Perturbation Based Stochastic Finite Element Analysis of the Structural Systems with Composite Sections under Earthquake Forces

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Abstract. This paper demonstrates an application of the perturbation based stochastic finite element method (SFEM) for predicting the performance of structural systems made of composite sections with random material properties. The composite member consists of materials in contact each of which can surround a finite number of inclusions. The perturbation based stochastic finite element analysis can provide probabilistic behavior of a structure, only the first two moments of random variables need to be known, and should therefore be suitable as an alternative to Monte Carlo simulation (MCS) for realizing structural analysis. A summary of stiffness matrix formulation of composite systems and perturbation based stochastic finite element dynamic analysis formulation of structural systems made of composite sections is given. Two numerical examples are presented to illustrate the method. During stochastic analysis, displacements and sectional forces of composite systems are obtained from perturbation and Monte Carlo methods by changing elastic modulus as random variable. The results imply that perturbation based SFEM method gives close results to MCS method and it can be used instead of MCS method, especially, if computational cost is taken into consideration.

Keywords : stochastic finite element method; stochastic perturbation technique; composite; stiffness matrix; monte carlo simulation.

1. Introduction

In recent years, the structural components and systems made of composite sections have received greater recognition in applications toward transportation systems, office/residential buildings, highway bridges, communication systems, offshore structures, etc. Such systems make use of each type of member in the most efficient manner to maximize the structural and economic benefits. Composites are lightweight and flexible in net shape fabrication. Therefore, the construction time, part redundancy and labor cost will be less for building composite structures due to ease of handling.

The traditional structural analyses are realized according to the assumption that geometrical and material characteristics of a structure are deterministic. However, there are some uncertainties about design values, certainly. These uncertainties can be illustrated geometrical characteristics (cross-sectional area, flexural inertia, length etc.), material characteristics (elastic modulus, poisson' ratio etc.), and magnitudes and distributions of loads. Because of these uncertainties, deterministic method can remain insufficient

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in a lot of structural system analysis. On the other hand, stochastic finite element method, which is one of the probabilistic analysis methods, increases its reliability day by day. This method is applied in several fields in civil engineering, especially, simple or semi-complex structural systems.

While analytic solutions to most of problem are restricted to simple linear elastic structures under dynamic loads, the recent research work is focused in obtaining numerical solutions that are more appropriate for handling realistic problems. Stochastic finite element methods (SFEM) belong to this second category (Kleiber and Hien 1992). SFEM approaches are based on the representation of stochastic fields as a series of random variables and various methodologies have been developed in order to achieve this objective. The response statistics are obtained using one of the stochastic finite element methods: the perturbation or Taylor expansion based methods (Kleiber and Hien 1992, Papadopoulos and Papadrakakis 1997, Kaminski 2006, Ghanem 1999), the weighted integral method (Deodatis 1991, Yamazaki *et al.* 1988), the Neumann expansion method (Yamazaki *et al* 1998)and the polynomial chaos expansion method (Ghanem and Spanos 1991). The perturbation method is the most widely used technique for analyzing uncertain system. In the perturbation based stochastic finite element analysis, only the first two moments of random variables need to be known, whereas statistical techniques such as the Monte Carlo simulation generally require knowledge of probability density functions that are usually not available in practice (Kleiber and Hien 1992).

As composites is a recent field of arousing interest, the studies about this field had been increased day by day. Although there is an extensive literature on deterministic analysis of stiffened laminated plates, shell, fiber reinforced polymer beams and on composite concrete slabs stiffened by steel beams (Sapountzakis 2004, Sapountzakis and Mokos 2007, Wang and Li 2007, Sapountzakis and Mokos 2007, Lal *et al* 2007, Vellascoa 2006) technical literature is not adequate on the stochastic dynamic analysis of structural systems made of composite sections. Sapountzakis and Mokos (2007) studied the deterministic dynamic analysis of 3-D composite beam elements subjected in dynamic twisting, bending, transverse or longitudinal arbitrary loading. (Ngah and Young 2007) studied an application of the spectral stochastic finite element method (SSFEM) for predicting the performance of a composite structure with variable material constitutive properties. It was observed the SSFEM is applicable over a wider range of material variability (standard deviation up to about 24% of mean). Antonio and Hoffbauer (2007) performed structural responses of statically loaded composite plate and shell structures with randomness in material properties. In addition, it has been observed that there are very limited works available regarding the perturbation based SFEM aspects of structures have composite sections. Very few researchers (2006), Kaminski and Kleiber 2000, Ganesan and Kowda 2005) studied the perturbation based stochastic finite element method with random variable material and geometrical properties of composite structures. Kaminski (2006) proposed to generalize nth order stochastic perturbation technique that can be applied to solve some boundary value or boundary initial problems in computational engineering with random parameters. He concluded that stochastic convergence of this methodology strongly depends on the coefficient of variation of the input random variable. Kaminski and Kleiber 2000 carried out the stochastic second order and second moment perturbation analysis for homogenization of the two-phase periodic composite structure. Ganesan and Kowda (2005) investigated the buckling of prismatic composite beam-columns with the objective of determining the mean values, mean square values, and standard deviation values of the buckling loads. The randomness in the material and geometric properties of the laminated beam-columns is modeled using homogeneous stochastic fields in space by them. The perturbation method is employed in the context of stochastic analysis.

The focus of the present paper is to perform the stochastic dynamic analysis of composite frame systems by using the perturbation based stochastic finite element method. For that reason, dynamic problems of beam-type structures with parameters described deterministically and/or stochastically proposed by

Kleiber and Hien (1992) were programmed in FORTRAN language by the authors and incorporated into a general-purpose computer program for dynamic deterministic and stochastic analysis of medium and large-scale three-dimensional frames. The program is modified for the stochastic dynamic analysis of composite frame systems based on the perturbation based stochastic finite element method and it is used in the stochastic dynamic analysis of the composite system. Then, this program is combined to Monte Carlo simulation. Two examples have been studied to demonstrate the efficiency and the range of applications of the developed method. During stochastic analysis, displacements and sectional forces of the systems are obtained from perturbation and Monte Carlo methods by using different uncertainties of material characteristics. The analysis results obtained from these two methods are compared with each other.

