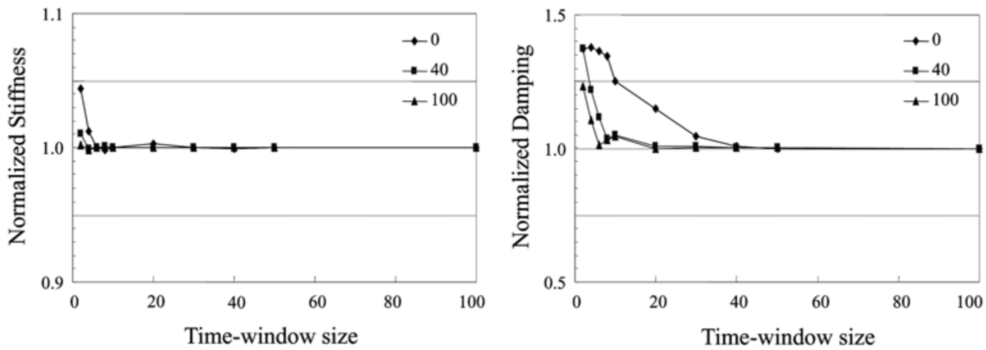

Fig. 3 Estimated parameters with the variation in time-window size under elastic behavior

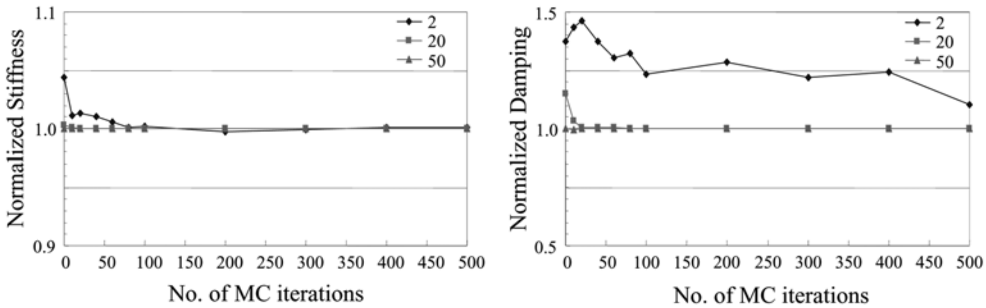

Fig. 4 Estimated parameters with the variation in the number of Monte Carlo iterations

Fig. 4 shows similar effects as Fig. 3 but from a different perspective. In Fig. 4, three lines in each figure were drawn with different sizes of time-window. Fig. 3 and Fig. 4 verify that a reasonable size of time-window together with sufficient Monte Carlo iterations can lead to a reliable estimation of structural parameters.

3.2. Moving average of time-varying parameters

To examine the proposed algorithm for damage assessment, damage was simulated by gradual decrease in stiffness and gradual increase in damping as drawn in Fig. 2. (Yoshida 2001). Therefore, the structure moved in and out of the plastic region during the damaging event. Fig. 5 shows the variations of stiffness and damping parameters in time estimated by the proposed SI algorithm. For the estimation, each time-window was opened with 10 data points and 40 Monte Carlo iterations were carried out with maximum 5% random noise in measured acceleration. In the figure of stiffness variation, two different cases were compared. The solid line is drawn when both stiffness and

