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Overturning of rocking rigid bodies under transient ground motions

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Abstract. In seismic prone areas it is possible to meet very different objects (equipment components, on shelf artefacts, simple architectural elements) that can be modelled as a rigid body rocking on a rigid foundation. The interest in their behaviour can have different reasons: seismological, in order to estimate the ground motion intensity, or more strictly mechanical, in order to limit the response severity and to avoid overturning. The behaviour of many rigid bodies subjected to twenty wide ranging acceleration recordings is studied here. The response of the blocks is described using kinematic and energy parameters. A condition under which a so called scale effect is tangible is highlighted. The capacity of the signals to produce overturning is compared to different ground motion parameters, and a good correlation with the Peak Ground Velocity is unveiled.

Key words: rocking rigid body; natural accelerograms; ground motion parameters; response measure; scale effect; structural dynamics.

1. Introduction

In seismic prone areas it is not uncommon to meet a whole set of simple building and archaeological structural elements (tombstones, boundary walls, columns), equipment and industrial components, on shelf stored boxes and artefacts, that, if adequately slender and with a non circular cross section (Koh and Mustafa 1990), lend themselves to be modelled as a rigid body rocking on a moving rigid foundation. In the last hundred and twenty years the response of such elements to seismic actions has attracted a certain interest in engineering literature. Initially such interest was motivated by seismological intents, like estimating the intensity of ground shaking through observation of overturned objects (see Omori 1900, Yim *et al.* 1980, Ishiyama 1982, 1984, Allen *et al.* 1986, Shenton and Jones 1991, and references therein). The resolution to explain counterintuitive behaviours, such as the toppling of structures stubbier than others was added later (Housner 1963). Then came the aim to estimate the vulnerability of architectural and archaeological objects, but also industrial equipment, exposed to earthquake risk. Nonetheless, although seismic danger was the reason for such attention, the driving forces usually considered to evaluate the response of a rocking rigid block were induced by simple pulses, steady-state harmonic functions or spectrum-compatible synthetic accelerograms. Rarer is the resort to strong ground motion recordings, however used in a

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limited number or over a limited set of blocks (e.g., Shi *et al.* 1996, Anooshehpoor *et al.* 1999, Liberatore and Spera 2001a, Makris and Black 2002, Makris and Konstantinidis 2003). In this paper the response of 240 rocking rigid blocks to a collection of 20 natural accelerograms will be studied. This choice has been motivated by the sensitivity of rocking bodies' response to the signals used, reported many times (e.g., Yim *et al.* 1980, Aslam *et al.* 1980), therefore requesting the use of earthquake ground motions. On the contrary, steady-state functions have been rejected since non-transient. Similarly, artificial accelerograms generated to agree to code spectra are not considered adequate, because they are usually based on the damped linear elastic single degree of freedom (SDOF) oscillator, whose behaviour is different under a number of points from the one of rocking rigid bodies (Makris and Konstantinidis 2003). Moreover, due to the highly geometric non linear behaviour of the oscillator considered and the consequent scatter in the results (Yim *et al.* 1980), the response of a wide ranging number of objects will be evaluated through the so called Housner model (Housner 1963).

2. Rocking rigid body model

Considering only plane motions, a rigid body with a non convex base, resting on a horizontal rigid foundation in motion can either move with the ground, rock, slide, slide-rock, or be completely detached from the base (Ishiyama 1982, 1984, Shenton and Jones 1991, Lipscombe and Pellegrino 1989, 1993).

The Housner model can only move with the ground or oscillate alternatively around the two inferior corners O and O' without the possibility of rebounds, as showed in Fig. 1. Therefore it is assumed that the static friction coefficient between block and base is high enough to prevent any sliding. Although it has been analytically demonstrated (Shenton 1996) that slide and slide-rock motions depend also on the amplitude of the excitation used, and not only on block and foundation characteristics, experimental tests have shown that for adequately slender objects sliding is negligible (Liberatore and Spera 2001a). Therefore, in the following pages sliding will be neglected.

Because the body is assumed infinitely rigid, a hypothesis valid whenever any displacement is mainly due to a rigid motion and not to a material deformation, the model has only one degree of freedom, and here is assumed as lagrangian coordinate the rotation θ, positive when anticlockwise.

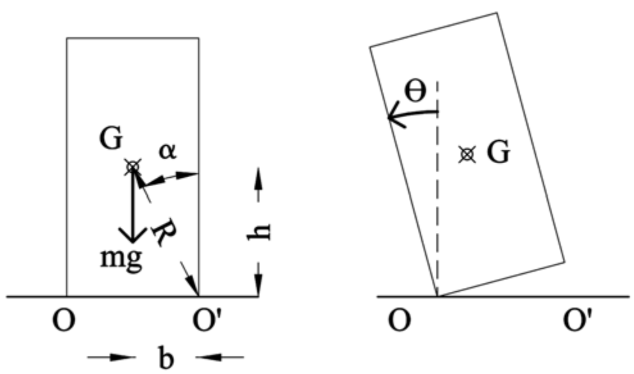

Fig. 1 Parameters describing Housner model of a rigid body and its displacement.

Considering a body symmetric with respect to a vertical axis, the equation of motion is (Housner 1963):

$$\ddot{\theta} = -p^2\left[\operatorname{sgn}(\theta)\sin(\alpha - |\theta|) + \frac{\ddot{x}_g}{g}\cos(\alpha - |\theta|)\right] \tag{1}$$

with the dots indicating derivation with respect to time, *sgn* signum function, $p = \sqrt{mR/I_0}$ frequency parameter, m mass of the body, I_O polar moment of inertia of the body relative to the corner, R distance of the centroid from the corner, $\alpha = \arctan(b/h)$ angle between R and the vertical passing through the corner, with b and h respectively horizontal and vertical projections of R, g gravity acceleration, $\ddot{x}_g$ a horizontal acceleration varying with time t acting on the ground (see Fig. 1).

Starting from zero initial conditions, Eq. (1) states that the block will initiate rocking only if the ground acceleration is greater than a threshold value $\ddot{x}_t = (b/h)g = \tan(\alpha)g$. Consequently, accelerograms with smaller amplitudes are "not felt" by a rocking rigid body with a perfectly plane base, while (due to the inescapable geometrical imperfections) experiments suggested that the actual threshold acceleration is usually lesser (Ishiyama 1984).

Considering a body undergoing rocking, when rotation θ becomes zero the body hits the foundation. Determining the type of motion after the impact is not an easy problem to solve, because the cardinal equations of impulsive dynamics do not set sufficient conditions. Different paths have been explored (Ishiyama 1982, Shenton and Jones 1991, Lipscombe and Pellegrino 1989, 1993, Sinopoli 1987, Augusti and Sinopoli 1992).

