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Optimization of the construction scheme of the cable-strut tensile structure based on error sensitivity analysis

Lian-meng Chen [1a], Dong Hu [1b], Hua Deng [2c], Yu-hong Cui [3d] and Yi-yi Zhou [*4]

[1] *College of Civil Engineering and Architecture, Wenzhou University, Wenzhou, 325035, China*
[2] *Space Structures Research Center, Zhejiang University, Hangzhou 310027, China*
[3] *Sanjian Construction Group, Wenzhou, 325000, China*
[4] *School of Civil Engineering and Architecture, Changzhou Institute of Technology, Changzhou 213002, China*

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Abstract. Optimization of the construction scheme of the cable-strut tensile structure based on error sensitivity analysis is studied in this paper. First, the element length was extracted as a fundamental variable, and the relationship between element length change and element internal force was established. By setting all pre-stresses in active cables to zero, the equation between the pre-stress deviation in the passive cables and the element length error was obtained to analyze and evaluate the error effects under different construction schemes. Afterwards, based on the probability statistics theory, the mathematical model of element length error is set up. The statistical features of the pre-stress deviation were achieved. Finally, a cable-strut tensile structure model with a diameter of 5.0 m was fabricated. The element length errors are simulated by adjusting the element length, and each member in one symmetrical unit was elongated by 3 mm to explore the error sensitivity of each type of element. The numerical analysis of error sensitivity was also carried out by the FEA model in ANSYS software, where the element length change was simulated by implementing appropriate temperature changes. The theoretical analysis and experimental results both indicated that different elements had different error sensitivities. Likewise, different construction schemes had different construction precisions, and the optimal construction scheme should be chosen for the real construction projects to achieve lower error effects, lower cost and greater convenience.

Keywords: cable-strut tensile structures; error sensitivity analysis; construction scheme; statistical analysis; model experiment

1. Introduction

The cable-strut tensile structure is a type of flexible structure composed of cables in tension and struts in compression. Benefiting from the high strength of the tension cables and the adjustable stiffness distribution by pre-stress, the cable-strut tensile structure has the advantages of large span, good economic performance and lightweight features. Well-known tensile structures include the

*Corresponding author, Ph.D., E-mail: 695018557@qq.com
[a] Professor, E-mail: wzchenlm@sina.com
[b] Master Candidate, E-mail: 690488763@qq.com
[c] Professor, E-mail: denghua@zju.edu.cn
[d] Senior Engineer, E-mail: 283662164@qq.com

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Olympic gymnastics hall in Seoul, the Redbird Arena and the Georgia Dome in the United States, the La Plata Stadium in Argentina, the Tao-Yuan County Arena in Taiwan, the Kuala Lumpur Stadium in Malaysia, and the Shenzhen Bao'an Stadium and the Foshan Century Lotus Stadium in China. The cable-strut tensile structure performs excellent bearing capacity, not only because of the high strength of the tension cables and the adjustable stiffness distribution by pre-stress but also because it makes full use of the pre-stress in providing stiffness and saving the material. Thus, accurate pre-stress distribution is a guarantee and premise of superior bearing performance. However, before construction, the pre-stress of the structure is zero. As construction progresses, the pre-stress will increase and will achieve the final pre-stress at the completion of the construction. Thus, the validity and rationality of the construction techniques determine the accuracy of the pre-stress distribution, which means that the construction techniques perform an 1 important role in the pre-stress distribution and guarantee excellent bearing capacity of the structure.

Over the last two decades, research on the construction technique of the cable-strut tensile structure has been conducted. Many construction schemes and form-finding methods for different equilibrium states during the whole construction procedure were investigated (Barnes 1999, Jeon and Lee 2000, Chen and Dong 2013, Coyetty and Guisset 1988, Deng *et al*. 2005, Dong and Yuan 2007, Greco and Cuomo 2012, Maurin and Motro 1998, Ye *et al*. 2012, Zhang and Ohsaki 2016). The existing methods were mostly based on theoretical calculation, in which construction errors were not considered. However, because of the complicated working conditions and other negative effects, construction errors are inevitable, including the element length error, installation deviation error, pinhole machining error, node or anchorage size error and temperature deviation error, resulting in deviations between the real and theoretical pre-stress distributions (Liu *et al*. 2012, Zhang *et al*. 2014).