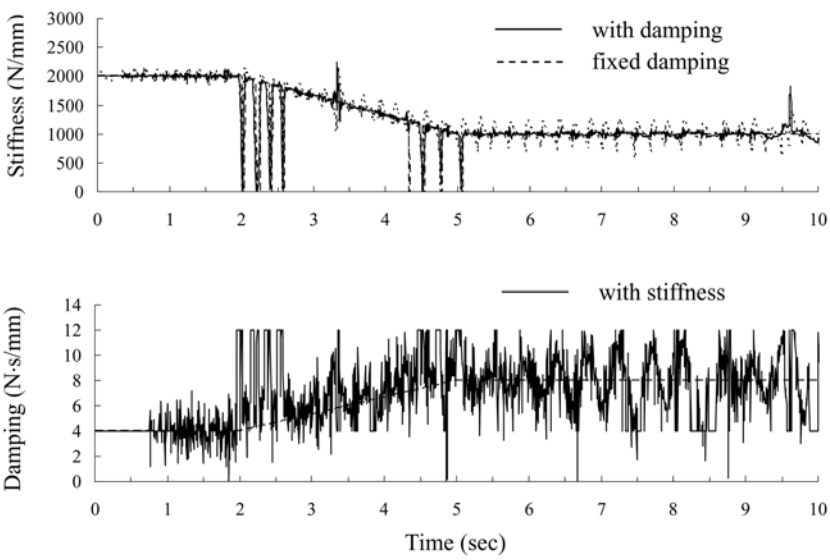

Fig. 5 Variations of estimated stiffness and damping in time

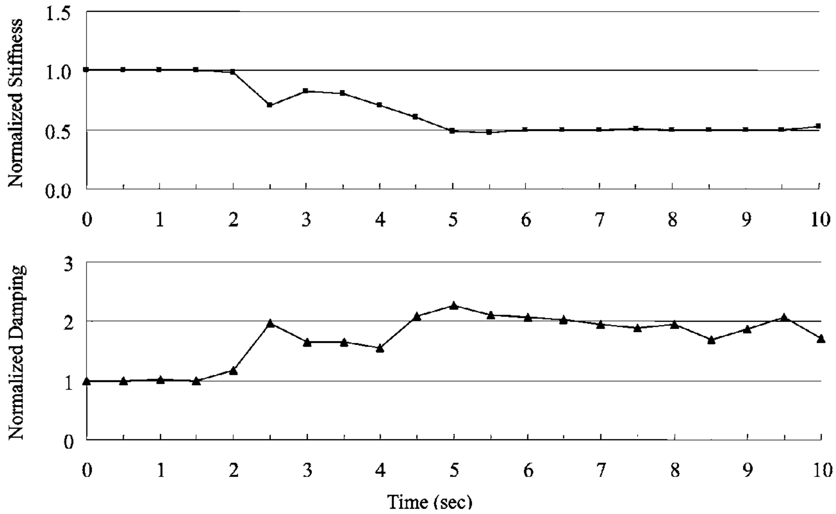

Fig. 6 Normalized moving average of stiffness and damping in time

damping were estimated synchronously at each time step. The dotted line is when only stiffness was estimated with a fixed damping of $c = 6.0$ N·s/mm as the average before and after the simulated damage shown in Fig. 2.

From Fig. 5, it can be observed that estimation of stiffness is much more stable than that of damping regardless of the fixity of the damping parameter. The estimation of stiffness clearly detected the damaging event during vibration and could assess damage severity reliably. Also, the time periods when the system jumped into plastic region around the damaging event could be correctly identified. In the case of fixed damping, the estimated stiffness oscillated around the actual value. If damping were assumed as a value away from the actual one, the estimation of stiffness could be more distorted. Even if a wild trend of damping change might be guessed from Fig. 5, the estimated damping fluctuated so violently. To chase the time-varying variation of structural parameters in a smooth way, a moving average was computed at every 0.5 seconds and drawn in Fig. 6 for both parameters. From Fig. 6, damage can be more easily identified even though detail variations of stiffness may not be seen as Fig. 5.

3.3. Recovery of nonlinear response

Fig. 7 compares the simulated nonlinear response with computed one using the estimated time-varying parameters. The time-varying stiffness and damping of Fig. 5 were used directly to compute displacement numerically from Newmark-β method. Therefore, no model for the nonlinear behavior was pre-defined in the current approach, which may be what set apart the developed algorithm from those already existing. Sharpness of chasing the recovery curve depended on the size of time-window. As the size of time-window decreases, a more sharp chasing could be obtained but with more fluctuations in estimating structural parameters as indicated in Fig. 3. Fig. 8 compared the simulated and computed acceleration time history of the simulated system. Since the system was SDOF as of a full measurement in space, the two responses matched well at all the time steps regardless of its elasto-plastic behavior.

