Steel and Composite Structures, *Vol. 19, No. 1 (2015) 191-208*
DOI: http://dx.doi.org/10.12989/scs.2015.19.1.191

Damage detection using finite element model updating with an improved optimization algorithm

Yalan Xu *1, Yu Qian [1], Gangbing Song [2] and Kongming Guo [2]

[1] *School of Electronic & Mechanical Engineering, Xidian University, Xi'an 710071, P.R. China*
[2] *Department of Mechanical Engineering, University of Houston, Houston, TX 77004, USA*

(*Received April 15, 2014, Revised December 22, 2014, Accepted December 30, 2014*)

Abstract. The sensitivity-based finite element model updating method has received increasing attention in damage detection of structures based on measured modal parameters. Finding an optimization technique with high efficiency and fast convergence is one of the key issues for model updating-based damage detection. A new simple and computationally efficient optimization algorithm is proposed and applied to damage detection by using finite element model updating. The proposed method combines the Gauss-Newton method with region truncation of each iterative step, in which not only the constraints are introduced instead of penalty functions, but also the searching steps are restricted in a controlled region. The developed algorithm is illustrated by a numerically simulated 25-bar truss structure, and the results have been compared and verified with those obtained from the trust region method. In order to investigate the reliability of the proposed method in damage detection of structures, the influence of the uncertainties coming from measured modal parameters on the statistical characteristics of detection result is investigated by Monte-Carlo simulation, and the probability of damage detection is estimated using the probabilistic method.

Keywords: damage detection; model updating; region truncation; uncertainty; probability

1. Introduction

Damage detection is critical in structural health monitoring of civil, mechanical and aerospace engineering. Being non-destructive nature, structural dynamics approaches have received increasing attention during past decades since any degradation of the structural properties results in changes of the dynamic characteristics such as modal parameters (i.e., modal frequencies, mode shapes), which can be estimated by experimental modal analysis (Cawley and Adams 1979, Doebling *et al*. 1998, Kosmatka and Ricles 1999, Shi *et al*. 2000, Carden and Fanning 2004, Meruane and Heylen 2011). In nondestructive dynamic methods for damage detection, the finite element (FE) model updating method performs well in locating damage and quantifying damage severity by using experimentally measured data and the FE numerical model.

In FE model updating, the physical parameters of a FE model is adjusted and updated such that numerically predicted features obtained from updated FE model are consistent with those obtained

*Corresponding author, Ph.D., E-mail: ylxu@mail.xidian.edu.cn

http://www.techno-press.org/?journal=scs&subpage=8 ISSN: 1229-9367 (Print), 1598-6233 (Online)

from measurements. FE model updating techniques can be sorted into two categories: direct methods and iterative methods. In direct methods (Yang and Chen 2009), individual components of mass and stiffness matrices are directly updated with low computational efforts, and the updated models usually reproduce the measured data exactly. The drawback of direct methods is that the updated models are generally difficult to be interpreted with physical significance. The iterative methods overcome the limitation of the direct methods with larger computational efforts (Friswell and Mottershead 1995, Link 1999, Mottershead *et al.*2011).