The previous studies showed that the pre-stress deviation is very sensitive to the performance of the bearing capacity (Gao *et al*. 2005, Peng and Wu 2004). Thus, it was important and necessary to evaluate the error sensitivity of different elements and the construction precision of different construction schemes. Over the last ten years, many error sensitivity analysis studies have been reported, most of which were based on the probability theory or orthogonal design method (Gao *et al*. 2005, Guo *et al*. 2009, Peng and Wu 2004, Wang *et al*. 2012, Zhang 2008, Zong *et al*. 2012). With the probability theory, the errors were modeled with a group of stochastic errors conforming to a certain distribution. In the orthogonal design method, the errors were modeled with some groups of deviation levels. The error sensitivity was then determined by evaluating their effects on the distribution of the initial pre-stress and structural static and dynamic behavior. Thus far, error sensitivity studies considering errors that conform not only to a certain distribution but also to the mathematical statistical law have not yet been reported. Likewise, further research on the construction of precision and error effects of those construction schemes and the optimization of construction schemes based on error sensitivity analysis have not been reported.

In this paper, the element length error sensitivity analysis and optimal construction scheme method are explored through theoretical analysis and experimental study. With the implementation of the proposed method, the error sensitivity of different elements can be analyzed, and elements with high error sensitivity whose lengths should be controlled exactly are found. Likewise, after analyzing and comparing the different error effects of different construction schemes, the optimal construction scheme should be chosen for the real construction projects to achieve low error effects, low cost and greater convenience.

2. Error sensitivity analysis method

To explore effective construction schemes that have the ability to control and reduce construction error effects, the error sensitivity of each element was analyzed in the first stage. Among all construction errors, the element length error is an important construction error and was taken as the main variable. The relationship between the element length change and element internal force change was established to evaluate the error effects caused by element length error, such as pre-stress deviation and shape deviation. In a cable-strut tensile structure, setting the element number as b, the node number as n, and the support constraint node number as c, the following equilibrium equation (Eq. (1)) (Pellegrino and Calladine 1986, Pellegrino 1993), physical equation (Eq. (2)) and geometric equation (Eq. (3)) can be established.

$$\boldsymbol{At} = \boldsymbol{P} \tag{1}$$

$$\boldsymbol{t} = \boldsymbol{M}(\boldsymbol{e} - \boldsymbol{e}_0) \tag{2}$$

$$\boldsymbol{Bd} = \boldsymbol{e} \tag{3}$$

where t is the internal force vector (b×1); P is the nodal load vector ((3n–c)×1); d is the node displacement vector ((3n–c)×1); e is the member expansion (b×1); e_0 is the initial member expansion (b×1); M is the element stiffness matrix (b×b); $M_{ii} = E_iA_i / l_i$ with E_i, A_i, and l_i as the elastic modulus, section areas and lengths for element i, respectively; A is the equilibrium matrix ((3n–c)×b); and B is the coordinate matrix (b×(3n–c)), $B = A^T$. Combining Eq. (1), Eq. (2) and Eq. (3) leads to

$$\begin{aligned}\boldsymbol{t} &= \boldsymbol{MA}^{\mathrm{T}}(\boldsymbol{AMA}^{\mathrm{T}})^{-1}\boldsymbol{P} + \boldsymbol{M}(\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{AMA}^{\mathrm{T}})^{-1}\boldsymbol{AM} - \boldsymbol{I})\boldsymbol{e}_0 \\ &= \boldsymbol{t}_P + \boldsymbol{t}_e\end{aligned} \tag{4}$$

$$\boldsymbol{d} = (\boldsymbol{AMA}^{\mathrm{T}})^{-1}(\boldsymbol{P} + \boldsymbol{AMe}_0) \tag{5}$$

Eq. (4) reflects the relationship among the initial defect length e_0, loading P and axial force t. Eq. (5) reflects the relationship among the initial defect length e_0, loading P and node displacement d. In the above equations, I is the unit matrix, t_P is the additional axial force caused by load P, and t_e is the additional axial force caused by the initial defect length e_0. Thus, when the load P is a constant number, the pre-stress deviations and shape deviations of the structure are caused by the original element length variation; when the member length changes by δe_0, the pre-stress and shape change are

$$\delta\boldsymbol{t} = \boldsymbol{M}(\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{AMA}^{\mathrm{T}})^{-1}\boldsymbol{AM} - \boldsymbol{I})\delta\boldsymbol{e}_0 = \boldsymbol{S}_t\delta\boldsymbol{e}_0 \tag{6}$$

$$\delta\boldsymbol{d} = (\boldsymbol{AMA}^{\mathrm{T}})^{-1}\boldsymbol{AM}\delta\boldsymbol{e}_0 = \boldsymbol{S}_d\delta\boldsymbol{e}_0 \tag{7}$$

where $S_t = M(A^T (AMA^T)^{-1}AM - I)$, S_t is the stress sensitivity matrix (b×b), and $(S_t)_{ij}$ reflects the magnification between the jth element's original length change and the ith element's pre-stress deviation. $S_d = (AMA^T)^{-1}AM$, S_d is the shape sensitivity matrix ((3n–c)×b), and $(S_d)_{ij}$ reflects the magnification between the jth element's original length change and the ith element's nodal displacement deviation. Eq. (6) represents the relationship between the element length deviation and the pre-stress deviation.

