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Simplified analytical model for flexural response of external R.C. frames with smooth rebars

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Abstract. In this paper an analytical model in a closed form able to reproduce the monotonic flexural response of external RC beam-column joints with smooth rebars is presented. The column is subjected to a constant vertical load and the beam to a monotonically increasing lateral force applied at the tip. The model is based on the flexural behavior of the beam and the column determined adopting a concentrated plasticity hinge model including slippage of the main reinforcing bars of the beam. A simplified bilinear moment-axial force domain is assumed to derive the ultimate moment associated with the design axial force. For the joint, a simple truss model is adopted to predict shear strength and panel distortion. Experimental data recently given in the literature referring to the load-deflection response of external RC joints with smooth rebars are utilized to validate the model, showing good agreement. Finally, the proposed model can be considered a useful instrument for preliminary static verification of existing external RC beam-column joints with smooth rebars for both strength and ductility verification.

Keywords: joint; beam; column; shear; flexure; smooth rebars

1. Introduction

Many reinforced concrete (RC) buildings currently present in the Mediterranean area (Italy, Greece, Spain, Turkey, etc.), constructed in the '50s-'70s, were designed before the advent of seismic codes, and show peculiar structural characteristics and material qualities. Among them a large number of old buildings have been identified as having potentially serious structural deficiencies with respect to earthquake resistance.

Typical structural deficiencies of existing RC buildings are the general lack of ductility as well as inadequate lateral strength: these issues have been recognized as the fundamental source of deficit in seismic performance of gravity- load-designed existing buildings, as a consequence of total absence of capacity design principles and poor reinforcement detailing (Priestley 1997). RC buildings constructed in the absence of seismic codes have both global deficiencies, connected to lack of regularity in plan and elevation, as well as to the possible onset of weak-column mechanism, with a tendency to develop soft-story mechanisms, and local deficiencies, connected to insufficient transverse reinforcement of beams, columns, joints, and to insufficient anchorage (Ehsani and Wight 1985, Hegger *et al.* 2003). In particular, at local level, inadequate protection of the panel zone region within beam-column joint subassemblies is expected as well as brittle failure mechanisms of structural elements. In most cases examined experimentally and theoretically failure was due to the shear collapse of external joints producing brittle response; in some other cases failure was due to flexural failure of the columns due to strong beam/weak column design (Attaalla 2004, Hegger *et al.* 2004). Generally, it is observed that typical structural deficiencies can be related to: - inadequate confining effects in the potential plastic regions; - insufficient amount, if any, of transverse reinforcement in the joint regions; - insufficient amount of column longitudinal reinforcement, when considering seismic lateral forces; - inadequate anchorage detailing, for both longitudinal and transverse reinforcement; - lapped splices of column reinforcement just above the floor level; - lower quality of materials (concrete and steel) when compared to current practice, in particular use of smooth (plain) bars for both longitudinal and transverse reinforcement and low-strength concrete.

In the scientific literature several studies have been proposed for the analysis of gravity-designed RC sub-assemblages subjected to lateral loading. Among these researches, Kim *et al.* (2007) proposed a statistical approach for the shear strength calculation of RC beam-to-column joints subjected to lateral force. The model was developed using an experimental database in conjunction with a Bayesian parameter estimation method. The experimental database consisted of RC beam-column connection subassemblies subjected to quasi-static cyclic lateral loading and experienced joint shear failure in some cases. Three types of joint shear strength models were developed: in the first model all possible influence

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parameters on joint shear strength were considered; the second model considered only those parameters left after a step-wise process that identifies and removes the least important parameters affecting the mechanism; finally, in the last model only the most relevant parameters for practical applications were taken into account. A similar analytical approach was used by Unal and Burak (2012) for the prediction of the shear strength of joint in case of cyclic loading. Prior to the development of the analytical formulation, numerous results of experimental test were collected establishing a database of geometric properties, material strengths, configuration details and test results of subassemblies. Therefore, statistical correlation method was used for determining the main parameters affecting joint shear capacity. Analytical equations were finally developed taking into account the effect of eccentricity, column axial load, wide beams and transverse beams, besides the key parameters such as concrete compressive strength, reinforcement yield strength, effective joint width and joint transverse reinforcement ratio.

More recently, Laterza *et al.* (2017) proposed a numerical modelling for the seismic assessment of old existing RC beam-column joints, providing two different approaches for internal and external joints both designed only for vertical loads. Such models have the advantage to be merely based on geometrical and mechanical properties of RC elements; moreover, they do not require excessive time computing and can be easily implemented in a general-purpose finite element program.

Finally, it could be noteworthy to mention that other researches are available in the literature dealing with analytical models for the response of longitudinal bar embedded in concrete taking into account the bond-slip phenomenon (Braga *et al.* 2012, D'Amato *et al.* 2012). Such studies are particularly appropriate for modeling bond-slip of smooth bars generally used in older reinforced concrete buildings and can be easily implemented in general-purpose nonlinear structural analysis software.

Based on this background of knowledges, this paper presents an analytical model for the prediction of the monotonic loading-deflection response of external beam-column joints that can be used for preliminary static verification of strength and ductility of existing external RC joints with smooth rebars. The model includes the flexural failure of the column, due to strong beam-weak column design, and the brittle shear collapse of the joint, in the absence of stirrups, due to the crushing of compressed concrete strut. The expressions derived can be reduced to a more compact formulation assuming few simplified hypotheses, allowing a hand computation for preliminary verification of the safety state.

2. Aim of research and range of variation of case studies

The main object of the present research is to propose a simple model for hand verification of the flexural behavior of external old type RC frames with smooth rebars representative of mid-rise building types designed to support vertical loads, constructed in the pre-70s period. With the aim of understanding the structural behavior of these RC frames, detailed experimental investigations on T subassemblages were carried out (Russo and Pauletta 2012, Calvi *et al.* 2001, Calvi *et al.* 2002, Braga *et al.* 2009).

The range of study cases was that in which buildings were only designed for gravity and so did not respect the criteria of plastic design. The beams were designed with the scheme of continuous beams for gravity loads and columns were designed for gravity loads considered as isolated member. This approach was based on elastic verification of RC cross-sections in which the compressive strength of concrete was limited to 6-7 MPa for beams and to 4-5 MPa for columns. The minimum compressive strength assumed, measured on cube specimens at 28 days of curing, was 12 MPa. The smooth steel rebars adopted had 280 MPa yielding stress. The steel rebars were straight at the ends or had hook anchorages. The dimensions of the cross-sections of the columns were at least 300×300 mm or 300×400 mm (for a number of floors limited to 3 or 4). The span of the floors and beams was at most 5000 mm. The longitudinal reinforcement of columns was constituted by 4 ϕ 12 mm (or 4 ϕ 14 mm) longitudinal bars and stirrups having 6 mm diameter were placed at pitch 250 mm. The beams had recurrent dimensions of 300×400 mm and 700×200 mm in the floor thickness. At least two bottom and two 12-mm top rebars were adopted. Additional reinforcement in the fixed sections and in the bottom central portion of the beams was adopted to verify the cross-sections (generally 2 or 3 rebars ϕ 14-16 mm diameter). The stirrups in the beams were designed to support 50% of shear, while the other 50% was supported by inclined rebars (inclination 45°). The stirrups were placed at a pitch of 330 mm.