Recently, sensitivity-based FE updating methods, which are iterative methods -- using experimentally measured modal data, have been successfully used for damage detection of structures. In the sensitivity-based methods, damage is identified by minimizing the differences between the numerical and experimental vibration data by updating the physical variables and using sensitivity derivatives of modal parameters with respect to physical variables. Friswell and Mottershead (1995) and Mottershead *et al.* (2011) elaborated on relevant techniques and a comprehensive review of sensitivity-based updating methods is given by Link (1999). Zhang *et al.* (2000) tested sensitivity-based model updating techniques to locate and quantify the damage in a model suspension bridge structure from the laboratory testing. Jaishi and Ren (2005, 2006) used either single-objective or multi-objective optimization technique to update the FE models of civil engineering structures in structural dynamics using the strain energy residual. A sensitivity-based algorithm using modal flexibility residuals is implemented and verified on simulated and experimental multi-cracked beams (Jaishi and Ren 2007). Fang *et al.* (2008) developed a sensitivity-based updating method to identify the damage in a tested reinforced concrete frame modeled with a bidimensional damage function to reduce the number of unknown variables. Bakir *et al.* (2007) used sensitivity-based FE model updating scheme in a reinforced concrete frame structure with a trust region Newton optimization algorithm. Ren and Chen (2010) developed a response surface-based FE model updating procedure for civil engineering structures in structural dynamics. Chakraborty and Sen (2014) investigated the application of adaptive response surface on the FE model updating by using moving lest-square method. Reynders *et al.* (2010) presented the application of FE model updating on structural damage identification by using OMAX data. Chellini *et al.* (2010) gave the FE model updating procedure used to detect, assess and quantify the structural damage of a high ductile steel concrete composite frame subjected to increasing seismic damage by means of pseudo dynamic and cyclic test. Shiradhonkar and Shrikhande (2011) gave a study of detecting and locating the damage in the beams with the aid of vibration based system identification and FE model updating method in which the modal parameters are identified using frequency domain decomposition and empirical transfer function estimation. Ribeiro *et al.* (2012) described the calibration of the numerical model of a bowstring-arch railway bridge based on modal parameters identified from an ambient vibration test using modal strain energy residuals in objective function and a genetic algorithm optimization algorithm. Sipple and Sanayei (2014) used FE model updating method for parameter estimation of the University of Central Florida's Grid Benchmark Structure, in which model updating was performed by using measured frequency response functions from the damaged structure to detect physical structural change.

It is well known that the success of sensitivity-based FE model updating methods depends on the accuracy of the FE model, the quality of modal test and identification, the chosen residuals in objective function and the capability of optimization algorithms (Gola *et al.* 2001, Natke 1998). Finding an optimization technique with high efficiency and fast convergence is one of the key issues for model updating. There are many global optimization methods that can be used for the process of FE model updating. These include gradient-based methods (trust region Newton method, sequential quadratic programming) and computationally intelligent algorithms (Ribeiro *et al*. 2012, Tu and Lu 2008) (e.g., genetic algorithm, evolutionary strategies, particle swarm optimization). Among these methods, the trust region Newton optimization method, which is a gradient-based algorithm, is often used in model updating procedure. In each iteration step of trust region approach, the searching step is limited within a 'trust region' to avoid unexpected large steps. The trust region is a sphere with a trust region radius, which restricts the design variables. Compared to the line search methods, the trust region approaches are more powerful, more reliable in convergence, but they are less straightforward and more expensive in computation. The researches by Shi and Xu 2009, Qiu and Chen 2012 focus on the studying of sub-problem and determination of parameters in trust region approaches, which are critical to the capability of optimization and the convergence to global optimum and have not yet been solved satisfactorily.

The objective of this paper is to develop a novel simple but computationally efficient optimization algorithm based on the sensitivity-based FE updating method. The original contribution of the paper is that the proposed algorithm integrates the Gauss-Newton method with the region truncation at each iteration step. Through this proposed method, not only the constraints are introduced instead of the penalty function, but also the design variables are restricted in truncated searching region. The proposed algorithm is illustrated by a numerically simulated 25-bar truss structure. The results are compared with those obtained from the trust region Newton method. Using the Monte-Carlo simulation, the reliability of the proposed method in damage detection of structures is also investigated by taking the modal parameter uncertainties into account.

2. General sensitivity-based FE updating method

2.1 Objective function and minimization problem

Generally, sensitivity-based FE model updating method can be posed as a minimization problem to find design variables set θ^* such that

$$\begin{aligned} & f(\theta^*) \leq f(\theta), \forall\theta; \\ & \underline{\theta}_i \leq \theta_i \leq \overline{\theta}_i, \quad i = 1, 2, \ldots, n \end{aligned} \tag{1}$$

where, $f(\theta)$ is the objective function and, $\overline{\theta}_i, \underline{\theta}_i$ are respectively the upper and lower bounds on the design variables θ_i.

