Structural Engineering and Mechanics, *Vol. 54, No. 2 (2015) 291-304*
DOI: http://dx.doi.org/10.12989/sem.2015.54.2.291

Detection and parametric identification of structural nonlinear restoring forces from partial measurements of structural responses

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(Received November 30, 2014, Revised February 14, 2015, Accepted March 16, 2015)

Abstract. Compared with the identification of linear structures, it is more challenging to conduct identification of nonlinear structure systems, especially when the locations of structural nonlinearities are not clear in structural systems. Moreover, it is highly desirable to develop methods of parametric identification using partial measurements of structural responses for practical application. To cope with these issues, an identification method is proposed in this paper for the detection and parametric identification of structural nonlinear restoring forces using only partial measurements of structural responses. First, an equivalent linear structural system is proposed for a nonlinear structure and the locations of structural nonlinearities are detected. Then, the parameters of structural nonlinear restoring forces at the locations of identified structural nonlinearities together with the linear part structural parameters are identified by the extended Kalman filter. The proposed method simplifies the identification of nonlinear structures. Numerical examples of the identification of two nonlinear multi-story shear frames and a planar nonlinear truss with different nonlinear models and locations are used to validate the proposed method.

Keywords: nonlinear structural systems; parametric identification; nonlinear restoring force; extended Kalman filter; partial measurements

1. Introduction

Parametric identification of structures is one of the important aspects of structural health monitoring. Structural nonlinearity occurs under strong dynamic loading. Also, structural nonlinearity is a useful indicator of the evolutions of structural damage under dynamic excitation (Yun and Shinozuka 1980, Jia *et al.* 2012, Zhu *et al.* 2012, Qarib and Adeli 2014) Therefore, it is necessary to detect, localize and identify structural nonlinearities using structural dynamic measurements for damage detection, performance evaluation and remaining service life forecasting of engineering structures (Li and Chen 2013, Yi *et al.* 2013, Lin *et al.* 2013, Kaloop *et al.* 2014, Wang *et al.* 2015). However, identification of nonlinear structural systems is far less established than that of linear systems due to the complexities of structural nonlinearities and the identification of many unknown structural parameters.

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http://www.techno-press.org/?journal=sem&subpage=8 ISSN: 1225-4568 (Print), 1598-6217 (Online)

The common identification methods and research needed of nonlinear structural dynamic systems were summarized by Kerschena *et al.* (2006), Lee *et al.* (2010), Qarib and Adeli (2014), etc. Previous identification methodologies can be mainly categorized into time domain or frequency domain approaches. Peng *et al.* (2008) investigated nonlinear parameter estimation for multi-degree-of-freedom nonlinear systems using nonlinear output frequency-response function; Carrella and Ewins (2011) proposed identifying and quantifying structural nonlinearities from measured frequency response functions. Aykan and Özgüven (2013) improved parametric identification of nonlinear elements by using incomplete frequency response function data. In the time domain identification studies, Yun and Shinozuka (1980) and Masri with his colleagues (Masri *et al.* 1987a, Masri *et al.* 1987b, Smyth *et al.* 1999, Masri *et al.* 2005) were the main pioneers for the identification approaches of nonlinear structural dynamic systems. Then, many researchers presented improved methodologies, e.g., Kumar *et al.* (2007) presented parametric identification of nonlinear dynamic systems using combined Levenberg-Marquardt and genetic algorithm; Kishore and Shankar (2009) proposed parametric identification of structures with nonlinearities using global and substructure approaches in the time domain; Zhang *et al.* (2008) developed a pattern recognition technique based on support vector for nonlinear system identification; Rochdi *et al.* (2009) studied parametric identification of nonlinear hysteretic systems; Jia *et al.* (2012) investigated restoring force identification for a nonlinear chain-like structure; Pai *et al.* (2013) presented nonlinearity identification by time-domain-only signal processing. Yang *et al.* (2005, 2006, 2009) proposed several approaches based on least squares estimation, sequential non-linear least-square estimation, extended Kalman filter, and adaptive quadratic sum-squares error, respectively for parametric identification of nonlinear structural systems. The authors (Lei *et al.* 2012a, Lei *et al.* 2013) have also developed algorithms for the identification of nonlinear structural parameters and nonlinear properties of rubber-bearings in base-isolated buildings under limited input and output measurements. However, in many prior studies, it is prerequisite that either the locations of structural nonlinearities are known or full measurements of structural responses are available. If the locations of structural nonlinearities are not clear and only partial measurements of structural responses are available, it is still difficult to detect and identify nonlinear structural systems due to the complexity of structural nonlinearity and the identification of many unknown structural parameters with incomplete measurement information.

In practice, it is impossible to deploy so many sensors to measure all response outputs of a structural system. Therefore, it is highly desirable to develop methods of parametric identification using partial measurements of structural responses for practical application. The extended Kalman filter (EKF) (Hoshiya and Saito 1984, Yang *et al.* 2006, Yuen *et al.* 2013) has been proved to be very useful for structural identification when only partial structural responses are measured. Also, EKF can be extended for the identification of nonlinear structures (Yang *et al.* 2006, Lei *et al.* 2012b, Lei *et al.* 2013). The EKF is based upon the principle of linearizing the nonlinear state transition function and observation function with Taylor series expansions. The derivation of the Jacobian matrices and the linearization approximations to the nonlinear functions can be nontrivial and lead to implementation difficulties if the EKF is directly used for the identification of a nonlinear structure in which the locations of structural nonlinearities are not prerequisite (Wu and Smyth 2007).