3. Proposed model

The case examined here is the one shown in Fig. 1(a)). It represents an external beam-joint-column system subjected to a constant vertical load N acting on the column and to a monotonically increasing lateral force F applied at the tip of the beam.

The bare model in Fig. 1 in the beam-column sub-assemblage reproduces the deformed shape that it should have in the real structure when subjected to lateral forces.

The analytical model presented below gives the overall beam tip force-displacement (F-δ) curve at the tip of the beam meeting the column at the joint, for any fixed value of the axial load N acting on the column.

The overall structure response is given (see Fig. 2), for each force level F, by adding the displacements due to the beam δ_b, to the column δ_c and to the joint δ_j. In particular, the beam displacement is obtained by modeling the beam as a cantilever including slippage of bars as suggested in Fib Bulletin No. 24. (2003), while the column is considered as a beam element simply supported and loaded by a constant axial force and a moment in the nodal region.

The joint is modeled as truss-structure. In the following sections, firstly the beam, column and joint displacements are evaluated and, secondly, they are put together in the F-δ curve of the system. No shear failure is considered in the present model.

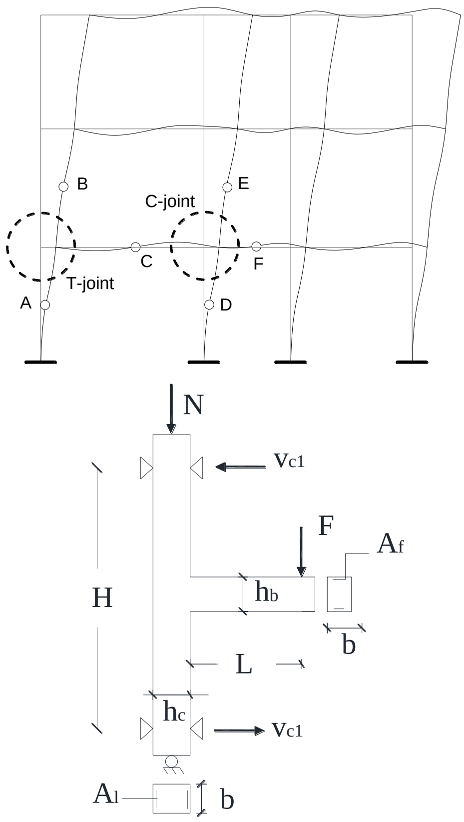
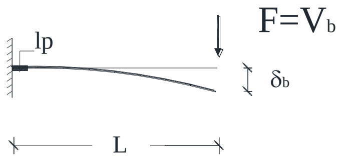
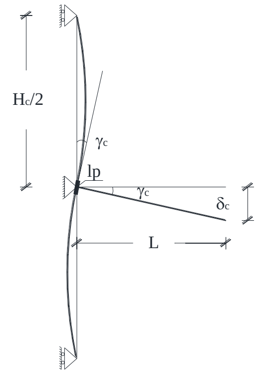


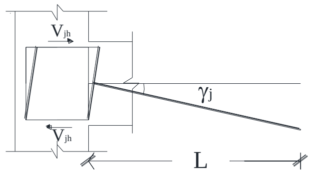
Fig. 1 Bare structure examined and beam-column specimen geometry with loading scheme



(a) beam



(b) column



(c) joint

Fig. 2 Contributions of single components in T subassemblages

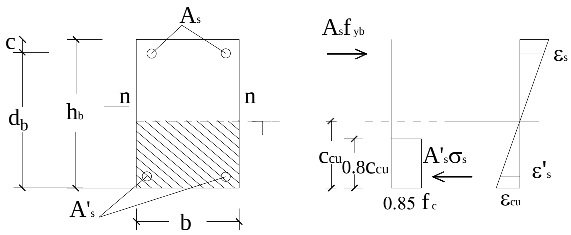


Fig. 3 Design assumptions for analysis of RC beams

The force F shown in Fig. 1 is related to the maximum beam moment $M_b=F{\cdot}L$, while the column is loaded by the abovementioned constant axial load N and by a linear moment whose maximum value is $M_b/2$. Actually, the case in which sections of the column are subjected to a moment equal to $M_b/2$ is a specific case of equal flexural stiffness above and below the joint; otherwise, the moment is distributed according to the relative stiffness. The shear force V_{c1} is related to F by a simple equilibrium equation ($F=\dfrac{V_{c1}\cdot H}{L}$). The inter-story drift is directly related to the displacements δ which correspond to the applied force F and it is calculated as δ/L.

3.1 Beam contribution

In flexure, with reference to the rectangular cross-section shown in Fig. 3, representing the section of the beam fixed into the column, using the translational and rotational equilibrium equations of internal forces, the neutral axis depth c_{cu} and the beam moment at the shear collapse of the joint M_{uy} can be obtained in the following form

$$c_{cu}=\left(A_s-\frac{\varepsilon_{cu}}{\varepsilon_{ys}}A_s^{'}\right)\frac{f_{yb}}{2\alpha\beta b\, f_c}+ \\ +\sqrt{\left(A_s-\frac{\varepsilon_{cu}}{\varepsilon_{ys}}A_s^{'}\right)^2\left(\frac{f_{yb}}{2\alpha\beta b\, f_c}\right)^2+\frac{\varepsilon_{cu}}{\varepsilon_{ys}}\frac{A_s^{'}c\, f_{yb}}{\alpha\beta b\, f_c}} \tag{1a}$$

$$M_{uy}=A_s\cdot f_{yb}\cdot\left(d-\frac{1}{2}\cdot\beta\cdot c_{cu}\right)+ \\ +A_s^{'}\frac{(c_{cu}-c)\varepsilon_{cu}}{c_{cu}}\frac{f_{yb}}{\varepsilon_{ys}}\left(\frac{1}{2}\cdot\beta\cdot c_{cu}-c\right) \tag{1b}$$

α and β being the stress block coefficients assumed equal to 0.85 and 0.80 for normal strength concrete, f_{yb} the working stress of the steel, A_s the area on the main rebars, f_c the compressive strength of concrete and d the effective depth of the beam.