3. Relationship between the pre-stress deviation of the passive cable and the element length error based on the exact control of the active cable pre-stress

In a real cable-strut tensile project, the structural members can be classified into two different types. The first type is the so-called “active member”, which helps lift the tensile structure. The active member’s length is adjusted during the construction procedure, and its internal force is recorded immediately by the jack. The second type is the ordinary “passive cable”, which has a predetermined fixed slack length between two predefined joints. The passive cable’s internal force is produced passively according to the balance equation and geometric equation. The pre-stress deviation is then classified into active cable pre-stress deviation δt_c and passive cable pre-stress deviation δt_u. Likewise, the element length error can also be classified into active cable length error δe_{c0} and passive cable length error δe_{u0}. The stress sensitivity matrix S_t is rewritten as

$$\begin{Bmatrix} \delta \boldsymbol{t}_c \\ \delta \boldsymbol{t}_u \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{S}_{cc} & \boldsymbol{S}_{cu} \\ \boldsymbol{S}_{uc} & \boldsymbol{S}_{uu} \end{Bmatrix} \begin{Bmatrix} \delta \boldsymbol{e}_{c0} \\ \delta \boldsymbol{e}_{u0} \end{Bmatrix} \tag{8}$$

During the construction procedure, the pre-stresses of the active cables were controlled exactly with the help of the jack. The pre-stress deviation was zero. According to Eq. (8), the length adjustment values of the active cable are written as

$$\begin{aligned} \delta \boldsymbol{e}_c &= -\boldsymbol{S}_{cc}^{-1} \delta \boldsymbol{t}_c \\ &= -\boldsymbol{S}_{cc}^{-1} [\boldsymbol{S}_{cc} \quad \boldsymbol{S}_{cu}] \delta \boldsymbol{e}_0 = -[\boldsymbol{I} \quad \boldsymbol{S}_{cc}^{-1} \boldsymbol{S}_{cu}] \delta \boldsymbol{e}_0 = -(\delta \boldsymbol{e}_{c0} + \boldsymbol{S}_{cc}^{-1} \boldsymbol{S}_{cu} \delta \boldsymbol{e}_{u0}) \end{aligned} \tag{9}$$

The length adjustment values of all cables are then written as

$$\delta \boldsymbol{e} = \{\delta \boldsymbol{e}_c \quad 0\}^{\mathrm{T}} \tag{10}$$

Thus, the pre-stress deviations of all passive cables are expressed as

$$\delta \boldsymbol{t}_u = [\boldsymbol{S}_{uc} \quad \boldsymbol{S}_{uu}] \delta \boldsymbol{e}_0 + \boldsymbol{S}_{uc} \delta \boldsymbol{e}_c = (\boldsymbol{S}_{uu} - \boldsymbol{S}_{uc} \boldsymbol{S}_{cc}^{-1} \boldsymbol{S}_{cu}) \delta \boldsymbol{e}_{u0} = \boldsymbol{R}_t \delta \boldsymbol{e}_{u0} \tag{11}$$

When the active cable number is s, $R_t = S_{uu} - S_{uc} S_{cc}^{-1} S_{cu}$ is the stress sensitivity matrix of the passive cables based on the condition that the pre-stress deviation of all active cables is zero. $(R_t)_{ij}$ reflects the magnification between the jth element’s original length change and the ith passive cable’s pre-stress deviation based on the internal force of all active cables. Comparing Eqs. (6) and (11), it can be found that the exact control for the active cable’s internal force will change the passive cable’s pre-stress deviation. Regarding different construction schemes, different active cables will be adopted and different error effects for the passive cables will be produced. After comparing the pre-stress deviation of different construction schemes, the optimal construction scheme can be chosen as the construction scheme, with minimum pre-stress deviation for all passive cables.

4. The statistical features of the pre-stress deviation

The element length error mainly comes from the original length calculation error and machining error. The original length calculation error is the deviation between the approximate curve equation and the real shape. It also comes from temperature change, humidity change, material property change and other factors. In this study, it is assumed that the errors are independent, that the probability of positive or negative error is the same and that according to the Lindberg–Levy central limit theorem, the total length error obeys the normal distribution $N\sim(\mu, \sigma^2)$, where μ is the average error and σ^2 is the error variance. For a random error variable obeying the normal distribution, the proportion of errors falling in $[\mu - 3\sigma, \mu + 3\sigma]$ is 99.74%. Assuming that the element length error control scope is in the range of $[a, b]$, then

$$\mu = \frac{a+b}{2} \qquad \sigma = \frac{1}{6}(b-a) \tag{12}$$

where a and b are the upper and lower limits of the length error, respectively. The manufacturing deviation of the cable-strut tensile structure can be found in the technical specification for the cable structure (JGJ257-2012) (as shown in Table 1). According to Eq. (12), the average value and standard deviation of the cable and strut length deviation distribution difference were obtained as shown in Table 2.

