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Effect of microtemperatures for micropolar thermoelastic bodies

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Abstract. In this paper we investigate the theory of micropolar thermoelastic bodies whose micro-particles possess microtemperatures. We transform the mixed initial boundary value problem into a temporally evolutionary equation on a Hilbert space and after that we prove the existence and uniqueness of the solution. We also approach the study of the continuous dependence of solution upon initial data and loads.

Keywords: micro-particles; microtemperatures; micropolar; semigroup; continuous dependence

1. Introduction

The study of elastic materials with microstructure was initiated by French Cosserat brothers 1909 and since then it has been investigated intensively in the literature. Eringen (1966) discussed the concept of micropolar continua which was similar to the Cosserat continua. He introduced a conservation law for the microinertia tensor as a particular case of micromorphic continua. Some fundamental results on micropolar bodies can be seen in Refs. Chirita and Ghiba (2012), Dyszlewicz (2004), Iesan (2004), Marin (1994), Marin (1995). The classical elasticity ignores that the response of the material to external stimuli heavily depends on the motions of its inner structure. So, it is not possible to consider this effect by ascribing only translational degrees of freedom to material points of the body. We recall that in the micropolar continuum theory, we have six degrees of freedom, instead of the three described within the classical elasticity. To describe the applied force on the surface element, a couple stress tensor together with classical stress tensor is introduced. Many papers devoted to the theory of microstretch elastic bodies were presented (see e.g., Eringen 1999). This approach is a generalization of the micropolar theory and it represents a particular case of the micromorphic theory as well. According to this approach each material point is equipped with three deformable directors. A body denotes a microstretch continuum if the directors fulfill only breathing-type microdeformations. Other materials with microstructure are studied in Refs. Marin (2010), Marin *et al*. (2014) and some considerations on waves for micropolar bodies are available in Sharma and Marin (2014), Straughan (2011), respectively.

The purpose of these approaches is to diminish the discrepancies between the classical elasticity and experiments, bearing in mind that the classical elasticity is unable to provide acceptable results when the effects of material microstructure were known to contribute significantly to the body's overall deformations. Also, the classical theory of elasticity cannot explain certain discrepancies that appear in problems dealing with elastic vibrations of high frequency and short wavelength.

Grot (see Grot 1969) is considered as the initiator of the theory of bodies with micro-temperatures. He used the approach of bodies with inner structure and developed a theory of thermodynamics of elastic bodies with microstructure whose microelements possess microtemperatures. In this case, the entropy production inequality is adapted to include the microtemperatures. Thus, the first-order moment of the energy equations was added to the well-known balance laws of a continuum with microstructure. Besides, the theory of thermoelasticity with microtemperatures was debated in various papers (see, for instance, Chirita *et al*. (2012), Iesan and Quintanilla (2000), Scalia and Svanadze (2009)).

An intelligent supersize finite element method was employed in the paper Kim *et al*. (2013) for the ultimate longitudinal strength analysis.

In the paper Takabatake (2012), the existence and effect of dead loads are proven by numerical calculations based on the Galerkin method. Some very recent results regarding micro-temperatures are presented in Othman *et al*. (2016a, b), Othman *et al*. (2016). In this paper we discuss the effect of microtemperatures on the main characteristics of the mixed initial boundary value problems for micropolar thermoelastic bodies. It is important to note that the presence of microtemperatures allows the transmission of heat as thermal waves at finite speed. This mixed problem is transformed into an abstract evolution equation on a suitable Hilbert space. After that, by utilizing the results from the theory of semi-groups of operators, we deduce the existence and the uniqueness of solution. Also, the

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continuous dependence of solution upon the initial data and loads is discussed.

2. Basic tools

Below we suppose that a bounded region B of R^3 is filled by a microstretch elastic body, referred to the reference configuration and a fixed system of rectangular Cartesian axes. Here $\overline{B}$ denotes the closure of B and ∂B represents the boundary of B, respectively. In this paper we let ∂B be a piecewise smooth surface and we denote by n_i the components of the outward unit normal to ∂B. The motion of the body is analyzed with respect to a fixed system of rectangular Cartesian axes Ox_i, where $i = 1,2,3$. Let u_i be the coordinates of the displacement vector and φ_i be the coordinates of the microrotation vector, respectively. In addition, let ϕ be the microstretch function and θ be the temperature measured from the constant absolute temperature T_0 of the body.

Below t_{ij} are the components of the stress tensor and m_{ij} represent the components of the couple stress tensor over B. Thus, the equations of motion for micropolar thermoelastic bodies are given by Iesan and Nappa (2005)

$$t_{ji,j} + \varrho F_i = \varrho \ddot{u}_i,$$
$$m_{ji,j} + \varepsilon_{ijk} t_{jk} + \varrho G_i = I_{ij} \ddot{\varphi}_j \tag{1}$$

The balance of the first stress moment has the form

$$\lambda_{i,i} - \sigma + \varrho L = J\ddot{\phi}. \tag{2}$$

In these equations F_i denote the components of the body force, G_i represent the components of the body couple, L is the generalized external body load, ϱ denotes the reference constant mass density, J and $I_{ij} = I_{ji}$ are the coeffcients of microinertia.

If T is the temperature within the body, we denote by θ the temperature measured from the constant absolute temperature T_0 in the body in its reference state, that is, $\theta = T - T_0$. We consider a generic microelement in the reference configuration and denote by (X_i') the coordinates of its center of mass. If (X_i) are the coordinates of an arbitrary point in the body, then we can assume that the absolute temperature in the body is a sum of the form

$$\theta + T_i(X_i' - X_i), \tag{3}$$

where the functions T_i are microtemperatures. We will denote by ϑ_i the microtemperatures measured from the microtemperatures T_i^0 in reference state, namely,$\vartheta_i = T_i - T_i^0$. The behavior of a micropolar thermoelastic body with microtemperatures can be characterized using the above mentioned variables u_i, φ_i, ϕ and the variables χ and τ_i, defined by

$$\chi = \int_{t_o}^{t} \theta dt,\ \tau_i = \int_{t_0}^{t} \vartheta_i dt, \tag{4}$$

in which, obviously, t_0 is a reference time. The components of the strain tensors $\varepsilon_{ij}, \mu_{ij}$ and γ_i are defined as

$$\varepsilon_{ij} = u_{j,i} + \varepsilon_{ijk}\varphi_k, \mu_{ij} = \varphi_{j,i}, \gamma_i = \phi_{,i}. \tag{5}$$

Here ε_{ijk} denotes the alternating symbol.