2. Formulation

In this section, stiffness matrix formulation of composite systems and perturbation based stochastic finite element dynamic analysis formulation of composite frame systems are given.

2.1 Stiffness matrix formulation of 3-D composite frame

The stiffness matrix formulation of the composite system is given according to References Sapountzakis and Mokos (2007), Pilkey (2002). For non homogeneous cross sections the elastic modulus, E, is a function of position [i.e., $E = E(y,z)$]. Let the reference modulus be given by E_r. Consider a prismatic 3-D beam element of length L with an arbitrarily shaped composite cross section consisting of materials in contact, each of which can surround a finite number of inclusions, with modulus of elasticity E_j, shear modulus G_j and mass density ρ_j, occupying the regions Ω_j ($j = 1,2,\ldots, K$) of the y, z plane (Fig. 1). K is number of materials. The materials of these regions are assumed homogeneous, isotropic and linearly elastic. These boundary curves are piecewise smooth, i.e., they may have a finite number of corners. Without loss of generality, it may be assumed that Cyz and Myz are the principal systems of axes through the cross section's centroid and shear center, respectively.

The nodal displacement vector in the local coordinate system, as shown in Fig. 1, can be written as

$$\{q_B\}^T = \{u_i,\ v_i,\ w_i,\ \theta_{xi},\ \theta_{yi},\ \theta_{zi},\ u_j,\ v_j,\ w_j,\ \theta_{xj},\ \theta_{yi},\ \theta_{zj},\} \tag{1}$$

where u_i, u_j, v_i, v_j, w_i and w_j, are axial displacement at joint i and j, y-direction transverse displacement at joint i and j, z-direction transverse displacement at joint i and j, respectively. θ_{xi}, θ_{xj}, θ_{yi}, θ_{yj}, θ_{zi} and θ_{zj} are rotation displacement in x-direction at joint i and j, y-direction rotation displacement at joint i and j and z-direction rotation displacement at joint i and j, respectively.

The nodal load vector in the local coordinate system, as shown in Fig. 1, can be written as

$$\{Q_\alpha\}^T = \{N_i,\ Q_{yi},\ Q_{zi},\ M_{xi},\ M_{yi},\ M_{zi},\ N_j,\ Q_{yj},\ Q_{zj},\ M_{xj},\ M_{yi},\ M_{zj},\} \tag{2}$$

where N_i, N_j, Q_{yi}, Q_{yj}, Q_{zi}, and Q_{zi} are axial force at joint i and j, y-direction shear force at joint i and j, z-direction shear force at joint i and j, respectively. M_{xi}, M_{xj}, M_{yi}, M_{yj}, M_{zi} and M_{zj} are torque in x-direction at joint i and j, y-direction bending moment at joint i and j and z-direction bending moment at joint i and j, respectively.

The nodal displacement and loads vectors given in Eqs. (1) and (2) are related with the 12 × 12 local

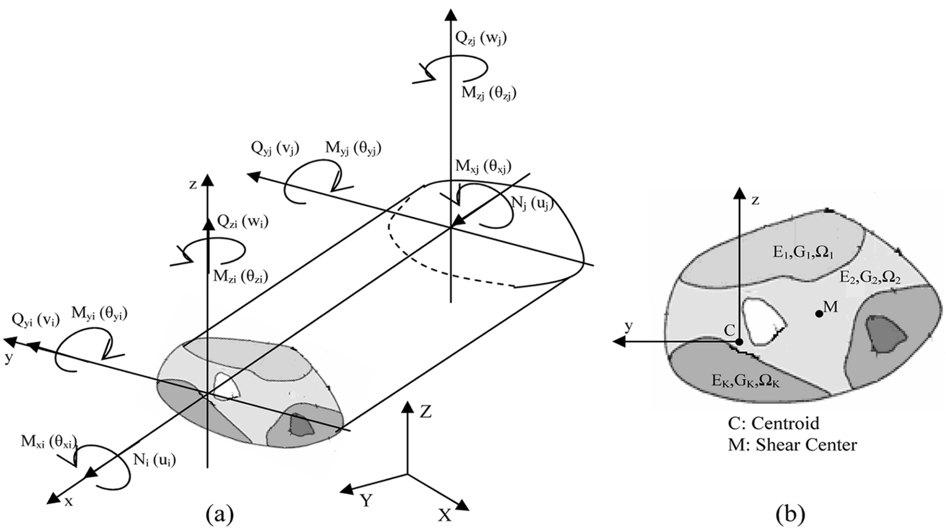

Fig. 1 Prismatic beam of an arbitrarily shaped composite cross section (a) and occupying the 2-D region Ω (b)

stiffness matrix of the spatial composite beam element written as

$$[K_{\alpha\beta}] = \begin{bmatrix} k_{11} & 0 & 0 & 0 & 0 & 0 & k_{17} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{22} & 0 & 0 & 0 & k_{26} & 0 & k_{28} & 0 & 0 & 0 & k_{2,12} \\ 0 & 0 & k_{33} & 0 & k_{35} & 0 & 0 & 0 & k_{39} & 0 & k_{3,11} & 0 \\ 0 & 0 & 0 & k_{44} & 0 & 0 & 0 & 0 & 0 & k_{4,10} & 0 & 0 \\ 0 & 0 & k_{53} & 0 & k_{55} & 0 & 0 & 0 & k_{59} & 0 & k_{5,11} & 0 \\ 0 & k_{62} & 0 & 0 & 0 & k_{66} & 0 & k_{68} & 0 & 0 & 0 & k_{6,12} \\ k_{71} & 0 & 0 & 0 & 0 & 0 & k_{77} & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{82} & 0 & 0 & 0 & k_{86} & 0 & k_{88} & 0 & 0 & 0 & k_{8,12} \\ 0 & 0 & k_{93} & 0 & k_{95} & 0 & 0 & 0 & k_{99} & 0 & k_{9,11} & 0 \\ 0 & 0 & 0 & k_{10,4} & 0 & 0 & 0 & 0 & 0 & k_{10,10} & 0 & 0 \\ 0 & 0 & k_{11,3} & 0 & k_{11,5} & 0 & 0 & 0 & k_{11,9} & 0 & k_{11,11} & 0 \\ 0 & k_{12,2} & 0 & 0 & 0 & k_{12,6} & 0 & k_{12,8} & 0 & 0 & 0 & k_{12,12} \end{bmatrix} \tag{3}$$