It is noteworthy to observe that, in practical applications related to old-type buildings, the contribution of the compressive reinforcement could be neglected, the amount of the latter being generally small with respect to the amount of tensile reinforcement. Therefore, under this assumption, Eq. (1) can be written in the following simplified form

$$c_{cu} = \frac{A_s \cdot f_{yb}}{\alpha \cdot f_c \cdot \beta \cdot b} \tag{2a}$$

$$M_{uy} = A_s \cdot f_{yb} \cdot \left(d - \frac{1}{2} \cdot \beta \cdot c_{cu}\right) \tag{2b}$$

The working stress f_{yb} of the beam longitudinal rebars present in Eqs. (1), (2) is related to the bond stress distribution and they are both unknown. These terms have been determined in the literature by Pauletta *et al.* (2015) for the case of ribbed bars anchored in beam-column joint having bended anchorages. In particular, on the basis of experimental results on 61 test specimens, Pauletta *et al.* (2015) showed that it is possible to obtain a single analytical expression giving the tensile stress trend in the beam longitudinal rebars when joint shear failure occurs. Specifically, the effective stress f_{yb} decreases with an increase in the mechanical percentage of beam tensile reinforcement $\omega_s = \frac{A_s \cdot f_y}{b \cdot h_b \cdot f_c}$ according to the following analytical law

$$\chi = \frac{f_{yb}}{f_y} = 0.63 \cdot (\omega_s)^{-0.21} \le 1 \tag{3}$$

f_y being the yielding stress of the steel, b and h_b the width and depth of the beam and A_s the longitudinal tensile reinforcement area.

The value of ω_s that gives $\frac{f_{yb}}{f_y} = 1$ is ω_{smax}=0.110.

By contrast, when smooth rebars had hook anchorage, as shown in Fabbrocino *et al.* (2004), yield stress is fully attained without loss of bond, but significant slippage occurs.

Assuming the contribution of compressive reinforcement to be negligible, the flexural strength given by Eq. (2b) does not exceed the moment capacity M_{uc} for compression failure, which is calculated as

$$M_{uc} = \alpha \cdot f_c \cdot \beta \cdot c_{cu} \cdot b \cdot \left(d - \frac{1}{2} \cdot \beta \cdot c_{cu}\right) \tag{4}$$

If strain hardening effects in longitudinal rebars are neglected the force F_b corresponding to the ultimate moment is expressed as $F_b = M_u / L$ with M_u the minimum value among those given by Eq. (2b) and Eq. (4). In the case of yielding of the main bars if we refer to $c_{cu}/d=0.2$ Eq. (2b) gives $\frac{M_{uy}}{b \cdot h_b^2 \cdot f_c} = 0.92 \cdot \omega_s$. It can be noteworthy to observe that the assumption $c_{cu}/d=0.2$ can be derived from practical applications and it results useful for the purpose of obtaining simplified analytical expressions from the model.

The neutral axis depth c_{cy} at yielding of the steel rebars can be found with the well-known expression

$$\frac{c_{cy}}{d} = \sqrt{(\rho \cdot n)^2 + 2 \cdot \rho} - \rho \cdot n \tag{5}$$

with n the ratio between the elastic modulus of steel and concrete E_s and E_c (modulus of elasticity of concrete assumed $E_c = 4200 \cdot \sqrt{f_c}$) and ρ the geometrical ratio of main bars $\rho = \frac{A_{sb}}{b \cdot d}$.

The yielding and the ultimate curvatures can be expressed as

$$\phi_y = \frac{\varepsilon_y}{d - c_{cy}} \tag{6}$$

$$\phi_{su} = \frac{\varepsilon_{su}}{d - c_{cu}} \quad M_u<M_c \tag{7}$$

$$\phi_{cu} = \frac{\varepsilon_{cu}}{c_{cu}} \quad M_u>M_c \tag{8}$$

The deflection δ_c and the force F_c at first cracking neglecting the reinforcement and the slippage of steel bars proves to be

$$\delta_c = \frac{2 \cdot \sigma_{ct} \cdot L^2}{3 \cdot E_c \cdot h_b} \tag{9}$$

$$F_c = \sigma_{ct} \cdot \frac{b \cdot h_b^2}{6 \cdot L} \tag{10}$$

with σ_{ct} the tensile strength of concrete assumed as in ACI 318 (2011) in the absence of experimentation.

The deflection of the beam at yielding δ_{by} can be assumed as

$$\delta_{by} = \frac{M_u \cdot L^2}{3 \cdot E_c \cdot J_n} + \theta_y^{slip} \cdot L \tag{11}$$

the moment of inertia being expressed as

$$J_n \cong \frac{b \cdot c_{cy}^{\ 3}}{3} + n \cdot A_f \cdot (d - c_{cy})^2 + n \cdot A_f' \cdot (c_{cy} - c)^2 \tag{12}$$

θ_y^{slip} being the plastic rotation at first yielding due to the slippage of the bar calculated as suggested in Fib Bulletin No. 24. (2003) in the form

$$\theta_y^{slip} = \frac{\varepsilon_y}{2 \cdot \left(d - \frac{c_{cy}}{3}\right)} \cdot \frac{\phi}{4} \cdot \frac{f_y}{f_{b,y}} \tag{13}$$

with $f_{b,y}$ the uniform bond stress along the development length L_b assumed $f_{b,y} = 0.2\sqrt{f_c^{'}}$ for smooth reinforcement as in Fib Bulletin No. 24. (2003) and ϕ the diameter of the bar.

At rupture we have

$$\delta_{bu} = \delta_{by} + \left(\phi_u - \phi_y\right) \cdot l_p \cdot \left(L - \frac{l_p}{2}\right) + \theta_u^{slip} \cdot L \quad (14)$$

l_p being the plastic hinge length assumed as suggested in Thom (1983) in the form $l_p=\alpha \cdot L+c_1$, where c_1 represents a correction for the tension shift effect which occurs due to diagonal cracking and α=the normalized strength increase from yield to ultimate tension steel (e.g., α=0.08 according to Priestley *et al.* 1997).

In Eq. (14) θ_u^{slip} is the plastic rotation at ultimate state due to the slippage of the bar calculated as suggested in Fib Bulletin No. 24. (2003) in the form

$$\theta_u^{slip} = \theta_y^{slip} + \frac{\left(\varepsilon_u - \varepsilon_y\right)}{\left(d - \frac{c_{cu}}{2}\right)} \cdot \frac{\phi}{4} \cdot \frac{f_u - f_y}{f_{b,u}} \quad (15)$$

with ε_u and f_u the ultimate strain and stress of longitudinal bar and $f_{b,u} = \lambda \cdot f_{b,y}$ and λ =1.2

3.2 Column contribution

For the cross-section analysis of the column, the scheme adopted is the one shown in Fig. 4. The moment-axial force domain adopted here has been simplified with respect to the effective domain by considering a bilinear domain (see Fig. 4). This simplification is conservative because this domain is internal to the effective one.