The objective function $f(\theta)$ is a sum of squared differences between FE computed and experimental eigenvalues and mode shapes, defined as

$$f(\theta) = r(\theta)^T W r(\theta) = \sum_{l=1}^{m} w_l \left[r_{l,f}^2(\theta) + r_{l,s}^2(\theta) \right] \tag{2}$$

where, $r(\theta)$ is a residual vector containing the differences between FE computed and experimental modal parameters, assembled by $r_{i,f}(\theta)$, $r_{l,s}(\theta)$ which are the residual nonlinear functions of the updating variables θ. The updating variable vector $\theta \in R^n$ is a set of physical parameters, which will be adjusted in order to minimize the objective function $f(\theta)$. W is a diagonal weighting matrix with each diagonal component inversely proportional to the modal frequency of the corresponding vibration mode, since the higher modal parameters are not measured and identified as accurately as the lower modal parameters. m denotes the number of vibration modes considered in the residual vector.

Since modal frequencies provide the global information about the dynamic behavior of the structure and mode shapes contain spatial information, a combination of residuals in eigenvalues and mode shapes is used to define the residual vector as

$$r(\theta)=\left[r_{1,f}(\theta),r_{1,s}(\theta),\ldots,r_{l,f}(\theta),r_{l,s}(\theta),\ldots,r_{m,f}(\theta),r_{m,s}(\theta)\right]^T \tag{3}$$

where, $r_{i,f}(\theta)$, $r_{l,s}(\theta)$ are the residuals in the lth eigenvalue and mode shape, and can be respectively expressed as

$$r_{l,f}(\theta)=\frac{\lambda_l(\theta)-\tilde{\lambda}_l(\theta)}{\tilde{\lambda}_l},\quad l=1,2,\ldots,m \tag{4a}$$

$$r_{l,s}(\theta)=\frac{\varphi_l^k(\theta)}{\varphi_l^p}-\frac{\tilde{\varphi}_l^k(\theta)}{\tilde{\varphi}_l^p(\theta)},\quad k\neq p, l=1,2,\ldots,m \tag{4b}$$

where, $\lambda_l(\theta)$, $\tilde{\lambda}_l(\theta)$ are the FE computed and experimental eigenvalues ($\lambda_l = 2\pi f_l$, f_l is the lth modal frequency), respectively. $\varphi_l(\theta)$, $\tilde{\varphi}_l(\theta)$ are respectively the FE computed and experimental mode shapes. In Eq. (4b), the superscript p indicates a reference component of the lth mode shape (with respect to which the other components of the mode shapes are normalized), the superscript k refers to the components that are used in the updating process. As we can see, modal parameters are scaled and the relative differences are taken in Eqs. (4a)-(4b) in order to obtain the similar weight in each modal residual.

2.2 Normalization of updating parameters

Instead of the absolute value of each updating parameter θ_i, its relative variation to the initial value θ_i^0 is chosen as a dimensionless updating parameter a_i

$$a_i=\frac{\theta_i^0-\theta_i}{\theta_i^{0,}},\quad \theta_i=\theta_i^0(1-a_i) \tag{5}$$

Using the normalized parameters, the problem of numerical ill-condition due to large relative differences in parameter magnitudes can be avoided. The objective of FE model updating problem is to find the value of vector a that minimizes the difference between the FE computed and experimental modal parameters. Hence, Eq. (2) becomes

$$f(a)=r(a)^T Wr(a)=\sum_{l=1}^{m} w_l\left[r_{l,f}^2(a)+r_{l,s}^2(a)\right] \tag{6}$$

As a result, the minimization problem can be mathematically formulated as

$$
\begin{aligned}
& f\left(a^*\right) \leq f(a), a; \\
& \underline{a}_i \leq a_i \leq \overline{a}_i, \quad i = 1, 2, \ldots, n
\end{aligned} \tag{7}
$$