Using a procedure analogous to that in Iesan and Quintanilla (2000), we obtain the constitutive equations for an anisotropic and homogeneous micropolar thermoelastic body with microtemperatures

$$t_{ij} = A_{ijmn}\varepsilon_{mn} + B_{ijmn}\mu_{mn} + a_{ij}\phi - \alpha_{ij}\dot{\chi} + D_{ijmn}\tau_{m,n},$$
$$m_{ij} = B_{ijmn}\varepsilon_{mn} + C_{ijmn}\mu_{mn} + b_{ij}\phi - \beta_{ij}\dot{\chi} \quad + E_{ijmn}\tau_{m,n},$$
$$\lambda_i = A_{ij}\gamma_j - d_{ij}\dot{\tau}_j + H_{ij}\chi_{,j},$$
$$\sigma = a_{ij}\varepsilon_{ij} + b_{ij}\mu_{ij} + \zeta\phi - \kappa\dot{\chi} + F_{ij}\tau_{i,j},$$
$$\varrho\eta = \alpha_{ij}\varepsilon_{ij} + \beta_{ij}\mu_{ij} + \kappa\phi + a\dot{\chi} + L_{ij}\tau_{i,j},$$
$$\varrho\eta_i = d_{ji}\gamma_j + B_{ij}\dot{\tau}_j + C_{ij}\chi_{,j}$$
$$S_i = H_{ji}\gamma_j - C_{ji}\dot{\tau}_j + K_{ij}\chi_{,j}$$
$$\Lambda_{ij} = D_{ijmn}\varepsilon_{mn} + E_{ijmn}\mu_{ij} + F_{ji}\phi - L_{ji}\dot{\chi} + G_{ijmn}\tau_{m,n} \tag{6}$$

In the above equations, the used notations have the following meanings: t_{ij}, m_{ij} and λ_i are the components of the stress, λ_i are the components of the internal hypertraction vector, σ is the generalized internal body load, η is the entropy per unit mass, η_i is the first entropy moment vector, S_i is the entropy flux vector and Λ_{ij} is the first entropy flux moment tensor.

Also, the quantities $A_{ijmn}, B_{ijmn}, \ldots, L_{ji}$ and G_{ijmn} are characteristic constitutive coeffi-cients and they obey to the following symme-try relations

$$A_{ijmn} = A_{mnij}, C_{ijmn} = C_{mnij}, A_{ij} = A_{ji},$$
$$B_{ij} = B_{ji}, K_{ij} = K_{ji}, a_{ij} = a_{ji}, b_{ij} = b_{ji},$$
$$D_{ijmn} = D_{jimn},\ E_{ijmn} = E_{jimn}, G_{ijmn} = G_{mnij} \tag{7}$$

If we denote by ξ_i the internal rate of production of entropy per unit mass and by H_i the mean entropy flux vector, then from the equation of energy we deduce the relation

$$\varrho\xi_i + S_i - H_i = 0, \tag{8}$$

wherein the meaning of S_i was exposed above.

Also, if we denote by s the external rate of supply of entropy per unit mass and by Q_i the first moment of the external rate of supply of entropy, we can write two more equations of energy

$$\varrho\dot{\eta} = S_{i,i} + \varrho s,$$
$$\varrho\dot{\eta}_i = \Lambda_{ji,j} + \varrho Q_i. \tag{9}$$

We substitute the geometric Eq. (5) and the constitutive Eq. (6) into equations of motion (1), in balance of first stress moment (2) and into equations of energy (9). Thus, we get a system of partial differential equations in which the unknown functions are $u_i, \varphi_i, \phi, \chi$ and τ_i, namely

$$A_{ijmn}\left(u_{m,nj} + \varepsilon_{mnk}\varphi_{k,j}\right) + B_{ijmn}\varphi_{n,mj} + a_{ij}\phi_{,j} - \alpha_{ij}\dot{\chi}_{,j} + D_{ijmn}\tau_{m,nj} + \varrho F_i = \varrho\ddot{u}_i,$$

$$B_{ijmn}\left(u_{m,nj} + \varepsilon_{mnk}\varphi_{k,j}\right) + C_{ijmn}\varphi_{n,mj} + b_{ij}\phi_{,j} - \beta_{ij}\dot{\chi}_{,j} +$$

$$E_{ijmn}\tau_{m,nj}+\varepsilon_{ijk}\big[A_{jkmn}(u_{m,n}+\varepsilon_{mnk}\varphi_k)+B_{jkmn}\varphi_{n,m}+a_{jk}\phi-\alpha_{jk}\dot{\chi}+D_{jkmn}\tau_{m,n}\big]+\varrho G_i=I_{ij}\ddot{\varphi}_j,$$
$$A_{ij}\phi_{,ij}-d_{ij}\dot{\tau}_{j,i}+H_{ij}\chi_{,ij}-a_{ij}(u_{j,i}+\varepsilon_{ijk}\varphi_k)$$
$$-b_{ij}\varphi_{j,i}-\zeta\phi-\kappa\dot{\chi}-F_{ij}\tau_{i,j}+\varrho L=J\ddot{\phi},$$
$$H_{ji}\phi_{,ij}-D_{ij}\dot{\tau}_{j,i}+K_{ij}\chi_{,ij}-\alpha_{ij}(\dot{u}_{j,i}+\varepsilon_{ijk}\dot{\varphi}_k)$$
$$-\beta_{ij}\dot{\varphi}_{j,i}-\kappa\dot{\phi}-a\ddot{\chi}=-\varrho s,$$
$$D_{ijmn}(u_{m,nj}+\varepsilon_{mnk}\varphi_{k,j})+E_{ijmn}\varphi_{n,mj}+F_{ji}\phi_{,j}-D_{ji}\dot{\chi}_{,j}$$
$$+G_{ijmn}\tau_{m,nj}-d_{ij}\dot{\phi}_{,j}-B_{ij}\ddot{\tau}_j=-\varrho Q_i \tag{10}$$

Here we used the notation $D_{ij}=C_{ij}+L_{ij}$.