The bilinear domain is constituted by the two linear branches, the first one of which connects the case of pure compression with the case of balanced failure and the second one connects the case of balanced failure with the case of pure flexure. Columns have an area of steel in tension equal to that in compression (A_l=A'_l). This approach has the advantage that the ultimate moment can be calculated with simple analytical expressions. In the case of pure compression and absence of buckling of longitudinal rebars (a hypothesis justified after a preliminary verification) the ultimate axial force is given by

$$N_u = \alpha \cdot f_c \cdot h_c \cdot b + 2 \cdot A_1 \cdot f_{yl} \quad (16)$$

Introducing the dimensionless axial force gives

$$n_u = \frac{N_u}{f_c \cdot h_c \cdot b} = \alpha + 2 \cdot \omega_l \quad (17)$$

$$\omega_s = \frac{A_l \cdot f_{yl}}{b \cdot h_c \cdot f_c} \quad (18)$$

In flexure, neglecting the compressed rebars gives

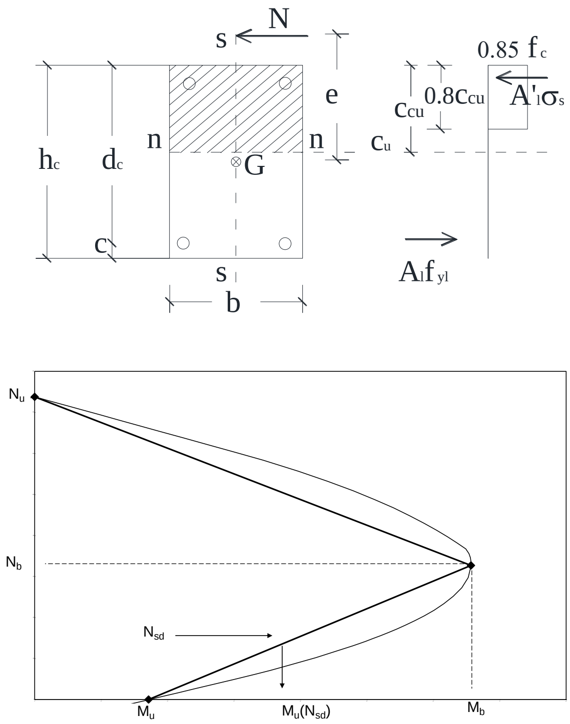


Fig. 4 Design assumptions for analysis of single RC column

$$\frac{M_u}{b \cdot d_c^2 \cdot f_c} \cong 0.9 \cdot \omega_l \quad (19)$$

For balanced failure, it is assumed that both the rebars in compression and in tension yield and, therefore, the equilibrium equations can be written in the following form

$$N_b = \beta \cdot c_{cb} \cdot b \cdot \alpha \cdot f_c \quad (20)$$

$$M_b = N_b \cdot e = \left(\alpha \cdot f_c \cdot \beta \cdot c_{cb} \cdot b\right) \cdot \left(\frac{h_c}{2} - \frac{\beta \cdot c_{cb}}{2}\right) + \\ +2 \cdot A_l \cdot f_{yl} \cdot \left(\frac{h_c}{2} - c\right) \quad (21)$$

With the neutral axis depth given by $\frac{c_{cb}}{h_c} = \left(\frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y}\right) \cdot \left(1 - \frac{c}{h_c}\right)$ with ε_{cu}=0.003 and ε_y=f_y/E_s

If N_b and M_b are dimensionless we have

$$n_b = \frac{N_b}{f_c \cdot h_c \cdot b} = \alpha \cdot \beta \cdot \frac{c_{cb}}{h_c} \quad (22)$$

$$m_b = \frac{M_b}{b \cdot d_c^2 \cdot f_c} = \left(0.5 \cdot \alpha \cdot \beta \cdot \frac{c_{cb}}{d_c}\right) \cdot \left(\frac{h_c}{d_c} - \beta \cdot \frac{c_{cb}}{d_c}\right) + \\ +2 \cdot \omega_l \cdot \left(\frac{h_c}{2 \cdot d_c} - \frac{c}{d_c}\right) \cdot \frac{h_c}{d_c} \quad (23)$$

The ultimate moment associated with the design axial force N_{sd} can be calculated in the following form

$$M_u\left(N_{sd}\right) = M_u + N_{sd} \cdot \left(\frac{M_b - M_u}{N_b}\right) \tag{24}$$

with $N_{sd} \leq N_b$ or in dimensionless form

$$m_u(n) = \frac{M_u(N)}{b \cdot d_c^2 \cdot f_c} = m_u + \frac{n}{n_b} \cdot \left(m_b - m_u\right) \tag{25}$$

with $n \leq n_b$.

If we assume $c/d_c=0.05$, $h_c/d_c=0.95$ and $f_{yl}=280$ MPa Eq. (25) yields the simplified expressions

$$m_u(n) = 0.9 \cdot \omega_l - 0.0765 \cdot \omega_l \cdot n + 0.176 \cdot n \tag{26}$$

with $n<n_b=0.653$. For balanced failure we have: $n_b = 0.623$ and $m_b = 0.125 + 0.85 \cdot \omega_l$

Graphical representation of the simplified moment-axial force domain is shown in Fig. 6 with the numerical domain deduced with the strips method.

Using Eq. (25) the force in the beam determined by the crisis of the column F_c can be expressed as

$$F_c = 2 \cdot \left[m_u + \frac{n}{n_b} \cdot \left(m_b - m_u\right)\right] \cdot \frac{b \cdot d_c^{\,2} \cdot f_c}{L} \tag{27}$$

To determine the load-deflection contribution due to the column, the rotation of the column in the loaded section γ_e is calculated as

$$\gamma_{cy} = \frac{\phi_y \cdot H}{6} \quad with \quad \phi_y = \frac{\varepsilon_y}{h_c - c_{cy}} \tag{28}$$

with the neutral axis depth given by

$$\begin{aligned} & c_{ce}^3 + 3 \cdot \left(e - \frac{h_c}{2}\right) \cdot c_{ce}^2 + \frac{6 \cdot n}{B_c} \cdot \left[\left(A_l^{'} + A_l\right) \cdot \left(e - \frac{h_c}{2} + d\right)\right] \cdot c_{ce} + \\ & - \frac{6 \cdot n}{B_c} \cdot \left[A_f^{'} \cdot c \cdot \left(e - \frac{h_c}{2} + c\right) + A_l \cdot h_c \cdot \left(e - \frac{h_c}{2} + d\right)\right] = 0 \end{aligned} \tag{29}$$

The deflection at the tip of the beam δ_{cy} due to the column at yielding proves to be

$$\delta_{cy} = \gamma_{cy} \cdot L \tag{30}$$

At rupture the rotation γ_{cu} proves to be

$$\gamma_{cu} = \gamma_{cy} + \left(\phi_u - \phi_y\right) \cdot \left(\frac{h_c}{2} - l_p^{\,c}\right) \tag{31}$$

with

$$\phi_u = \frac{\varepsilon_{cu}}{c_{cu}} \tag{32}$$

In Eq. (31) the length l_p^c is assumed as previously done for the beam. Finally, the ultimate displacement δ_{cu} is

$$\delta_{cu} = \delta_{cy} + \gamma_{cu} \cdot L \tag{33}$$

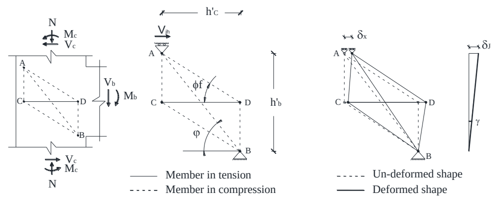


Fig. 5 Strut-and-tie model adopted for beam-to-column joint

3.3 Joint contribution

The model proposed to describe the response of the joint region is the one shown in Fig. 5. It is a multiple truss constituted by one tie, corresponding to the transversal reinforcement (stirrup) inside the joint, and three struts modeling the compressed concrete zones. Among the struts, the main is the one connecting points A and B in Fig. 5.