Experiments on adequately slender bodies, characterised by usual values of the static friction coefficient, showed that the type of motion after impact will be a rotation around the corner opposite to the rotation one (Liberatore and Spera 2001a). Moreover, coupling this condition with the classical ones of impulsive dynamics it is possible to demonstrate that some kinetic energy is lost, and the angular velocity after the impact is a fraction e of the one before. In literature e is frequently called coefficient of restitution, a designation not completely proper when the impact regards a finite extension surface (Shenton and Jones 1991). Energy dissipation measured by the coefficient of restitution has also been increased or substituted by means of an equivalent viscous damping (Iyengar and Manohar 1991, Hogan 1992a,b). The value of e depends on the position of the point where the resultant impact impulse is applied, being minimum if such point is coincident with the centre of the base, and maximum if coincident with the corner. If this is the case, e is (as will be assumed in the rest of the paper) a function of the sole height to thickness ratio (Housner 1963). Such conjecture is always true only for concave base blocks, while for plane base blocks it is valid only on average, although with some scatter (Liberatore and Spera 2001b), and provided that the body considered is not too stubby (i.e., h/b not less than 3.5, Tocci 1996).

The trigonometric terms of Eq. (1) have been often linearised, in the literature, implying however an error the more pronounced the smaller the oscillations (Allen and Duan 1995). In this paper, considering that the integration will be performed numerically, no linearisation will take place.

Numerical integration has been carried out in a state-space formulation using a Runge-Kutta algorithm, of variable order and time step (The MathWorks 2003). Numerical sensitivity of the solution has been reported several times (e.g., refer to Yim *et al.* 1980, Suherman *et al.* 1997). Therefore, the numerical solution has been compared to the closed form one available for the case of free vibrations (Pompei *et al.* 1998). The comparison has been carried out in Fig. 2 where $\dot{\theta}_r$, reference angular velocity necessary to bring a body initially at rest to the position of unstable

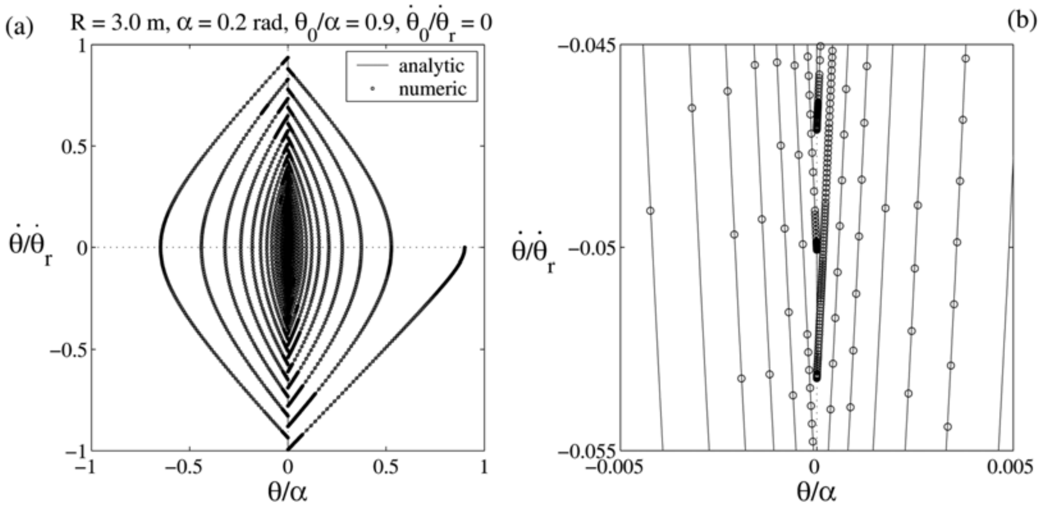

Fig. 2 Comparison of analytic (closed form) solution and numeric integration for free vibration. (b) is a zoom of (a).

equilibrium $\theta = \alpha$, is equal to:

$$\dot{\theta}_r = \sqrt{\frac{3}{2}\frac{g}{R}(1-\cos\alpha)} \quad (2)$$

To achieve the accuracy of Fig. 2 the default error tolerances of the differential equation solver have been sensibly reduced.

3. Recorded accelerograms used

To evaluate the response of the model to seismic ground motions, twenty natural accelerograms, part of a much wider database described elsewhere (Decanini and Mollaioli 1998a, Mollaioli *et al.* 2002), have been chosen. Their most significant features are reported in Table 1. Such signals vary with reference to event, magnitude, duration, distance and position between source and recording station, soil type, and so on. They have been selected to represent quite a wide-ranking set of ground motions in terms of amplitude, duration, frequency content, and sequence of pulses.

Beside signals like El Centro 1940 and Taft 1952, frequently recurring in the technical literature, some others have been considered:

- near fault recorded accelerograms, with forward (LucN80W, KJM000, RRS228, Syl360, and so on), backward (Joshua90) or neutral (BCr230) directivity;
- long duration signals, with different distance from the source (1St280, LlolIN10, CalitWE, SecreN27);
- accelerograms recorded during events with deep energy release source and great distance from the causative fault (Bucar0);
- signals registered in different soil conditions: from stiff, S1, to soft, S3 (Decanini and Mollaioli 1998a).

Table 1 Recorded accelerograms used and their main features

	Event	Date	M_W[a]	Station	D[c] km	Soil[d]	PGA[e] g	PGV[e] cm/s	PGD[e] cm	Δt[f] s	Record
1	Imperial Valley, CA, USA	1940-V-19	7.0	El Centro Array #9	6.4	S2	0.35	29.8	13.3	53.8	40ElC180
2	Kern County, CA, USA	1952-VII-21	7.4	Taft Lincoln School	40	S1	0.18	17.5	9	54.4	Taft111
3	San Fernando, CA, USA	1971-II-9	6.6	Pacoima Dam, abutment	3.2	S1	1.17	114.4	44.5	41.78	Pac164
4	Friuli, Italy	1976-V-6	6.5	Tolmezzo	16	S2	0.35	30.8	5.1	36.5	TolmezWE
5	Romania	1977-III-4	7.5	Bucarest, Romania Building Research Institute	150	S3	0.21	73.6	24.4	16.2	Bucar0
6	Imperial Valley, CA, USA	1979-X-15	6.5	Bonds Corner	2.8	S2	0.78	45.9	14.9	37.595	BCr230
7	Imperial Valley, CA, USA	1979-X-15	6.5	El Centro Array #7	0.2	S2	0.46	109.3	44.5	36.795	IVC230
8	Irpinia, Italy	1980-XI-23	6.8[b]	Calitri	20.7	S2	0.18	18.7	5.1	68.9	CalitWE
9	Michoacan, Mexico	1985-IX-19	8.1[b]	Secretaria comunicacion and tran. Texcoco lake bed zone	389	S3	0.17	59.8	94	135.2	SecreN27
10	Nahanni, Canada	1985-XII-23	6.8	Site 1	0.1	S1	1.10	46.1	14.6	20.545	1St280
11	Chile	1985-III-3	7.8[b]	Llolleo	33	S2	0.71	41.9	77.6	49.3	LlollN10
12	Loma Prieta, CA, USA	1989-X-18	6.9	Los Gatos Presentation Center	0.1	S1	0.56	94.8	41.2	24.95	LGPC000
13	Landers, CA, USA	1992-VI-28	7.3	Joshua Tree Fire Station	11.3	S2	0.28	43.2	14.5	80	Joshua90
14	Landers, CA, USA	1992-VI-28	7.3	Lucerne Valley	1.8	S1	0.64	146.5	262.7	40	LucN80W
15	Northridge, CA, USA	1994-I-17	6.7	Rinaldi Receiving Station	0.1	S2	0.84	166.1	28.8	14.945	RRS228
16	Northridge, CA, USA	1994-I-17	6.7	Sylmar - Olive View Med Parking Lot Free Field	2	S2	0.84	129.6	32.7	39.98	Syl360
17	Kobe, Japan	1995-I-16	6.9	KJMA	1	S2	0.82	81.3	17.7	47.98	KJM000
18	Kobe, Japan	1995-I-16	6.9	Takatori	1.8	S3	0.61	127.1	36	40.95	Tak000
19	Kocaeli, Turkey	1999-VIII-17	7.4	Yarimca Petrokimya Tesisleri	2.6	S3	0.35	62.2	51	34.995	YPT330
20	ChiChi, Taiwan	1999-IX-20	7.6	TCU129	1.2	S1	1.01	60	50.4	89.995	TCU129W