The $k_{\alpha\beta}$ coefficients of stiffness matrix of Eq. (3) can be written as:

$$k_{11} = -k_{71} = k_{17} = -k_{77} = \frac{E_r A_x}{L} \tag{4}$$

$$k_{22} = -k_{28} = -k_{82} = k_{88} = \frac{12 E_r G_r A I_z}{G_r A L^3 + 12 a_y E_r I_z L} \tag{5}$$

$$k_{26} = k_{2,12} = k_{62} = -k_{68} = -k_{86} = -k_{8,12} = k_{12,2} = -k_{12,8} = \frac{6E_rG_rAI_z}{G_rAL^2 + 12a_yE_rI_z} \tag{6}$$

$$k_{33} = -k_{39} = -k_{93} = k_{99} = \frac{12E_rG_rAI_y}{G_rAL^3 + 12a_zE_rI_yL} \tag{7}$$

$$k_{59} = -k_{3,11} = -k_{53} = -k_{35} = k_{95} = k_{9,11} = -k_{11,3} = k_{11,9} = \frac{6E_rG_rAI_z}{G_rAL^2 + 12a_zE_rI_y} \tag{8}$$

$$k_{44} = -k_{4,10} = -k_{10,4} = k_{10,10} = \frac{G_rI_t}{L} \tag{9}$$

$$k_{55} = k_{11,11} = \frac{4E_rG_rI_yAL^2 + 12a_zE_r^2I_y^2}{G_rAL^3 + 12a_zE_rI_yL} \tag{10}$$

$$k_{5,11} = k_{11,5} = \frac{2E_rG_rI_yAL^2 + 12a_zE_r^2I_y^2}{G_rAL^3 + 12a_zE_rI_yL} \tag{11}$$

$$k_{66} = k_{12,12} = \frac{4E_rG_rI_yAL^2 + 12a_yE_r^2I_z^2}{G_rAL^3 + 12a_yE_rI_zL} \tag{12}$$

$$k_{6,12} = k_{12,6} = \frac{2E_rG_rI_zAL^2 + 12a_yE_r^2I_z^2}{G_rAL^3 + 12a_yE_rI_zL} \tag{13}$$

where E_r, G_r, A, L are the modulus of elasticity of the reference material, the shear modulus of reference material, composite cross section area, composite element length, I_y, I_z are the bending moments of inertia of the composite cross section with respect to y and z axes, respectively.

$$I_y = \sum_{j=1}^{K} \frac{E_j}{E_r}\int_{\Omega_j} z^2 d\Omega_j, \qquad I_z = \sum_{j=1}^{K} \frac{E_j}{E_r}\int_{\Omega_j} y^2 d\Omega_j \tag{14a,b}$$

$$A = \sum_{j=1}^{K} \frac{G_j}{G_r}\int_{\Omega_j} d\Omega_j, \qquad A_x = \sum_{j=1}^{K} \frac{E_j}{E_r}\int_{\Omega_j} d\Omega_j \tag{15a,b}$$

where ignoring the torsional warping, I_t is the polar moment of inertia of the composite cross section given as:

$$I_t = \sum_{j=1}^{K} \frac{E_j}{E_r} \int_{\Omega_j} (y^2 + z^2) d\Omega_j \tag{16}$$

It is worth noting that the reduction of Eqs. (14a,b), (15b), (16) using the modulus of elasticity E_r and of Eq. (15a) using the shear modulus G_r of the reference material, could be achieved using any other material, considering it as reference material.

The shear deformation coefficients; a_y, a_z are established equating the approximate formula of the shear strain energy per unit with the exact one given and are obtained as (Pilkey 2002):

$$a_y = \frac{A}{E_r \Delta^2} \sum_{j=1}^{K} \int_{\Omega_j} E_j((\nabla\psi)_j - d) \cdot ((\nabla\psi)_j - d) d\Omega_j \tag{17a}$$

$$a_z = \frac{A}{E_r \Delta^2} \sum_{j=1}^{K} \int_{\Omega_j} E_j((\nabla\phi)_j - h) \cdot ((\nabla\phi)_j - h) d\Omega_j \tag{17b}$$

where $(\nabla)_j \equiv i_y(\partial/\partial_y) + i_z(\partial/\partial_z)$ is a symbolic vector with i_y, i_z the unit vectors along y and z axes, respectively, Δ is given from

$$\Delta = 2(1 + \upsilon) I_y I_z \tag{18}$$

υ is the Poisson ratio of the cross section materials, **d** and **h** are vectors defined as

$$d = \upsilon I_y \frac{y^2 - z^2}{2} i_y + (\upsilon I_y yz) i_z \tag{19a}$$

$$h = (\upsilon I_y yz) i_z + \left(\upsilon I_z \frac{z^2 - y^2}{2}\right) i_z \tag{19b}$$

where ψ, ϕ are stress functions (Sapountzakis and Mokos 2007).

2.2 Stochastic finite element method (SFEM)

In the stochastic finite element method (SFEM), the deterministic finite element formulation is modified using the perturbation technique or the partial derivative method to incorporate uncertainty in the structural system. Since the basic variables are stochastic, every quantity computed during the deterministic analysis, being a function of the basic variables, is also stochastic. Therefore, the efficient way to arrive at the stochastic response may be to keep account of the stochastic variation of the quantities at every step of the deterministic analysis in terms of the stochastic variation of the basic variables.

The basic idea is conceptually simple. However, the implementation in actual analysis may not be simple since it involves the computation and assembly of large matrices of partial derivatives of the various quantities in terms of the basic variables. Furthermore, devising methodologies to transform the spatially correlated random fields into the uncorrelated random fields renders the implementation more complicated. There are two fundamental ways to solve the stochastic problem (i) analytical approach and (ii) numerical approach. Among analytical approaches, the perturbation method is widely used because of its simplicity. Numerical method such as Monte Carlo Simulation is generally applicable to all types' stochastic problems and is often used to verify the results obtained from analytical methods. A detailed discussion of these methods is presented below:

2.2.1 Perturbation based stochastic finite element method formulation

The perturbation method is the most widely used technique for analyzing uncertain system. This method consists of expanding all the random variables of an uncertain system around their respective mean values via Taylor series and deriving analytical expression for the variation of desired response quantities such as natural frequencies and mode shapes of a structure due to small variation of those random variables. The basic idea behind the perturbation method is to express the stiffness and mass matrices and the responses in terms of Taylor series expansion with respect to the parameters centered at the mean values. Generally, the Taylor series is expanded only to the first order. That is why this method is often referred to as first-order perturbation method.