As discussed above, this study assumes that the errors are independent, the probability of positive or negative error is the same, and the error obeys the normal distribution $N\sim(\mu_{ei}, \sigma_{ei})$, where μ_{ei} and σ_{ei} are the average value and standard deviation for the ith element length error, respectively. According to Eq. (6) and probability statistics theory, when each element length

Table 1 Allowable error of element length

Element	Element length L (m)	Allowable deviation ΔL (mm)
Cable	≤ 50	± 15
	$50 < L \leq 100$	± 20
	> 100	$L/5000$
Strut	≤ 5	± 5
	$5 < L \leq 10$	± 10
	> 10	± 15

Table 2 Error characteristic of element length

Element	Length L (m)	Average μ (mm)	Standard deviation σ (mm)
Cable	≤ 50	0	5
	$50 < L \leq 100$	0	6.67
	> 100	0	$L/15000$
Strut	≤ 5	0	1.67
	$5 < L \leq 10$	0	3.33
	> 10	0	5

error is distributed independently, the distribution of the initial pre-stress deviation of each element also obeys the normal distribution $N\sim(\mu_{ti}, \sigma_{ti})$, where μ_{ti} and σ_{ti} are the average value and standard deviation of the ith element internal force, respectively. Among them are the following

$$\mu_{ti} = \sum_{j=1}^{b} S_{tij} \mu_{ej} \tag{13}$$

$$\sigma_{ti}^2 = \sum_{j=1}^{b} S_{tij}^2 \sigma_{ej}^2 \tag{14}$$

According to Eq. (13) and Eq. (14), the pre-stress deviation effects caused by the element length error, namely the element length error sensitivity, can be achieved and used to evaluate the length error sensitivity of each type of element. Likewise, the distribution of initial pre-stress deviation of passive cables also obeys the normal distribution $N\sim(\mu_{ti}, \sigma_{ti})$ when the pre-stress in the active cables are controlled exactly, where μ_{ti} and σ_{ti} are the average value and standard deviation of the ith passive cable internal force, respectively. Among them are the following

$$\mu_{ti} = \sum_{j=1}^{b-s} R_{tij} \mu_{ej} \tag{15}$$

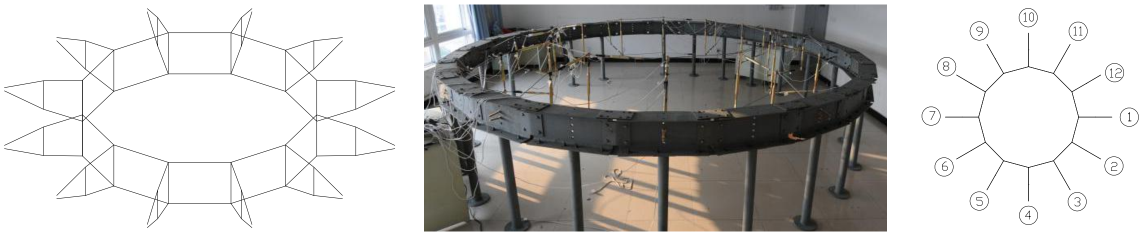
$$\sigma_{ti}^2 = \sum_{j=1}^{b-s} R_{tij}^2 \sigma_{ej}^2 \tag{16}$$

According to Eqs. (15) and (16), the pre-stress deviation effects in passive cables caused by the element length error can also be achieved and can be used to evaluate the error effects under different construction schemes. After comparing the pre-stress deviation effects of different construction schemes, the optimal construction scheme is achieved, with lower pre-stress deviation and greater convenience.

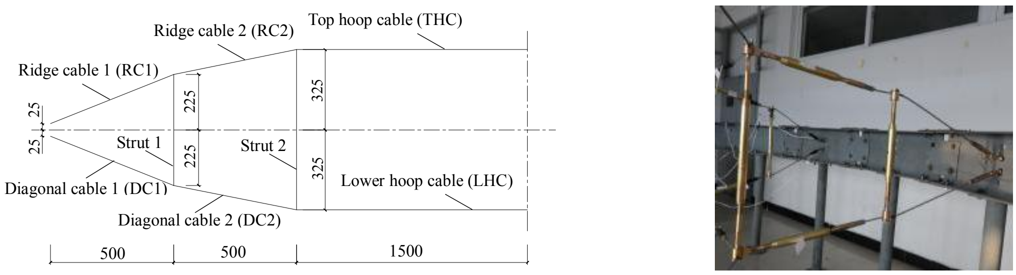
5. Experimental study

5.1 Model design

To test the proposed method, a cable-strut tensile structure model with a diameter of 5 m, as shown in Fig. 1, was fabricated in the laboratory. It was composed of twelve pieces of symmetrical cable-strut units (as shown in Fig. 1(a)), and each unit included tension cables and compression struts. The tension cables were divided into three categories: hoop cables, ridge cables and diagonal cables, as shown in Fig. 1(b). Each type of cable was comprised of high-strength steel wire, a cable end and a sleeve, which was used for adjustment of the cable lengths. The compression struts consisted of strut 1 and strut 2, with a section parameter of Φ15×3. All struts also had sleeves, which were used for adjustment of the strut lengths. During the experiment, the resistance foil strain gauge BX120-5AA was employed to measure the member internal force, and a static data logger was used to collect the readings of the strain gauges.