The truss is restrained by a support in B and by a sliding support in A, which allows the horizontal displacement. The truss is therefore a determined structure for external loading. The horizontal force V_{jh} applied in A (see Fig. 5) represents the shear force that the joint is able to transmit.

Being V_j the angular distortion of the joint, the V_{jh}-γ_j curve can be calculated on the basis of the deformed configuration. Once γ_j and V_{jh} are known, it is possible to obtain the corresponding force F_j and displacement δ_j at the end of the beam with the following expressions

$$F_j = V_{jh} \cdot \frac{h_b}{L} \tag{34}$$

$$\delta_j = \gamma_j \cdot L \tag{35}$$

The geometric characteristics of the truss are the angles between the struts and the horizontal direction and the member lengths and sections shown in Fig. 5. According to Hwang and Lee (1999) the strut width can be expressed as

$$a_s = \sqrt{a_b^2 + a_c^2} \tag{36}$$

a_b and a_c being the compressed zone lengths of the beam and column respectively.

On the other hand, when failure is approaching, the joint compressed zone is negligible $\left(a_b \approx 0\right)$, and thus $a_s=a_c$.

The column compressed length due to the elastic bending moment is given by Park and Paulay (1975)

$$a_c = \left(0.25 + 0.85 \cdot n\right) \cdot h_c \tag{37}$$

The strut width b_s can be assumed equal to the joint width.

The strut angle φ shown in Fig. 7 is the following

$$\tan\varphi = \frac{h_b}{h_c} \tag{38}$$

All the other geometric quantities can be calculated on the basis of h_b', h_c' which are correlated by simple geometrical relations to the strut angles. The strut strength can be expressed as

$$N_u = \psi \cdot f_c \cdot a_c \cdot b \tag{39}$$

ψ being an efficiency factor adopted in strut-and-tie models to take into account the reduction in concrete strength in biaxial compression. Here it is assumed the expression given in Campione (2015) in the form

$$\psi = \frac{1}{1 + 0.66 \cdot \tan\varphi} \tag{40}$$

The static solution of the truss in Fig. 5 can be found after solving the simple truss assuming the stirrups to have yielded.

From the scheme in Fig. 5 it results that the axial force in the main strut is the sum of N_v and N_{st} which are the contribution due to the force V_{jh} and to the yielded stirrups, respectively.

Such contributions can be evaluated as

$$N_V = V_{jh} \cdot \sqrt{1 + \left(\frac{h_b}{h_c}\right)^2} = V_{jh} \cdot \cos\varphi \tag{41}$$

$$N_{st} = -\frac{n_{st} \cdot A_{st} \cdot f_{yst}}{\sin\phi}\left[\frac{1}{2} \cdot \frac{h_b}{h_c} + \tan\phi\right] = \\ = -n_{st} \cdot A_{st} \cdot f_{yst} \cdot \cos\phi \tag{42}$$

Being n_{st}, A_{st} and f_{yst} the number, the area and the yielding stress od stirrups respectively

The axial force in the main strut proves to be

$$N = N_v + N_{st} = \frac{V}{\cos\varphi} - n_{st} \cdot A_{st} \cdot f_{yst} \cdot \cos\varphi \tag{43}$$

Assuming $N=N_u$, utilizing Eq. (37) and solving Eq. (43) with respect to V_{jh} it results

$$V_{jh} = V_u = \psi \cdot f_c \cdot (0.25 + 0.85 \cdot n) \cdot h_c \cdot b \cdot \cos\phi + \\ + n_{st} \cdot A_{st} \cdot f_{yst} \cdot (\cos\phi)^2 \tag{44}$$

And in dimensionless form, taking into account Eq. (38) it results

$$\frac{V_u}{b \cdot h_c \cdot f_c} = \left[\psi \cdot (0.25 + 0.85 \cdot n) + \omega_{st}\right] \cdot \\ \cdot \cos\left(\arctan \cdot \frac{h_b}{h_c}\right) \tag{45}$$

$$\omega_{st} = \frac{n_{st} \cdot A_{st} \cdot f_{yst}}{b \cdot h_c \cdot f_c} \tag{46}$$

In the case here developed no stirrups are considered and Eq. (46) becomes

$$\frac{V_u}{b \cdot h_c \cdot f_c} = [\psi \cdot (0.25 + 0.85 \cdot n)] \cdot \cos\left(\arctan \cdot \frac{h_b}{h_c}\right) \tag{47}$$

The axial force in the main compressed strut proves to be

$$\eta = \frac{N_u \cdot \sqrt{h_b^2 + h_c^2}}{E^* \cdot (0.25 + 0.85 \cdot n) \cdot h_c \cdot b} \tag{48}$$

with

$$E^* = \frac{\psi \cdot f_c}{\varepsilon_o} \tag{49}$$

in which ε_0 is the concrete strain corresponding to f_c. Therefore, the distortion of the joint at rupture is

$$\gamma_j = \frac{\eta \cdot \cos\varphi}{h_b} \tag{50}$$

The displacement at the tip of the beam is

$$\delta_{jy} = \gamma_j \cdot L \tag{51}$$

Substituting Eqs. (47)-(50) into Eq. (51) it results

$$\delta_{jy} = \frac{\varepsilon_o \cdot L}{\psi} \cdot \cos\varphi \cdot \sqrt{1 + \left(\frac{h_c}{h_b}\right)^2} \tag{52}$$

By using Eqs. (34) and (44) it is possible to obtain the ultimate force F associated with the joint failure $V_j=V_u$ in the form

$$F_j = \left[\psi \cdot \cos\varphi \cdot (0.25 + 0.85 \cdot n) \cdot h_c \cdot b \cdot f_c'\right] \cdot \frac{h_b}{L} \tag{53}$$