If we take into account the Dirichlet problem associated to the system of Eq. (10), then the boundary conditions have the form

$$u_i=\bar{u}_i,\varphi_i=\bar{\varphi}_i,\phi=\bar{\phi},\chi=\bar{\chi},$$
$$\tau_i=\bar{\tau}_i\ on\ \partial B\times(0,\infty) \tag{11}$$

where $\bar{u}_i,\bar{\varphi}_i,\bar{\phi},\bar{\chi},\bar{\tau}_i$ are known functions.

For a boundary value problem of Neumann type, the boundary conditions (11) are replaced as

$$t_{ji}n_j=\bar{t}_i,m_{ji}n_j=\bar{m}_i,\lambda_jn_j=\bar{\lambda},$$
$$S_jn_j=\bar{S},\Lambda_{ji}n_j=\bar{\Lambda}_i\ on\ \partial B\times(0,\infty) \tag{12}$$

where, also, the functions $\bar{t}_i,\bar{m}_i,\bar{\lambda},\bar{S}$ and $\bar{\Lambda}_i$ are given. In the following we restrict our considerations only on the Dirichlet problem.

The mixed initial boundary value problem associated to the system (10) is complete if we consider the initial conditions, namely

$$u_i(x,0)=u_i^0(x),\dot{u}_i(x,0)=u_i^1(x),$$
$$\varphi_i(x,0)=\varphi_i^0(x),\ \dot{\varphi}_i(x,0)=\varphi_i^1(x),$$
$$\phi(x,0)=\phi^0(x),\dot{\phi}(x,0)=\phi^1(x),$$
$$\chi(x,0)=\chi^0(x),\dot{\chi}(x,0)=\chi^1(x),$$
$$\tau_i(x,0)=\tau_i^0(x),\dot{\tau}_i(x,0)=\tau_i^1(x) \tag{13}$$

for any $x\in B$. Here the functions $u_i^0,u_i^1,\varphi_i^0,\varphi_i^1,\phi^0,\phi^1,\chi^0,\chi^1,\tau_i^0$ and τ_i^1 are prescribed.

3. Qualitative results of the solutions

Below we investigate the existence and uniqueness of solution of the mixed initial boundary value problem in our context. Also, we obtain the continuous dependence of solution with respect to the initial data and charges.

We assume that the functions, that appear in the equations and the conditions formulated in Section 2, are sufficiently regular on their domain of definition to allow the mathematical operations.

For the next result of uniqueness, we need the following auxiliary result.

Theorem 1. Among the variables that characterize the deformation of a thermoelastic micropolar body with microtemperatures the following equality holds true

$$t_{ij}\varepsilon_{ij}+m_{ij}\mu_{ij}+\lambda_i\phi_{,i}+\sigma\phi+\varrho\eta\dot{\chi}+\varrho\eta_i\dot{\tau}_i+S_i\chi_{,i}+\Lambda_{ij}\tau_{i,j}=$$
$$A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+2B_{ijmn}\varepsilon_{ij}\mu_{mn}+2a_{ij}\varepsilon_{ij}\phi+2D_{ijmn}\varepsilon_{ij}\tau_{m,n}$$
$$+C_{ijmn}\mu_{ij}\mu_{mn}+2b_{ij}\mu_{ij}\phi+2E_{ijmn}\mu_{ij}\tau_{m,n}+A_{ij}\phi_{,i}\phi_{,j}$$
$$+2H_{ij}\phi_{,i}\chi_{,j}+\zeta\phi^2+2F_{ij}\tau_{i,j}\phi+K_{ij}\chi_{,i}\chi_{,j}$$
$$+G_{ijmn}\tau_{m,n}\tau_{i,j}+a\dot{\chi}^2+B_{ij}\dot{\tau}_i\dot{\tau}_j \tag{14}$$

Proof. Multiply each equation of the system of the constitutive Eq. (6), namely $t_{ij}\cdot\varepsilon_{ij},m_{ij}\cdot\mu_{ij},\lambda_i\cdot\phi_{,i},\sigma\cdot\phi,\varrho\eta\cdot\dot{\tau}_i,S_i\cdot\chi_{,i}$ and $\Lambda_{ij}\cdot\tau_{i,j}$. Then, we add the equalities which are obtained and considering the relations of symmetry (7) we obtain the desired equality (14).

The quadratic form U is defined as follows

$$U=\frac{1}{2}[A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+2B_{ijmn}\varepsilon_{ij}\mu_{mn}+2a_{ij}\varepsilon_{ij}\phi$$
$$+2D_{ijmn}\varepsilon_{ij}\tau_{m,n}+C_{ijmn}\mu_{ij}\mu_{mn}+2b_{ij}\mu_{ij}\phi$$
$$+2E_{ijmn}\mu_{ij}\tau_{m,n}+A_{ij}\phi_{,i}\phi_{,j}+2H_{ij}\phi_{,i}\chi_{,j}+$$
$$\zeta\phi^2+2F_{ij}\tau_{i,j}\phi+K_{ij}\chi_{,i}\chi_{,j}+\quad G_{ijmn}\tau_{m,n}\tau_{i,j}] \tag{15}$$

We can state and prove the uniqueness of the solution of the mixed initial boundary value problem considered in the previous section.