(a) The whole model



(b) Element of each symmetrical unit

Fig. 1 A cable-strut tensile structure model

5.2 FEA model in ANSYS

The element length error sensitivity of the model was also simulated by finite element analysis in the ANSYS software program, in which the element length change was simulated by the temperature change. The temperature change ΔT was described as

$$\Delta T = \Delta L / \alpha L \tag{17}$$

where ΔL = 3 mm is the element length change in this study, $a = 1.2\times10^{-5}$ is the linear expansion parameter, and L is the designed element length.

5.3 Model research

Ridge cable 1 (denoted as RC1), diagonal cable 1 (denoted as DC1), strut 1, strut 2, the top hoop cable (denoted as THC) and the lower hoop cable (denoted as LHC) in unit 4 were each elongated by 3 mm. The variations of internal forces in all elements in unit 4, unit 1 and unit 10 were measured, as shown in Tables 3-8. A positive value indicates that the element force increases, and a negative value indicates that the element force decreases. The following are shown: (1) Any element length change could cause force variations in all elements of the structure, which indicated that the tensile structure had strong overall balance performance. (2) The internal force changes of all elements caused by the element length change of ridge cable 1 and diagonal cable 1 were similar, which indicated that their error sensitivities were similar. In the same way, strut 1 and strut 2, and the top hoop cables and the lower hoop cables also had similar error sensitivity. In comparison, the error sensitivity of the hoop cables was the most sensitive, whereas those of the ridge cables and diagonal cables were less sensitive, and the struts were least sensitive. (3) The

Table 3 The element internal force changes caused by diagonal cable elongation of 3 mm in unit 4 (kN)

Unit No.	Element		DC1	DC2	RC1	RC2	Strut 1	Strut 2	THC	LHC
	Pre-stress		1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
4	Numerical ANSYS value	Axial force	1.48	1.40	1.34	1.27	-0.26	-0.26	2.41	2.68
		Deviation %	-15.50	-15.67	-16.05	-16.00	-16.18	-15.66	-15.74	-14.52
	Measured values	Axial force	1.65	1.28	1.50	1.42	-0.31	-0.31	2.21	2.40
		Deviation %	-5.92	-23.04	-6.00	-5.90	0.00	0.00	-22.89	-23.44
	Measured error %		11.35	-8.74	11.98	12.02	19.30	18.56	-8.49	-10.43
1	Numerical ANSYS value	Axial force	1.50	1.42	1.34	1.27	-0.26	-0.26	2.41	2.68
		Deviation %	-14.55	-14.64	-15.72	-15.81	-15.03	-15.04	-15.71	-14.54
	Measured values	Axial force	1.34	1.56	1.40	1.42	-0.31	-0.31	2.68	2.95
		Deviation %	-23.29	-6.00	-12.00	-6.00	0.00	0.00	-6.17	-6.00
	Measured error %		-10.20	10.16	4.45	11.70	17.71	17.78	11.32	10.00
10	Numerical ANSYS value	Axial force	1.50	1.42	1.35	1.27	-0.26	-0.26	2.41	2.68
		Deviation %	-14.51	-14.61	-15.71	-15.80	-15.01	-15.02	-15.70	-14.50
	Measured values	Axial force	1.44	1.56	1.41	1.15	-0.31	-0.31	2.58	2.93
		Deviation %	-17.75	-6.20	-11.54	-24.00	0.00	0.00	-9.88	-6.76
	Measured error %		-3.78	9.86	4.96	-9.73	17.68	17.70	6.90	9.06

Table 4 The element internal force changes caused by ridge cable elongation of 3 mm in unit 4 (kN)