Since the deterministic equations are valid for the Monte-Carlo simulation analysis as well, then the essential differences are observed in case of perturbation-based analysis. Let us consider a deterministic equation of motion in the form of

$$M_{\alpha\beta}\ddot{q}_\beta + C_{\alpha\beta}\dot{q}_\beta + K_{\alpha\beta}q_\beta = Q_a \tag{20}$$

where $K_{\alpha\beta}$, $M_{\alpha\beta}$, $C_{\alpha\beta}$ denote the stiffness matrix, mass matrix and damping matrix, $\ddot{q}_\beta$, $\dot{q}_\beta$, q_β denote the acceleration, velocity, displacement, respectively. The stochastic perturbation based approach consists usually of the up to the second order equations obtained starting from the deterministic ones.

The perturbation stochastic finite element equations describing dynamic response of the single random variable system for zeroth, first and second order (Kleiber and Hien 1992):

Zeroth-order equation ($\in^0$ terms, one system of N linear simultaneous ordinary differential equations for $q^0_\alpha(b^0_l;\tau)$, $\alpha = 1, 2, \ldots, N$)

$$M^0_{\alpha\beta}(b^0_l)\ddot{q}^0_\beta(b^0_l;\tau) + C^0_{\alpha\beta}(b^0_l)\dot{q}^0_\beta(b^0_l;\tau) + K^0_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau) = Q^0_\alpha(b^0_l;\tau) \tag{21}$$

First-order equations, rewritten separately for all random variables of the problem ($\in^1$ terms, $\overline{N}$ systems of N linear simultaneous ordinary differential equations for $q^{,\rho}_\alpha(b^0_l;\tau)$, $\rho = 1, 2, \ldots, \overline{N}$, $\alpha = 1, 2, \ldots, N$

$$\begin{aligned} M^0_{\alpha\beta}(b^0_l)\ddot{q}^{,\rho}_\beta(b^0_l;\tau) + C^0_{\alpha\beta}(b^0_l)\dot{q}^{,\rho}_\beta(b^0_l;\tau) + K^0_{\alpha\beta}(b^0_l)q^{,\rho}_\beta(b^0_l;\tau) &= Q^{,\rho}_\alpha(b^0_l;\tau) \\ -[M^{,\rho}_{\alpha\beta}(b^0_l)\ddot{q}^0_\beta(b^0_l;\tau) + C^{,\rho}_{\alpha\beta}(b^0_l)\dot{q}^0_\beta(b^0_l;\tau) + K^{,\rho}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)] & \end{aligned} \tag{22}$$

Second-order ($\in^2$ terms, one system of N linear simultaneous ordinary differential equations for $q^2_\alpha(b^0_l;\tau)$, $\alpha = 1, 2, \ldots, N$)

$$\begin{aligned} M^0_{\alpha\beta}(b^0_l)q^{,\rho}_\beta(b^0_l;\tau) + C^0_{\alpha\beta}(b^0_l)q^{,\rho}_\beta(b^0_l;\tau) + K^0_{\alpha\beta}(b^0_l)q^{,\rho}_\beta(b^0_l;\tau) &= \{Q^{,\rho\sigma}_\alpha(b^0_l;\tau) \\ -2[M^{,\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau) + C^{,\sigma}_{\alpha\beta}(b^0_l)q^\sigma_\beta(b^0_l;\tau) + K^{,\sigma}_{\alpha\beta}(b^0_l)q^\sigma_\beta(b^0_l;\tau)] & \end{aligned}$$

$$-[M^{,\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau) + C^{,\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau) + K^{,\rho\sigma}_{\alpha\beta}(b^0_l)q^0_\beta(b^0_l;\tau)]\}Cov(b_r, b_s) \tag{23}$$

$$(\cdot)^{(0)} = (\cdot)|_{\{\beta\} = \{\beta\}^{(0)}}, \qquad (\cdot)^{(,\rho)}_i = \frac{\partial}{\partial\beta_i}(\cdot)|_{\{\beta\} = \{\beta\}^{(0)}}, \qquad (\cdot)^{(,\rho\sigma)}_i = \frac{\partial^2}{\partial\beta_i^2}(\cdot)|_{\{\beta\} = \{\beta\}^{(0)}} \tag{24}$$

where b^0_l is the vector of nodal random variables, q_α is the vector of nodal displacement-type variables, τ is forward time variable, $\overline{N}$ is the number of nodal random variables. $M^0_{\alpha\beta}$, $C^0_{\alpha\beta}$ and $K^0_{\alpha\beta}$ are system mass matrix, damping matrix and system stiffness matrix, respectively. Q^0_α, q^0_β and $Cov\,(b_r, b_s)$ are load vector, displacement and the covariance matrix of the nodal random variable, respectively. N is the number of degrees of freedom in the system. $(\cdot)^0$ is zeroth-order quantities, taken at means of random variables, $(\cdot)^{,\rho}$ is first partial derivatives with respect to nodal random variables, $(\cdot)^{,\rho\sigma}$ is second partial derivatives with respect to nodal random variables.

The first two statistical moments for the random fields $b_r(x_k)$, $r = 1,2,...,R$, are defined as

$$E[b_r] = b^0_r = \int_{-\infty}^{+\infty} b_r p_1(b_r)db_r \tag{25}$$

$$Cov(b_r, b_s) = S^{rs}_b = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(b_r - b^0_r)(b_s - b^0_s)p_2(b_r, b_s)db_r db_s \tag{26}$$

r, $s = 1,2..., R$

The latter definition can be replaced by

$$S^{rs}_b = \alpha_{b_r}\alpha_{b_s}b^0_r b^0_s \mu_{b_r b_s} \tag{27}$$

with

$$\alpha_{b_r} = \left[\frac{Var(b_r)}{b}\right]^{1/2} \qquad \mu_{b_r b_s} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} b_r b_s p_2(b_r, b_s)db_r db_s \tag{28}$$

where, $E[b_r]$, $Cov(b_r, b_s)$, $Var(b_r)$, $\mu_{b_r b_s}$, α_{b_r}, $p_1(b_r)$ and $p_2(b_r, b_s)$ denote the spatial expectation, covariance, variance, correlation functions, the coefficients of variation, probability density function (PDF) and the joint PDF, respectively. R is the random fields, which can represent randomness in the cross sectional area, length of truss and beam members, thickness of plate and shell elements, elastic modulus, and mass density of the material, etc.