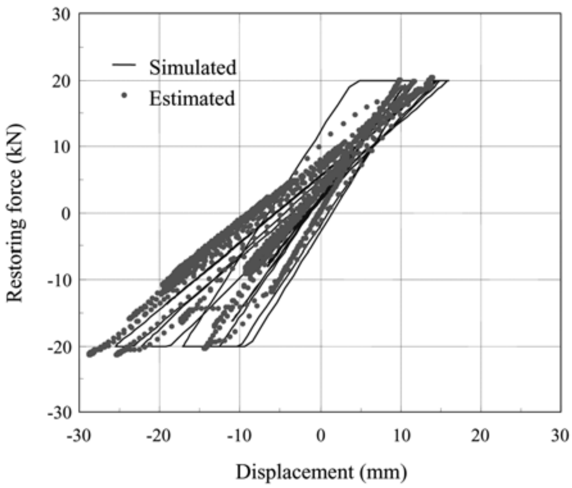

Fig. 7 Recovered nonlinear response of restoring force v.s displacement

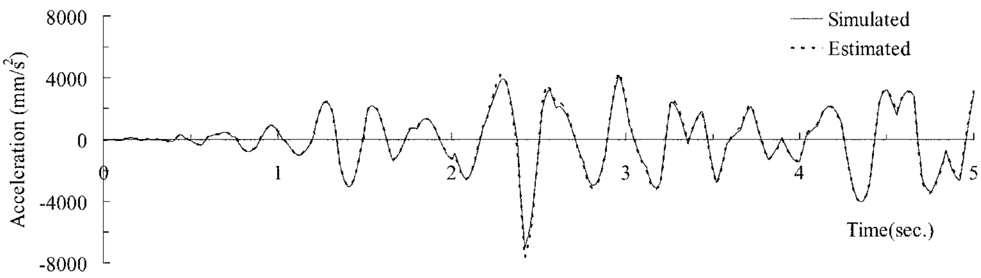

Fig. 8 Comparison of simulated and estimated acceleration responses

3.4. Damage assessment of a shear building with incomplete measurement

The developed algorithm was examined through a simulation study on a ten-story shear building with sparse and noisy measured acceleration data. The simulated properties of the building were as described in Table 1 and damage was simulated as a gradual change between 3 and 6 second as summarized in Table 2. Acceleration time histories were assumed as measured at every floor except the 3rd, 6th, and 9th floor of the shear building with maximum 5% random noise. To satisfy the identifiability condition of Eq. (14), each time-window was set with 20 data points. Moving average of each parameter was computed at

Table 1 Simulated properties of the ten-story shear building

Story	Mass ($kN\ s^2/m$)	Damping (KN s/m)	Stiffness (kN/m)	Yeilding displacement (mm)	Restoring force (kN)	2nd Stiffness (kN/m)
1~7	25	25	25000	20	500	5000
8~10	20	20	20000		400	4000

Story	1	2	3	4	5	6	7	8	9	10
Behavior	Bilinear		Elastic							
Damage	Damaged		Undamaged							

Table 2 Simulated damage for the study

Story	Stiffness (kN/m)		Ratio(%)	Damping (kN s/m)		Ratio(%)
1	25000	12500	-50	25	50	+100
2		18750	-25		37.5	+50

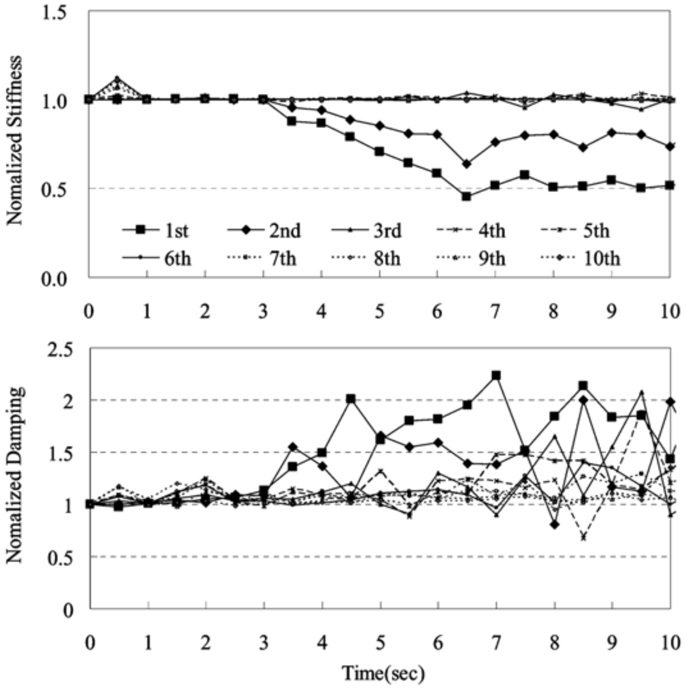

Fig. 9 Moving average of estimated stiffness and damping in time

every 0.5 second as shown in Fig. 9. From the first figure showing stiffness variation, damage in the 1st and 2nd story can be clearly identified as the damaging event between 3 and 6 second. Even though relatively large fluctuations are noticed in the damping, increase of damping in the 1st and 2nd stories can be also observed.