3.4 Load-deflection response of the structure

After calculating the beam load-deflection curve, the column moment-rotation curve and the joint shear-distortion curve, the total response of the structure, in terms of deflection δ of the beam, can be determined by summing the deflection contribution of each component for a fixed load stage F (in equilibrium with the system) with the following relationship

$$\delta(F) = \delta_b(F) + \delta_c(F) + \delta_j(F) \tag{54}$$

Fig. 6 shows two examples of application of the proposed model in terms of load-deflection curves. The first case examined (named "case (*a*)") refers to a beam-to-column sub assemblage with: - beam having cross-section 300×400 mm, length L=2500 mm, bottom reinforced with 2 smooth rebars having 12 mm and top reinforcement 2 having diameter 16 mm; - column having cross-section 300×300 mm, length H=3000 mm, reinforced with 2

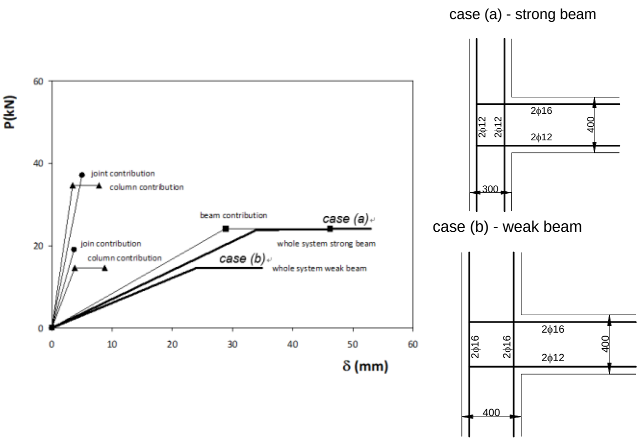


Fig. 6 Theoretical load-deflection response of RC external joint

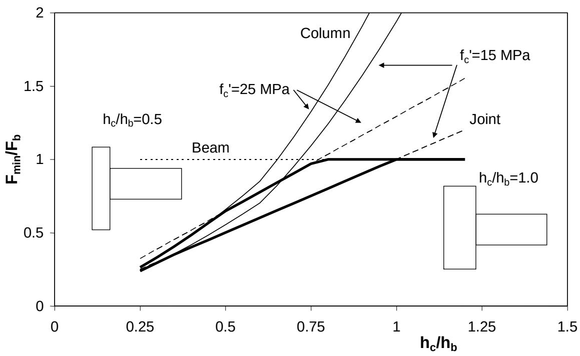


Fig. 7 Dimensionless strength of subassemblages with h_c/h_b variation

smooth rebars having diameter 12 mm, both in tension and compression. The concrete has compressive strength f_c=9.95 MPa (0.83 R_{ck}=12 MPa) and the steel rebars have yielding stress f_y=280 MPa. The tensile strength of the concrete is neglected.

The second example (named "case (*b*)") is similar to the first one but the column has the dimensions 300×400 mm and is reinforced with 2 smooth rebars having a diameter of 16 mm, both in tension and compression. These examples represent two typical cases of existing RC frames designed in the Mediterranean area without seismic detail and referring to a top floor and bottom floor. The comparison shows that in the case of columns and beams both 300×400 mm, failure is governed by the beam crisis, the joint and columns having overstrength with respect to the beam. Conversely, for columns with a 300×300 mm cross-section, the failure is attained by the column and the lower strength of the joint region. These examples show the effectiveness of the proposed model in predicting the structural response of assemblages in spite of a preliminary verification of the ductility of subassemblages and before more detailed and complex finite element nonlinear analyses.

Fig. 7 shows the dimensionless strength domain with variation in the h_c/h_b ratios for two fixed values of f_c=15 and 25 MPa. The strength is the lowest one between F_b, F_c and F_j. The cases examined refer to subassemblies with beams designed for a gravity load with an area of influence

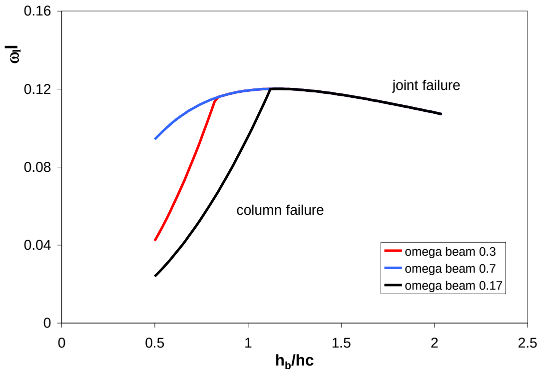


Fig. 8 Variation of mechanical ratio of longitudinal rebars in column with ratio h_b/h_c

of floors of 5 m and a dead load of 60 kN/m^2. The steel grade was 280 MPa and the percentages of steel respected the limit of 0.8% of the area of the concrete transverse cross-section.

From the graph it emerges clearly that for h_c/h_b lower than 1 failure is due to the column and the joint. Most existing buildings designed according to these rules do not respect the principles of ductile design and the risk of brittle failure under a seismic attack is very high.

To check whether the capacity design is satisfied, it should be respected the condition $F_b<F_c<F_y$, being F_b, F_c and F_y the forces in the beam, column and joint respectively. These equations for c/d=0.05, steel grade 280 MPa and n=0.238, give

$$0.56\cdot\omega_s\cdot\left(\frac{h_b}{h_c}\right)^2\le\omega_l\le 0.28\cdot\left(\frac{h_b}{h_c}\cdot\frac{\cos\varphi}{1+0.66\cdot\frac{h_b}{h_c}}\right)\qquad(55)$$

Fig. 10 shows the variation of ϖ_l with h_c/h_b for different values of ϖ_b. It is interesting to observe that for low mechanical ratio of the beam reinforcement, the failure of column is attained for low mechanical ratio of main steel, up to h_b/h_c lower than one.

For higher value of the mechanical ratio, the joint failure occurs before the column failure, resulting thus in a more brittle behavior. By increasing the mechanical ratio of beam reinforcement, the ratio h_b/h_c decreases and a limit of 0.5 is observed.

4. Experimental validation

The proposed model is validated both for the shear strength prevision and for load-deflection response of subassemblages.

For shear strength previsions data collected by Park and Mosalam (2012) have been utilised. They refer to 62 tests utilised in joint without stirrups. The shear strength of the 62 considered exterior RC beam-column joints has been evaluated by means of the procedure provided by: Bakir and Boduroğlu (2002), Vollum and Parker (2008), Park and Mosalam (2012) and current model. Analytical expressions

Table 1 Analytical expressions available for shear strength of external R/C joints without stirrups