[a]M_W = moment magnitude.
[b]M_S = surface waves magnitude.
[c]D = Distance from the surface projection of the source.
[d]Soil: S1 = stiff, S2 = intermediate, S3 = soft (Decanini and Mollaioli 1998a).
[e]Peak Ground: PGA = Acceleration, PGV = Velocity, PGD = Displacement.
[f]Δt = Duration.

4. Computed rocking curves

For each accelerogram, "rocking" curves have been computed for 240 parallelepiped blocks, meaning for 30 values of α and eight values of R. The latter has been assumed equal to: 1.5, 2 … 5 m, the former to: 0.01, 0.02 … 0.3 rad. The upper bound of α has been chosen so that the hypotheses at the base of the model previously discussed are still valid, at least approximately. For purpose of comparison between different accelerograms the set of α values has been kept equal for all the accelerograms, thus involving (due to the threshold acceleration) that some blocks under certain recordings are not set into motion. The curves have been represented with the angle α as abscissa, and different curves for the values of R considered. This has been done because α is approximately equal to the static load multiplier that causes overturning. Therefore, in the rocking curves, for a given accelerogram and a particular block, it is possible to check the outcome of a dynamic analysis against the prediction of a static one.

As for the ordinates, different quantities have been evaluated, in order to establish which better represents the severity of the response. These are: the absolute maximum rotation, velocity, mechanical energy E (sum of potential and kinetic energies) and the dissipated energy E_D.

All these quantities have been normalised: the displacement by the angle α, the velocity by $\dot{\theta}_r$

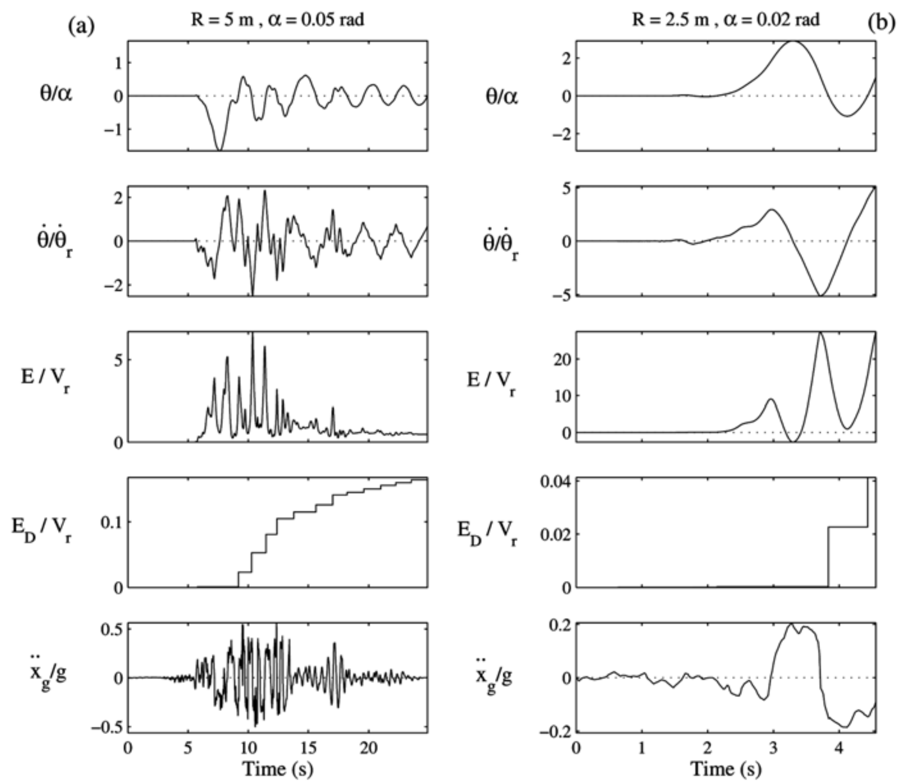

Fig. 3 Sample time histories of, from top to bottom, normalised angular displacement and velocity, mechanical energy, dissipated energy and ground acceleration. Accelerograms: (a) LGPC000, (b) BUCAR0 (refer to Table 1).

and the two energies by the difference in potential energy V_r between the values for $\theta = \alpha$ and $\theta = 0$. Therefore, V_r is equal to:

$$V_r = mgR(1 - \cos\alpha) \tag{3}$$

Starting from rest, V_r is a measure of the energy that is necessary to bring the block to the verge of overturning under gravity only. Similarly, the two quantities α and $\dot{\theta}_r$ have been chosen so that the exceeding the unity value of normalised rotation or angular velocity, while the other parameter is nil, implies overturning under the sole gravity forces.