The damage ratio as the ratio of averaged parameters before and after the damaging event was computed at every story for each parameter and drawn in Fig. 10. From the figure, it can be observed

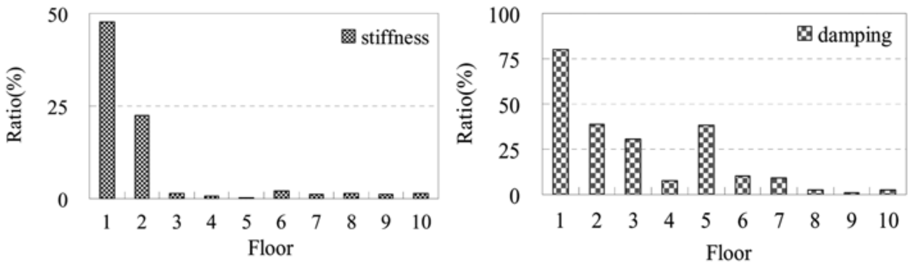

Fig. 10 Damage ratio of each floor computed from stiffness and damping

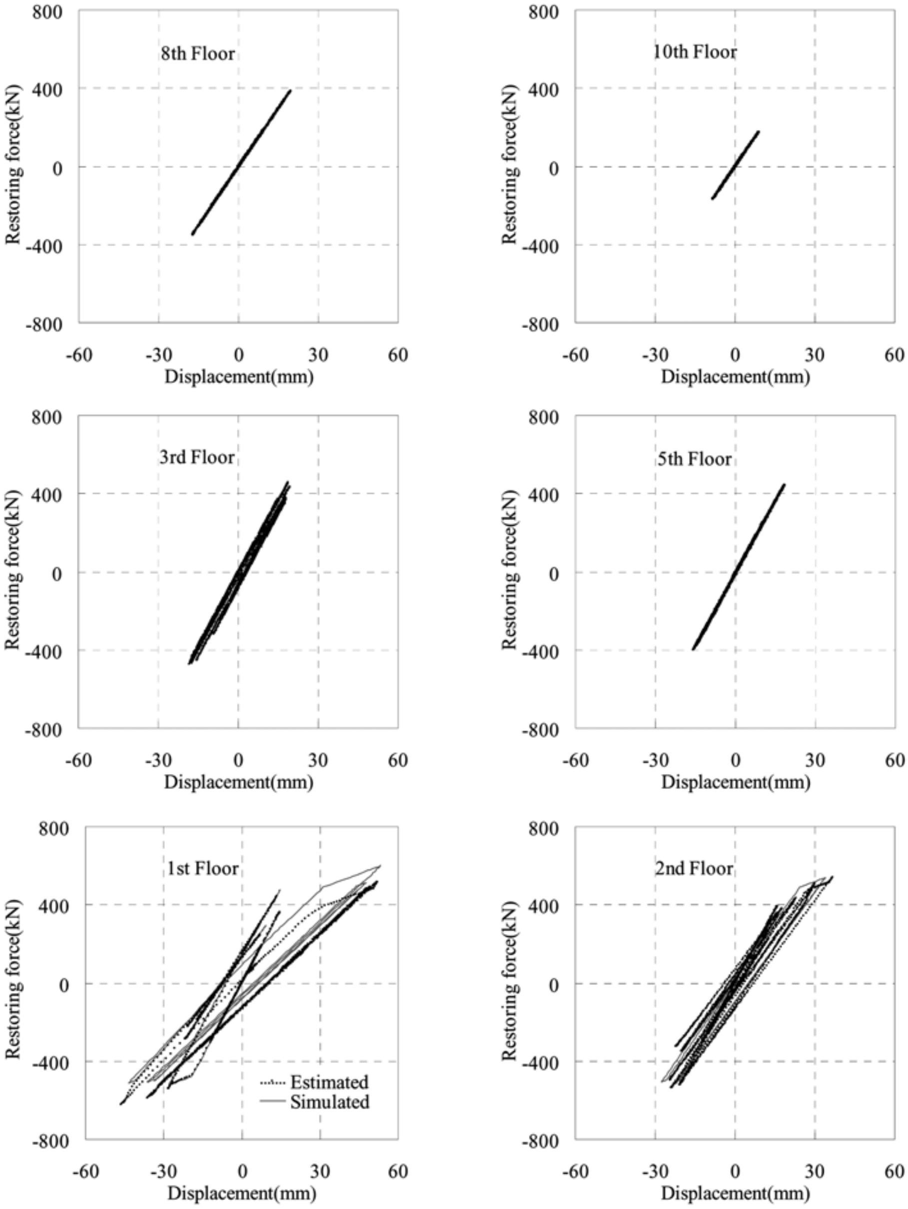

Fig. 11 Recovered responses of restoring force v.s displacement at some floors

that damage can be more reliably detected and assessed by stiffness than damping. However, as lessoned by Fig. 5, a synchronous estimation of damping and stiffness is still needed to identify stiffness more reliably.

Fig. 11 compares recovered restoring responses with the simulated actual ones at some floors. From the figure, it can be observed that bi-linear behaviors of the 1st and 2nd floors can be chased with minor error but with good agreement in overall.

4. Application to a laboratory test data

