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Probabilistic study of the influence of ground motion variables on response spectra

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Abstract. Response spectra of earthquake ground motions are important in the earthquake-resistant design and reliability analysis of structures. The formulation of the response spectrum in the frequency domain efficiently computes and evaluates the stochastic response spectrum. The frequency information of the excitation can be described using different functional forms. The shapes of the calculated response spectra of the excitation show strong magnitude and site dependency, but weak distance dependency. In this paper, to compare the effect of the earthquake ground motion variables, the contribution of these sources of variability to the response spectrum's uncertainty is calculated by using a stochastic analysis. The analytical results show that earthquake source factors and soil condition variables are the main sources of uncertainty in the response spectra, while path variables, such as distance, anelastic attenuation and upper crust attenuation, have relatively little effect. The presented formulation of dynamic structural response in frequency domain based only on the frequency information of the excitation can provide an important basis for the structural analysis in some location that lacks strong motion records.

Keywords: probabilistic; Fourier amplitude; response spectra; design earthquake

1. Introduction

Among all sources of uncertainty stemming from the material properties, the design assumptions, and the earthquake-induced ground motion, the latter seems to be the most unpredictable (Kappos 2002) and it has a significant effect on the variability of structural response (Padgett and Desroches 2007). For earthquake-resistant design and for seismic assessment of existing structures, the earthquake-induced ground motion is generally represented in the form of a response spectrum (Bommer and Acevedo 2004). Ground motion and the corresponding response spectrum are significantly influenced by the energy release mechanism of the earthquake, the source-to-site distance, the travel path between the source and the site, and the local soil conditions.

Past studies on the influence of the ground motion variables to the response spectra and structural response have been limited by the amount of available strong motion data. Since earthquake magnitude (M), source-to-site distance (R), and soil profile (A) at the site of interest are the most common variables related to a seismic event, it is evident that the selection of recorded ground

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motion involves identifying these characteristic, leading to (M, R, A) record set. Bommer and Scott (2000) quantitatively assessed the influence of soil profile to the selection procedure of recorded ground motion for seismic design. They noticed that the (M, R, A) selection process drastically reduced the number of records suitable for selection when compared to the simple (M, R) variable pair. Bommer and Acevedo (2004) considered earthquake magnitude as an important earthquake record selection variable, or at least as an initial criterion for use in the selection process, while the role of the source-to-site distance has not been established.

Katsanos *et al.* (2010) reviewed the effect of rupture mechanism, seismotectonic environment, type of faulting, source path and directivity of seismic waves, and their influence on strong motions selection. Kappos and Kyriakakis (2000) and Boore and Atkinson (2007) demonstrated the effect of different seismotectonic environment and different type of faulting on strong motion selection, respectively. As a result, it seems logical to infer that it could be beneficial to combine different data sets so as to increase the scarcity of available records for dynamic analyses.

In structural engineering, a probabilistic analysis is a reasonable method to approach problems that involve stochastic variables or processes. If a dynamic system is nonlinear, an exact probabilistic solution is obtainable only under the ideal situation of Gaussian white noise excitation. This provides a basis for obtaining approximate solutions for realistic and non-stationary excitations, such as strong motion. In random vibration theory, the probability density for the response of a system under Gaussian white noise excitation can be calculated based on the frequency information of excitation. This information only requires computation of the Fourier amplitude spectrum of excitation. In this study, a stochastically based seismological model (Brune 1970, Boore 2003) is used to generate Fourier Amplitude Spectra (FAS) of excitation. One of the essential characteristics of the seismological method is that it takes the information that is known about the various factors affecting ground motions (source, path, and site) and filters it into simple functional forms. The calculation of the response spectrum based on FAS information of ground motion provides an important basis for the probabilistic study of the influence of ground motion variables on response spectrum.

The solutions of many earthquake engineering problems involve dynamic analysis by using ground motion time series. For identifying earthquake scenarios that can be derived through seismic hazard analysis, there are two methodologies, namely the deterministic and probabilistic approach. The deterministic seismic hazard and the process of disaggregating the seismic hazard in a probabilistic analysis are made to develop one or a few design earthquakes that can be used for detailed analysis and decision making (Bazzurro and Cornell 1999). This paper studies the effect of the variability in the earthquake ground motion variables (such as the earthquake's magnitude, source-to-site distance, soil conditions, and other earthquake source and path variables) on the linear and nonlinear response spectrum.

2. Methodology

Excitation input function displays a wider power spectrum density function in comparison with the corresponding response function. The power spectrum density function of the input for structural dynamic systems is assumed to be wide band-limited noise. As the input process is filtered through the oscillating system, only a narrow band of frequencies around the oscillator's natural frequency is transmitted, and thus, the output displays a narrow band power spectrum (Manolis and Koliopoulos 2001).

Strong ground motion is a typical example of a non-stationary stochastic process. With the adopted simplification, a non-stationary stochastic process is modeled as the product of a stationary stochastic process $Y(t)$ and a deterministic envelope function; this envelope function conveys the time-dependent characteristics of each harmonic component of $Y(t)$. This allows the properties of the non-stationary stochastic process to be separated in the sense that the randomness is attributed to $Y(t)$, while the statistical time evolution is attributed to the envelope function. This expression is easily used to obtain the statistics of the non-stationary process from the corresponding statistics of $Y(t)$ (Solnes 1997).