Theorem 2. We assume that the following assumptions are met

1. ϱ,I_{ij},J and the constitutive coefficient a are strictly positive;
2. the symmetry relations (7) hold true;
3. the quadratic form U defined in (15) is positive semi-definite;
4. the constitutive coefficients B_{ij} are compo-nents of a positive definite tensor. Then, the mixed initial boundary value problem consists of Eq. (10), the initial conditions (13) and boundary conditions (11) admits at most one solution.

Proof. As in the proof of Theorem 1 we start by multiplying each equation of the system of the constitutive Eq. (6) as follows: $t_{ij}\cdot\dot{\varepsilon}_{ij},m_{ij}\cdot\dot{\mu}_{ij},\lambda_i\cdot\dot{\phi}_{,i},\sigma\cdot\dot{\phi},\varrho\dot{\eta}\cdot\dot{\tau}_i,S_i\cdot\dot{\chi}_{,i}$ and $\Lambda_{ij}\cdot\dot{\tau}_{i,j}$. Then, we add the equalities which are obtained and considering the relations of symmetry (7) and the quadratic form U from (15) we report the following equality

$$t_{ij}\dot{\varepsilon}_{ij}+m_{ij}\dot{\mu}_{ij}+\lambda_i\dot{\phi}_{,i}+\sigma\dot{\phi}+\varrho\dot{\eta}\dot{\chi}+\varrho\dot{\eta}_i\dot{\tau}_i+S_i\dot{\chi}_{,i}$$
$$+\Lambda_{ij}\dot{\tau}_{i,j}=\frac{\partial}{\partial t}(U+\tfrac{1}{2}a\dot{\chi}^2+\tfrac{1}{2}B_{ij}\dot{\tau}_i\dot{\tau}_j) \tag{16}$$

Now, we take into account the geometric Eq. (5), the equations of motion (1), the balance of first stress moment (2) and the equations of energy (9). So, the following equality is obtained

$$t_{ij}\dot{\varepsilon}_{ij}+m_{ij}\dot{\mu}_{ij}+\lambda_i\dot{\phi}_{,i}$$
$$+\sigma\phi+\varrho\dot{\eta}\dot{\chi}+\varrho\dot{\eta}_i\dot{\tau}_i+S_i\dot{\chi}_{,i}+\Lambda_{ij}\dot{\tau}_{i,j}$$

$$\doteq \left(t_{ij}\dot{u}_i + m_{ij}\dot{\varphi}_i + \lambda_j\dot{\phi} + S_j\dot{\chi} + \Lambda_{ij}\dot{\tau}_i\right)_{,j}$$
$$+\varrho\left(F_i\dot{u}_i + G_i\dot{\varphi}_i + L\dot{\phi} + s\dot{\chi} + Q_i\dot{\tau}_i\right) -$$
$$-\varrho\ddot{u}_i\dot{u}_i - I_{ij}\ddot{\varphi}_i\dot{\varphi}_j - J\ddot{\phi}\dot{\phi} \quad (17)$$

It is easy to see that Eqs. (16) and (17) provide the equality

$$\frac{1}{2}\frac{\partial}{\partial t}\left(2U + \varrho\dot{u}_i\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + J\dot{\phi}^2 + a\dot{\chi}^2 + B_{ij}\dot{\tau}_i\dot{\tau}_j\right)$$
$$= (t_{ij}\dot{u}_i + m_{ij}\dot{\varphi}_i + \lambda_j\dot{\phi} + S_j\dot{\chi} + \Lambda_{ij}\dot{\tau}_i)_{,j} +$$
$$\varrho(F_i\dot{u}_i + G_i\dot{\varphi}_i + L\dot{\phi} + s\dot{\chi} + Q_i\dot{\tau}_i) \quad (18)$$

The equality (18) is integrated over the domain B such that, with the help of the divergence theorem, we obtain

$$\frac{1}{2}\frac{\partial}{\partial t}\int_B \left(2U + \varrho\dot{u}_i\dot{u}_i + I_{ij}\dot{\varphi}_i\dot{\varphi}_j + J\dot{\phi}^2 + a\dot{\chi}^2\right.$$
$$\left.+B_{ij}\dot{\tau}_i\dot{\tau}_j\right)dV = \int_{\partial B}(t_{ji}\dot{u}_i + m_i\dot{\varphi}_i + \lambda_j\dot{\phi} + S_j\dot{\chi}$$
$$+\Lambda_{ij}\dot{\tau}_i)n_j dA + \int_B \varrho(F_i\dot{u}_i + G_i\dot{\varphi}_i + L\dot{\phi} + s\dot{\chi}$$
$$+Q_i\dot{\tau}_i)dV \quad (19)$$

where n_i are the components of the outward unit normal of the surface ∂B.

We will mark with "*" the difference of two solutions of the mixed problem consisting of (10), (13) and (11), that is,

$$u_i^* = u_i^2 - u_i^1, \varphi_i^* = \varphi_i^2 - \varphi_i^1,$$
$$\phi^* = \phi^2 - \phi^1, \chi^* = \chi^2 - \chi^1, \tau_i^* = \tau_i^2 - \tau_i^1$$