The proposed method was examined to detect and assess damage with laboratory experiments, too. A three-story shear building shown in Fig. 12 was used for the experiments and accelerations were measured at the center of each floor plate. The sectional and material properties of the columns and slabs of the test model are given in Table 3.

Experiments were carried out separately before and after damage imposed by bolt loosening as shown in Fig. 13 because it was difficult to impose damage during the vibration in a laboratory test.

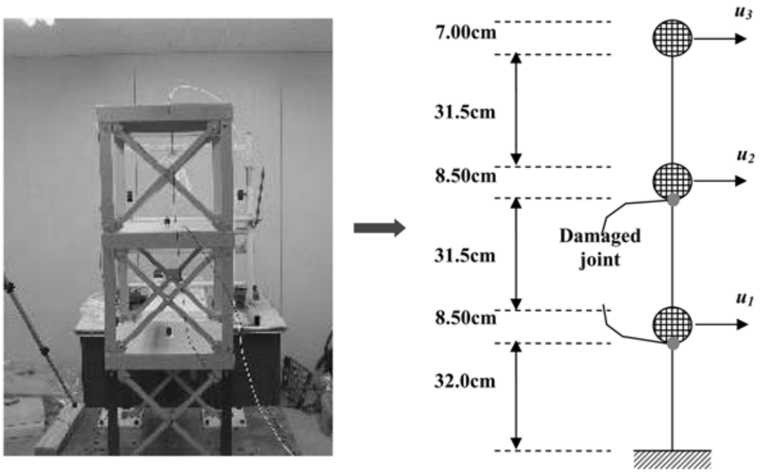

Fig. 12 Test shear building model and locations of imposed damage

Table 3 Sectional and material properties of the columns and slabs of the test model

Floor	Mass (N·s^2/cm)	Thickness (cm)	Area (cm^2)	CTC Length (cm)
1	0.1432	0.400	2.00	36.25
2	0.1376	0.300	1.50	40.00
3	0.1368	0.300	1.50	39.25

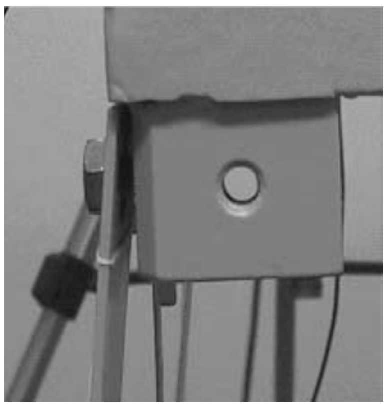

Fig. 13 Imposed damage by loosening bolt connection at a joint

Table 4 Identified natural frequencies of intact and damaged cases

Mode	Intact (Hz)	Damaged (Hz)	Ratio (%)
1	2.39	1.88	-27.13
2	6.50	5.93	-9.61
3	8.94	7.28	-22.80