2.3 Gradient of objective function

The nonlinear optimization problem as shown in Eq. (7) can be solved with a gradient-based iterative optimization method. Therefore, the gradient matrix needs to be computed in each iterative step. Taking the first derivative of objective function in Eq. (6) with respect to the updating parameter a_i gives

$$
\frac{\partial f(a)}{\partial a_i} = 2\sum_{l=1}^{m} w_l \left[r_{l,f}(a) \frac{\partial r_{l,f}(a)}{\partial a_i} + r_{l,s}(a) \frac{\partial r_{l,s}(a)}{\partial a_i} \right], \quad i = 1, 2, \ldots, n \tag{8}
$$

In its matrix form, Eq. (8) can be expressed as

$$
\left[\frac{\partial f(a)}{\partial a_1}, \frac{\partial f(a)}{\partial a_2}, \cdots, \frac{\partial f(a)}{\partial a_n} \right] = 2WS(a)^T r(a) \tag{9}
$$

where, $r(a) = [r_{1,f}(a),\ r_{1,s}(a), \ldots,\ r_{m,f}(a),\ r_{m,s}(a)]^T$, and $S(a)$ is the sensitivity matrix, which can be obtained by

$$
S(a) = \begin{bmatrix}
\frac{\partial r_{1,f}(a)}{\partial a_1} & \frac{\partial r_{1,f}(a)}{\partial a_2} & \cdots & \frac{\partial r_{1,f}(a)}{\partial a_n} \\
\frac{\partial r_{1,s}(a)}{\partial a_1} & \frac{\partial r_{1,s}(a)}{\partial a_2} & \cdots & \frac{\partial r_{1,s}(a)}{\partial a_n} \\
\vdots & \vdots & \vdots & \vdots \\
\frac{\partial r_{m,f}(a)}{\partial a_1} & \frac{\partial r_{m,f}(a)}{\partial a_2} & \cdots & \frac{\partial r_{m,f}(a)}{\partial a_n} \\
\frac{\partial r_{m,s}(a)}{\partial a_1} & \frac{\partial r_{m,s}(a)}{\partial a_2} & \cdots & \frac{\partial r_{m,s}(a)}{\partial a_n}
\end{bmatrix} \tag{10}
$$

where, $\frac{\partial r_{l,f}(a)}{\partial a_i}$, $\frac{\partial r_{l,s}(a)}{\partial a_i}$ are the first-order derivatives of the residuals in the lth eigenvalue and mode shape with respect to each updating parameter a_i, which can be expressed as

$$
\frac{\partial r_{l,f}(a)}{\partial a_i} = \frac{1}{\tilde{\lambda}_l} \frac{\partial \lambda_l(a)}{\partial a_i} \tag{11a}
$$

$$
\frac{\partial r_{l,s}(a)}{\partial a_i} = \frac{1}{\varphi_l^p} \frac{\partial \varphi_l^k}{\partial a_i} - \frac{\varphi_l^k}{(\varphi_l^p)^2} \frac{\partial \varphi_l^p}{\partial a_i} \tag{11b}
$$

The modal sensitivities with respect to the updating parameter a_i can be calculated using the following expressions (Mottershead *et al*. 2011)

$$\frac{\partial \lambda_l}{\partial a_i} = \varphi_l^T \left[\frac{\partial K}{\partial a_i} - \lambda_l \frac{\partial M}{\partial a_i} \right] \varphi_l \tag{12a}$$

$$\frac{\partial \varphi_l}{\partial a_i} = \sum_{q=1}^{m} \beta_{lq} \varphi_q \tag{12b}$$

where, β_{lq} is determined by using the generalized eigenvalue problem and orthogonalization properties of eigenvectors. Provided that the eigenvectors have been normalized to unit modal masses, one can get (Mottershead *et al*. 2011, Jaishi and Ren 2006)

$$\beta_{lq} = \begin{cases} \varphi_q^T \left[\left(\frac{\partial K}{\partial a_i} - \lambda_l \frac{\partial M}{\partial a_i} \right) / \left(\lambda_l - \lambda_q \right) \right] \varphi_j, & q \neq l \\ -\frac{1}{2} \varphi_l^T \frac{\partial M}{\partial a_i} \varphi_l, & q = l \end{cases} \tag{13}$$