In this paper, an identification method is proposed for detection and parametric identification of structural nonlinear restoring forces using only partial measurements of structural responses. First, an equivalent linear structural system is proposed for a nonlinear structure and the locations of structural nonlinearities are detected based on the identification results of the equivalent linear system and those of the original nonlinear structure under weak excitation. Then, the parameters of structural nonlinear restoring forces at the locations of identified structural nonlinearities together with the rest linear part structural parameters are identified by the extended Kalman filter. Numerical simulation examples of the identification of a nonlinear multi-story shear frame and a truss with different nonlinear models and locations are used to validate the proposed method.

2. Detection for the locations of structural nonlinearities in structural systems

The equation of motion for a nonlinear structural system can be written as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{F}[\boldsymbol{x}(t),\dot{\boldsymbol{x}}(t),\boldsymbol{\theta}]=\boldsymbol{B}\boldsymbol{f}(t) \tag{1}$$

where $\boldsymbol{M}$ is the mass matrix of the structure, $\ddot{\boldsymbol{x}}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\boldsymbol{x}(t)$ are the vectors of the structural acceleration, velocity and displacement responses, respectively; $\boldsymbol{\theta}$ is the unknown structural parameters vector; $\mathrm{F}[\dot{\boldsymbol{x}}(t), \boldsymbol{x}(t), \boldsymbol{\theta}]$ is the vector of nonlinear restoring forces; $\boldsymbol{B}$ and $\boldsymbol{f}(t)$ are the external force influence matrix and external force vector, respectively. Usually, $\boldsymbol{M}$ can be assumed known as the mass of a structure can be estimated quite accurately.

2.1 Identification of the nonlinear structure under weak external excitation

Under weak external excitation, the nonlinear restoring forces in the structural system become linear ones. Thus, the equation of motion of the structural system reduces to

$$\boldsymbol{M}\ddot{\boldsymbol{x}}_l(t)+\boldsymbol{C}\dot{\boldsymbol{x}}_l(t)+\boldsymbol{K}\boldsymbol{x}_l(t)=\boldsymbol{B}\boldsymbol{f}(t) \tag{2}$$

where $\ddot{\boldsymbol{x}}_l(t)$, $\dot{\boldsymbol{x}}_l(t)$and $\boldsymbol{x}_l(t)$ are the vectors of the acceleration, velocity and displacement responses of the linear structure, respectively. $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices of the linear structure, respectively.

The extended state vector of the above linear structure is defined as $\boldsymbol{X}_l=\left\{\boldsymbol{x}_l^T,\dot{\boldsymbol{x}}_l^T,\boldsymbol{\theta}_l^T\right\}^T$ with $\boldsymbol{\theta}_l$ being a vector of linear stiffness and damping parameters. Then, the equation for the extended state vector of the linear structure can be derived as

$$\dot{\boldsymbol{X}}_l=\begin{Bmatrix}\dot{\boldsymbol{x}}_l\\ \boldsymbol{M}^{-1}\left[\boldsymbol{B}\boldsymbol{f}(t)-\boldsymbol{K}\boldsymbol{x}_l-\boldsymbol{C}\dot{\boldsymbol{x}}_l\right]\\ 0\end{Bmatrix}=\boldsymbol{g}_l(\boldsymbol{X}_l,\boldsymbol{f}) \tag{3}$$

The availability of acceleration data is usually ensured since this is what is commonly measured. Therefore, the discrete form of equation for measured acceleration vector can be expressed as

$$\boldsymbol{y}_{l,k+1}=-\boldsymbol{D}\boldsymbol{M}^{-1}(\boldsymbol{C}\dot{\boldsymbol{x}}_{l,k+1}+\boldsymbol{K}\boldsymbol{x}_{l,k+1}+\boldsymbol{B}\boldsymbol{f}_{k+1})+\boldsymbol{v}_{k+1}=\boldsymbol{h}_l(\boldsymbol{X}_{l,k+1},\boldsymbol{f}_{k+1})+\boldsymbol{v}_{k+1} \tag{4}$$

where $\boldsymbol{y}_{l,k+1}$ is the measured linear acceleration response vector at time $t=(k+1)\times\Delta t$ (Δt is the sampling time step), $\boldsymbol{D}$ denotes the location of accelerometers, $\boldsymbol{v}_{k+1}$ is the measurement noise assumed as Gaussian white noise.

Based on the extended Kalman filter (EKF), the extended state vector can be identified as

$$\hat{\boldsymbol{X}}_{l,k+1|k+1} = \tilde{\boldsymbol{X}}_{l,k+1|k} + \boldsymbol{K}_{k+1}[\boldsymbol{y}_{l,k+1} - \boldsymbol{h}_l(\tilde{\boldsymbol{X}}_{l,k+1|k}, \boldsymbol{f}_{k+1})] \tag{5}$$

$$\tilde{\boldsymbol{X}}_{l,k+1|k} = \hat{\boldsymbol{X}}_{l,k+1|k} + \int_{t_k}^{t_{k+1}} \boldsymbol{g}_l(\hat{\boldsymbol{X}}_l, \boldsymbol{f})dt \tag{6}$$

$\hat{\boldsymbol{X}}_{l,k+1|k+1}$ is the estimated state vector $\boldsymbol{X}_l$ at time $t=(k+1)\times\Delta t$ and $\boldsymbol{K}_{k+1}$ is the Kalman Gain matrix at the time instant (Hoshiya and Saito 1984, Lei *et al*. 2012b).