If a dynamic system is nonlinear or if non-stationary random excitations are present, then a mathematically exact solution is not always possible. If the excitation is Gaussian white noise, the state space vector is a Markov process, and the joint probability density of the state space variables can be obtained as the corresponding Fokker-Planck-Kolmogorov equation (Solnes 1997, Lin and Cai 2004). The exact solutions under the ideal situation of Gaussian white noise excitations provide a basis for obtaining approximate solutions for realistic excitations, such as strong ground motions. The following presentation will focus on oscillators with a differential equation of dynamic equilibrium with the following form

$$\ddot{y}_{(t)} + q(y, \dot{y}) = f_{(t)} \tag{1}$$

where $q(y, \dot{y})$ is a nonlinear differential function of the response y and its derivative $\dot{y}$, and $f_{(t)}$ is a zero mean stationary normal stochastic process. It should be noted here that if $q(y, \dot{y})$ is linear in regard to $\dot{y}$ and if the nonlinearity in y is odd with respect to zero, the mean response value is zero and the response distribution is symmetric around the mean (Manolis and Koliopoulos 2001).

By assuming that the condition that makes the Markov process assumptions valid is met, it can be shown that the joint probability density of the response and its time derivative, $p(y, \dot{y})$, are governed by the Fokker-Planck-Kolmogorov equation (Lin and Cai 2004, Sun 2006).

Assume that the loading $f_{(t)}$ of the oscillator is stationary band-limited white noise with a power spectral function $S_f(\omega)$. The Duffing oscillator with linear damping and nonlinear (cubic) stiffness is governed by an equation of motion of the following type

$$\ddot{y}_{(t)} + 2\xi\Omega\dot{y}_{(t)} + \Omega^2 y_{(t)} + \nu y^3_{(t)} = f_{(t)} \tag{2}$$

where ζ, Ω and ν are the damping coefficient, the natural circular frequency and a coefficient that guarantees a nonlinear system and nonlinear stiffness function, respectively. In this oscillator, the response derivative $\dot{y}_{(t)}$ exhibits a normal first-order probability density function $p(\dot{y})$, while the non-normal probability density of the response $y_{(t)}$ is given as follows (Manolis and Koliopoulos 2001, Lin and Cai 2004)

$$p(y) = C\exp\left(-\frac{4\xi\Omega}{\pi S_f(\omega)}\int_0^y(\Omega^2 y_{(t)} + \nu y^3_{(t)})dy\right) = C\exp\left(-\frac{2\xi\Omega}{\pi S_f(\omega)}(\Omega^2 y^2 + \nu y^4/2)\right) \tag{3}$$

The value of the constant C ensures that the area under $p(y)$ is equal to unity. In this equation, ω is the circular frequency and $S_f(\omega)$ is a power spectral density (PSD) function with a ground motion input that is defined as follows

$$S_f(\omega) = |F(\omega)|^2/T_w \tag{4}$$

where $F(\omega)$ is the Fourier amplitude spectrum (FAS) of ground motion acceleration and T_w is the earthquake ground motion duration. The time duration is related to earthquake size and propagation distance. Here, a simplified form of the distance-dependent term (0.15 R) was adopted and rupture duration part is assumed to be predicted by π/ω_c (Boatwright and Choy 1992), where ω_c is the corner frequency (Eq. (7)). There is a vast amount of research aimed at predicting the amplitude of the Fourier spectra, especially in the engineering seismology field. For instance, ground motion descriptions are always given in terms of FAS, which comes from the use of theoretical models of the radiated spectrum with the attenuation, diminution, and amplification functions. This approach has been used in the past to predict peak motion values and response spectra (Boore 2003).

Brune (1970) assumes that the far-field accelerations on an elastic half-space are band-limited, finite-duration Gaussian noise and that the source spectra are described by a single corner frequency model whose corner frequency depends on the earthquake's size. The far-field Fourier amplitude spectrum, $F(\omega)$, that has been used in seismological models can be broken into contributions from the earthquake source model (point-source); the typical geometric, anelastic whole path and upper crust attenuation; and site amplification functions

$$|F(\omega)| = \frac{R_P F_S P}{4\pi\rho\beta^3}\frac{1}{R}E(\omega)An(\omega)P(\omega)A(\omega) \tag{5}$$

where R is the distance, R_p is the wave radiation factor (taken here as 0.55), F_S is the free surface amplification factor (taken to be 2), and P is the factor that partitions the energy into orthogonal directions (taken to be $\sqrt{2}/2$). ρ is the density of the rock within the top 10 km of the Earth's crust (typically 2.8 ton/m^3), and β is the shear-wave velocity in the vicinity of the source (Brune 1970). $E(\omega)$ is Brune's source spectrum, which is given by the following

$$E(\omega) = \frac{M_0\omega^2}{1+(\omega/\omega_c)^2} \tag{6}$$

M_0 is the seismic moment, and ω_c is the corner frequency, which is given as follows

$$\omega_c = (2\pi)\times 4.9\times 10^6\beta_s(\Delta\sigma/M_0)^{1/3} \tag{7}$$

where, in this equation, the stress drop $\Delta\sigma$ has units of bars, ω_c has units of Hz, β_s has units of km/s, and M_0 has units of dyne-cm. The seismic moment M_0 is often expressed in terms of the moment magnitude M_w, which is defined as follows (Kanamori 1977)

$$M_w = \frac{2}{3}\log M_0 - 10.7 \tag{8}$$

The loss of energy along the wave's travel path is very complex. By definition, the $An(\omega)$ factor includes all of the losses that have not been accounted for by the geometrical attenuation factor and is defined by the exponent expression, which is given as follows (Boore 2003)

$$An(\omega) = \exp\left(\frac{-0.5\,\omega R}{\beta\cdot Q_0\left(\frac{1}{2\pi}\omega\right)^n}\right) \tag{9}$$

where Q_0 and n are the regional dependent factors of the wave transmission quality factor Q, which is defined by the exponent expression.