The procedure explained above for the analytical computation of sensitivity may not be easy when the study is carried out using the FE model of structures with a large amount of degrees of freedom. In that case finite difference approximation is one of the alternatives computing the eigensensitivity. In the approach, the sensitivity matrix is approximated using the forward difference of the function with respect to each parameter considered. From Eqs. (12a) and (13), the derivatives of the structural stiffness and mass matrices with respect to the updating parameter a_i are required, which can be obtained by

$$\frac{\partial K}{\partial a_i} = \frac{K(a + \Delta a_i e_i) - K(a)}{\Delta a_i} \tag{14a}$$

$$\frac{\partial M}{\partial a_i} = \frac{M(a + \Delta a_i e_i) - M(a)}{\Delta a_i} \tag{14b}$$

$$\Delta a_i = \frac{\Delta D}{100} \left(\overline{a}_i - \underline{a}_i \right) \tag{14c}$$

where, Δa_i is the step length (ΔD is a forward difference step size, which is generally taken as 0.2 and $\overline{a}_i, \underline{a}_i$ are the upper and lower limit for the updating parameter respectively) and e_i is the vector with its ith element equal to 1, and all other elements equal to zero.

3. The proposed optimization algorithm

Optimization algorithms can seek approximate solutions by iterative computing. One of the techniques to solve non-linear optimization problems is to expand the model vector into a Taylor series about the current parameters. The quadratic model $m(a)$ is defined by a truncated Taylor series of $f(a)$

$$\min_a m(a) = f(a_0) + [\nabla f(a_0)]^T r(a) + \frac{1}{2} r^T(a) [\nabla^2 f(a_0)]^T r(a) \tag{15}$$

where, a denotes a step vector form, and $f(a_0)$, $\nabla f(a_0)$, $\nabla^2 f(a_0)$ are respectively the values of the function, the gradient and the Hessian of $f(a)$ for the current parameter a_0. The gradient and Hessian of $f(a)$ are

$$\nabla f(a) = \sum_{l=1}^{m} \{ w_l [r_{l,f}(a) \nabla r_{l,f}(a) + r_{l,s}(a) \nabla r_{l,s}(a)] \} = WS^T(a) r(a) \tag{16a}$$

$$\nabla^2 f(a) = WS^T(a) S(a) + \sum_{l=1}^{m} \{ w_l [r_{l,f}(a) \nabla^2 r_{l,f}(a) + r_{l,s}(a) \nabla^2 r_{l,s}(a)] \} \approx WS^T(a) S(a) \tag{16b}$$

In order to convert the constrained minimization problem Eq. (7) to the unconstrained problem and limit the design variables in each iteration step, the region truncation method is introduced to improve the gradient-based optimization algorithm Eq. (15) to guarantee that the searching step is always limited within an acceptable region to avoid unexpected large steps, just as that in the trust region method. In the region truncation-based optimization method, the region is updated to control the searching direction vector Δd_k in each iterative step and the variable constraints are included in the region boundaries instead of using penalty functions. Using the Gauss-Newton method, the solution to Eq. (7) can be given as

$$a_{k+1} = a_k + \Delta d_k, \quad \Delta d_k = \begin{cases} \overline{\sigma}_k, & \Delta d_k \geq \overline{\sigma}_k \\ (S^T(a_k) WS(a_k))^{-1} S^T(a_k) Wr(a), & \underline{\sigma}_k \leq \Delta d_{kk} \leq \overline{\sigma}_k \\ \underline{\sigma}_k, & \Delta d_k \leq \underline{\sigma}_k \end{cases} \tag{17}$$

where $\overline{\sigma}_k = \alpha I_k$, $\underline{\sigma}_k = -\alpha I_k$, $\alpha \in (0,1)$, $I_k = \min\{|a_k - \overline{a}|, |a_k - \underline{a}|\}$, I_k is the distance vector of the current iterative point to the nearest constraints, and $\overline{\sigma}_k$, $\underline{\sigma}_k$ are the truncated region boundaries vectors. The optimization algorithm can be implemented by the following steps.