Therefore, the linear structural parameters can be identified by using the EKF and the partial measurements of structural responses under weak excitation.

2.2 Identification of an equivalent linear structure under strong external excitation

Under strong external excitation, structural nonlinear effect becomes significant. An equivalent linear system for the nonlinear structural system is introduced with the following equation of motion (Jia *et al*. 2012).

$$\boldsymbol{M}_e\ddot{\boldsymbol{x}}_e(t) + \boldsymbol{C}_e\ddot{\boldsymbol{x}}_e(t) + \boldsymbol{K}_e\boldsymbol{x}_e(t) = \boldsymbol{Bf}(t) \tag{7}$$

in which $\ddot{\boldsymbol{x}}_e(t)$, $\dot{\boldsymbol{x}}_e(t)$ and $\boldsymbol{x}_e(t)$ are the vectors of the acceleration, velocity and displacement responses of the equivalent linear system, respectively; $\boldsymbol{M}_e$, $\boldsymbol{C}_e$ and $\boldsymbol{K}_e$ are the equivalent linear mass, damping and stiffness matrices, respectively; Usually, the mass matrix is hardly changed when nonlinear behavior appears, so $\boldsymbol{M}_e=\boldsymbol{M}$.

Analogously, the equivalent linear stiffness and damping parameters can be identified using the extended Kalman filter (EKF) with the partial measurements of the nonlinear structural responses.

$$\hat{\boldsymbol{X}}_{e,k+1|k+1} = \tilde{\boldsymbol{X}}_{e,k+1|k} + \boldsymbol{K}_{e,k+1}[\boldsymbol{y}_{k+1} - \boldsymbol{h}_e(\tilde{\boldsymbol{X}}_{e,k+1|k}, \boldsymbol{f}_{k+1})] \tag{8}$$

$$\tilde{\boldsymbol{X}}_{e,k+1|k} = \hat{\boldsymbol{X}}_{e,k+1|k} + \int_{t_k}^{t_{k+1}} \boldsymbol{g}_e(\hat{\boldsymbol{X}}_e, \boldsymbol{f})dt \tag{9}$$

where

$$\boldsymbol{g}_e(\boldsymbol{X}_l, \boldsymbol{f}) = \begin{Bmatrix} \dot{\boldsymbol{x}}_l \\ \boldsymbol{M}^{-1}[\boldsymbol{Bf}(t) = \boldsymbol{K}_e\boldsymbol{x}_i - \boldsymbol{C}_e\dot{\boldsymbol{x}}_l] \\ 0 \end{Bmatrix} \tag{10a}$$

$$\boldsymbol{h}_e(\boldsymbol{X}_{l,k+1}, \boldsymbol{f}_{k+1}) = -\boldsymbol{DM}^{-1}(\boldsymbol{C}\ddot{\boldsymbol{x}}_{l,k+1} + \boldsymbol{Kx}_{l,k+1} + \boldsymbol{Bf}_{k+1}) \tag{10b}$$

$\hat{\boldsymbol{X}}_{e,k+1|k+1}$ is the estimated state vector $\boldsymbol{X}_e$ at time $t=(k+1)\times\Delta t$ and $\boldsymbol{X}_e = \left\{\boldsymbol{x}_e^T, \dot{\boldsymbol{x}}_e^T, \boldsymbol{\theta}_e^T\right\}^T$ and $\boldsymbol{\theta}_e$ is the parametric vector of the equivalent linear structural system.

Then, the locations of structural nonlinearity can be identified by comparing the differences between the identified structural parameters of equivalent linear system and those of the linear structure as illustrated in the following numerical examples.

2.3 Parametric Identification of the structural nonlinear restoring forces

After the identification of the locations of structural nonlinearities, parametric identification of the nonlinear structural system can be conducted by the EKF using the partial measurements of the nonlinear structural responses under strong external excitations.

In this case, the extended state vector is defined as $\boldsymbol{X}=\{\boldsymbol{x}^T, \dot{\boldsymbol{x}}^T, \boldsymbol{\theta}^T\}^T$ where $\boldsymbol{\theta}$ is the parametric vector containing the parameters of nonlinear restoring forces in the identified locations of structural nonlinearities and the rest linear part structural parameters.