Also, we will mark with "*" the other quantities which correspond to the above differences. Because of linearity, these differences satisfy the equations of motion (1), the balance of first stress moment (2) and the energy Eq. (9) but with null body loads. Also, the initial conditions become homogeneous, that is, for any $x \in B$,

$$u_i^*(x,0) = 0, \dot{u}_i^*(x,0) = 0,$$
$$\varphi_i^*(x,0) = 0, \dot{\varphi}_i^*(x,0) = 0, \phi^*(x,0) = 0,$$
$$\dot{\phi}^*(x,0) = 0, \chi^*(x,0) = 0, \dot{\chi}^*(x,0) = 0,$$
$$\tau_i^*(x,0) = 0, \dot{\tau}_i^*(x,0) = 0, \quad (20)$$

and, certainly, the boundary conditions become null

$$u_i^* = 0, \varphi_i^* = 0, \phi^* = 0, \chi^* = 0,$$
$$\tau_i^* = 0 \; on \; \partial B \times (0,\infty), \quad (21)$$

and

$$\varepsilon_{ij}^*(x,0) = 0, \mu_{ij}^*(x,0) = 0, \phi_{,i}^*(x,0) = 0,$$
$$\chi_{,i}^*(x,0) = 0, \tau_{i,j}^*(x,0) = 0, x \in B. \quad (22)$$

Taking into account these considerations, the relation (19) written for these differences, becomes

$$\int_B (2U^* + \varrho\dot{u}_i^*\dot{u}_i^* + I_{ij}\dot{\varphi}_i^*\dot{\varphi}_j^* + J\left(\dot{\phi}^*\right)^2 +$$
$$a(\dot{\chi}^*)^2 + B_{ij}\dot{\tau}_i^*\dot{\tau}_j^*)dV = 0, t \geq 0 \quad (23)$$

Based on the hypothesis 3 of the theorem and by using (22) we deduce that the quadratic form U written for the differences becomes null. From (23) we deduce

$$\int_B [\varrho\dot{u}_i^*\dot{u}_i^* + I_{ij}\dot{\varphi}_i^*\dot{\varphi}_j^* + J\left(\dot{\phi}^*\right)^2 +$$
$$a(\dot{\chi}^*)^2 + B_{ij}\dot{\tau}_i^*\dot{\tau}_j^*]dV = 0 \quad (24)$$

Considering the hypothesis 1 of the theorem regarding the amounts ϱ, I_{ij}, J and a and the hypothesis 4 regarding the tensor B_{ij}, from (24) we must have

$$\dot{u}_i^* = 0, \dot{\varphi}_i^* = 0, \dot{\phi}^* = 0, \dot{\chi}^* = 0,$$
$$\dot{\tau}_i^* = 0, on \; B \times (0,\infty)$$

such that, if we take into account (20), we deduce

$$u_i^* = 0, \varphi_i^* = 0, \phi^* = 0, \chi^* = 0,$$
$$\tau_i^* = 0, on \; B \times (0,\infty).$$

So, the proof of the theorem is complete.

We shall prove now a result of the existence of solution for the mixed initial boundary value problem mentioned above, but in the case where the boundary conditions are homogeneous, that is

$$u_i = \varphi_i = \phi = \chi = \tau_i = 0, on \; \partial B \times (0,\infty), \quad (25)$$

Taking into account that the system of governing equations and conditions for the investigated problem are more sophisticated, it is necessary a new approach for the existence of solution. Thus, we will transform the problem into an abstract evolution equation on a Hilbert space suitable chosen.

Using the usual Hilbert spaces $W_0^{1,2}$ and L^2, we consider the Hilbert space $\mathcal{H}$ defined by

$$\mathcal{H} = \mathbf{W}_0^{1,2} \times \mathbf{L}^2 \times \mathbf{W}_0^{1,2} \times \mathbf{L}^2 \times W_0^{1,2} \times L^2$$
$$\times W_0^{1,2} \times L^2 \times \mathbf{W}_0^{1,2} \times \mathbf{L}^2$$

where we used the notation $\mathbf{W}_0^{1,2} = W_0^{1,2} \times W_0^{1,2} \times W_0^{1,2}$, or, shorter, $\mathbf{W}_0^{1,2} = [W_0^{1,2}]^3$. Also, $L^2 = [L^2]^3$. For more details about Hilbert and Sobolev spaces see Adams (1975). On the space $\mathcal{H}$ we define the following scalar product

$$\langle, (u_i, U_i, \varphi_i, \Psi_i, \phi, \Phi, \chi, \mu, \tau_i, \nu_i)\rangle =$$
$$\frac{1}{2}\int_B (\varrho U_i U_i^* + I_{ij}\Psi_i\Psi_i^* + J\mu\mu^* + a\mu\mu^* + B_{ij}\nu_i\nu_i^*)dV$$
$$+\frac{1}{2}\int_B [A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}^* + B_{ijmn}\left(\varepsilon_{ij}\mu_{mn}^* + \varepsilon_{ij}^*\mu_{mn}\right) +$$
$$a_{ij}\left(\varepsilon_{ij}\phi^* + \varepsilon_{ij}^*\phi\right) + D_{ijmn}\left(\varepsilon_{ij}\tau_{m,n}^* + \varepsilon_{ij}^*\tau_{m,n}\right) +$$
$$C_{ijmn}\mu_{ij}\mu_{mn}^* + b_{ij}\left(\mu_{ij}\phi^* + \mu_{ij}^*\phi\right)$$
$$+E_{ijmn}\left(\mu_{ij}\tau_{m,n}^* + \mu_{ij}^*\tau_{m,n}\right) + A_{ij}\phi_{,i}\phi_{,j}^*$$
$$H_{ij}\left(\phi_{,i}\chi_{,j}^* + \phi_{,i}^*\chi_{,j}\right) + \zeta\phi\phi^* + F_{ij}$$
$$(\tau_{i,j}\phi^* + \tau_{i,j}^*\phi) +]dV \quad (26)$$

We can prove that the norm induced by this scalar product is equivalent to the original norm on the Hilbert space $\mathcal{H}$.