- Step 1: Let $k = 1$, and let $0 < \alpha < 1$, ε be pre-specified constants;
- Step 2: Let $a_k \in R^n$ be given;
- Step 3: Evaluate the approximation of objective function with Eq. (15), and if $m(a) < \varepsilon$ then stop;
- Step 4: Determinate the truncation region with Eqs. (17a) and (17b);
- Step 5: Generate the searching direction $\Delta d_k = (S^T(a_k) WS(a_k))^{-1} S^T(a_k) Wr(a)$, and if Δd_k is within the region, then step 7;
- Step 6: Otherwise, Δd_k is taken to be the region boundary vectors and go to step 7;
- Step 7: Calculate $a_{k+1} = a_k + \Delta d_k$, replace a_k with a_{k+1}, and go to step 3.

4. Probability-based damage detection

In the formulation above, the experimentally measured modal frequencies and mode shapes are considered to be exact and deterministic. In reality, however, there are always uncertainties in the measured modal parameters. The uncertainties may be introduced by the inherent variability in

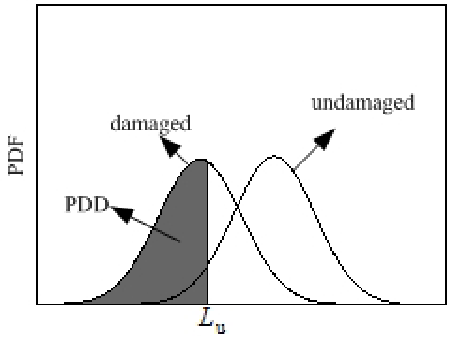


Fig. 1 PDD and PDFs

structural parameters and arise from the measurement noise and modal identification techniques. The uncertain measured data may lead to unreliable and false prediction of structural damage, and, as a result, it is necessary to detect the damage of structures in the probabilistic frame. In this work, the uncertainties are described as random variables with mean values and standard deviations.

Integrating with the Monte-Carlo methods, the statistics (means and standard deviations) of the updating parameters a_i (i = 1, 2,..., n), which are related to the damage severity of a structural, can be obtained in the case that the uncertainties in modal parameters are taken into accounted. Using the probabilistic method, the probability of damage detection (PDD) can be estimated by comparing the probability distributions of undamaged variables a_{di} and damaged a_{ui}. Assuming that both a_{ui} and a_{di} are normally distributed, their probability density functions (PDF), $p(a_{ui})$ and $p(a_{di})$, can be obtained as shown in Fig. 1, where L_u is the lower bound of the parameter a_{ui} in the undamaged state.

The lower bound L_u can be computed by

$$L_u = \mu(a_{ui}) - \varphi(1-\alpha)\sigma(a_{ui}) \tag{18}$$

where α is the confidence level α, $\phi(1-\alpha)$ is the $1-\alpha$ quantile of the standard normal distribution for a single-side confidence interval with α confidence level, which indicates that the undamaged parameter falls in the range of $[L_u, \infty]$ with the probability of α.

Further, the PDD of the structure can be calculated by

$$PDD = \int_{-\infty}^{L_u} p(a_{di}) da_{di} \tag{19}$$

where, the PDD has a value between 0 and 1. The closer the PDD is to 1, and the higher the probability of damage existence. On the other hand, the closer the PDD is to 0, and the lower the probability of damage existence.