The attenuation (or diminution) operator $P(\omega)$ in Eq. (5) accounts for the path independent loss of high-frequencies in the ground motions.

$$P(\omega) = \exp\left(-\frac{\omega\kappa}{2}\right) \tag{10}$$

where κ is the attenuation parameter that accounts for the high-frequency cutoff (Anderson and Hough 1984). The term site effect is generally used to refer to wave propagation in the immediate vicinity of the site and not to propagation effects, which refer to the complete path from the source to the receiver. The boundary between a site effect and a propagation effect is not always clear, but it is useful to discuss them separately. In Eq. (5), $A(\omega)$ is the upper crust amplification factor and is a function of the shear-wave velocity versus the depth. The geometrical attenuation can be defined from the developed trilinear attenuation model in accordance with the regional crustal thickness (Boore 2003); here, it is assumed to be $1/R$ for simplicity.

3. Stochastic analysis

Stochastic structural dynamics is a subject that deals with uncertainty in the response of engineering structures. The cause of the response uncertainty may be the unpredictability of the excitations, where each uncertain parameter is assumed to be described as a random variable. To study the stochastic response spectra, the earthquake's magnitude M_w, distance R, static stress drop $\Delta\sigma$, quality factor Q, high-frequency attenuation parameter κ, and amplification factors $A(\omega)$ were modeled as random variables. Each random variable is modeled as follows (Sakurai *et al.* 2001)

$$X = \mu_X(1 + \alpha_X) \tag{11}$$

where μ_x is the mean value and α_x is a random variable with a zero mean. The perturbation method, which is based on a Taylor series expansion of the function of random variables, is used to evaluate the mean and variance of the system's response. The overall variance in the response of the single degree of freedom (SDOF) is affected by the variances in each of the random variables. The relative contributions of the variances in the random variables and the covariance between these random variables to the variance of the system can be approximated by the following equation (IASSAR 1997, Yazdani and Takada 2009)

$$\mathrm{Cov}[U, U] = \sum_{i=1}^{n}\sum_{j=1}^{n} U_i^1 (U_j^1)^T E[\alpha_i \alpha_j] \tag{12}$$

where n is the number of random variables, and U_i^1 and U_j^1 are the coefficient vectors of the first-order rates of change. The covariance matrix of random variables α with zero mean is defined by the following equation

$$\mathrm{Cov}[\alpha_i \alpha_j] = E[\alpha_i \alpha_j] = V_i V_j \rho_{ij} \tag{13}$$

where V_i and V_j are the coefficients of variation (COV) of the random variables α_i and α_j, respectively, and ρ_{ij} is the correlation coefficient of α_i and α_j. For simplicity, ρ_{ij} is taken as 1.0 for each random variable to itself; as 0.5 for the earthquake magnitude to stress drop, earthquake magnitude to attenuation parameter, attenuation parameter to quality factor, attenuation parameter to amplification factor, attenuation parameter to stress drop; and as 0.0 for the other variables (Boore and Joyner 1997, Atkinson and Silva 1997, Mohammadioun and Serva 2001, Franceschina *et al.* 2006).

The overall variance in the response spectrum is affected by the variances in each of the source, path, and site variables. The COV of these variables plays an important role in the variation of the response. The determination of the ground motion variables for pervious earthquake records invariably carries a high degree of uncertainty (Atkinson and Silva 1997), and the specification of these variables can involve a significant degree of expert judgment. The previous studies revealed that the variance of the moment magnitude and the source-to-site distance are less than other variables (EPRI 1993, Boore and Joyner 1997, Benz *et al.* 1997, Mohammadioun and Serva 2001, Silva *et al.* 2002, Bilici *et al.* 2009, Ates *et al.* 2009, Yazdani 2010). In this study, three coefficient of variation values of 0.12, 0.15, and 0.2 were assumed for the Q factor, attenuation parameter, stress drop, and amplification factor variables, while the coefficient was fixed at 0.10 and 0.02 for the distance and the moment magnitude, respectively.

Table 1 shows the chosen mean value set for the earthquake ground motion variables in this study for the calculation of the ground motion's Fourier amplitude spectra, which is used to obtain the response spectra. The expected values of response spectra are calculated from the deterministic and probabilistic analyses. Fig. 1 indicates the minor mismatches between the deterministic response spectrum and the mean values in the probabilistic method. This figure shows that the standard deviation of the response spectra's amplitude increases when the coefficients of variation of the ground motion variables are increased from 0.12 to 0.20. In this figure, the COV is fixed at 0.10 and 0.02 for the distance and moment magnitude.

For an earthquake ground motion with a moment magnitude of $Mw7$, a distance of 40 km, and hard rock site conditions, the mean values of the probabilistic response spectra are illustrated in Fig. 2 for different coefficients of structural nonlinearity. Fig. 2 shows the expected reduction in amplitudes of the response spectra when the coefficient of structural nonlinearity is increased.

Table 1 Set of earthquake ground motion variables

Variables (random variables)	Mean value
Earthquake magnitude, M_w	6, 7, 8
Distance, R (km)	20, 40, 80, 120
Density, ρ_s (gr/cm^3)	2.8
Shear-wave velocity, β_s (km/s)	3.5
Stress drop, $\Delta\sigma$ (bar)	100
Duration, T_w (s)	$\pi/\omega_c + 0.15R$
Quality factor, $Q(\omega)$	$Q = 72.9\ \omega^{0.56}$ (Atkinson and Silva 1997)
High-frequency attenuation parameter, κ (s)	0.05
Geometrical attenuation	R^{-1}
Amplification factor, $V(\omega)$	Very hard rock, NEHRP class C and D (Boore and Joyner 1997)