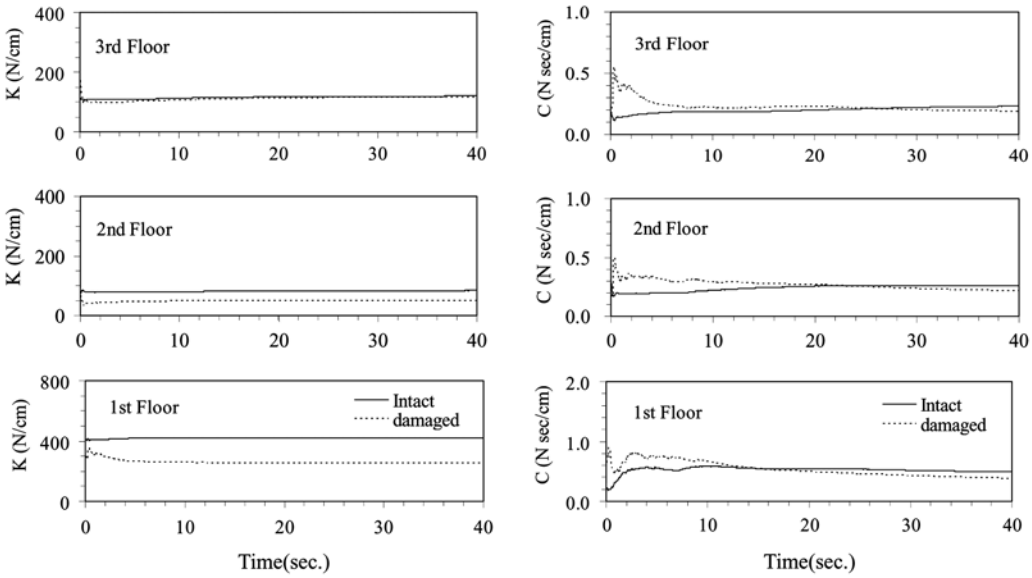

Fig. 14 Moving average of stiffness and damping in time for the intact and damaged cases

Accelerations were measured from free vibration induced by sudden release of a static loading. A steal block of 12.78 kg was used as a static loader, and was applied at the top floor in the horizontal direction. Accelerations were measured in the time period of 80 seconds with the sampling rate of 50 Hz. Acceleration data measured during the early 40 seconds were used in the study. The lowest three natural frequencies obtained from measured acceleration are compared in Table 4 for the cases of intact and damaged, respectively.

For the current study, only the acceleration data measured at the 2nd and 3rd floors were used for the identification to examine a case of incomplete measurement in space. The identified moving average of stiffness and damping of each floor were drawn in Fig. 14. Each time-window was opened with 20 data points and 40 Monte Carlo iterations were carried out with assumed 5% maximum proportional white noise for estimating parameters at each time step. As time step moves on, averaged parameters converged fast to certain values in most cases. Convergence of damping was slower especially in the initial stage. When comparing the identified values before and after damage, a clear reduction in the stiffness of the 1st story and a smaller reduction in the 2nd story were observed. However, damping properties were almost consistent regardless of the imposed damage, which may represent an actual situation of damage by joint loosening without member failure.

The recovered restoring relation at each floor is drawn in Fig. 15. From the figures, it can be observed that all the three members behave elastically during the vibration regardless of the imposed damage by loosening bolts. The reduction of stiffness at the 1st and 2nd stories can be easily identified from the