The equation for the extended state vector is expressed as

$$\dot{\boldsymbol{X}} = \begin{Bmatrix} \dot{\boldsymbol{x}} \\ \boldsymbol{M}^{-1}[\boldsymbol{B}\boldsymbol{f}(t) - \boldsymbol{F}[\boldsymbol{x}(t), \dot{\boldsymbol{x}}(t), \boldsymbol{\theta}]] \\ 0 \end{Bmatrix} = \boldsymbol{g}_n(\boldsymbol{X}, \boldsymbol{f}) \tag{11}$$

The discrete form of equation for measured acceleration vector is written as

$$\boldsymbol{y}_{k+1} = -\boldsymbol{D}\boldsymbol{M}^{-1}(\boldsymbol{B}\boldsymbol{f}_{k+1} - \boldsymbol{F}[\boldsymbol{x}_{k+1}, \dot{\boldsymbol{x}}_{k+1}, \boldsymbol{\theta}_{k+1}) + \boldsymbol{v}_{k+1} = \boldsymbol{h}_n(\boldsymbol{X}_{k+1}, \boldsymbol{f}_{k+1}) + \boldsymbol{v}_{k+1} \tag{12}$$

Analogously, the extended state vector of the nonlinear structure can be recursively estimated by EKF as

$$\hat{\boldsymbol{X}}_{k+1|k+1} = \tilde{\boldsymbol{X}}_{k+1|k} + \boldsymbol{K}_{k+1}[\boldsymbol{y}_{k+1} - \boldsymbol{h}_n(\tilde{\boldsymbol{X}}_{k+1|k}, \boldsymbol{f}_{k+1})] \tag{13}$$

$$\tilde{\boldsymbol{X}}_{k+1|k} = \hat{\boldsymbol{X}}_{k|k} + \int_{t_k}^{t_{k+1}} \boldsymbol{g}_n(\hat{\boldsymbol{X}}, \boldsymbol{f})dt \tag{14}$$

Usually, structural nonlinear restoring forces only exist in the locations of structural nonlinearities while the other structural restoring forces are linear ones. Since structural nonlinearities only exist in the limited components in local part of a whole structure, the numbers of unknown parameters in the nonlinear restoring forces are greatly reduced due to the detection of structural nonlinearities. The computational burden of the derivation of the Jacobian matrices is also reduced. Therefore, proposed method not only simplifies the identification of nonlinear structures but also ensures identification convergence in an inverse problem.

3. Numerical validations

To validate the proposed method for the detection and parametric identification of structural nonlinear restoring forces using only partial measurements of structural responses, numerical simulation examples of the identification of two nonlinear multi-story shear frames and a truss with different nonlinear models and locations are used.

3.1 Detection and parametric identification of nonlinear multi-story shear frames

The numerical examples of the identification of two nonlinear multi-story shear frames with different nonlinear models and locations are used to validate the proposed method. The frames are subjected to the El Centro earthquake ground excitation with different levels of peak ground acceleration (PGA), respectively. The numerical calculation responses are treated as “measured responses” for the identification problem. Also, the influence of measurement noise is considered by superimposition of noise process with the computed response quantities. In the examples, the acceleration responses are added by white noises with 5% noise-to-signal ratio in root mean square (rms).

Table 1 Comparisons of the equivalent linear and linear parameters of the six-story shear frame

Story No.	k_{ei} (kN/m)	k_{li} (kN/m)	c_{ei} (kN·s/m)	c_{li} (kN·s/m)
1st	**182.96**	**237.77**	**0.58**	**0.19**
2nd	240.18	240.01	0.16	0.19
3rd	245.06	240.71	0.18	0.19
4th	236.82	239.98	0.18	0.19
5th	237.61	240.18	0.20	0.20
6th	240.52	240.03	0.18	0.20

3.1.1 Identification of a six-story shear frame with nonlinear force in Bouc-Wen model

The linear structural parametric values of the six-story shear frame are selected as: mass of each floor m_i=600 kg, each story stiffness k_i=240kN/m, each story damping coefficients c_i=0.20 kN·s/m (i=1,2,…,6). Only the acceleration responses 1st, 3rd, 4th and 6th floors are measured and used for structural identification. The extended state vector of the linear structure is defined as

$$\boldsymbol{X}_l = \left\{ \boldsymbol{x}_l^T \ , \dot{\boldsymbol{x}}_l^T , \boldsymbol{\theta}_l^T \right\}^T ; \ \ \boldsymbol{\theta}_l^T = \left\{ k_{l1}, k_{l2}, ..., k_{l6}, \mathrm{c}_{l1}, c_{l2}, ..., c_{l6} \right\} \tag{15}$$

in which, k_{li} and c_{li} (i=1,2,…,6) are the i-th linear stiffness and damping parameters, respectively.

The linear structural parameters of the six-story shear frame can be identified by the EKF with the partial measurements of acceleration responses of the frame under weak earthquake ground excitation. The identification results are shown in Table 1.

When the above six-story shear frame is subjected to strong earthquake excitation, structural nonlinearities occur in the frame. In this example, Bouc-Wen model (Wen 1987), which has been widely used for the description of nonlinear hysteretic forces, is adopted. It is assumed that story nonlinear hysteretic restoring force in Bouc-Wen model exists in the 1st story. In the Bouc-Wen hysteretic model, the vector of inter-story hysteretic drift z_1 can be described by

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_0 - (1-\alpha_1)\{\beta_1 |\dot{x}_1 - \dot{x}_0| |\dot{z}_1|^{n_1 - 1}(z_1) - \gamma_1(\dot{x}_1 - \dot{x}_0)|\dot{z}_1|^{n_1} \tag{16}$$

in which, β_1, γ_1 and n_1 are the Bouc-Wen hysteretic parameters, α_1 is the ratio of post-yielding stiffness to pre-yielding stiffness. In the example, these parameters are selected as: α_1=0.5, β_1=500 s^2/m^2, γ_1=5000 s^2/m^2, n_1=2.