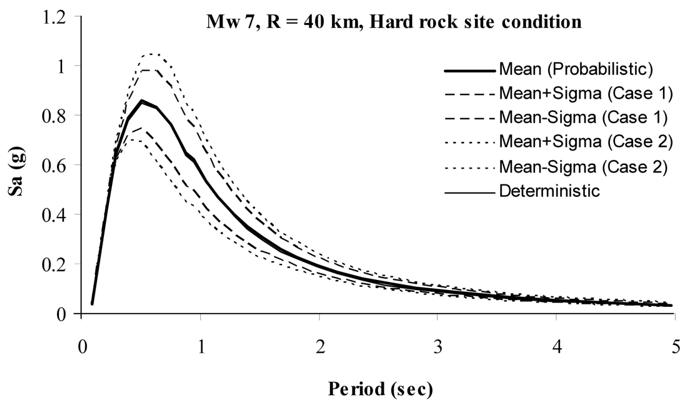

Fig. 1 Comparison of the mean value obtained with the probabilistic and deterministic methods. The dashed lines show the effect of the standard deviation for the following cases: Case 1) $V_{Mw} = .02$, $V_R = 0.10$, $V_Q = V_{Amp.} = V_{Kappa} = V_{Stress\ drop} = 0.12$ and Case 2) $V_{Mw} = 0.02$, $V_R = 0.10$, $V_Q = V_{Amp.} = V_{Kappa} = V_{Stress\ drop} = 0.20$

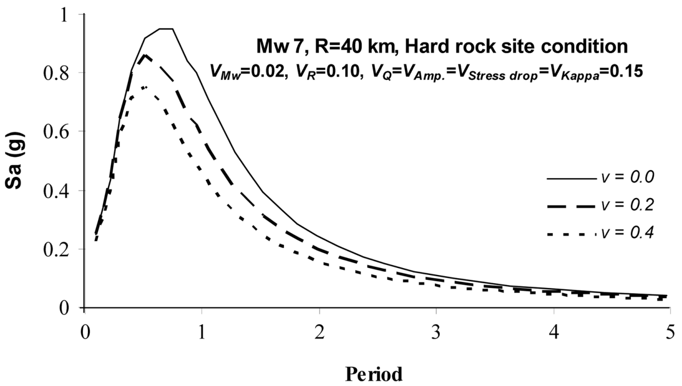

Fig. 2 The probabilistic mean values of the response spectra for different coefficients of structural nonlinearity

The effect of the source-to-site distance on the response spectra can be examined by comparing the response spectra with the same source, path, and site variables at different distances. Fig. 3 compares the mean values of the linear and nonlinear acceleration response spectra at four distances. The spectral acceleration initially gradually increases with increasing period, reaches a peak value, and slowly decreases at high periods. In both traces of this figure, the spectral amplitude of each distance group steadily decreases with increasing distance. These results are similar to those of previous studies that were done based on recorded ground motions (Tehranizadeh and Hamedi 2002, Su *et al.* 2006), where these results are consistent with geometric attenuation in the seismological method.

Seed *et al.* (1976) showed that the soil condition substantially affects the average response spectra. They demonstrated that the spectral amplifications are much higher for deep cohesionless soil deposits and soft to medium clay deposits than for stiff site conditions and rock. Mohraz (1976) showed that, for low and intermediate frequency regions, the spectral bounds for rock deposits are

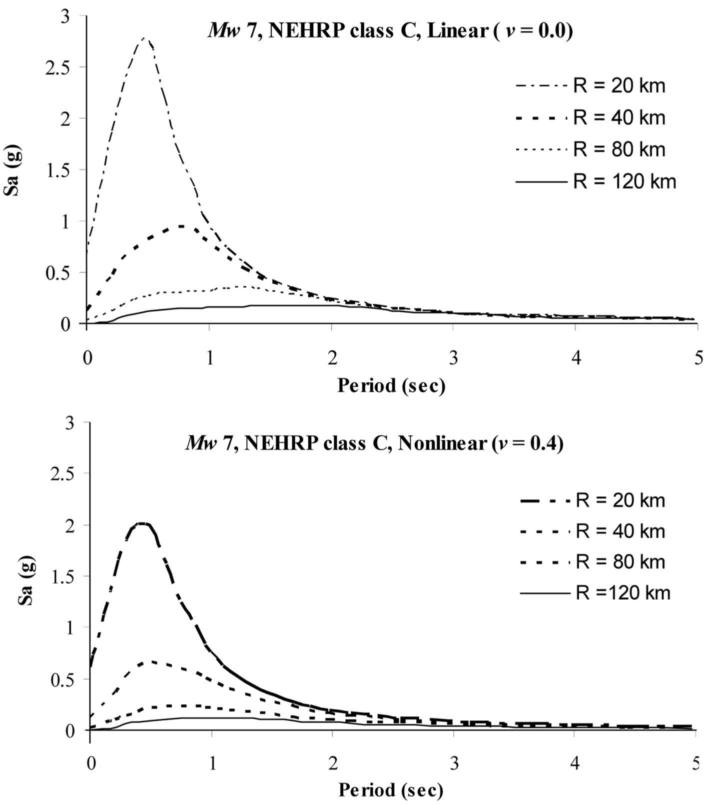

Fig. 3 Comparison of the linear and nonlinear acceleration response spectra in four different distances when the COV of the moment magnitude, distance, and other variables are assumed to equal 0.02, 0.10, and 0.12, respectively

substantially lower than those for alluvium deposits. Fig. 4 shows the effect of the soil condition on the response spectra. This figure shows that the mean amplitude of the linear response spectrum for stiff soil (NEHRP class D) is slightly larger than dense soil and soft rock (NEHRP class C) for periods more than about 0.5 s; however, the converse is observed for short periods, which verifies the results obtained by Seed *et al.* (1976) and Ambraseys *et al.* (2005). Fig. 4 indicates that for large periods, all soil types have nearly identical values.