5. Numerical studies

In order to demonstrate the proposed method and investigate the reliability of the proposed optimization algorithm in damage detection, the numerical simulations of a 25-bar planar truss

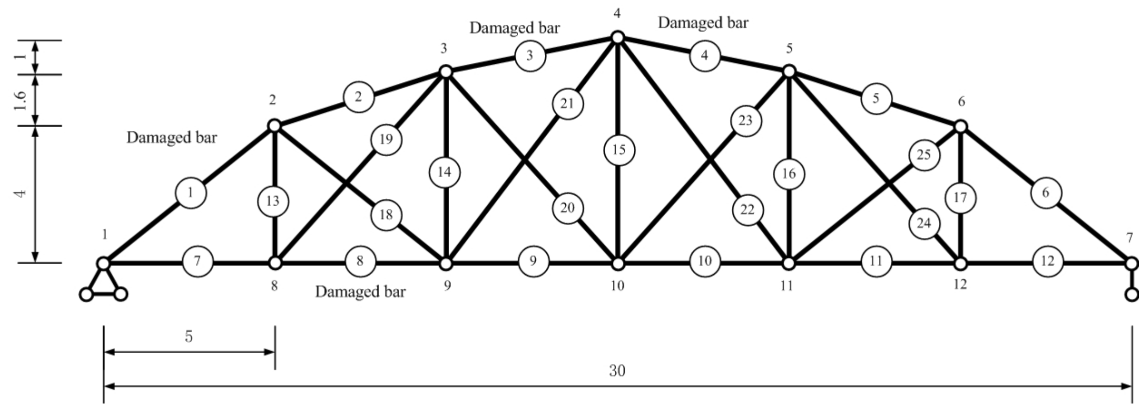


Fig. 2 Schematic diagram of a 25-bar planar truss

structures (Fig. 2) without and with damages are conducted. The mass density of bar material is 7.8×10^3 kg/m^3, and Young's elastic modulus is 2.1×10^{11} N/m^2.

Analytically modal analysis is first carried out to obtain the initial reference modal frequencies and mode shapes by using FE model. Damages are introduced by reducing the elastic moduli of bar elements 1, 3, 4, 8 by 30%, 15%, 10%, 5%, respectively. Modal analysis is then again carried out on the damaged truss to obtain the damaged modal parameters. The comparison of damaged and initial modal parameters of the truss before updating is shown in Table 1. Fig. 3 shows the first ten mode shapes for the initial model and the damaged truss. Here, the index MAC (modal assurance criterion) is introduced to indicates the correlation between two sets of mode shapes, and it is defined as

$$MAC(\varphi_i, \tilde{\varphi}_j) = \frac{\left|\varphi_i^T \tilde{\varphi}_j\right|^2}{(\varphi_i^T \varphi_i)(\tilde{\varphi}_j^{\mathrm{T}} \tilde{\varphi}_j)} \tag{20}$$

where, MAC values change between 0 and 1. A MAC value close to 1 indicates a good correlation and a MAC close to 0 indicates a poor correlation. The maximum relative error in modal frequency is about 6% (Mode No. 3 and 5) and the minimum MAC is about 73% (Mode No. 8 and 9) due to damages.

5.1 Deterministic analysis

The sensitivity–based FE updating with both the proposed optimization method and the trust region method is performed to identify the damage severity and location. The moduli of elasticity of bar elements are chosen as updating parameters. Instead of the absolute value of each elastic modulus, the relative reduction in modulus of elasticity which represents the severity of damage (SoD) in each bar is defined as

$$\mathrm{SoD} = 1 - \frac{E_j^{\mathrm{damaged}}}{E_j^{\mathrm{undamaged}}} \tag{21}$$

With the first 10 modal parameter of damaged truss, the SoDs of four damaged bar (elements 1,

Table 1 Comparison of damaged and initial modal parameters of truss before updating