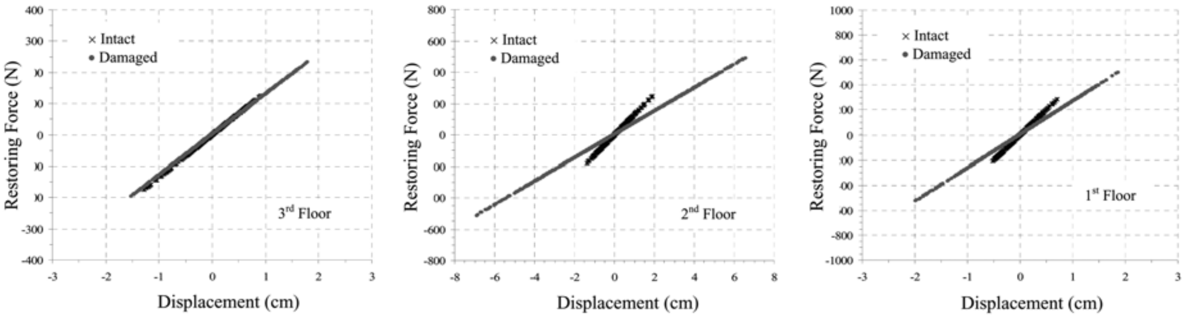

Fig. 15 Comparison of recovered responses for the intact and damaged cases

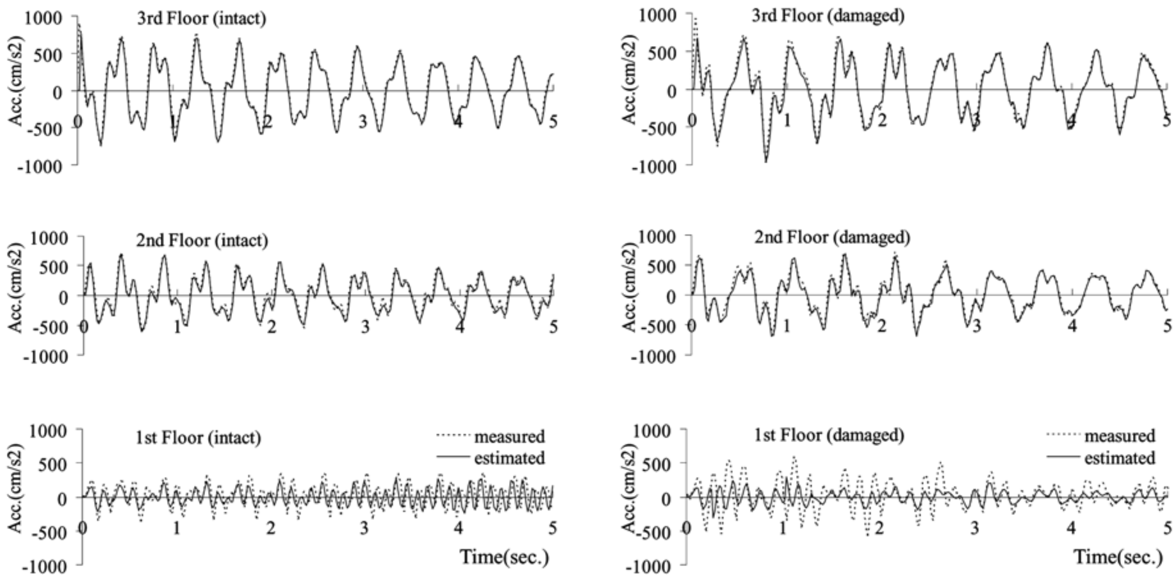

Fig. 16 Comparison of acceleration histories at each floor for the intact and damaged cases

slope inclinations. Another observation from Fig. 15 is that the lateral vibrating displacements became larger at every floor after the damage was imposed. Increase of vibrating displacement at the 3rd floor was relatively smaller compared with those of the 1st and 2nd floors, which matched with the locations of bolt loosening.

Fig. 16 compares the measured and computed acceleration time histories at each floor for the intact and damaged cases, respectively. For both cases, the error in accelerations of the 1st floor was relatively larger compared with those of the other floors because accelerations were measured only at the 2nd and 3rd floors to identify structural parameters.

5. Conclusions

A nonlinear time-domain SI method was proposed to assess damage in a shear building by synchronously estimating time-varying stiffness and damping parameters using acceleration data. Incompleteness of measured data in space and that in state were considered in developing the algorithm. To chase nonlinear dynamic response in time, an incremental form of vibrational motion was utilized as the governing equation. A response error between measured and computed acceleration increment at limited DOF was minimized to estimate optimal structural parameters.

A time-window scheme together with Monte Carlo iterations was applied to estimate structural parameters with sparse and noisy measured acceleration. A simulation study showed that estimation of stiffness and damping could be improved with a larger size of time-window and with more Monte Carlo iterations. A reasonable time-window size and number of Monte Carlo iterations could be adjusted by the identifiability criterion. In the simulation studies with damage as gradual changes in structural parameters under earthquake excitation, the damaging event and damage severity could be identified reliably by moving average of the estimated parameters in time. The idea of moving average has been verified as useful from all the sample studies examined in the paper. However, nonlinear behavior was recovered reliably by using the estimated time-varying parameters instead of moving averaged values because sharpness of chasing the nonlinearity was dependent on the size of time-window. Estimation of stiffness was more reliable to assess damage compared with that of damping. In the current research, viscous damping was utilized and its variation was estimated separately at each floor of a shear building. The estimated time-varying damping parameters oscillated violently in the examples but the application of moving average could calm it down effectively with minor error in the estimation.