Several studies have illustrated that the earthquake's magnitude has a strong effect on the shape of the response spectra (Sabetta and Pugliese 1996, Su *et al.* 2006, Ambraseys *et al.* 2005). Larger-size earthquakes have proportionately more low frequency energy and less high frequency energy relative to smaller ones (Su *et al.* 2006). Fig. 5 shows the effect of the magnitude on the response spectra. The response spectra were normalized by the peak response spectra to compare the spectral shape. For short-period structures, the response values decrease slightly with an increase in the earthquake's magnitude. For structures with periods smaller than 0.2 s, the magnitude of an earthquake has minimal influence on the response spectrum. Medium- and high-rise structures are more sensitive to earthquakes with high magnitudes. As shown in Fig. 5, the response spectra of larger earthquakes have proportionately more long-period energy and less short-period energy than those of smaller ones, which is consistent with different recorded ground motions (Su *et al.* 2006)

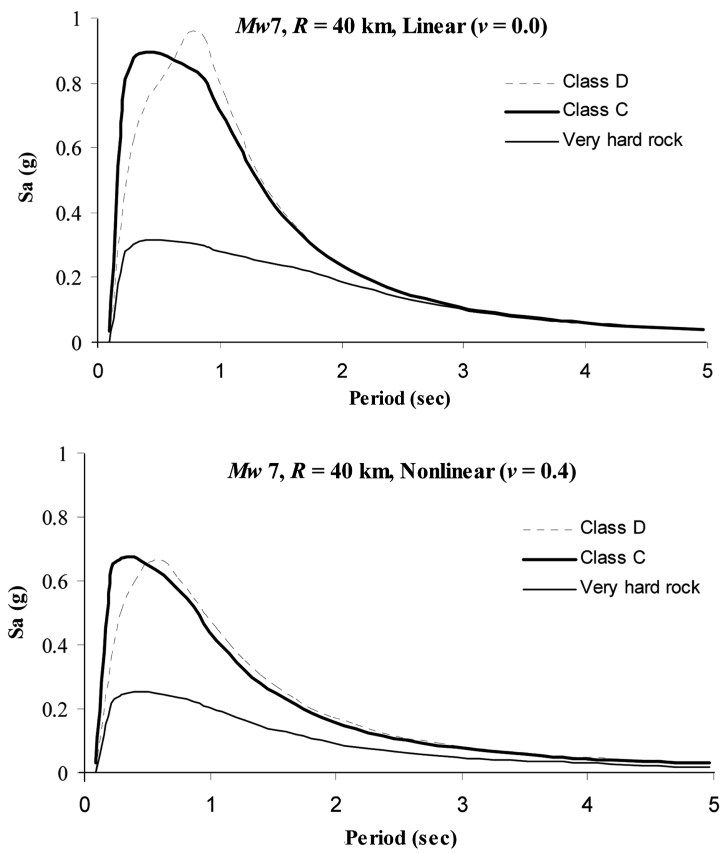

Fig. 4 Comparison of the linear and nonlinear response spectra for different site conditions when the COV for the moment magnitude, distance, and other variables are assumed to equal 0.02, 0.10, and 0.15, respectively

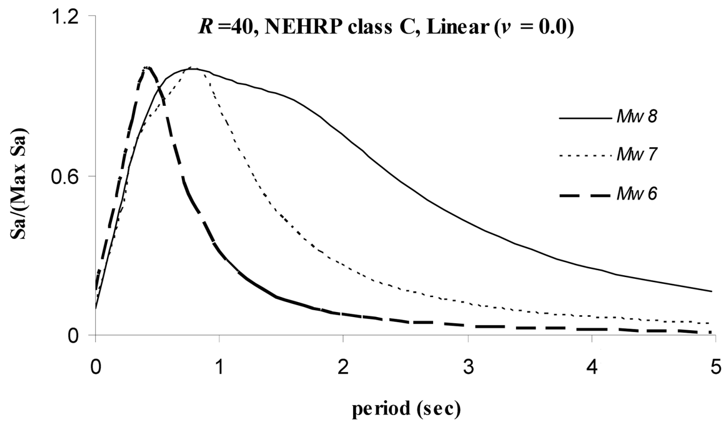

Fig. 5 Effect of the earthquake's magnitude on the response spectra when the COV of the moment magnitude, distance, and other variables are assumed to equal 0.02, 0.10, and 0.20, respectively. The response spectra were normalized by the peak response spectra to compare the spectral shape

and is predicted by seismic scaling laws (Boore 2003).

These results reveal the application of the presented formulation of response spectrum based on frequency domain information of the input excitation.

4. Results and discussions

In structural engineering, problems involving unpredictable or stochastic variables and, in these cases, a probabilistic analysis may be the most rational way of approaching the problem. Based on the presented formulation, the response spectrum can be analytically related to the earthquake ground motion variables. Fig. 6 shows the contributions of the variances to the variables and the covariance between them relative to the variance in the response for different coefficients of structural nonlinearity. This figure shows that there is little variation in the variables' contribution as a function of the structure's coefficient of nonlinearity. Fig. 6 indicates that the magnitude and site amplification are the main sources of uncertainty affecting the probabilistic response of the structures.

Fig. 7 shows the contributions of the variance of the variables and the covariance between them relative to the variance in the response in different distances at different structural periods of 0.3, 0.6, and 1.0 s. In this figure, the COVs of the moment magnitude, the distance, and other earthquake variables are assumed to be 0.02, 0.10 and 0.15, respectively. The COVs in the amplitude of the response spectra at periods of 0.3, 0.6, and 1.0 s are approximately 0.2, 0.25 and 0.2, respectively. As mentioned in Fig. 3, the spectral values decrease as the distance increases for all periods, but Fig. 7 indicates that, at far distances, the relative contribution of the distance is slightly larger than for near distances. Fig. 7 shows that the relative contribution of the earthquake's magnitude is larger in the near distance than in the far distance.