First, an equivalent linear frame is established and identified by the extended Kalman filter using the measured partial nonlinear structural responses. The identification results of equivalent linear parameters are shown and compared with those of the linear farme in Table 1.

In Table 1, it is noted that large differences exist between the values of k_{e1}, c_{e1} and k_{l1}, c_{l1} as marked by values in bolds. Based on these differences, the location of structural nonlinearity is identified.

Then, the parameters of the Bouc-Wen model for the 1st story hysteretic nonlinear restoring force can be identified by using the EKF with the extended state vector defined as

$$\boldsymbol{X} = [x_1, ..., x_6, \dot{x}_1, ..., \dot{x}_6, z_1, k_1, ..., k_6, c_1, ..., c_6, \beta_1, \gamma_1]^{\mathrm{T}} \tag{17}$$

The identification results of the nonlinear structural parameters are shown in Table 2. Compared with their actual values, it is shown that the identification results are accurate.

Table 2 Parametric identification results of the nonlinear six-story shear frame

Story No.	k_i (kN/m)	c_i (kN·s/m)	β_i (s^2/m^2)	γ_i (kN·s/m)
1st	239.50	0.18	515.68	4929.19
2nd	239.91	0.20		
3rd	239.37	0.20		
4th	239.91	0.19		
5th	239.27	0.20		
6th	239.87	0.21		

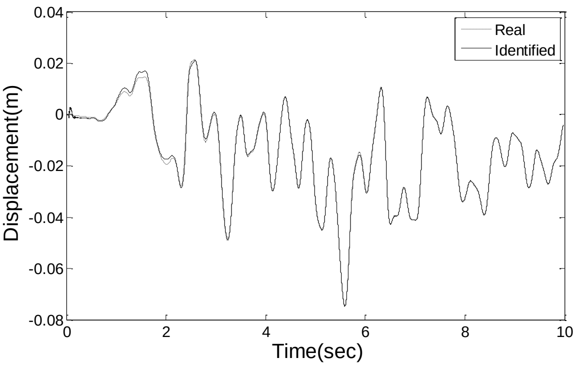

Fig. 1(a) Displacement of the 1st story

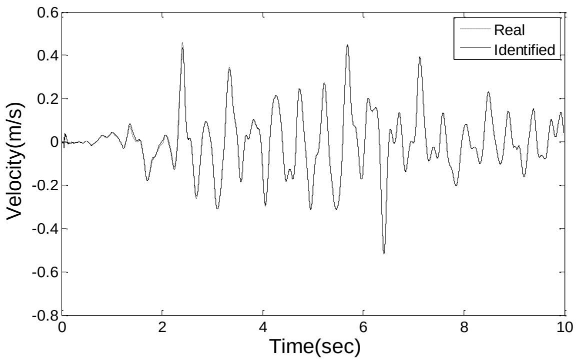

Fig. 1(b) Velocity of the 1st story

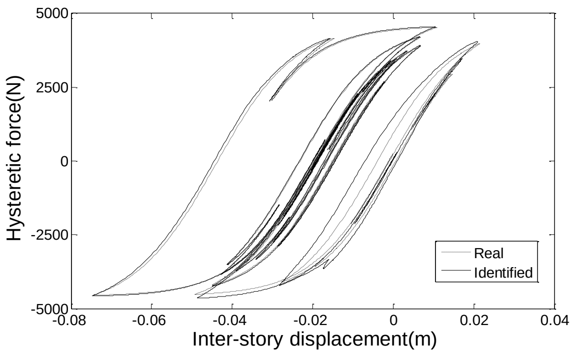

Fig. 2(a) Hysteretic force in the 1st story

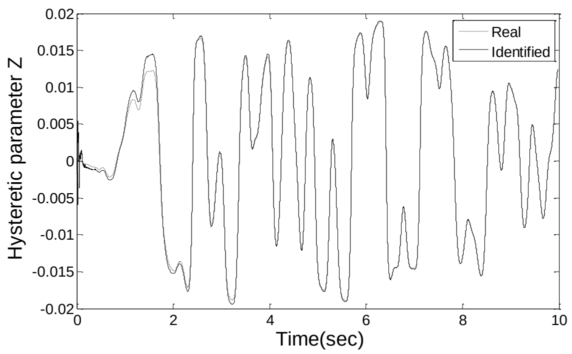

Fig. 2(b) Hysteretic parameter z_1

Figs. 1(a)-(b) show the comparisons of the identified structural displacement and velocity responses with the real responses, respectively. In Fig. 2(a)-(b), the identified hysteretic restoring force (hysteretic loop) together with inter-story hysteretic drift z_1 of the 1st story of the frame are compared with the real ones, respectively. It is demonstrated that the identification results are in good agreement with the real ones.