Some sample time histories are presented in Fig. 3. From (a) it is possible to observe that normalised displacement, velocity and mechanical energy may cross the threshold of the unity value without the block necessarily overturning. As a matter of fact, a favourable combination of input and mechanical energies can lead to the excitation bringing back the body to bounded oscillations. These behaviours have been observed more frequently for very slender bodies, whose potential and dissipated energies are small fractions of the input one. Due to the possibility of the rotation safely

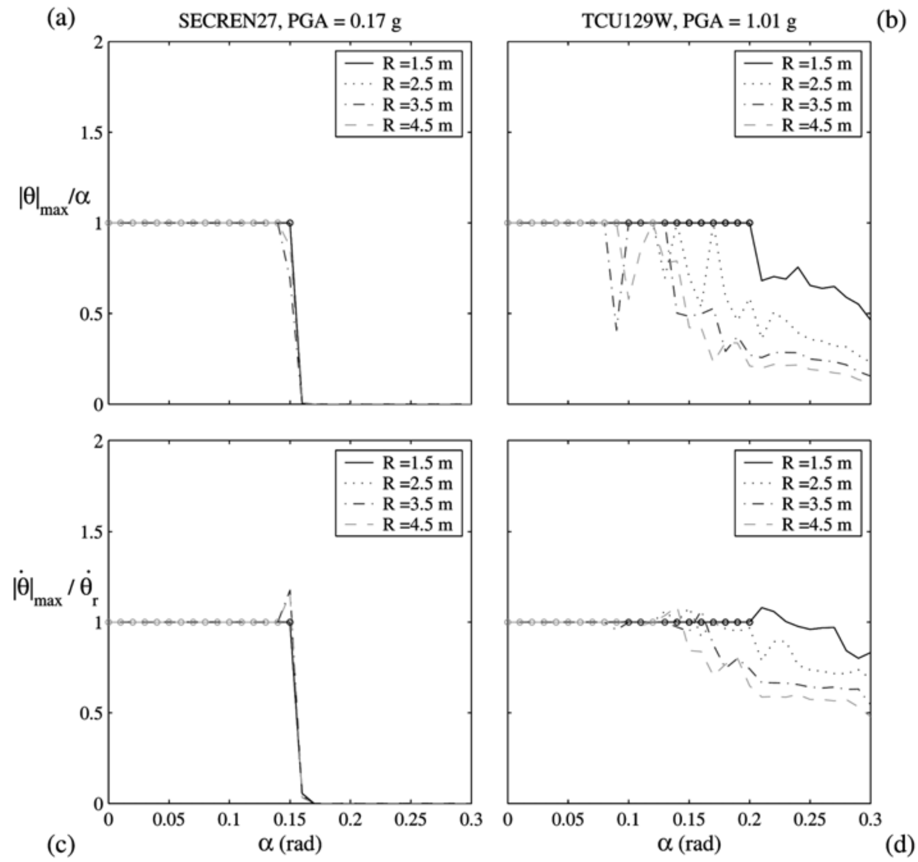

Fig. 4 Rocking curves as a function of α and R in terms of kinematic parameters: (a) and (b) normalised maximum absolute rotation, (c) and (d) normalised maximum absolute angular velocity, for different accelerograms: (a) and (c) SecreN27, (b) and (d) TCU129W, both unscaled (refer to Table 1). The circle indicates an overturning.

exceeding the value of α the integration has been extended up to $\theta = \pi/2$ (as in Ishiyama 1982, Virgin *et al.* 1996).

The mechanical energy can assume, although quite rarely, negative values as shown in Fig. 3(b). This happens when, due to a very large rotation, the height of the centroid on ground level is lower than the one in the rest position, while the angular velocity is next to zero. Again, such phenomena have been observed almost exclusively for very slender bodies.

Sample rocking curves, as a function of α, are represented in Fig. 4 and in Fig. 5. For the sake of representation, only four values of R are displayed and the abscissa values start from zero (corresponding theoretically to an infinite slender body), assuming fictitiously a unity normalised displacement, velocity and mechanical energy and a zero dissipated energy. Due to the possibility of crossing the unity threshold without overturning, this has been marked with a circle. Such crossings are more frequent, in ascending order, for the angular velocity and the mechanical energy than for rotation. Therefore the latter is closer related to overturning. However, it is possible to observe a common trend: while α increases (i.e., the block becomes stubbier) rotation, velocity, and mechanical energy tend to decrease.

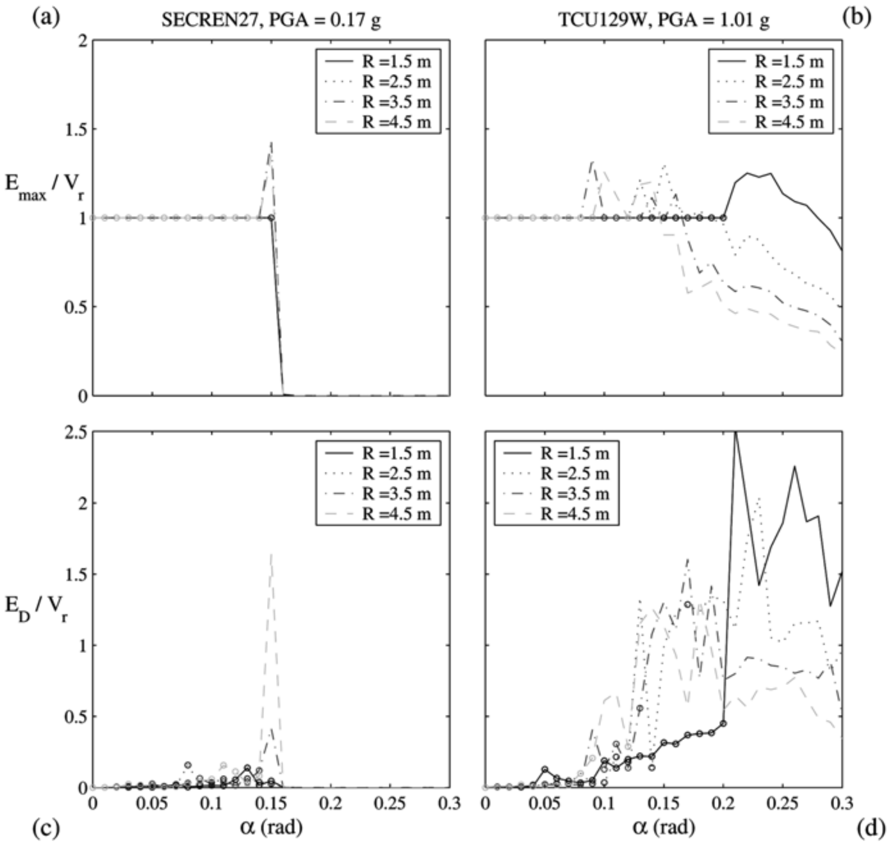

Fig. 5 Rocking curves as a function of α and R in terms of energy parameters: (a) and (b) normalised maximum mechanical energy, (c) and (d) normalised dissipated energy, for different accelerograms: (a) and (c) SecreN27, (b) and (d) TCU129W, both unscaled (refer to Table 1). The circle indicates an overturning.

With reference to the dissipated energy, it is possible to note that it is a parameter poorly related to the severity of the response. As a matter of fact, the highest values of dissipated energy will be registered for blocks set into motion for the longest time but not overturned, whereas toppled elements will not have the time to dissipate the same level of kinetic energy, although suffering a much more serious outcome. Such remarks are valid for any parameter computed over the entire time history, instead than in a single, critical instant. Of course this is also due to the characteristics of the model that does not take into account any progressive damage of the system in the equation of motion.