Author	Expression of V_u (MPa)
Bakir and Bodurog˘lu (2002)	$\dfrac{0.71\cdot\left(100\cdot\dfrac{A_f}{B_b\cdot h_b}\right)^{0.4289}}{\left(\dfrac{h_b}{h_c}\right)^{0.61}}\cdot h_c\cdot b\cdot\sqrt{f_c^{'}}$
Vollum and Parker (2008)	$0.642\cdot\beta\cdot\left[1+0.555\cdot\left(2-\dfrac{h_b}{h_c}\right)\right]\cdot h_c\cdot b_b\cdot\sqrt{f_c^{'}}$
Park and Mosalam (2012)	$(1.413\cdot k)\cdot h_c\cdot b_b\cdot\sqrt{f_c^{'}}\cdot\cos\varphi$ $k=\left[0.4+\dfrac{0.6}{1.413\cdot\cos\varphi}\cdot\left(\dfrac{SJ_j-0.466\cdot\cos\varphi\cdot\sqrt{f_c^{'}}}{1-0.33\cdot\sqrt{f_c^{'}}}\right)\right]\leq 1$ $SJ_i=\dfrac{A_s\cdot f_y}{B_b\cdot h_c\cdot\sqrt{f_c^{'}}}\cdot\left(1-0.85\cdot\dfrac{h_b}{L_c}\right)$
Proposed model	$\dfrac{\cos\varphi}{1+0.66\cdot\dfrac{h_b}{h_c}}\cdot(0.25+0.85\cdot n)\cdot h_c\cdot b\cdot f_c^{'}$

Table 2 Comparison between experimental and analytical results for 62 data of Park and Mosalam (2012) referred to R/C joints without stirrups

Author	Average	Standard deviation	Correlation factor
Without stirrups			
Bakir Boduroglu (2002)	1.760	0.623	0.354
Vollum and Parker (2008)	1.083	0.327	0.302
Park and Mosalam (2012)	0.851	0.209	0.245
Proposed model	1.080	0.201	0.228

Table 3 Comparison between existing models referred to R/C joints without and with stirrups

Author	Safe	Unsefe	whole	% Safe
Bakir Boduroglu (2002)	58	4	62	93
Vollum and Parker (2008)	17	45	62	27
Park and Mosalam (2012)	19	43	62	30
Proposed model	50	12	62	80

of the above mentioned closed form models are given in Table 1.

The average ratios between the experimental shear strength and the calculated values are given in Table 2, where the corresponding, standard deviation and correlation factor values are also reported.

Fig. 9 shows variation of the experimental shear strength and the calculated values of Park and Mosalam (2012).

The cases considered for experimental validation are those of Russo and Pauletta (2012), Calvi *et al.* (2002) and Braga *et al.* (2009).

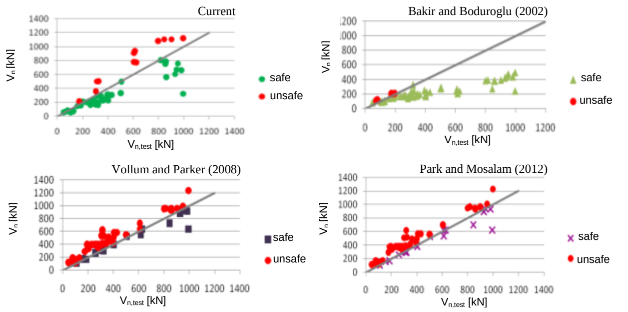


Fig. 9 Comparison between experimental and theoretical shear strength of joint for RC external beam column without stirrups

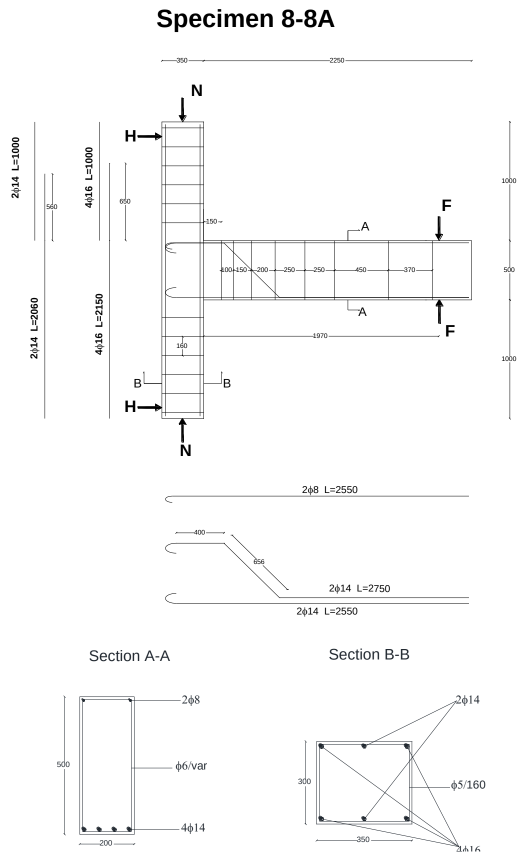


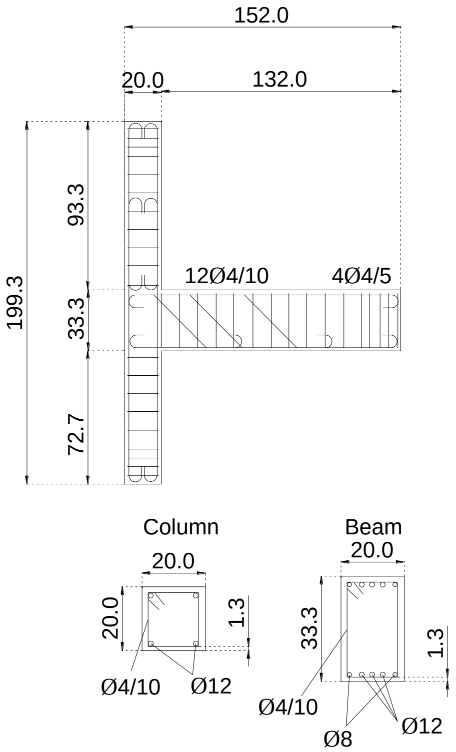
Fig. 10 External RC joint tested in Russo and Pauletta (2012)

For the same models Table 3 gives the number and percentage of safe and unsafe prediction.

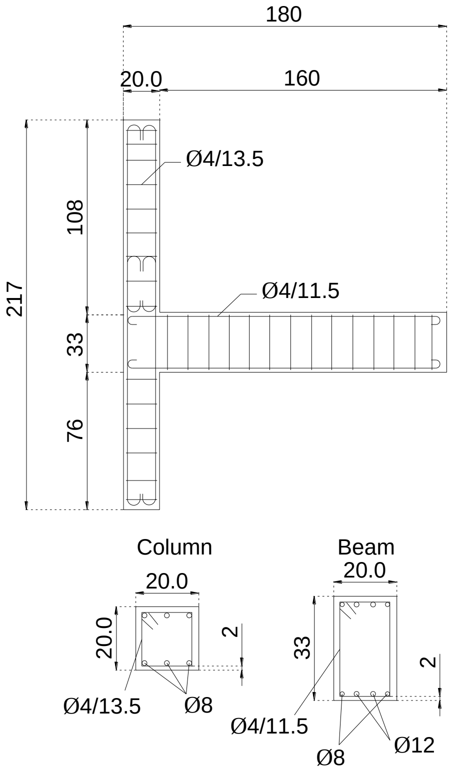
First case examined is that shown in Fig. 10. The first case examined (Russo and Pauletta 2012) refers to an experimental investigation on external (T-Joint) R/C beam-column joints built using concrete with low strength and smooth reinforcing bars, without hoops in the panel zone.