3.1.2 Identification of a ten-story shear frame with nonlinear forces in Dahl models

The linear structural parametric values of the ten-story shear frame are selected as: m_i=600 kg, k_i=240 kN/m, and c_i=0.20 kN s/m (i=1,2,…,10). Partial measurements of the acceleration responses 1st, 2nd, 4th, 6th, 8th and 10th floors are used for structural identification. The linear

Table 3 Comparisons of the equivalent linear and linear parameters of the ten-story shear frame

Story No.	k_{ei} (kN/m)	k_{li} (kN/m)	c_{ei} (kN·s/m)	c_{li} (kN·s/m)
1st	246.40	239.59	**0.88**	**0.21**
2nd	235.25	241.09	0.21	0.18
3rd	237.45	240.11	0.18	0.22
4th	239.84	238.29	0.19	0.18
5th	237.89	239.12	0.21	0.20
6th	241.25	241.44	0.22	0.20
7th	237.28	240.25	0.24	0.21
8th	244.21	238.63	**0.89**	**0.20**
9th	238.98	240.63	0.24	0.20
10th	240.18	238.98	0.20	0.22

structural parameters of the frame can be identified by the EKF with the partial measurements of acceleration responses of the frame under weak earthquake excitation. The identification results are shown in Table 3 with k_{li} and c_{li} being the i-th identified stiffness and damping parameters, respectively.

Under strong earthquake excitation, structural nonlinearities occur in the frame. In this example, identification of story nonlinear restoring forces in Dahl models (Dahl 1976) is studied. In the Dahl hysteretic model, the nonlinear restoring force is expressed as

$$F_i = k_{0i}(x_i - x_{i-1}) + c_{0i}(\dot{x}_i - \dot{x}_{i-1}) + f_{di} z_i + f_{0i} \tag{18}$$

in which $k_{0i,}$ c_{0i}, f_{di}, f_{0i} are model parameters, and z_i is the Coulomb dry friction described by

$$\dot{z}_i = \sigma_i(\dot{x}_i - \dot{x}_{i-1})(1 - z_i \cdot \mathrm{sgn}(\dot{x}_i - \dot{x}_{i-1}) \tag{19}$$

In the numerical example, It is assumed that story nonlinear restoring forces in Dahl models exist in both the 1st and the 8th stories with the parameters set as: k_{01}=k_{08}=0.02 kN/m, c_{01}=c_{08}=0.01 kNs/m, f_{d1}=0.06 kN, f_{d8}=0.05 kN, f_{01}= f_{08}=0 kN, and σ_1=1000 s/m, σ_8=800 s/m.

An equivalent linear frame is established and identified by the extended Kalman filter using the measured partial nonlinear structural responses. The identification results of equivalent linear parameters are shown and compared with those of the linear frame in Table 3.

From the comparisons in Table 3, it is noted that there are large differences between the identified equivalent damping and linear damping parameters in the 1st and 8th stories, as marked by values in bolds. Therefore, the locations of the two structural nonlinearities are identified.

Then, parametric identification of the ten-story shear frame can be conducted by using the EKF with the extended state vector defined as

$$\boldsymbol{X} = [x_1,\ldots,x_{10},\dot{x}_1,\ldots,\dot{x}_{10},k_1,\ldots,k_{10},c_1,\ldots,c_{10},k_{01},c_{01},f_{d1},\sigma_1,k_{08},c_{08},f_{d8},\sigma_8]^{\mathrm{T}} \tag{20}$$

The identification results of the nonlinear structural parameters are shown in Table 4. Compared with their actual values, it is shown that the identification results are accurate.

In Figs. 3(a)-(d), the identified two nonlinear restoring forces and the two parameters z_1 and z_8 of the 1st and 8th stories of the frame are compared with the real results, respectively. It is shown that the identification results are in good agreement with the real ones.

Table 4 Parametric identification results of the nonlinear ten-story shear frame

Story No	k_i (kN/m)	c_i (kN·s/m)	k_{0i} (kN/m)	c_{0i} (kN·s/m)	f_{di} (kN)	σ_i (s/m)
1st	239.91	0.20	0.018	0.19	0.061	984.29
2nd	239.97	0.21				
3rd	239.98	0.19				
4th	239.04	0.20				
5th	240.58	0.20				
6th	240.72	0.20				
7th	239.67	0.19				
8th	240.25	0.20	0.018	0.19	0.048	817.63
9th	239.61	0.20				
10th	240.46	0.20				