In the curves of Fig. 4(b) it is possible to observe a “scale effect”: the larger (i.e., with higher R) of two blocks with same aspect ratio (i.e., angle α) usually experiences a smaller maximum rotation, as well as velocity and mechanical energy. However, comparing Fig. 4(b) with Fig. 4(a), one can note that this effect is not always evenly tangible. To account for this diversity it is useful to recur to the reference potential energy V_r: let us consider two blocks with same α, but diverse R.

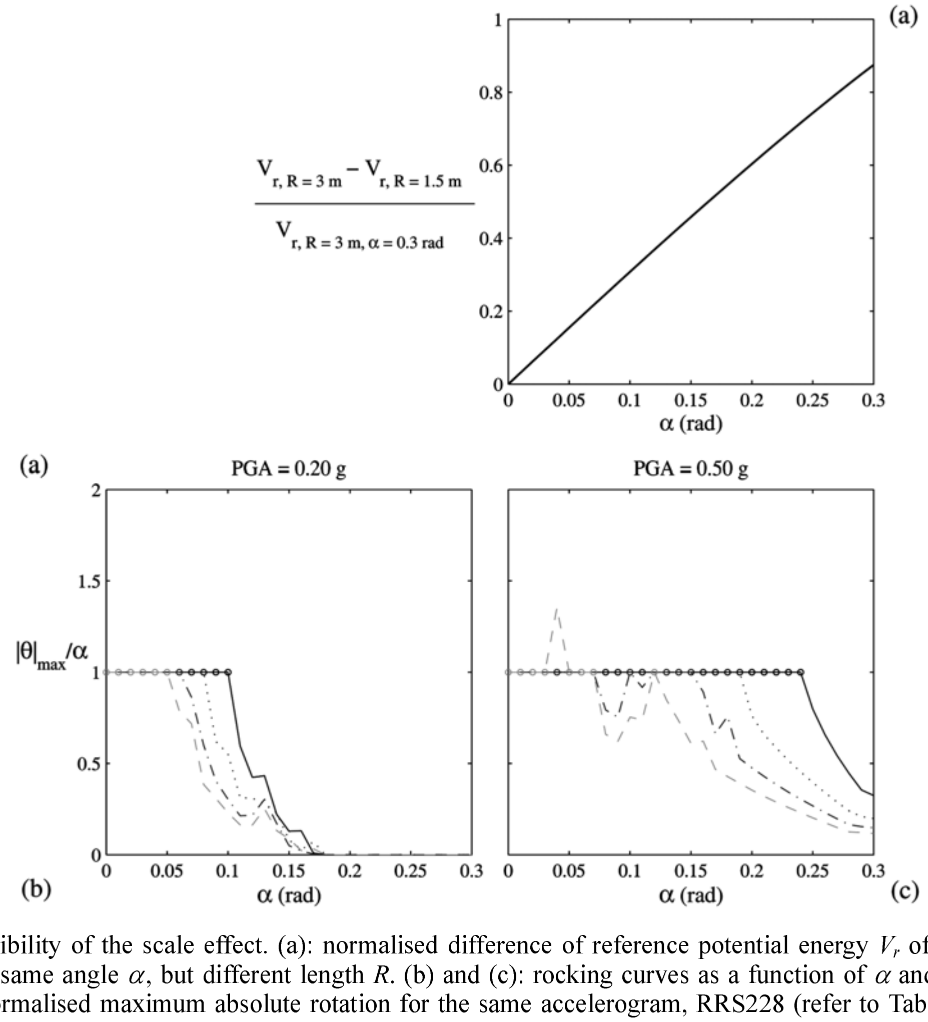

Fig. 6 Tangibility of the scale effect. (a): normalised difference of reference potential energy V_r of two blocks with same angle α, but different length R. (b) and (c): rocking curves as a function of α and R in terms of normalised maximum absolute rotation for the same accelerogram, RRS228 (refer to Table 1), scaled to different PGA: 0.2 g (b) and 0.5 g (c). The circle indicates an overturning.

The difference between the respective energies of Eq. (3) is a measure of the difference of input energy that is necessary to supply to bring both blocks to the unstable position. Such difference becomes more and more relevant the bigger α (refer to Fig. 6(a)). But to have stubby blocks set into motion it is necessary to use high amplitude accelerograms, such as that in Fig. 4(b). This explanation has been confirmed scaling up a signal. Whereas for a certain PGA the maximum rotation was lightly dependent upon R, an increase of the signal amplitude led to a much more pronounced dependency from the size of the blocks. This is shown in Fig. 6(b) and (c) where, once the accelerogram has been scaled up, the curves of equal size become further apart from each other.

5. Searching for a parameter of the dangerousness of a signal

There are many synthetic parameters proposed to measure the ground motion destructiveness potential (for a review of many of them refer to e.g., Decanini and Mollaioli 1998b). Some are based solely on the signal's characteristics, and upon its integration in time and frequency domains,