All the equations, solved consequently for zeroth, first and second order displacements, velocities and accelerations, make it possible to compute the first two probabilistic moments of the output in the form of expected values and cross-covariances of the structural response.

2.2.2 Monte carlo method

The Monte Carlo method is a quite versatile mathematical tool capable of handling situations where all other methods fail to succeed; in structural dynamics, it has attracted intense attention only recently following the widespread availability of inexpensive computational systems. This computational availability has triggered an interest in developing sophisticated and efficient simulation algorithms. Shinozuka 1972 had a pioneering role in introducing the method to the field of structural dynamics. Shinozuka used the Monte Carlo Simulation for simulating a random process as the superposition of a large number of sinusoids having a uniformly distributed random phase angle. Zhang and Ellingwood (1996) used this method to obtain the effects of random material properties. However, in most of the studies, the Monte Carlo Simulation was used to verify the results obtained from approximate methods (Stefanou and Papadrakakis 2004).

The Monte Carlo Simulation generates a set of random values of X according to its probability distribution function. The set can be written as $X = \{x_1, x_2, ..., x_N\}$, where N is the number of simulation. For each values of X, the stiffness and mass matrices are computed. At the end of N simulations, we have a random set of displacement and stress values $\{\{q_\beta\}_1, \{q_\beta\}_2, \{q_\beta\}_3, \ldots, \{q_\beta\}_N\}, \{\{\sigma\}_1, \{\sigma\}_2, \{\sigma\}_3, \ldots, \{\sigma\}_N\}$ for X^i. From this finite set of solutions, the expected values of displacement and stress are computed using the following formulas:

$$\mu_{\{q_\beta\}} = \frac{1}{N}\sum_{i=1}^{N}\{q_\beta\}_i \tag{29}$$

$$\mu_{\{\sigma\}} = \frac{1}{N}\sum_{i=1}^{N}\{\sigma\}_i \tag{30}$$

3. Numerical examples

On the basis of the analytical and numerical procedures presented in the previous sections, a FORTRAN program has been written and representative examples have been studied to demonstrate the efficiency and the range of applications of the developed method. During stochastic analysis, displacements and sectional forces of the systems are obtained from perturbation and Monte Carlo methods by using different uncertainties of material characteristics (Elastic modulus). The analysis results obtained from these two methods are compared with each other.

For two examples, the elastic modulus was chosen as random variable. Coefficient of variation (COV) was selected as $\alpha = 0.1$ for this random variable (kleiber and Hien 1992). The perturbation based SFEM is very efficient for low material variability. How it was supposed before Kleiber and Hien 1992, Kaminski 2006, higher than 0.1~0.15 coefficients of variation (COV) of input random variables demand higher order Taylor expansion in equilibrium equations and higher order expansion of the solution. The respective expectation and correlation function (Kleiber and Hien 1992) for the elastic modulus E_ρ are assumed as follows:

$$E[E_\rho] = 3.0 \times 10^7 \qquad \lambda = 10$$

$$\mu(E_\rho, E_\sigma) = \exp\left(-\frac{|x_\rho - x_\sigma|}{\lambda l}\right)$$

where x_p, l and λ are ordinates of the element midpoints (n random variable, $\rho,\sigma = 1,2,...,n$), structural member length and decay factor.

Example 1. A frame system of four spans and five stories consisting of beams and columns that have composite cross section subjected to earthquake ground motion (Fig. 2) is selected as first application (Fig. 3) for the generalized perturbation-based stochastic finite element method. The span length of the composite frame system is $l = 40$ m and story height is $h = 3.0$ m. The composite columns (Fig. 3b) and composite beams (Fig. 3c) are consisting of a concrete part ($E_c = 3.0 \times 10^7$ kPa, $G_c = 1.25 \times 10^7$ kPa,

ρ_c = 2,500 kg/m^3, υ = 0.20) (reference material) stiffened by a steel one (E_s = 2.1 × 10^8 kPa, G_s = 8.75 × 10^7 kPa, ρ_s = 7,850 kg/m^3). The column have a box shaped closed composite cross section as shown in Fig. 3(b). The cross section properties are computed as $A_E = A_G$ = 0.2144 m^2, I_y = 0.00374 m^4, I_z = 0.00113 m^4, I_t = 0.040 m^4. The composite beams are formed as a box shaped composite cross section, with uniform Poisson's ratio υ = 0.20 and damping ratios ξ = 0.05. The cross section properties is computed as $A_E = A_G$ = 0.1618 m^2, I_y = 0.00266 m^4, I_z = 0.00173 m^4, I_t = 0.0351 m^4 (Fig. 3c). The