Unit no.	Element		DC1	DC2	RC1	RC2	Strut 1	Strut 2	THC	LHC
	Pre-stress		1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
4	Numerical ANSYS value	Axial force	1.49	1.42	1.33	1.25	-0.26	-0.26	2.40	2.69
		Deviation %	-14.64	-14.58	-16.91	-17.10	-16.18	-15.67	-15.95	-14.35
	Measured values	Axial force	1.65	1.28	1.40	1.42	-0.31	-0.31	2.70	2.89
		Deviation %	-5.92	-23.04	-12.00	-5.90	0.00	0.00	-5.72	-7.81
	Measured error %		10.21	-9.91	5.91	13.51	19.30	18.58	12.16	7.63
1	Numerical ANSYS value	Axial force	1.50	1.42	1.34	1.27	-0.26	-0.26	2.40	2.69
		Deviation %	-14.33	-14.41	-15.98	-16.09	-15.06	-15.07	-15.98	-14.32
	Measured values	Axial force	1.44	1.46	1.31	1.42	-0.31	-0.31	2.16	2.57
		Deviation %	-17.47	-6.00	-18.00	-6.00	0.00	0.00	-24.68	-18.00
	Measured error %		-3.63	9.87	-2.37	12.07	17.75	17.81	-10.35	-4.30
10	Numerical ANSYS value	Axial force	1.50	1.42	1.34	1.27	-0.26	-0.26	2.41	2.69
		Deviation %	-14.32	-14.40	-15.95	-16.06	-15.03	-15.05	-15.94	-14.30
	Measured values	Axial force	1.54	1.35	1.41	1.51	-0.31	-0.31	2.30	2.29
		Deviation %	-11.83	-18.61	-11.54	0.00	0.00	0.00	-19.77	-27.02
	Measured error %		2.91	-4.91	5.26	19.15	17.71	17.74	-4.55	-14.84

Table 5 The element internal force changes caused by strut 1 elongation of 3 mm in unit 4 (kN)

Unit no.	Element		DC1	DC2	RC1	RC2	Strut 1	Strut 2	THC	LHC
	Pre-stress		1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
4	Numerical ANSYS value	Axial force	1.80	1.70	1.65	1.56	-0.33	-0.32	2.95	3.23
		Deviation %	3.00	2.71	3.31	2.99	6.47	1.49	3.04	2.76
	Measured values	Axial force	1.75	1.79	1.60	1.42	-0.31	-0.32	2.86	3.14
		Deviation %	0.00	7.68	0.00	-5.90	0.00	3.42	0.00	0.00
	Measured error %		-2.91	4.84	-3.21	-8.63	-6.08	1.90	-2.95	-2.68
1	Numerical ANSYS value	Axial force	1.80	1.70	1.64	1.56	-0.32	-0.32	2.95	3.23
		Deviation %	2.75	2.63	3.03	2.90	2.99	2.94	3.04	2.75
	Measured values	Axial force	1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
		Deviation %	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Measured error %		-2.64	-2.53	-2.91	-2.78	-2.88	-2.80	-2.95	-2.68
10	Numerical ANSYS value	Axial force	1.80	1.70	1.64	1.56	-0.32	-0.32	2.95	3.23
		Deviation %	2.74	2.63	3.03	2.91	2.98	2.93	3.03	2.75
	Measured values	Axial force	1.75	1.66	1.69	1.51	-0.31	-0.30	2.86	3.14
		Deviation %	0.00	0.00	5.77	0.00	0.00	-3.52	0.00	0.00
	Measured error %		-2.66	-2.55	2.67	-2.81	-2.88	-6.25	-2.94	-2.67

Table 6 The element internal force changes caused by strut 2 elongation of 3 mm in unit 4 (kN)

Unit no.	Element		DC1	DC2	RC1	RC2	Strut 1	Strut 2	THC	LHC
	Pre-stress		1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
4	Numerical ANSYS value	Axial force	1.80	1.71	1.65	1.56	-0.32	-0.34	2.95	3.23
		Deviation %	2.91	2.87	3.19	3.13	1.54	8.58	3.07	2.79
	Measured values	Axial force	1.75	1.66	1.60	1.51	-0.30	-0.30	2.86	3.14
		Deviation %	0.00	0.00	0.00	0.00	-3.33	-3.42	0.00	0.00
	Measured error %		-2.83	-2.79	-3.09	-3.04	-4.80	-11.05	-2.97	-2.71
1	Numerical ANSYS value	Axial force	1.80	1.70	1.64	1.56	-0.32	-0.32	2.95	3.23
		Deviation %	2.78	2.66	3.05	2.93	3.01	2.97	3.07	2.78
	Measured values	Axial force	1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
		Deviation %	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Measured error %		-2.67	-2.56	-2.93	-2.81	-2.90	-2.83	-2.97	-2.71
10	Numerical ANSYS value	Axial force	1.80	1.70	1.64	1.56	-0.32	-0.32	2.95	3.23
		Deviation %	2.77	2.66	3.05	2.94	3.01	2.96	3.06	2.78
	Measured values	Axial force	1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
		Deviation %	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Measured error %		-2.69	-2.58	-2.95	-2.83	-2.90	-2.86	-2.97	-2.70