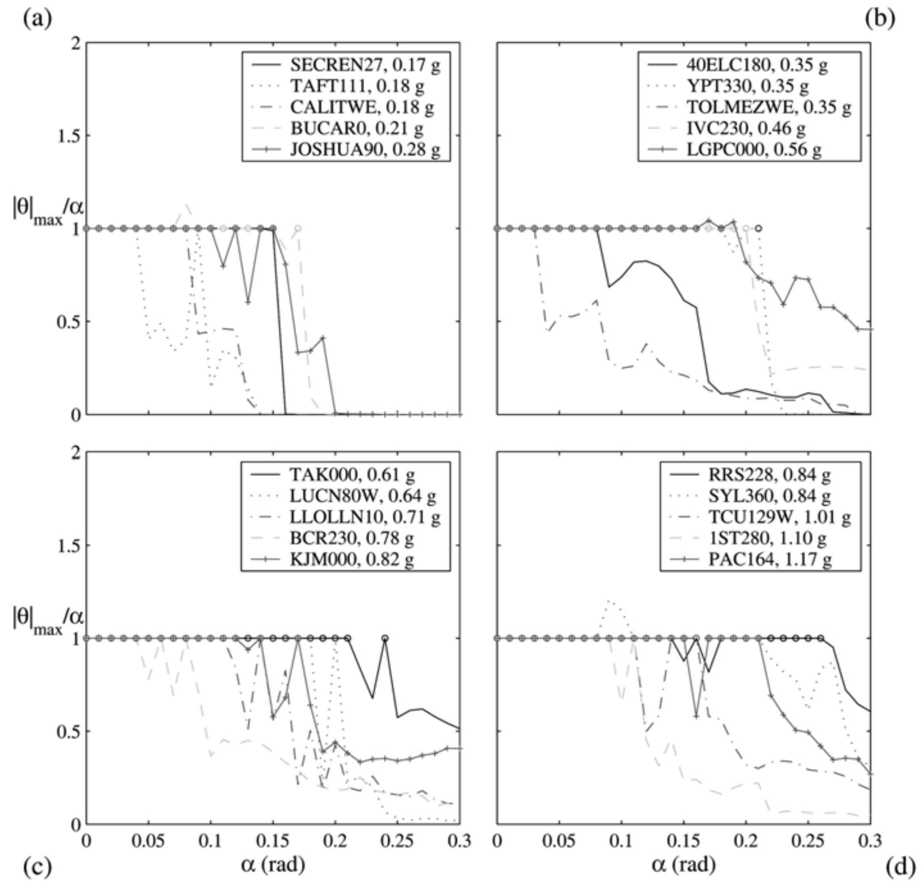

Fig. 7 For R = 3 m, rocking curves in terms of normalised maximum absolute rotation to different signals (refer to Table 1), sorted by PGA in ascending order from (a) to (d), as reported in each legend. The circle indicates an overturning.

some others refer to the response of a set of SDOF oscillator. The latter depend of course on the oscillator's dynamic features, the most widespread being the damped linear elastic. Recent studies have confirmed how much the rocking rigid bodies' behaviour is different from the one of this oscillator (Makris and Konstantinidis 2003). Therefore, to evaluate which signal is more dangerous, i.e., capable to determine the highest number of overturnings, it is necessary to look directly to the rigid body's response. Moreover, it is crucial to take into account a set of blocks both to represent those objects whose behaviour during an earthquake will be a rocking one, and to account for the scatter in the response, which can change dramatically for small changes in the input motion or in the system parameters (Aslam *et al.* 1980, Yim *et al.* 1980, Plaut *et al.* 1996).

Among the accelerograms' synthetic parameters, probably the first and most widespread is the Peak Ground Acceleration (PGA). As aforementioned, in the Housner model the rigid-softening stiffness makes the PGA very important: in fact if the amplitude of the recording is smaller than the threshold acceleration the motion cannot start. In Fig. 7, for a given R, the rocking curves to different accelerograms are sorted by PGA.

Defining α_{lim} as the maximum value of α among those of the overturned blocks, two different extreme behaviours may be observed. The first one is characterised by a tangent of α_{lim} (approximately equal to α_{lim} in the range considered) well below PGA/g, therefore highlighting the existence of a safety reserve, with respect to the prediction of a static analysis, sometimes quite noteworthy. Among such signals is Taft111, one of the most used in early earthquake engineering. The second one shows a tangent of α_{lim} almost equal to the normalised PGA, therefore meaning that nearly the whole set of blocks set into motion is overturned. Among such recordings are SecreN27, Bucar0, RRS228. If, however, the comparison is made not between overturned over rocked blocks but between overturned over whole set of blocks considered, from Fig. 7 it is possible to affirm that a signal with low PGA will not be extremely dangerous, since, due to the threshold acceleration, it will not be able to set stubby blocks into rocking. Meanwhile, a high PGA is not necessarily a guarantee of toppling, because a block can be set into motion without automatically overturning. An example of the latter behaviour is the recording 1St280 which has the second highest PGA but is the 15th considering the number of overturnings (refer again to Fig. 7).

In order to compare more promptly signals' parameter to blocks' response, an Overturning Index I_{Over} has been defined, for each value of R, as:

$$I_{Over}(R) = \frac{n_{Over}}{n_{Tot}} \tag{4}$$

with n_{Over} number of overturned blocks and n_{Tot} total number of blocks of equal R (30 in this paper). To account for the scatter in the response, the value of I_{Over} reported in Fig. 8 and in Fig. 9 has been computed as the mean (and interquartile range, to assess an incidental skew scatter) over a population of eight values of R, normally distributed around a mean equal to 3.0 m with a standard deviation equal to 0.05 m. The standard deviation has been kept small in order to make the influence of the scale effect negligible. Consequently, each value of I_{Over} displayed, being the result of eight R curves each constituted by 30 α values (ranging as before between 0.01 and 0.3 rad), represents the outcome of 240 time histories. An example of this kind of representation is in Fig. 8(a), where it is very clear that the PGA is not a strongly correlated parameter to the ultimate response of the family of blocks considered here. Thus, it is apparent that the sole PGA is not sufficient to convey the destructiveness potential of the ground motion towards a rocking block, and it is necessary to account for other features of the accelerograms.

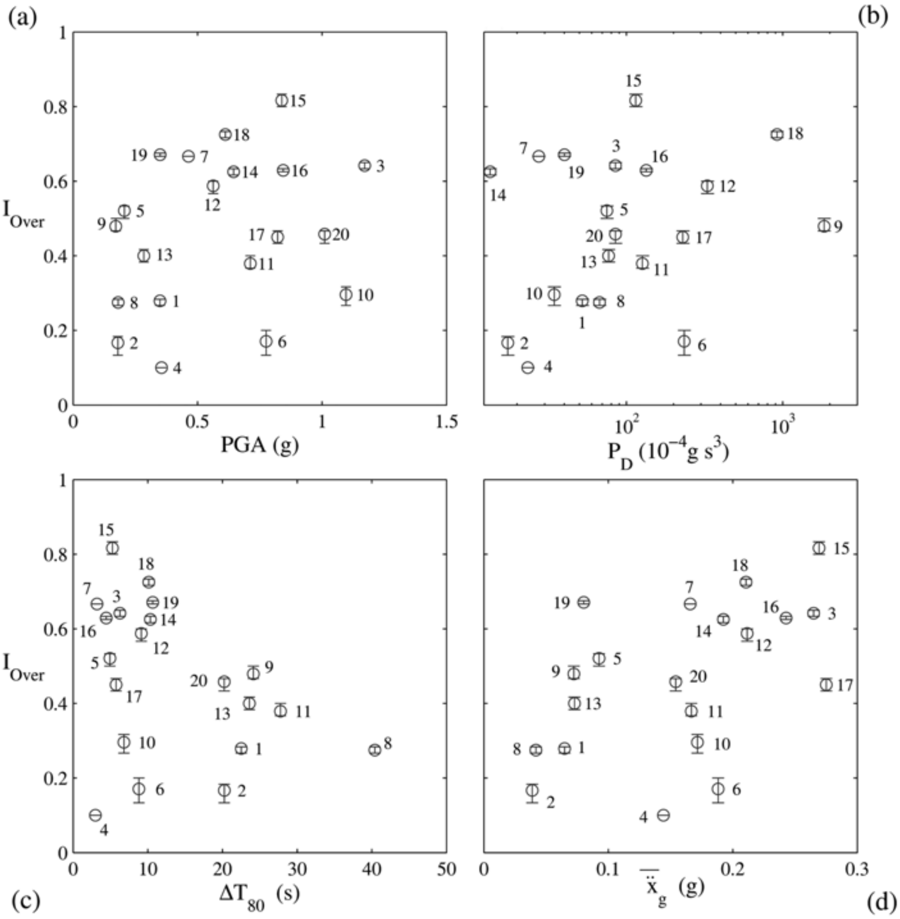

Fig. 8 For $\overline{R}$ = 3 m and unscaled PGA, correlation between the mean value and the interquartile range of the Overturning Index of Eq. (4) and PGA (a), destructiveness potential of Eq. (5) (b), Significant Duration (c), root mean square acceleration (d). Numbering refers to signals as reported in Table 1.