Fig. 5 indicates that the earthquake's magnitude influences the response spectrum's shape and value. Fig. 8 shows the relative contribution of the variances that were computed for the three different magnitudes. This figure indicates how the relative contribution of the earthquake's

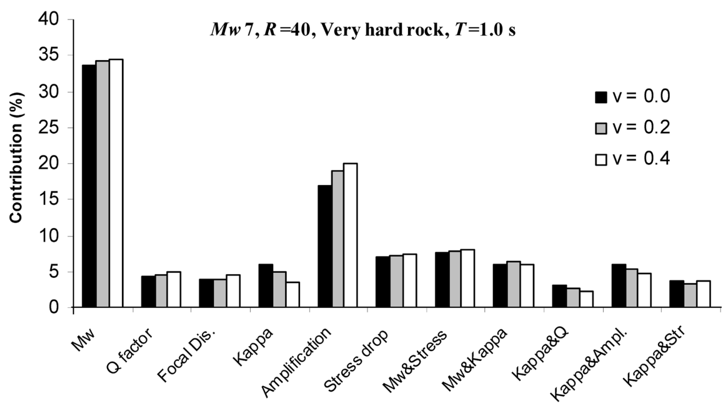

Fig. 6 Relative contributions of the variances in variables and the covariance between them to the variance in the response in the SDOF system when the distance takes the value of 40 km and M_w 7 at a period of 1 s for different nonlinearity when the COV of the moment magnitude, distance, and other variables are assumed to equal 0.02, 0.10, and 0.15, respectively

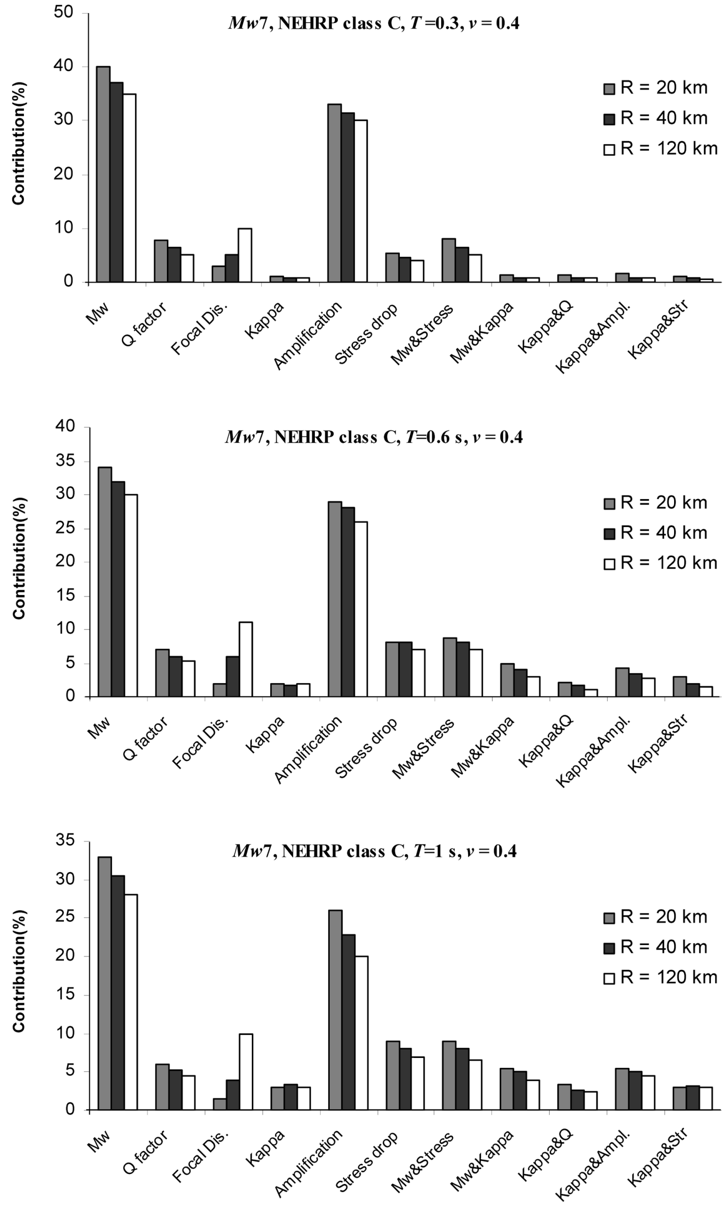

Fig. 7 The relative contributions to the response for distances of 20, 40 and 120 km at periods of 0.3, 0.6, and 1.0 s when the COV of the moment magnitude, distance, and other variables are assumed to equal 0.02, 0.10, and 0.15, respectively

magnitude changes as the magnitude grows, which shows a steady increase in the relative contribution.