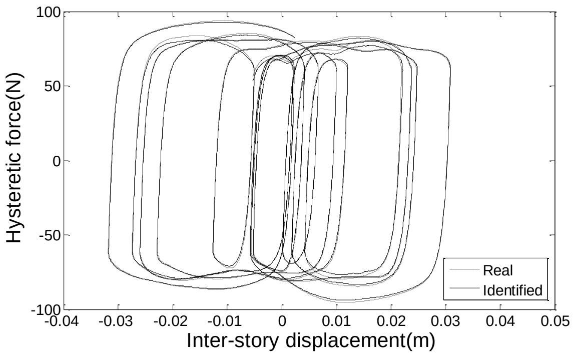

Fig. 3(a) Nonlinear force in the 1st story

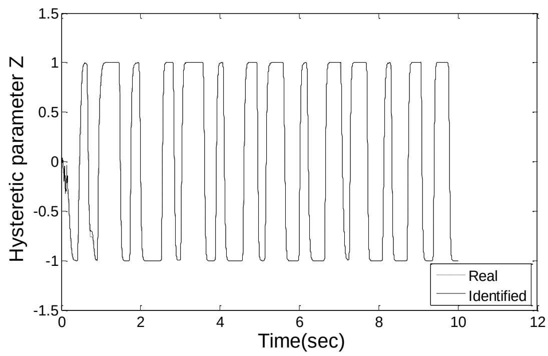

Fig. 3(b) Nonlinear force parameter z_1

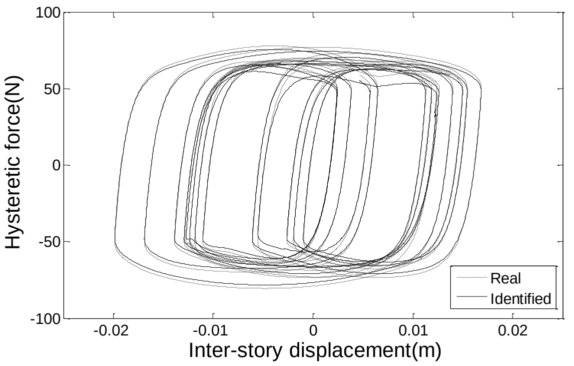

Fig. 3(c) Nonlinear force in the 8th story

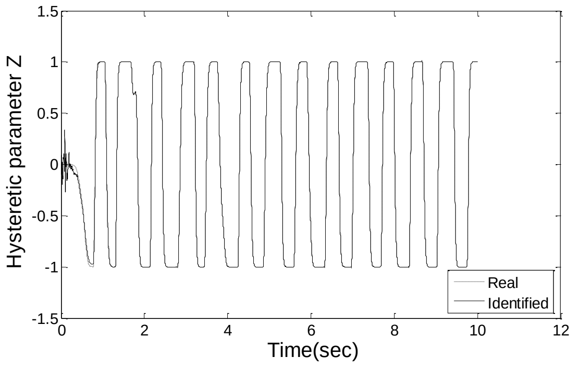

Fig. 3(d) Nonlinear force parameter z_8

3.2 Detection and parametric identification of a nonlinear truss

To further validate the proposed method for the parametric identification of other types of nonlinear structures, the identification of a nonlinear truss shown in Fig. 4 is used. It is assumed that all bars in the truss have uniform cross sections and the length of each horizontal bar is 2*m* while the length of each inclined bar is $\sqrt{2}$ m. The finite element model of the truss has 11 bar

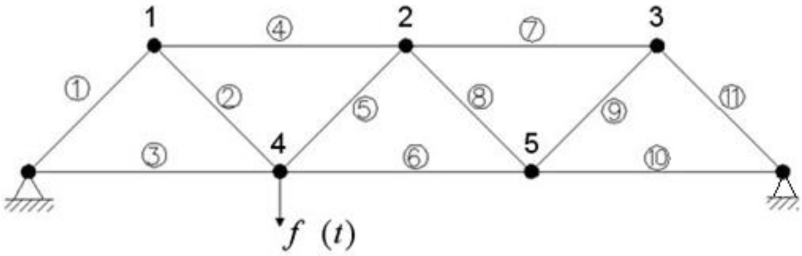

Fig. 4 A plan nonlinear truss

members and 10 degrees-of-freedom (DOFs). The structural parameters are: cross section area A=7.854×10^{-5} m^2, Young's module E=2×10^8 Pa and mass density of the truss member ρ=7850 kg/m^3. The truss is subjected to an external white noise excitation in the vertical direction at node 4.

Let $\bar{\boldsymbol{K}}_i$ be the local linear stiffness matrix of the i-th element in the local coordinate system i=1, 2,…,11). Then, the linear element matrices can be transformed into the corresponding element matrices in the global coordinate system as

$$\boldsymbol{K_i} = \boldsymbol{T^T}\bar{\boldsymbol{K}}_i\boldsymbol{T} \tag{21}$$

where $\boldsymbol{T}$ is the transformation matrix. The global linear stiffness matrix $\boldsymbol{K}$ can be obtained based on assembling all the elements, i.e.

$$\boldsymbol{K} = \sum_{\boldsymbol{i}=1}^{\boldsymbol{i}=11}\boldsymbol{K_i} = \sum_{\boldsymbol{i}=1}^{\boldsymbol{i}=11} k_{\boldsymbol{i}}\boldsymbol{S_i} \tag{22}$$

where $k_i=EA/l_i$is defined as the stiffness of i-th truss element and $\boldsymbol{S}_i$ is a matrix of the i-th element. In this example, mass is concentrated on each node. The linear viscous damping is adopted. It is assumed that linear viscous damping for each horizontal bar is c_i=0.028 kN·s/m (i=3,4,6,7,10) and for each inclined bar c_i=0.020 kN·s/m (i=1,2,5,8,9).

To consider partial measurements of structural responses, it is assumed that only acceleration responses in the horizontal directions at node 1, 2, 4, 5 and in the vertical directions of node 1 and 3 are measured. All the measured acceleration responses are simulated by superimposing the theoretically computed responses with the stationary white noises with 5% noise in rms.

When the plane truss is subjected to strong external excitation, structural nonlinearity occurs. It is assumed that the restoring force in bar element 1 become nonlinear one described by the Bouc-Wen model with the parameters as: α_1=0.5, β_1=200000 s^2/m^2, γ_1=400000 s^2/m^2, n_1=2.

The linear structural parameters of the truss can be identified by the EKF with the partial measurements of acceleration responses of the truss subject to weak external excitation, as shown in Table 5 with k_{li} and c_{li} being the i-th identified stiffness and damping parameters, respectively.