Now, with a suggestion given by (10), we introduce the operators

$$A_i^1 u = \frac{1}{\varrho} A_{ijmn} u_{m,nj},$$

$$A_i^2 \varphi = \frac{1}{\varrho} \left[A_{ijmn} \varepsilon_{mnk} \varphi_{k,j} + B_{ijmn} \varphi_{n,mj} \right],$$

$$B_i^1 \phi = \frac{1}{\varrho} a_{ij} \phi_{,j}, \; C_i^1 \mu = \frac{1}{\varrho} \alpha_{ij} \mu_{,j},$$

$$D_i^1 \tau = \frac{1}{\varrho} D_{ijmn} \tau_{m,nj},$$

$$A_i^3 u = \frac{1}{I_{ij}} \left(B_{ijmn} u_{m,nj} + \varepsilon_{ijk} A_{jkmn} u_{m,n} \right),$$

$$A_s^4 \varphi = W_{si} \left[A_{ijmn} \varepsilon_{jmn} \varphi_j + B_{ijmn} \varepsilon_{jmn} \varphi_{n,m} + C_{ijmn} \varphi_{n,mj} \right],$$

$$B_s^2 \phi = W_{si} \left(b_{ij} \phi_{,j} + a_{jk} \varepsilon_{ijk} \phi \right),$$

$$C_s^2 \mu = -W_{si} \left(\beta_{ij} \mu_{,j} + \varepsilon_{ijk} \alpha_{jk} \mu \right),$$

$$D_s^2 \tau = W_{si} \left(E_{ijmn} \tau_{m,nj} + \varepsilon_{ijk} D_{jkmn} \tau_{m,n} \right),$$

$$E\phi = \frac{1}{J} \left(A_{ij} \phi_{,ij} - \zeta \phi \right), F\nu = -\frac{1}{j} d_{ij} \nu_{j,i},$$

$$G\chi = \frac{1}{J} H_{ij} \chi_{,ij}, Hu = -\frac{1}{J} a_{ij} u_{j,i},$$

$$K\varphi = -\frac{1}{J} \left(a_{ij} \varepsilon_{ijk} \varphi_k + b_{ij} \varphi_{j,i} \right),$$

$$L\mu = \frac{1}{J} \kappa \mu, M\tau = -\frac{1}{J} F_{ij} \tau_{i,j}, N\chi = \frac{1}{a} K_{ij} \chi_{,ij},$$

$$P\phi = \frac{1}{a} H_{ij} \phi_{,ij}, Q\nu = \frac{1}{a} D_{ij} \nu_{j,i}, R^1 \mathrm{v} = \frac{1}{a} \alpha_{ij} \nu_{i,j},$$

$$R^2 \Psi = \frac{1}{a} (\alpha_{ij} \varepsilon_{ijk} \Psi_k + \beta_{ij} \Psi_{j,i}),$$

$$S\Phi = -\frac{1}{a} \kappa \Phi, A_s^5 u = \Gamma_{si} D_{ijmn} u_{m,nj},$$

$$A_s^6 \varphi = \Gamma_{si} \left(D_{ijmn} \varepsilon_{mnk} \varphi_{k,j} + E_{ijmn} \varphi_{n,mj} \right),$$

$$W_s \phi = \Gamma_{si} F_{ij} \phi_{,j}, X_s \mu = -\Gamma_{si} D_{ij} \mu_{,j},$$

$$Y_s \tau = \Gamma_{si} G_{ijmn} \tau_{m,nj},$$

$$Z_s \Phi = -\Gamma_{si} d_{ji} \Phi_{,j} \tag{27}$$

in which the matrices W_{si} and Γ_{si} are defined by means of the equations

$$W_{si} J_{ir} = \delta_{sr}, \Gamma_{si} B_{ir} = \delta_{sr}.$$

If we denote by $\mathcal{T}$ the matrix operator which has as components the operators defined in (27), then the mixed initial boundary value problem is transformed into a Cauchy problem associated to an evolutionary equation, namely

$$\frac{d\mathcal{U}}{dt} = \mathcal{T}\mathcal{U}(t) + \mathcal{F}(t),$$

$$\mathcal{U}(0) = \mathcal{U}_0 \tag{28}$$

In order to use the theoretical results that follow, we have to take as domain for the operator $\mathcal{T}$, that is, $D(\mathcal{T})$, the next set

$$\left(\mathbf{W}_0^{1,2} \times \mathbf{W}^{2,2} \right) \times \mathbf{W}_0^{1,2} \times \left(\mathbf{W}_0^{1,2} \cap \mathbf{W}^{2,2} \right) \times$$
$$\mathbf{W}_0^{1,2} \times (W_0^{1,2} \cap W^{2.2}) \times W_0^{1,2} \times (W_0^{1,2} \cap W^{2,2})$$
$$\times W_0^{1,2} \times (W_0^{1,2} \cap W^{2,2}) \times W_0^{1,2} \times (W_0^{1,2} \cap W^{2,2})$$
$$\times W_0^{1,2} \times (\mathbf{W}_0^{1,2} \cap \mathbf{W}^{2,2}) \times \mathbf{W}_0^{1,2} \times (\mathbf{W}_0^{1,2} \cap \mathbf{W}^{2,2}) \times \mathbf{W}_0^{1,2}$$

Also, the unknown matrix function $\mathcal{U}$, the initial data $\mathcal{U}_0$ and the matrix of charges $\mathcal{F}$ are defined by

$$\mathcal{U} = (u_i, v_i, \varphi_i, \Psi_i, \phi, \Phi, \chi, \mu, \tau_i, \nu_i),$$
$$\mathcal{U}_0 = (u_i^0, v_i^0, \varphi_i^0, \Psi_i^0, \phi^0, \Phi^0, \chi^0, \mu^0, \tau_i^0, \nu_i^0),$$
$$\mathcal{F} = (0, \mathcal{F}_i, 0, G_i, 0, L, 0, s, 0, Q_i).$$

In the next theorem, we will show, a property of the operator $\mathcal{T}$ which is needed to prove the existence of the solution of the abstract problem (28).

Theorem 3. We assume that the following assumptions are fulfilled:

1. ϱ, I_{ij}, J and the constitutive coefficient a are strictly positive;
2. The symmetry relations (7) hold true;
3. The quadratic form U defined in (15) is positive definite;
4. The constitutive coefficients B_{ij} are components of a positive definite tensor. Then, the operator $\mathcal{T}$ is dissipative.