Table 7 The element internal force changes caused by top hoop cable elongation of 3 mm in unit 4 (kN)

Unit no.	Element		DC1	DC2	RC1	RC2	Strut 1	Strut 2	THC	LHC
	Pre-stress		1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
4	Numerical ANSYS value	Axial force	1.30	1.23	1.14	1.08	-0.23	-0.23	2.03	2.33
		Deviation %	-25.80	-25.84	-28.76	-28.83	-27.17	-26.69	-28.89	-25.82
	Measured values	Axial force	1.44	1.28	1.21	1.24	-0.27	-0.27	2.21	2.16
		Deviation %	-17.75	-23.04	-24.00	-17.70	-13.30	-13.66	-22.89	-31.25
	Measured error %		10.85	3.77	6.68	15.63	19.05	17.76	8.43	-7.32
1	Numerical ANSYS value	Axial force	1.30	1.23	1.14	1.08	-0.23	-0.23	2.04	2.33
		Deviation %	-25.78	-25.83	-28.79	-28.88	-27.25	-27.20	-28.79	-25.77
	Measured values	Axial force	1.44	1.26	1.21	1.15	-0.27	-0.27	2.16	2.57
		Deviation %	-17.47	-24.00	-24.00	-24.00	-13.28	-11.60	-24.68	-18.00
	Measured error %		11.24	2.50	6.75	6.89	19.23	21.49	5.78	10.47
10	Numerical ANSYS value	Axial force	1.30	1.23	1.14	1.08	-0.23	-0.23	2.04	2.33
		Deviation %	-25.76	-25.82	-28.73	-28.82	-27.24	-27.15	-28.72	-25.75
	Measured values	Axial force	1.44	1.45	1.23	1.24	-0.28	-0.27	2.30	2.29
		Deviation %	-17.75	-12.40	-23.07	-18.00	-10.92	-14.07	-19.77	-27.02
	Measured error %		10.79	18.09	7.95	15.22	22.45	17.98	12.57	-1.71

Table 8 The element internal force changes caused by lower hoop cable elongation of 3 mm in unit 4

Unit no.	Element		DC1	DC2	RC1	RC2	Strut 1	Strut 2	THC	LHC
	Pre-stress		1.75	1.66	1.60	1.51	-0.31	-0.31	2.86	3.14
4	Numerical ANSYS value	Axial force	1.29	1.23	1.14	1.08	-0.23	-0.22	2.05	2.32
		Deviation %	-26.07	-26.11	-28.42	-28.48	-27.13	-27.64	-28.43	-26.22
	Measured values	Axial force	1.44	1.28	1.21	1.24	-0.27	-0.27	2.21	2.40
		Deviation %	-17.75	-23.04	-24.00	-17.70	-13.30	-13.66	-22.89	-23.44
	Measured error %		11.25	4.16	6.17	15.07	18.97	19.32	7.73	3.77
1	Numerical ANSYS value	Axial force	1.29	1.22	1.14	1.08	-0.23	-0.23	2.05	2.32
		Deviation %	-26.12	-26.18	-28.37	-28.45	-27.19	-27.15	-28.38	-26.11
	Measured values	Axial force	1.44	1.26	1.21	1.24	-0.27	-0.27	2.33	2.39
		Deviation %	-17.47	-24.00	-24.00	-18.00	-13.28	-11.60	-18.51	-24.00
	Measured error %		11.74	2.99	6.14	14.64	19.14	21.41	13.79	2.85
10	Numerical ANSYS value	Axial force	1.29	1.23	1.14	1.08	-0.23	-0.23	2.05	2.32
		Deviation %	-26.05	-26.12	-28.35	-28.43	-27.18	-27.10	-28.35	-26.04
	Measured values	Axial force	1.44	1.35	1.32	1.15	-0.28	-0.27	2.30	2.29
		Deviation %	-17.75	-18.61	-17.30	-24.00	-10.92	-14.07	-19.77	-27.02
	Measured error %		11.23	10.19	15.43	6.20	22.35	17.90	11.98	-1.33

element force changes in unit 4 were similar to those in other units, exhibiting no phenomenon in which the force changes in the nearer position to unit 4 were more obvious than those in the farther position to unit 4, which again indicated that the structure had strong overall balance performance. (4) The measured values and computed results of the error sensitivity analysis were almost consistent, but there were still various errors, such as the element length error, the geometric error in the boundary platform, and the adjustment length error. Thus, this method is an effective way to increase the experimental precision by considering errors when building the numerical model to simulate the real experimental model.