Fig. 4 shows the effect of the soil conditions on the response spectra. This figure displays the mean values for very hard rock ($V30 = 2880$ m/s), NEHRP class C, and NEHRP class D. Fig. 9 shows the relative contributions of the response at periods of 0.3 and 1.0 s for three groups of soil

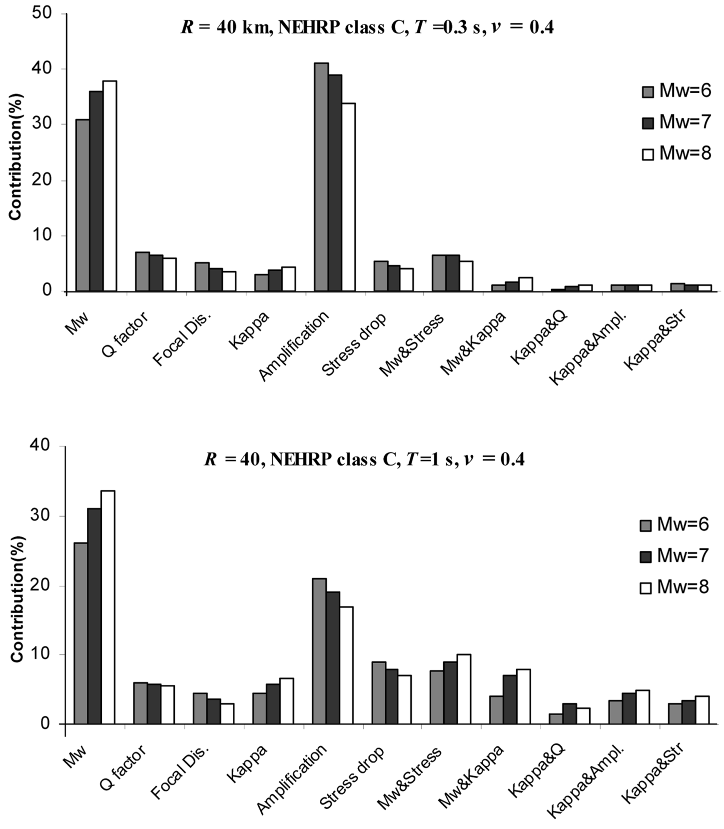

Fig. 8 The relative contribution that was computed for three different magnitudes when the COV of the moment magnitude, distance, and other variables are assumed to equal 0.02, 0.10, and 0.15, respectively

conditions. The figure indicates that the relative contributions of the variables, especially at medium periods, are not more sensitive to local site categories.

The study of seismic wave attenuation is useful to predict the earthquake's ground motion in seismic hazard analysis. The attenuation of the material is often modeled by multiplying by the anelastic path $An(\omega)$ and by using a high-cut filter $D(\omega)$. The high-cut filter process is described by "attenuation parameter" and has often been used to refer more specifically to the distance-independent attenuation operator. The anelastic attenuation's effect is described through the Q factor, which is the distance-dependent operator. Fig. 7 shows that the relative contribution of attenuation parameter is not a function of distance, but the relative contribution of the Q factor varies with distances. The attenuation parameter that is used to account for the high-frequency cutoff is dependent on the earthquake's magnitude (Atkinson and Silva 1997). Fig. 8 illustrates that the relative contribution of attenuation parameter is consistent with the relative contribution of the magnitude.

As a simple way of capturing the variance of Q, the attenuation operator is made up of three piecewise-continuous line segments (Boore 2003). The outer lines are specified by slopes and intercepts at specified reference frequencies, and the middle line joins the outer lines between the

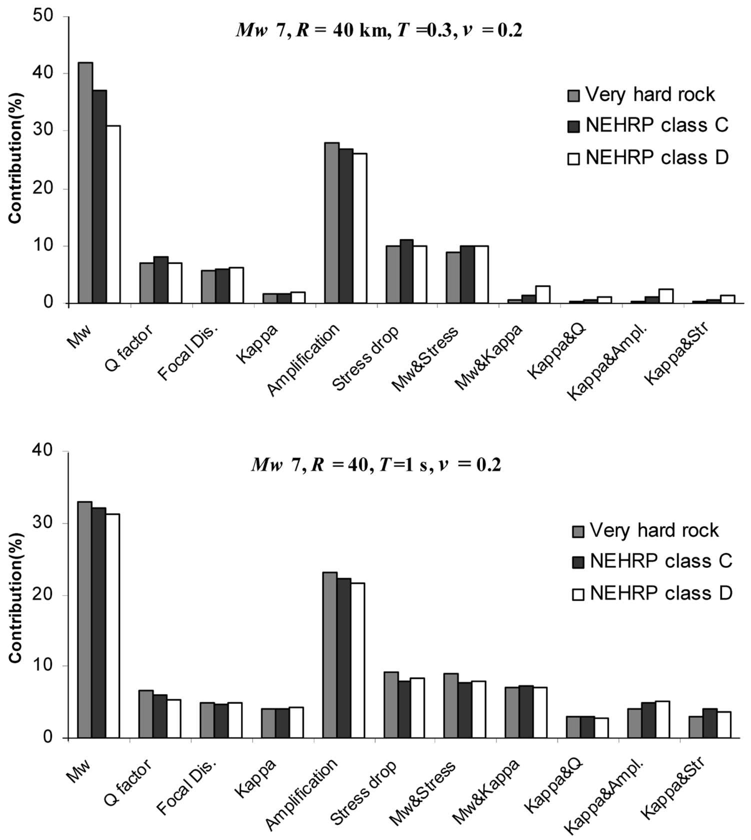

Fig. 9 The relative contributions at periods of 0.3 and 1.0 s for three groups of soil conditions when the COV of the moment magnitude, distance, and other variables are assumed to be equal to 0.02, 0.10, and 0.15, respectively

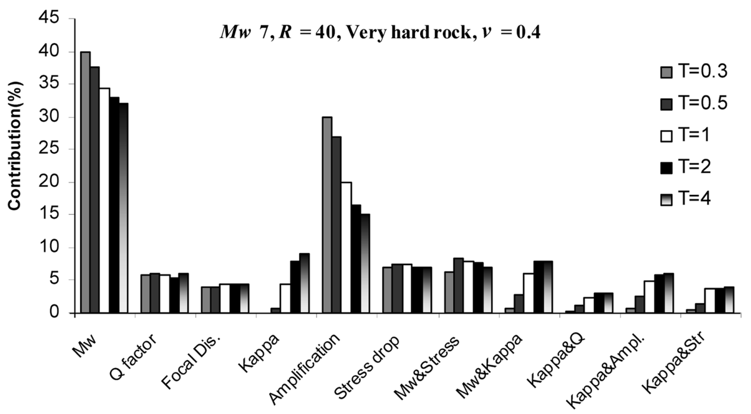

Fig. 10 The relative contributions to the response in the SDOF system at different periods when the COV of the moment magnitude, distance, and other variables are assumed to equal 0.02, 0.10, and 0.15, respectively