Proof. In fact, we have to prove that

$$\langle \mathcal{T}\mathcal{U}, \mathcal{U} \rangle \leq 0, \forall \mathcal{U} \in D(\mathcal{T}). \tag{29}$$

Let us consider $\mathcal{U}$, an arbitrary element in the domain of the operator $\mathcal{T}$. Taking into account the definition of the scalar product (26) and the expressions of the operators defined in (27), we obtain

$$\langle \mathcal{T}\mathcal{U}, \mathcal{U} \rangle = -\int_{\partial B} \left(t_{ji} U_i + m_{ij} \Psi_i + \lambda_j \Phi + S_j \mu + \Lambda_{ij} \nu_i \right)$$
$$n_j dA + \int_B \left[A_{ijmn} \varepsilon_{ij} \varepsilon_{mn}^* + B_{ijmn} \left(\varepsilon_{ij} \mu_{mn}^* + \varepsilon_{ij}^* \mu_{mn} \right) \right.$$
$$+ a_{ij} \left(\varepsilon_{ij} \phi^* + \varepsilon_{ij}^* \phi \right) + D_{ijmn} \left(\varepsilon_{ij} \tau_{m,n}^* + \varepsilon_{ij}^* \tau_{m,n} \right)$$
$$+ C_{ijmn} \mu_{ij} \mu_{mn}^* + b_{ij} \left(\mu_{ij} \phi^* + \mu_{ij}^* \phi \right)$$
$$+ E_{ijmn} \left(\mu_{ij} \tau_{m,n}^* + \mu_{ij}^* \tau_{m,n} \right) + A_{ij} \phi_{,i} \phi_{,j}^*$$
$$+ H_{ij} \left(\phi_{,i} \chi_{,j}^* + \phi_{,i}^* \chi_{,j} \right) + \zeta \phi \phi^* + F_{ij} \left(\tau_{i,j} \phi^* + \tau_{i,j}^* \phi \right)$$
$$\left. + K_{ij} \chi_{,i} \chi_{,j}^* + G_{ijmn} \tau_{m,n} \tau_{i,j}^* \right] dV \tag{30}$$

The integrand in the last integral from (30) is a quadratic form which corresponds to the elements $\omega = (u_i, \varphi_i, \phi, \chi, \tau_i)$ and $\omega^* = (U_i, \Psi_i, \Phi, \mu, \nu_i)$, that is, this integral is of the form

$$\int_B W(\omega, \omega^*) dV$$

$$= \int_B W\left((u_i, \varphi_i, \phi, \chi, \tau_i), (U_i, \Psi_i, \Phi, \mu, \nu_i) \right) dV$$

Keep in mind this observation and apply the divergence theorem in the first integral in (30) to get

$$\langle \mathcal{T}\mathcal{U}, \mathcal{U} \rangle$$

$$= -\int_B \left(t_{ji} U_{i,j} + m_{ji} \Psi_i + \lambda_j \Phi_{,j} + S_j \mu_{,j} + \Lambda_{ij} \nu_{i,j} \right) dV$$

$$+\int_B W\big((u_i,\varphi_i,\phi,\chi,\tau_i),(U_i,\Psi_i,\Phi,\mu,\nu_i)\big)dV=0,$$

which concludes the proof of the theorem.

The property of the operator $\mathcal{T}$ which will be proved in the following theorem is essential to characterize the solution of the problem (28).

Theorem 4. Suppose that the conditions of Theorem 3 are satisfied. Then the operator $\mathcal{T}$ satisfies the range condition.

Proof. Let $\mathcal{U}^*$ be an element of the Hilbert space $\mathcal{H}$, such that

$$\mathcal{U}^*=(u_i^*,U_i^*,\varphi_i^*,\Psi_i^*,\phi^*,\Phi^*,\chi^*,\mu^*,\tau_i^*,\nu_i^*).$$

The affirmation of the statement of the theorem is equivalent to showing that the equation $\mathcal{T}\mathcal{U}=\mathcal{U}^*$ has a solution $\mathcal{U}\in D(\mathcal{T})$. In view of operators (27), we use the vector notations

$$\boldsymbol{A}^1=(A_i^1),\boldsymbol{A}^2=(A_i^2),\boldsymbol{A}^3=(A_i^3),$$
$$\boldsymbol{A}^4=(A_s^4),\boldsymbol{A}^5=(A_s^5),\boldsymbol{A}^6=(A_s^6),$$
$$\boldsymbol{B}^1=(B_i^1),\boldsymbol{B}^2=(B_s^2),\boldsymbol{C}^1=(C_i^1),$$
$$\boldsymbol{C}^2=(C_s^2),\boldsymbol{D}^1=(D_i^1),\boldsymbol{D}^2=(D_s^2),$$
$$\boldsymbol{W}=(W_s),\boldsymbol{X}=(X_s),\ \ \boldsymbol{Y}=(Y_s),\boldsymbol{Z}=(Z_s) \qquad (31)$$

Taking into account the operators from (27) and the notations (31), the system of Eq. (10) can be rewritten in the form

$$U=u^*,$$
$$A^1u+A^2\varphi+B^1\phi+C^1\mu+D^1\tau=U^*,$$
$$\Psi=\varphi^*,$$
$$A^3u+A^4\varphi+B^2\phi+C^2\mu+D^2\tau=\Psi^*,$$
$$\Phi=\phi^*$$
$$Hu+E\phi+G\chi+L\mu+M\tau+F\nu=\Phi^*,$$
$$\mu=\chi^*,$$
$$RU+P\phi+S\Phi+N\chi+Q\nu=\mu^*,$$
$$\nu=\tau^*,$$
$$A^5u+A^6\varphi+W\phi+Z\Phi+X\mu+Y\tau=\nu^* \qquad (32)$$