Then, an equivalent linear truss is established and identified by the extended Kalman filter using the measured partial nonlinear structural responses. The identification results of equivalent linear parameters are shown and compared with those of the linear truss in Table 5. From the comparisons in Table 5, it is noted that there are large differences between the identified equivalent stiffness and damping and linear ones in the 1st bar element, as marked by values in bolds. Therefore, the locations of the two structural nonlinearities are identified.

Table 5 Comparisons of the equivalent linear and linear parameters of the truss

Bar No.	k_{ei} (kN/m)	k_{li} (kN/m)	c_{ei} (kN·s/m)	c_{li} (kN·s/m)
1st	**7.62**	**11.09**	**0.062**	**0.028**
2nd	11.04	11.38	0.028	0.030
3rd	6.92	7.78	0.020	0.021
4th	8.45	7.86	0.018	0.022
5th	11.17	11.43	0.028	0.030
6th	7.64	7.67	0.019	0.021
7th	7.84	8.00	0.021	0.021
8th	11.19	10.91	0.027	0.028
9th	11.10	10.77	0.027	0.029
10th	8.62	7.93	0.020	0.020
11th	11.65	10.71	0.026	0.025

Table 6 Parametric identification results of the nonlinear truss

Bar No.	k_i (kN/m)	c_i (kN·s/m)	β_i (s^2/m^2)	γ_i (kN·s/m)
1st	11.11	0.031	210102.56	408573.38
2nd	11.39	0.030		
3rd	7.79	0.021		
4th	7.91	0.021		
5th	11.42	0.030		
6th	7.71	0.019		
7th	7.86	0.020		
8th	11.06	0.027		
9th	11.05	0.027		
10th	7.90	0.020		
11th	11.12	0.026		

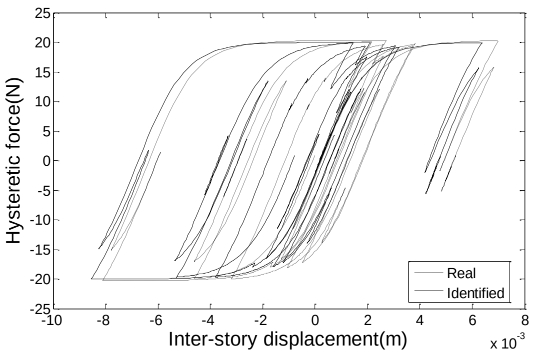

Fig. 5(a) Hysteretic force in the 1st bar

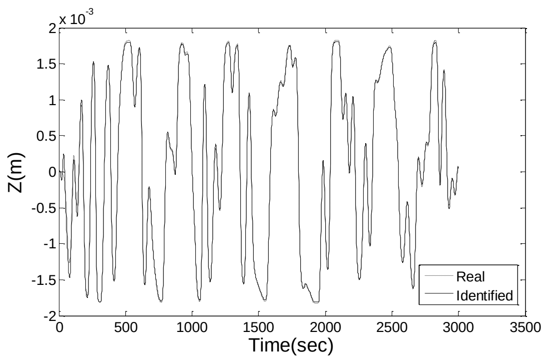

Fig. 5(b) Hysteretic parameter z_1

Finally, parametric identification of the truss can be conducted by using the EKF and the measured partial nonlinear structural responses. The identification results of the nonlinear structural parameters are shown in Table 6. Compared with their actual values, it is shown that the identification results are accurate.

Figs. 5(a)-(b) compare the identified nonlinear hysteretic restoring force (hysteretic loop) and the hysteretic parameters z_1 of the 1st bar element in the truss are compared with the real results, respectively. It is shown that the identification results are in good agreement with the real ones.

4. Conclusions

In this paper, an identification method is proposed for detection and parametric identification of structural nonlinear restoring forces using only partial measurements of structural responses. First, an equivalent linear structural system is proposed for a nonlinear structure and the locations of structural nonlinearities are detected based on the identification results of the equivalent linear system and those of the original nonlinear structure under weak excitation. Then, the parameters of structural nonlinear restoring forces at the locations of identified structural nonlinearities together with the linear part structural parameters are identified by the extended Kalman filter. As structural nonlinearities only exist in local areas of a whole structure, the numbers of unknown parameters in the nonlinear restoring forces are greatly reduced due to the detection of structural nonlinearities. Several numerical simulation examples of the identification of nonlinear multi-story shear frames and a truss with different nonlinear models and locations have demonstrated the effectiveness of the proposed method.

In this paper, measurement noises are assumed as Gaussian white noises. For non-Gussian noises, other complex methodologies such as the Particular Filter (PF) can be utilized. The accuracy of identification results by the proposed algorithm dependents on the numbers and optimal points of measurements. The numbers and optimal locations of measured partial structural responses, which are not investigated in this manuscript, can be a future study. Also, the identification of nonlinear restoring forces without mathematical models and the effect of miss matches of the models need further studies. Moreover, it is important to investigate the identification of other complex nonlinear structural systems and the experimental validation of the proposed algorithm. The relevant work is being undertaken by the authors.

Acknowledgments

This research described in this paper is financially supported by the Natural Science Foundation of China (NSFC) through the Grant No. 51378445 and the Fujian Provincial Science and Technology Key Project No. 2013Y0079.

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