5.4 Element length error sensitivity analysis of the model structure based on the proposed method

According to the proposed element length error sensitivity analysis method, the mathematical models of the element length error can be established based on Eq. (12) and Table 2. The sensitivity matrix S_t and statistical characteristics of the pre-stress deviation were achieved based on Eq. (6), Eqs. (13) and (14), as shown in Table 9 (in the column labeled "No active cable"), which indicated the following: (1) Ridge cable 1 and diagonal cable 1 had similar length error sensitivity because of symmetry and because the gravity effects were not considered. In the same way, strut 1 and strut 2, ridge cable 2 and diagonal cable 2, and the top hoop cables and lower hoop cables also had similar length error sensitivity. They were almost consistent with the numerical analysis results in ANSYS and the experimental results in model testing. (2) Each element had different element length error sensitivity; in comparison, the error sensitivity of the hoop cables was the most sensitive, whereas those of the ridge cables and diagonal cables were less sensitive, and the struts were least sensitive; this was also consistent with the numerical analysis results and experimental results. (3) The error standard deviation ratios among the hoop cable, ridge cable (diagonal cable) and strut were $\sigma_{\mathrm{HC}} : \sigma_{\mathrm{RC(DC)}} : \sigma_{\mathrm{Strut}} = 8.43 : 4.45 : 0.87$, which was almost consistent with the numerical analysis results and experimental results. As previously mentioned, the element length sensitivity calculation results by the method proposed in this paper were almost consistent with the numerical analysis results of the finite element analysis model in ANSYS and the experimental results, which indicated that the proposed error sensitivity analysis method was accurate and that the design of the model was effective.

5.5 Error effect analysis of three different construction schemes

In this study, three different construction schemes of the model structure were analyzed. The first one was to tension the ridge cable 1. For simplicity, it was denoted as scheme 1. The next one was to tension the diagonal cable 1. The third one was to tension both ridge cable 1 and diagonal cable 1. Likewise, they were denoted as scheme 2 and scheme 3. According to Eq. (11), the sensitivity matrix R_t was established. According to Eqs. (15) and (16), the statistical characteristics of the passive cable pre-stress deviation, based on the pre-stress in the active cables were exactly controlled, can also be obtained for the above three construction schemes, as shown in Table 9, which indicated the following: (1) With the help of exactly controlling the pre-stress in some active cables, the pre-stress deviation level was reduced, and the construction precision was obviously increased, compared with the construction condition with no active cables. (2) Because of symmetry and because the gravity effects were not considered, scheme 1 and scheme 2 had similar construction precisions. (3) Compared with scheme 1 and scheme 2, scheme 3 had higher construction precision, with lower error standard deviation. However, it featured tension in all

Table 9 The statistical features of the pre-stress deviation under different construction schemes

Construction scheme element	No active cable		Scheme 1		Scheme 2		Scheme 3	
	μ_{ti}	σ_{ti}	μ_{ti}	σ_{ti}	μ_{ti}	σ_{ti}	μ_{ti}	σ_{ti}
Diagonal cable 1	0	4.70	0	0.138	0	0	0	0
Diagonal cable 2	0	4.45	0	0.137	0	0.031	0	0.031
Ridge cable 1	0	4.70	0	0	0	0.138	0	0
Ridge cable 2	0	4.45	0	0.031	0	0.137	0	0.031
Strut 1	0	0.87	0	0.015	0	0.015	0	0.008
Strut 2	0	0.87	0	0.018	0	0.018	0	0.012
Top hoop cable	0	8.43	0	0.047	0	0.230	0	0.047
Lower hoop cable	0	8.43	0	0.230	0	0.047	0	0.047

outer cables, and the total number of active cables was 24. Thus, the scheme 3 construction procedure was more complicated and the construction cost was higher. Nevertheless, the increase level was not obvious.

6. Conclusions

According to the various construction errors and resulting in deviations between the real and theoretical pre-stress, theoretical analysis and experimental study have been performed to analyze and control construction error in this paper. First, the element length was taken as the variable, and the fundamental relationship between the element length change and element internal force change was set up based on the balance equation, geometric equation and physical equation, which can evaluate the error sensitivity of different elements. By setting all pre-stress in the active cables to zero, the equation between the pre-stress deviation in the passive cables and element length error was obtained, which allows for the analysis and comparison of the error effects under different construction schemes. Based on the probability statistics theory, the mathematical model of the element length error was then set up, and the statistical features of the pre-stress deviation were determined under different construction schemes. Finally, a cable-strut tensile structure model with a diameter of 5.0 m was fabricated. The element length errors were simulated by adjusting the element lengths, and each member in one symmetrical unit was elongated by 3 mm to explore the error sensitivity of each type of element. An error sensitivity numerical analysis was also carried out by the finite element analysis model in ANSYS, in which the element length change was simulated by imposing appropriate temperature changes. The theoretical analysis and experimental results both indicated that different elements had different error sensitivity. Likewise, different construction schemes had different construction precision, and the optimal construction scheme should be chosen for the real construction to achieve lower error effects, lower cost and greater convenience.

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