Among the synthetic parameters of a signal is the so-called destructiveness potential factor, P_D. The equation proposed in Araya and Saragoni (1984) is:

$$P_D = \frac{I_A}{\nu_o^2} \tag{5}$$

with ν_0 number of time axis crossings per time unity, and I_A Arias Intensity (Arias 1970), equal to:

$$I_A = \frac{\pi}{2g}\int \ddot{x}_g^2 dt \tag{6}$$

The correlation between P_D and I_{Over} is showed in Fig. 8(b), where for the sake of illustration a logarithmic scale has been used for abscissa. The agreement is not faultless because ν_0 tends to exaggeratedly exalt some accelerograms, as the no.9 – SecreN27, and to belittle some other, e.g., no.14 – LucN80W. Modifying Eq. (5), dividing Arias Intensity by ν_0 instead of its square, led to little improvement. Moreover, correlation with Arias Intensity proved even worse. The destructiveness potential of Eq. (5) tends to exalt, through v_0, the signals characterized by low frequencies, while accounting for amplitudes and duration, through Arias Intensity. The duration, however, weakly affects the block response because the model does account neither for any cumulative damage, nor for any sliding, that in stubbier and multiblock structures can lead to collapse due to degradation of geometric configuration (Sinopoli and Sepe 1993). Even the use of the so called Significant Duration Δt_{80}, i.e., the time interval between the instants t_{10} and t_{90} when 10% and 90% of Arias Intensity are released (Trifunac and Brady 1975), did not lead to a better agreement, as showed in Fig. 8(c).

Other quantities related to the significant duration where considered too. This is the case of the root mean square acceleration $\bar{\ddot{x}}_g$ (Housner and Jennings 1964):

$$\bar{\ddot{x}}_g = \sqrt{\frac{1}{\Delta t_{80}} \int_{t_{10}}^{t_{90}} \ddot{x}_g^2 dt} \tag{7}$$

reported in Fig. 8(d), and of the Characteristic Intensity I_a (Park *et al.* 1985):

$$I_a = \bar{\ddot{x}}_g^{1.5} \Delta t_{80}^{0.5} \tag{8}$$

not plotted here. Both correlations proved meagre. This is also the case of the magnitude, and not only because the accelerograms considered were recorded at different distances from the fault, since in Makris and Black (2002), where only near-fault accelerograms were used to shake two different blocks, no good agreement was found.

An additional line of inquiry considered the predominant period T_p of the recording. A first simplified comparison has been outlined with reference to the soil type. A reasonably good correlation has been obtained if the signals are scaled to a common PGA, with soft soils being most dangerous. However if the excitations are unmodified the correlation deteriorates sharply. Evidently the scaling, by praising the differences between frequency contents and soothing those between amplitudes, strongly affects the block's response. Yet, soil type can only partially influence the predominant period of the accelerogram, therefore this has been directly computed as the period associated with the peak of the Fourier amplitude spectrum of the accelerogram (Ojeda and Escallon 2000). Other definitions of T_p present in the literature (Ojeda and Escallon 2000, Miranda and Ruiz-Garcia 2002), although usually leading to similar values, are not fit for the present discussion since referred to different oscillators or to the definition of site characteristics on the base of amplification ratios. The comparison of a predominant period, as early defined, versus overturning index is presented in Fig. 9(a), where it is evident that no clear correlation is recognizable. However, it is possible to remark that signals with a high T_p are usually capable of overturning the large part of the blocks set into motion, as even more evident when all the signals are scaled to the same PGA, and in agreement with the sensitivity to low frequencies observed experimentally and numerically on multiblock columns (Psycharis *et al.* 2000, Papantonopoulos *et al.* 2002). Yet, neglecting the amplitude an important piece of information is lost. Other attempts based on the Fourier Transform were therefore made, using for example the ratio between absolute amplitude and frequency of the Power Spectral Density's centroid, but without marked enhancement. Accelerograms' phase spectra did not yield better results.

Finally the correlation with other parameters, such as Peak Ground Displacement (PGD) and Peak Ground Velocity (PGV) has been investigated. This has been done only for the non scaled signals.