In the next step, from (32) we get a new system of equations in which the main unknowns are $(u,\varphi,\phi,\chi,\tau)$ and other variables pass on the right-hand side, in the role of "free terms". The resulting system is

$$A^1u+A^2\varphi+B^1\phi+D^1\tau=U^*-C^1\chi^*,$$
$$A^3u+A^4\varphi+B^2\phi+D^2\tau=\Psi^*-C^2\chi^*,$$
$$Hu+E\phi+G\chi+M\tau=\Phi^*-L\chi^*-F\tau^*,$$
$$P\phi+N\chi=\mu^*-Ru^*-S\phi^*-Q\tau^*$$
$$A^5u+A^6\varphi+W\phi+Y\tau=\nu^*-Z\phi^*-X\chi^* \qquad (33)$$

Now we introduce the notations

$$\tilde{u}=A^1u+A^2\varphi+B^1\phi+D^1\tau,$$
$$\tilde{\varphi}=A^3u+A^4\varphi+B^2\phi+D^2\tau,$$
$$\tilde{\phi}=Hu+E\phi+G\chi+M\tau,$$
$$\tilde{\chi}=P\phi+N\chi,$$
$$\tilde{\tau}=A^5u+A^6\varphi+W\phi+Y\tau, \qquad (34)$$

such that the following scalar product $\langle(\tilde{u},\tilde{\varphi},\tilde{\phi},\tilde{\chi},\tilde{\tau}),(u,\varphi,\phi,\chi,\tau)\rangle$ is a bounded bilinear form on $W_0^{1,2}$. Moreover, by direct calculations we obtain

$$\langle(u,\varphi,\phi,\chi,\tau),(u,\varphi,\phi,\chi,\tau)\rangle$$
$$=\int_B [A_{ijmn}\varepsilon_{ij}\varepsilon_{mn}+2B_{ijmn}\varepsilon_{ij}\mu_{mn}+2a_{ij}\varepsilon_{ij}\phi+$$
$$2D_{ijmn}\varepsilon_{ij}\tau_{m,n}+C_{ijmn}\mu_{ij}\mu_{mn}+2b_{ij}\mu_{ij}\phi+$$
$$2E_{ijmn}\mu_{ij}\tau_{m,n}+A_{ij}\phi_{,i}\phi_{,j}+2H_{ij}\phi_{,i}\chi_{,j}+$$
$$\zeta\phi^2+2F_{ij}\tau_{i,j}\phi+K_{ij}\chi_{,i}\chi_{,j}+G_{ijmn}\tau_{m,n}\tau_{i,j}]dV \qquad (35)$$

such that, based on the assumptions of the theorem, we infer that this bilinear form is coercive on space $W_0^{1,2}$.

Clearly, the functions from the right-hand side of the system (33), namely $U^*-C^1\chi^*,\Psi^*-C^2\chi^*,\Phi^*-L\chi^*-F\tau^*,\mu^*-Ru^*-S\phi^*-Q\tau^*$, and $\nu^*-Z\phi^*-X\chi^*$, are functions which belong to the space $W^{1,2}$. So, we met the conditions to apply the Lax-Milgram theorem, which ensures the existence of functions $\mathcal{U}=(u,\varphi,\phi,\chi,\tau)$ as a solution of the system (33), and this, in turn, ensure the existence of solution for the system (32). Thus, the proof of the theorem is completed.

Based on Theorem 3 and Theorem 4 we deduce that the operator $\mathcal{T}$ satisfies the requirements of the Lumer-Phillips corollary of the known Hille-Yosida theorem (see Pazy 1983). That is, we have the following result.

Theorem 5. We assume that the following assumptions are fulfilled

1. ϱ, I_{ij}, J and the constitutive coefficient a are strictly positive;
2. The symmetry relations (7) hold true;
3. The quadratic form U defined in (15) is positive definite;
4. The constitutive coefficients B_{ij} are components of a positive definite tensor.

Then the operator $\mathcal{T}$ generates a semigroup of contracting operators on Hilbert space $\mathcal{H}$.

With the help of the same Lumer-Phillips corollary, we deduce the following result of uniqueness.

Theorem 6. Suppose that the conditions of Theorem 5 are satisfied. Moreover, we assume that

$$F_i,G_i,L,s,Q_iC^1([0,\infty),L^2)\cap C^0([0,\infty),W_0^{1,2})$$

and the initial data $\mathcal{U}_0$ belongs to the domain of the operator $\mathcal{T}$.

Then the abstract problem (28) admits the only one solution $\mathcal{U}(t)\in C^1([0,\infty),\mathcal{H})$.

A final result to characterize the solution of the abstract problem (28) (as such, of the solution to our mixed initial boundary value problem), which also is obtained by means of Lumer-Phillips corollary, is a result regarding the continuous dependence of the solutions with respect to initial data and loads.

Theorem 7. Suppose that the conditions of Theorem 5 are satisfied. Then the solution $\mathcal{U} = (u, \varphi, \phi, \chi, \tau)$ of problem (28) depends continuously with regard to the initial data $\mathcal{U}_0$ and the loads F_i, G_i, L, s, Q_i, that is,

$$|\mathcal{U}(t)| \leq |\mathcal{U}_0| + \int_0^t \|(F_i, G_i, L, s, Q_i)\| \, ds$$

4. Conclusions

For a more valuable characterization of the modern materials,in our study, we proposed to take into account the intimate structure of materials together with the fact that microparticles possess microtemperatures. In our problem because the number of initial conditions and the boundary conditions increased we investigate complex differential equations. By using some powerful techniques of the semi-group theory, we analyzed these differential equations, namely, we described the qualitative results of the mixed problem within the content of thermoelastic micropolar bodies. We benefited from the flexibility of the theory of semi-groups which allows that behind a single equation of evolution we have a great number of partial differential equations. With this tool, we proved the existence and uniqueness results together with the continuous dependence of solutions with regard to the initial data and loads.

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