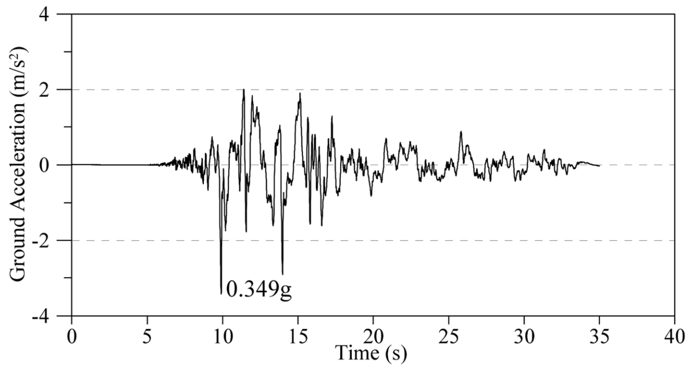

Fig. 2 Acceleration time history of Kocaeli earthquake (YPT330), 1999

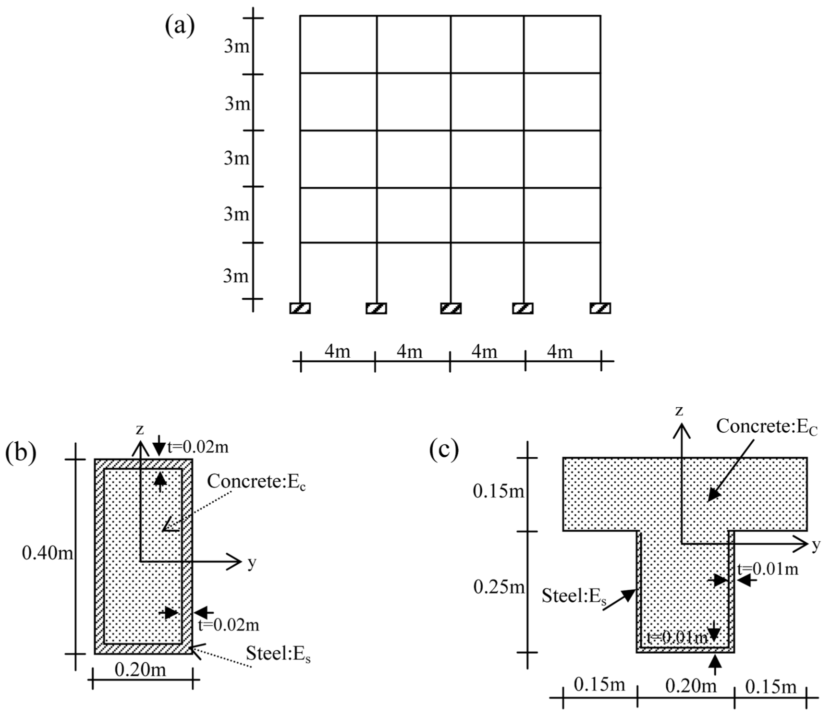

Fig. 3 (a) The dimensions of the composite frame systems in Example 1. (b) The dimensions of column cross-section. (c) The dimensions of beam cross-section shear deformation coefficient for two sections is selected as $a_y = a_z = 0$. It is supposed that the unit mass of beams is 40.0 kN/m with its self-weight and sum of live and dead load coming from slabs.

For the dynamic analysis, YPT330 component of Yarimca station records of 1999 Kocaeli Earthquake (Fig. 2) is utilized as ground motion (peer 2007). This ground motion continued up 35.0 s is applied to the system in a horizontal direction. The dynamic responses of the composite frame system are obtained for a time interval of 0.005 s.

The composite frame system is modeled by 155 stochastic finite elements of equal length (155 random variables, ρ, σ= 1,2,...,155; x_p are ordinates of the element midpoints). Monte Carlo Simulation (MCS) method was simulated for 10,000 simulation.

Absolute values of maximum displacements and sectional forces are determined according to perturbation based stochastic finite element method and Monte Carlo Simulation for composite frame system and the analysis results compared with each other.

Fig. 4 presents horizontal displacements along the left border of composite frame system according to MCS and perturbation based SFE methods. As shown in Fig. 4, the displacement values obtained from the perturbation method are close to those acquired from MCS method. The minimum absolute differences between these two methods for horizontal displacement value are about 3.1%, however, maximum differences are about 3.7%.

Maximum moments in the top joint of columns in every floor for the frame system are plotted in Fig. 5. The maximum moments occurs in the columns at second and fourth axes for this frame system (Fig. 3). It is seen from Fig. 5 that values obtained from MCS and perturbation methods are much closer to each other. While minimum differences between the moments of these two methods are 4.8%, maximum differences are about 5.1%.

The last comparison for example 1 is about axial forces obtained from this frame system. Fig. 6 presents maximum axial forces of the columns in every floor for the frame system. Maximum axial forces occur in the columns at border axes for frame system. Likely to other results, axial forces obtained from MCS and perturbation method is closer to each other, too. Minimum differences for axial forces

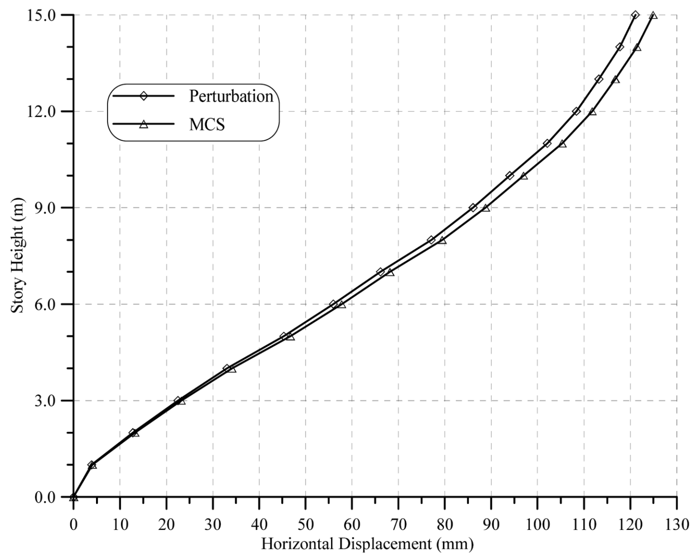

Fig. 4 Horizontal displacement of the frame system along story height

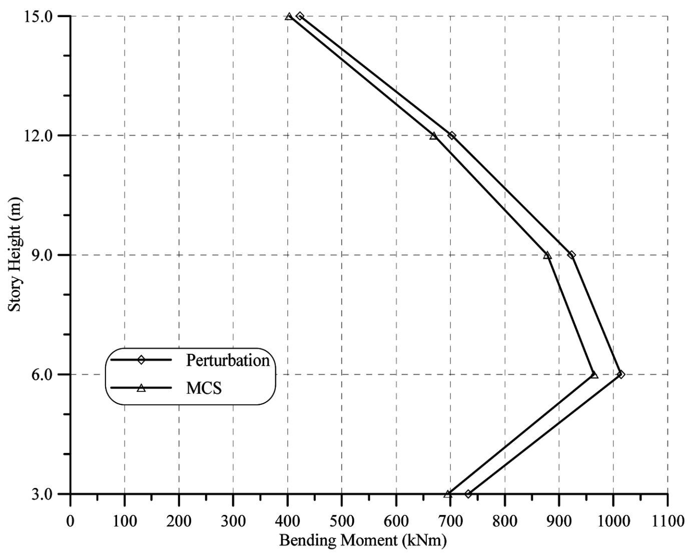

Fig. 5 Maximum absolute bending moment of the top joint of the column of each story for the frame system