The tests were performed by increasing cyclic horizontal displacements up to collapse. The experimental results show that seismic response of this kind of structures is mainly influenced by bond slips of longitudinal bars, and that the shear collapse regards the external joints rather the internal ones.

All specimens were cast using concrete with average cubic compressive strength equal to 25 MPa, while the



(a) Braga *et al.* (2009)



(b) Calvi *et al.* (2002)

Fig. 11 External RC joint tested in Braga *et al.* (2009) and Calvi *et al.* (2002)

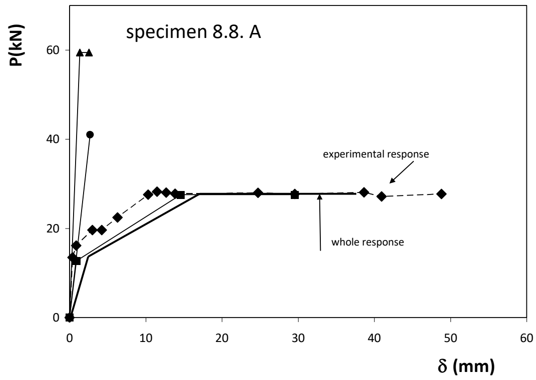
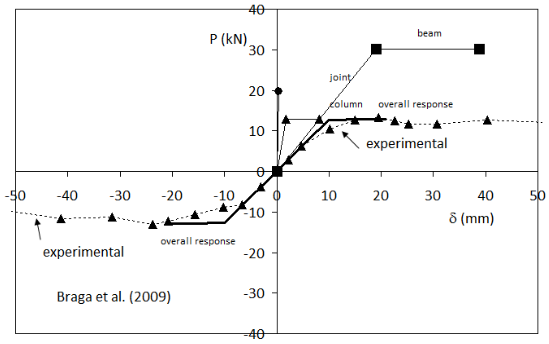
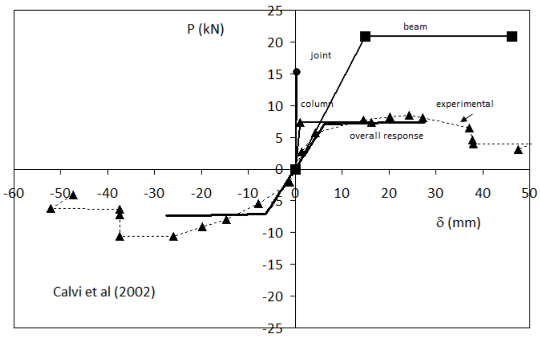


Fig. 12 Theoretical load-deflection response of RC external joint for specimen tested in Russo and Pauletta (2012)



(a) Braga *et al.* (2009)



(b) Calvi *et al.* (2002)

Fig. 13 Theoretical load-deflection response of RC external joint for specimen tested in Braga *et al.* (2009) and Calvi *et al*. (2002)

yield strength of the steel bars was 380 MPa for the 5 mm, 14, 16 mm diameter bars, respectively.

The tests were carried out in a quasi-static way applying a time-history of displacements to the beam tip and, therefore, the force F is present also on the top of the column when the same force acts on the beam from the bottom to the top.

The second case examined (Calvi *et al.* 2002 of Fig. 11) refers to quasi-static cyclic tests performed on six gravity-load-designed beam-column subassemblies. Six one-way beam-column subassembly specimens, 2/3 scaled, were tested, representing the following typologies: - two exterior knee-joints (specimens L); - two exterior tee-joints (specimens T); - two interior cruciform joints (specimens C). Steel smooth bars, with mechanical properties (allowable stress 160 MPa) similar to those typically used in that period, were adopted for both longitudinal and transverse reinforcements.

The main mechanical characteristics of the concrete and reinforcing steel were the following: average cylindrical and cube compression strength equal to 23.9 MPa and 29.1 MPa, respectively; yielding and ultimate stress of the steel reinforcement equal to 385.6 MPa and 451.2 MPa, respectively, for 8 mm diameter rebars, and 345.9 MPa and 458.6 MPa, respectively, for 12 mm diameter rebar. The test setup for the different specimens was intended to reproduce the configuration of a beam-column subassembly in a frame subjected to reversed cyclic lateral loading. A constant value of axial force due to the gravity loads was applied at 100 kN. The axial load was applied by means of a vertical hydraulic jack, acting on a steel plate connected to the column base plate by vertical external post-tensioned bars. No simulated gravity loads were applied to the beam elements.

The third case examined (Braga *et al.* 2009 of Fig. 11) refers to an experimental investigation on four internal (C-Joint) and external (T-Joint) R/C beam-column joints built using concrete with low strength and smooth reinforcing bars, without hoops in the panel zone. The tests were performed by increasing cyclic horizontal displacements up to collapse.

The experimental results show that seismic response of this kind of structures is mainly influenced by bond slips of longitudinal bars, and that the shear collapse regards the external joints rather the internal ones. Failure mechanisms observed (column plastic hinging for internal joints, shear failure for external joints) point out the vulnerability of these structures due to the soft story mechanism. All specimens were cast using concrete with average cubic compressive strength equal to 17.5 MPa, while the yield strength of the steel bars was 350 MPa, 325 MPa and 345 MPa, for the 8 mm, 12 mm and 18 mm diameter bars, respectively. The tests were carried out in a quasi-static way applying a time-history of displacements to the upper column. A vertical load was applied at the head of the upper column equal to 120 kN. The proposed model is applied to the reference dataset and the results are shown in Fig. 12 and Fig. 13. They show good agreement both in terms of mechanisms of failure than of ultimate load and corresponding displacements.

5. Conclusions

In the present paper an analytical model for predicting the flexural response of reinforced concrete beam-column external joints under monotonic loading is presented.

Yielding of main steel and crushing of concrete in the beam, in the column and in the beam-column joint were identified in order to determine the corresponding loads and displacement and to plot the simplified load-deflection curves of the sub-assemblages subjected at the tip of the beam to monotonically increasing lateral force and in the column to a constant vertical load.

The original contribution consists in including in a simple analytical model the main aspects regarding the structural behavior of external beam-joint-column such as slippage on longitudinal bars of beam and brittle failure of joint. The flexural failure of the columns, due to strong beam/weak column design, and the brittle collapse of the external joint in the absence of stirrups due to crushing of the compressed strut are included in the model. The expressions derived, allowing simple hand computation, can be considered a useful instrument for a preliminary verification of the safety state. On the whole, the model gives a physical interpretation of the flexural behaviour of beam-column sub assemblages up to rupture and shows good agreement with the experimental results available in the literature.

Finally, prescriptions on mechanical ratios of steel rebars for ductile design of subassemblages are derived and discussed.

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