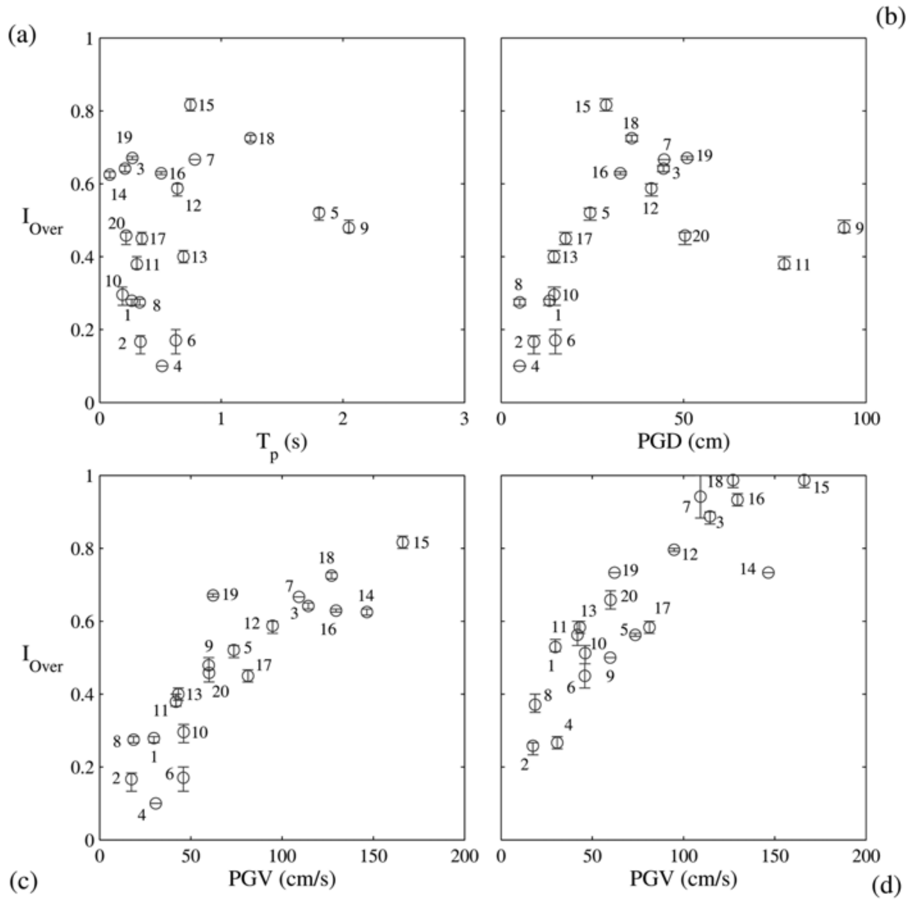

Fig. 9 For unscaled PGA, correlation between the mean value and the interquartile range of the Overturning Index of Eq. (4) and predominant period ($\overline{R}$ = 3 m) (a), PGD ($\overline{R}$ = 3 m) (b), PGV ($\overline{R}$ = 3 m) (c) and PGV ($\overline{R}$ = 1.5 m) (d). In figure (b) the PGD of the record LucN80W is not represented, because out of range. Numbering refers to signals as reported in Table 1.

Even if this is the case, such values must be considered carefully (Gregor and Bolt 1997). In fact they are affected by the type of recording (accelerogram or velocigram), the instrument used to record the signal and its dynamic features, the filtering process, etc. These reasons discouraged the attempt to compute their values for scaled signals. Even the figures of Table 1 have been obtained from the issuing fonts of the recordings. Nonetheless, the PGD values of the accelerograms TolmezWE, CalitWE, SecreN27 and LucN80W should be considered warily.

The comparison of PGD and I_{Over} shown in Fig. 9(b) demonstrates a correlation worse than the preceding ones.

In Fig. 9(c) the same comparison has been realized considering the PGV. In this case it is possible to witness a far better agreement, certainly the best one among those attempted here. Almost as good are the correlations with the maximum incremental velocity, IV, that is the maximum area under an acceleration pulse, proposed by Anderson and Bertero (1987), and the Fajfar Intensity I_v (Fajfar *et al.* 1989):

$$I_v = PGV\Delta t_{80}^{0.25} \tag{9}$$

two velocity based parameters. However, the latter, for reasons already discussed regarding the role of the duration, does not bear a marked improvement with reference to the simpler PGV.

The good correlation of blocks' overturning and PGV is a result of some interest, since the ground velocity can be assessed more robustly than the displacement, or even directly measured using the proper instrument. As a matter of fact, such findings are in agreement with Makris and Black (2002), where the best parameter for predicting a rocking block's overturning was established in the product of amplitude and duration of the most dominant velocity pulse of the record, again therefore a velocity related quantity. Moreover, velocity based intensity parameters proved well related with collapse or damage of more complicated masonry assemblies (Casolo 2001, de Felice and Giannini 2001).

In order to check if the results obtained are valid only for the assumed values of R, a complete new set of numerical analyses has been performed, considering the same values of α while R was part of a Gaussian distributed population with mean 1.5 m and standard deviation 0.025 m. The plot PGV versus I_{Over} is presented in Fig. 9(d), and the correlation holds. A higher number of overturning is noticeable, as expected, since the scale has been reduced. However, the increase in the number of toppling is small for signals 9, 5, 19, 2 and 8 (all having relatively moderate PGA, see Table 1) and more pronounced for the others, thus confirming as already noted that the scale effect is more tangible with high amplitude signals.

To explain the good correlation between peak ground velocity and overturning index it is useful to consider a very simple ground motion: a half-sine pulse of acceleration. For this motion a straightforward equation relates PGA and PGV:

$$PGV = \frac{PGV}{\omega} \tag{10}$$

with ω being the circular frequency of the ground motion. The minimum non dimensional acceleration amplitude, given the non dimensional frequency, necessary to overturn a certain block, was correctly determined by Shi *et al.* (1996), for the case of the linearised equation of motion. This is:

$$\frac{PGA}{g\alpha} = \sqrt{1 + \left(\frac{\cos\psi + e^{-\frac{p}{\omega}(\pi-\psi)}}{\cos\psi}\frac{\omega}{p}\right)^2} \tag{11}$$

with $\psi = \arcsin(\alpha g/PGA)$ phase angle necessary to have a ground acceleration equal to the threshold one when t = 0. Considering Eq. (11) it is possible to observe that a certain combination of amplitude and frequency is necessary to overturn a block, and an increase in the frequency requests an appropriate increase in the amplitude. Therefore, the sole frequency or amplitude is not significant for the overturning of a block, since high amplitudes might not be enough if associated with high frequencies, and the same is true if low frequencies are coupled with low amplitudes. Vice versa the PGV seems to have the capability to grasp both amplitude and frequency, so that to have a certain peak ground velocity if the frequency is increased the amplitude will also have to be increased (refer to Eq. (10)), in a way qualitatively similar to the one stated by Eq. (11).

6. Conclusions

In this paper the response of a rocking rigid body, the so called Housner model, to twenty recorded accelerograms has been studied. The use of such excitations have been chosen due to the limited number of studies on this matter, due to the seismic related interest to the response of block like structures, and to the undeniable physical meaningfulness of the excitations considered.

Different parameters have been used to measure the response of the system. Rotation was the closer related to toppling, whereas angular velocity and mechanical energy can more frequently (and largely) cross the instability threshold without any overturning. The dissipated energy proved itself badly correlated to the severity of the response.

The consideration of the rocking curves, in terms of maximum absolute rotation, angular velocity and mechanical energy, as functions of the angle α and the dimension R, has confirmed that (although with some scatter) a system is safer the stubbier and the bigger it is. The latter behaviour, also known as "scale effect", is not equally tangible varying the accelerogram. It has been demonstrated here that this effect is much more pronounced the less a block is slender, and therefore is predominantly emphasized by high amplitude excitations.

Exactly the absolute peak amplitude, the PGA, is one of the parameters most frequently used in earthquake engineering to express seismic hazard (Chen and Scawthorn 2003). For a rocking rigid body it certainly has no little importance, due to the existence of a threshold acceleration. However, it is not completely satisfactory, and the same is true for duration related quantities. A comparison between response and predominant period was partially adequate only for signals scaled to a common peak, thus underlining the sensitivity to low frequencies, already reported for similar types of structures. Also the comparison with the peak ground displacement and velocity has been carried out. Especially the first, which incidentally did not lead to encouraging results, must be considered sceptically due to the uncertainties in PGD evaluation. On the contrary the PGV brought to a much more interesting outcome, confirmed also by a different choice of the system parameters. This is due to its characteristic to summarize both amplitude and frequency, as necessary when overturning of a rocking body is under inquiry. Therefore PGV should be used when assessing seismic hazard for rocking objects.

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