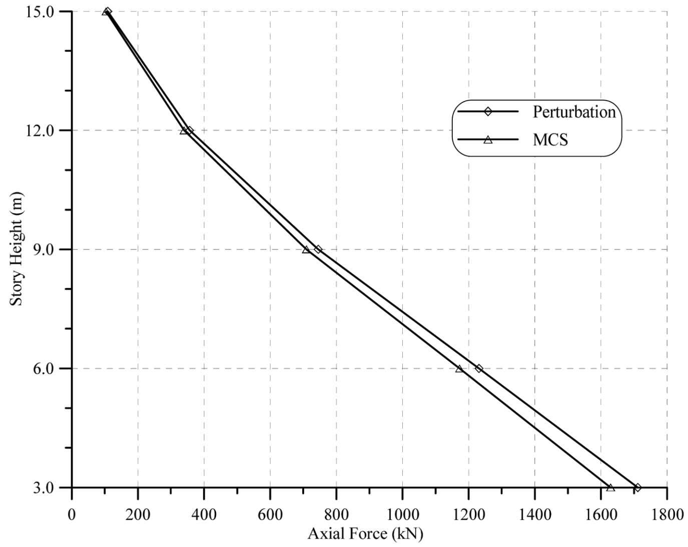

Fig. 6 Maximum axial force of the columns of each story for the frame system

are about 4.5%, while maximum ones are about 4.7%.

Example 2. A two-dimensional bridge that has three piers and four spans, subjected to earthquake ground motion (Fig.2), is chosen as second application (Fig. 7) for the generalized perturbation-based stochastic finite element method. The bridge's piers and decks are consisting of composite cross section made from concrete and steel. The bridge's length l = 90 m and its height is h = 12 m. The composite piers (Fig. 7b) and composite decks (Fig. 7c) are consisting of a concrete part ($E_c = 3.0 \times 10^7$ kPa, $G_c = 1.25 \times 10^7$ kPa, $\rho_c = 2{,}500$ kg/m^3, $\upsilon = 0.20$) stiffened by a steel (reference material) one ($E_s = 2.1 \times 10^8$ kPa, $G_s = 8.75 \times 10^7$ kPa, $\rho_s = 7{,}850$ kg/m^3). The piers have a box shaped closed composite cross section as

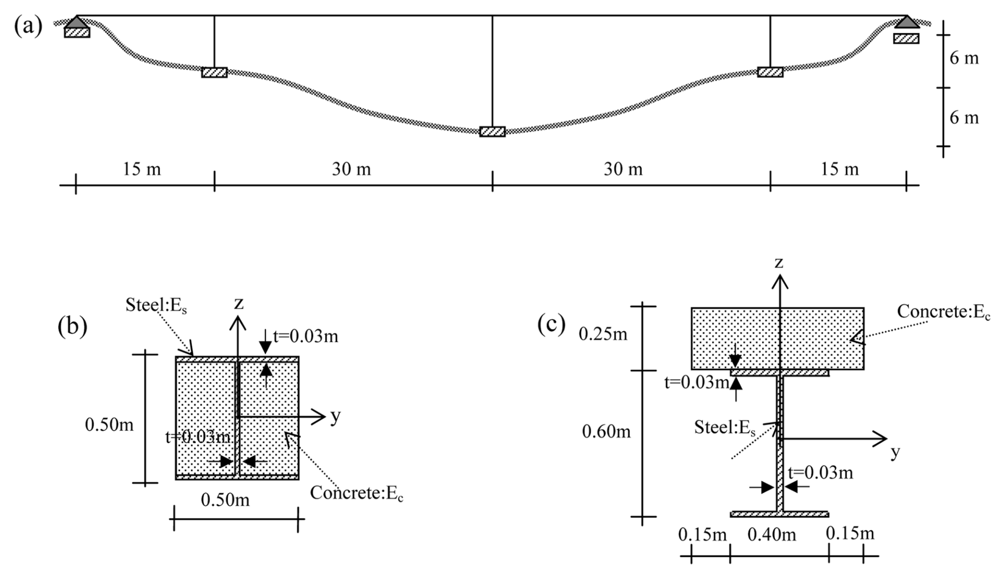

Fig. 7 (a) The dimensions of the bridge in Example 2. (b) The dimension of piers of the bridge. (c) The dimensions of deck of the bridge.

shown in Fig. 7b. The cross section properties is computed as $A_E = A_G$ = 0.061 m^2, I_y = 0.00187 m^4, I_z = 0.00110 m^4, I_t = 0.0034 m^4. The composite decks are formed as a box shaped composite cross section, with uniform Poisson's ratio $\upsilon = 20$ and damping ratios $\xi = 0.05$. The cross section properties are computed as $A_E = A_G$ = 0.0652 m^2, I_y = 0.00572 m^4, I_z = 0.00134 m^4, I_t = 0.0081 m^4 (Fig. 7c). The shear deformation coefficient for two sections is selected as $a_y = a_z = 0$. In addition, it is supposed that the unit mass of beams is 110.0 kN/m with its self-weight and sum of live and dead load coming from slabs.

For the dynamic analysis, YPT330 component of Yarýmca station records of 1999 Kocaeli Earthquake (Fig. 2) is utilized as ground motion (peer 2007). The earthquake motion continued up 35.0 s is applied to the bridge in a vertically direction. The dynamic responses of the composite bridge are obtained for a time interval of 0.005 s.

The composite bridge is modeled by 114 stochastic finite elements (114 random variables, ρ,σ= 1,2,..., 114; x_ρ are ordinates of the element midpoints). Monte Carlo Simulation (MCS) method was simulated for 10,000 simulation.

Absolute values of maximum vertical displacements and sectional forces are determined according to perturbation based SFEM and MCS method for this composite bridge system, and the analysis results compared with each other.

In Fig. 8 vertical displacements are plotted along the decks of composite bridge system according to MCS and perturbation methods. As shown in Fig. 8, the displacement values obtained from the perturbation method are close to those acquired from MCS method. The minimum absolute differences between these two methods for vertical displacement value are about 0.88%, however, maximum differences are about 1.06%.