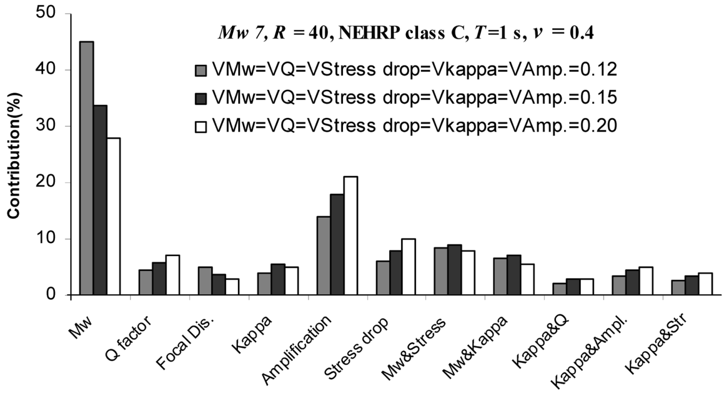

Fig. 11 The relative contributions of the nonlinear response in the SDOF system when coefficient of variation values of 0.12, 0.15, and 0.2 are assumed for the Q factor, attenuation parameter, stress drop, and amplification factor variables, while the coefficient is fixed at 0.10 and 0.02 for the distance and moment magnitude, respectively. The COV in the amplitude of the response spectra at a period of 1.0 s takes 0.18, 0.2 and 0.23, respectively, for the three different COVs in earthquake variables

frequencies of approximately 0.2 and 5 Hz. Fig. 10 demonstrates that the relative contributions of the Q factor do not vary with the structural period in the 0.3 to 4 s range.

Also, Fig. 10 illustrates that the relative contribution of the earthquake's magnitude at short periods is more pronounced than at high periods, and the contribution of the distance and stress drop are not dependent on the frequency. This figure also demonstrates that the relative contribution of the amplification is more sensitive to the period than other variables, and the contribution is more significant in short periods (high frequency) than in high periods (low frequency).

Fig. 11 compares the effect of different COV for different variables. The COV in the response amplitude increases slightly as the COV in the earthquake variables increases. This figure indicates that increasing these coefficients of variation gives the expected increase in the relative contribution of the moment magnitude and focal distance, while reducing the relative contribution of the amplification and other variables.

These analytical results show that path variables, such as the source-site distance as well as both the anelastic and upper crust attenuation, have relatively little effect. By assessment the recorded ground motion, Stafford *et al.* (2008) concluded that there are no systematic differences between ground motions in Western North America versus those in Europe and Middle East. As a result, it seems logical to infer that it could be beneficial to combine these two data sets so as to increase the available records.

5. Conclusions

When the excitation is given in terms of a stochastic process, the response of the mechanical system is also a stochastic process. If the excitation process is Gaussian and the system is linear, a fairly complete theory exists to evaluate the statistical properties of the response. In nonlinear random vibration, an exact probabilistic solution is possible only if the system's response can be modeled as a diffusive Markov process. The exact solution for the Duffing oscillator in this study provides a basis for obtaining approximate solutions for realistic and non-stationary excitations, such as earthquake ground motions.

Most studies on the influence of ground motion variables on the structural response were based on combined data sets that came from different earthquakes and that were recorded in different regions. Thus, inclusion of criteria using site and seismotectonic environment features may significantly reduce the acceptable number of required records.

In some locations where there is a lack of sufficient recorded data, the well-known stochastic models are customarily used for generating strong motion for earthquake design. The stochastic point source model that is based on a static corner frequency, which was used here despite some theoretical deficiencies, gives similar results as the dynamic corner frequency version for medium, far away from the fault and for ground motion frequencies of most interest to engineers ($f > 0.6$ Hz) (Atkinson *et al.* 2009, Boore 2009). One of the deficiencies of this method is that it cannot assume the effect of the fault mechanism, but Ambraseys *et al.* (2005) described that the average effect of different fault mechanisms is not large. As mentioned in the results, the influence of the local soil condition is important. These amplification functions may be different for small and large as well as distant and close earthquakes, which reflect the non-linear soil response. In this study, the possibility of non-linear phenomena in soil during the considered events or the effect of a 2D site condition has not been taken into account. As mentioned by Castellaro *et al.* (2008) and Lee and Trifunac (2010), the average 30-m shear wave velocity, V_{30} may be a deficient parameter to characterize the local site condition, but its effect is considered here in light of its widespread usage in the building codes.

Despite these deficiencies, formulating the response spectrum based on the frequency information of excitation opens the door for wider use of seismological theory to understand the relationship between the response spectra and the seismological variables of interest.

The shape of the response spectra change as the magnitude grows, which shows that medium- and high-rise structures are more sensitive to an earthquake's magnitude than short period structures. It can be seen that the relative contribution of the distance at far distances is more important than at near distances. The amplitude of the response spectrum for stiff soil is slightly larger than for dense soil for medium and high periods, while the reverse holds for short periods.

For design, analysis, retrofit, or other seismic risk decisions, a few design earthquakes can be used wherein the earthquake's threat is characterized by the magnitude, distance, and other variables. The analytical results show that the earthquake source factors and soil condition variables are the main source of uncertainty, while path variables (such as distance, anelastic attenuation and upper crust attenuation) have relatively little effect.

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