The proposed SI algorithm was applied to laboratory test data collected separately before and after imposing artificial damage because damage could not be simulated during free vibration in the laboratory experiments. However, damage could be reliably assessed by comparing the identified time-varying structural parameters from both cases. Damping parameters estimated from the SI algorithm were consistent regardless of the damage of bolt loosening without any member failure. Linear behavior and larger lateral dynamic displacements were accompanied with bolt loosening.

Acknowledgements

This study has been a part of a research project supported by Korea Ministry of Construction and Transportation (MOCT) via the Infra-Structures Assessment Research Center. The authors are grateful to Prof. H.S. Lee of Seoul National University, Seoul, Korea for allowing them to use the laboratory test data.

References

Banan, M. R., Banan, M. R. and Hjelmstad, K. D. (1995), "Time-domain parameter estimation algorithm for structures II: numerical simulation studies", *J. Eng. Mech.*, ASCE, **121**(3), 435-447.

Douglas, B. M. and Reid, W. H. (1982), "Dynamic tests and system identification of bridges", *J. Struct. Div.*, ASCE, **108**(ST10), October, 2295-2312.

Hjelmstad, K. D., Banan, M. R. and Banan, M. R. (1995), "Time-domain parameter estimation algorithm for structures I: computational aspects", *J. Eng. Mech.*, ASCE, **121**(3), 424-434.

Hjelmstad, K. D. and Shin, S. (1996), "Crack identification in a cantilever beam from modal response", *J. Sound Vib.*, **198**(5), 527-545.

Jang, J. H., Yeo, I., Shin, S. and Jang, S. P. (2002), "Experimental investigation of SI-based damage assessment of structures", *J. Struct. Eng.*, ASCE, **128**(5), 673-682.

Kang, J. S., Park, S.-K, Shin, S. and Lee, H. S. (2005), "Structural system identification in time domain using measured acceleration", *J. Sound Vib.*, **288**, 215-234.

Kim, C.-Y., Jung, D.-S., Kim, N.-S., Kwon, S.-D. and Feng, M. Q. (2003), "Effect of vehicle weight on natural frequencies of bridges measured from traffic-induced vibration", *J. Earthq. Eng. Eng. Vib.*, **2**(1), June, 109-115.

Kim, J. T. Yun, C. B. and Park, J. H. (2004), "Temperature effects on vibration-based damage detection in model plate-girder bridges", *The 5th Korea-Japan Seminar/Workshop on System Identification and Structural Health Monitoring*, Dongguk University, Jan. 30-31, 159-168.

Li, B. (2000), "Reconstruction of linear and nonlinear continuous time system models from input/output data using the Kernel invariance algorithm", *J. Sound Vib.*, **233**, 877-896.

Lin, C. C., Soong, T. T. and Natke, H. G. (1989), "Real time system identification of degrading structures", *J. Eng. Mech.*, ASCE, **116**(10), 2258-2274.

Loh, C. H. and Lin, C. Y. (2000), "Time domain identification of frames under earthquake loadings", *J. Eng. Mech.*, ASCE, **126**(7), 693-703.

Raghavendrachar, M. and Aktan, A. E. (1992), "Flexibility by multi-reference impact testing for bridge diagnostics", *J. Struct. Eng.*, ASCE, **118**(8), 2186-2203.

Salawu, O. S. and Williams, C. (1995), "Bridge assessment using forced-vibration testing", *J. Struct. Eng.*, ASCE, **121**(2), February, 161-173.

Shin, S. (1994), "Damage detection and assessment of structural systems from measured response", Ph.D. Thesis, University of Illinois at Urbana-Champaign.

Tiwari, R. (2000), "On-line identification and estimation of nonlinear stiffness parameters of bearings", *J. Sound Vib.*, **234**(5), 906-910.

Yoshida, I. (2001), "Damage detection using Monte Carlo filter based on non-Gaussian noise", *Proceedings of Nonlinear System Identification and Structural Health Monitoring*, DPRI, Kyoto University, 281-292.

Yoshida, I. and Sato, T. (2004), "Damage detection by adaptive Monte Carlo filter", *The 5th Korea-Japan Seminar/Workshop on System Identification and Structural Health Monitoring*, Dongguk University, Jan. 30-31, 209-216.

Yun, C. B. and Shinozuka, M. (1980), "Identification of nonlinear structural dynamic systems", *J. Struct. Mech.*, ASCE, **8**(2), 187-203.