Maximum moments of deck spans along for the bridge system are plotted in Fig. 9. It is seen from Fig. 9 that values obtained from MCS and perturbation methods are closed to each other. While minimum differences between the moments of these two methods are 5.9%, maximum differences are about 6.6%.

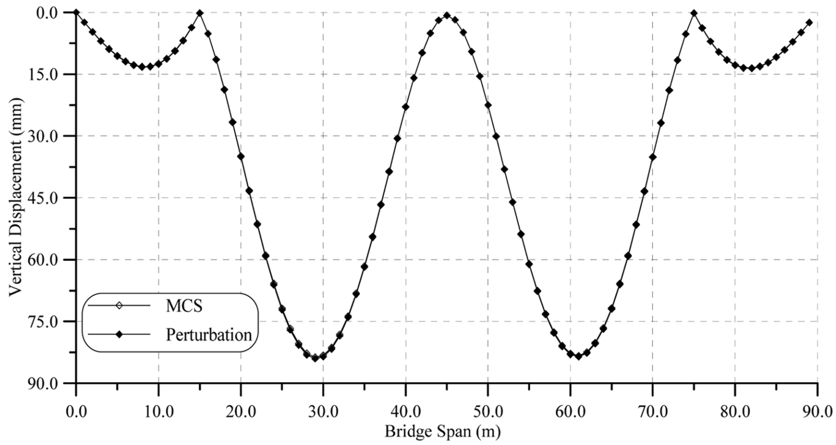

Fig. 8 Vertical displacements of the bridge along the bridge span

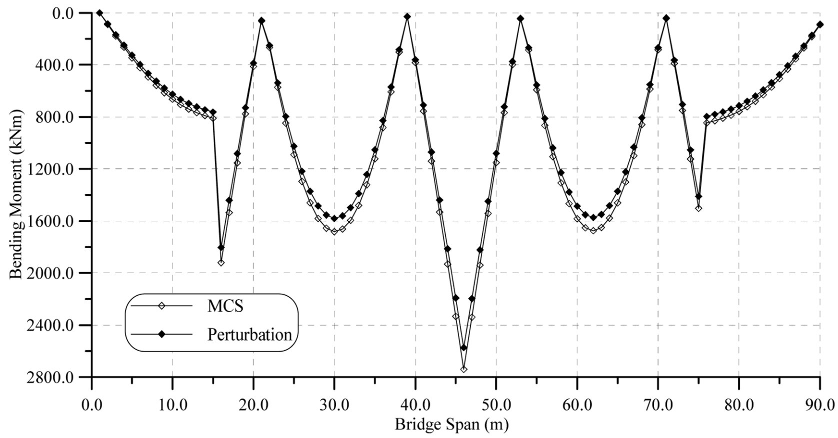

Fig. 9 Maximum absolute bending moment in the first joint of the each element of bridge's deck

The last comparison for example 2 is about shear forces obtained from this bridge system. The changing of the shear forces of the bridge along the decks is plotted in Fig. 10. Likely to other results, shear forces obtained from MCS and perturbation method is closer to each other, too. Minimum differences for axial forces are about 5.8%, while maximum ones are about 6.5%.

If it is mentioned the results obtained from these two examples; for the analysis of this bridge system (Fig. 7) presented its numerical properties, it needs about 13 seconds for perturbation based stochastic analysis, however, it needs about 11 hours for MCS analysis with the PC which have Intel Pentium (R) 2.40 GHz CPU and 768 MB RAM. On the other hand, for the analysis of the frame system (Fig. 3), it needs about 15 seconds for perturbation based stochastic analysis, and, it needs about 15 hours for MCS analysis.

The responses obtained shows that selected correlation function suitable for these problems for chosen coefficient of variation (COV) value ($\alpha = 0.10$).

It is seen also from the analysis results that dynamic response values for MCS are greater than those obtained from perturbation based SFEM for chosen bridge system. However, for the frame system,

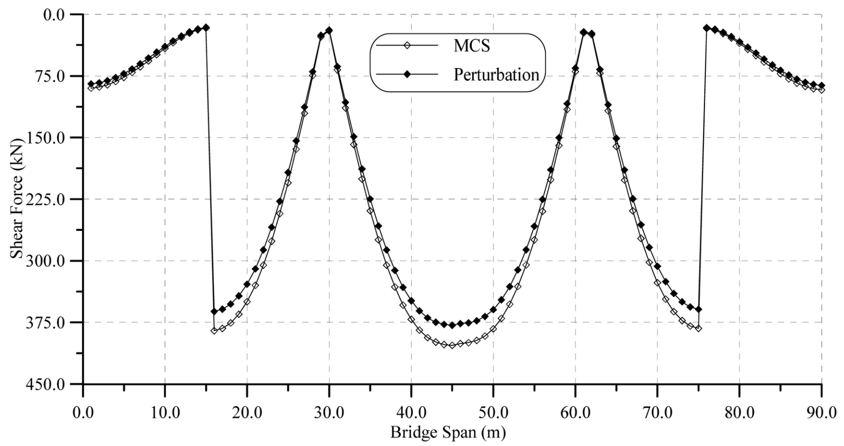

Fig. 10 Maximum absolute shear forces in the first joint of the each element of bridge's deck.

responses from perturbation based SFEM are higher, except for horizontal displacement.

4. Conclusions

In this paper, the comparison of the perturbation based SFEM and MCS method for dynamic responses of structural systems made of composite sections subjected to ground motion is performed with two examples, and some conclusions are drawn for the systems as follows:

The presented numerical technique is well suited for computer-aided analysis for structural systems made of composite cross section. It was seen that this technique is very suitable for chosen coefficient of variation (COV) value ($\alpha = 0.10$).

At two types of structural systems made of composite sections modeled in this study, perturbation based SFEM gives close results to MCS method for displacements, moments, axial forces and shear forces. Therefore, it can be said that perturbation method could be used instead of Monte Carlo Simulation method.

The dynamic response values obtained by the MCS method are higher than those of the perturbation based SFEM for chosen bridge system. However, this situation for frame system is on the contrary, except for horizontal displacements.

Finally, it should be mentioned that the approach proposed should turn out useful in generating material data for the efficient SFEM analysis of various composites because of its relatively low computational cost it should also find applications.

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