Mode No.	Modal frequency (Hz)			MAC %
	Damaged truss	Initial model	Relative error %	
1	9.9863	10.2441	2.5815	99.8844
2	22.4781	23.1095	2.8090	98.7180
3	30.3913	32.2470	6.1060	98.7731
4	60.2776	60.5244	0.4094	99.4083
5	70.4548	74.7424	6.0856	98.3333
6	90.8862	91.3322	0.4907	99.7418
7	106.9459	107.3259	0.3553	98.9470
8	115.6163	116.4521	0.7229	72.2158
9	117.1497	119.2319	1.7774	73.1560
10	123.4195	123.6693	0.2024	99.1147

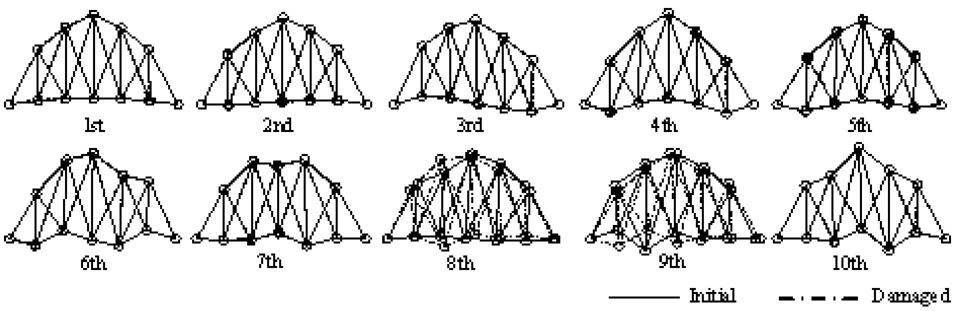
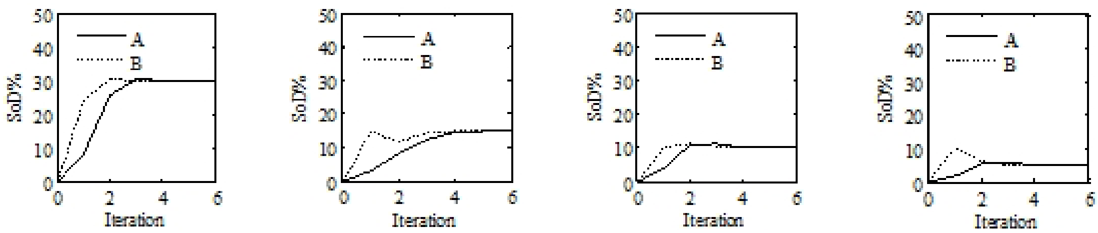


Fig. 3 The first 10 mode shapes for initial model and damaged truss

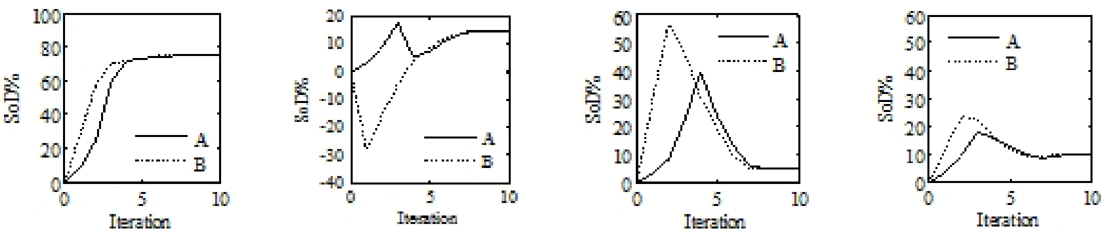
3, 4, 8) and one undamaged bar (element 10) are chosen and adjusted in the updating process of the analytical FE model. With both frequency and mode shape residuals in objective function, in which each modal residual has the same weight for assuming that the first 10 modal parameters are tested with accuracy, the tuning on modal parameters can be achieved with negligible error. The comparison of the iterative process and convergence of SoDs between the proposed method and trust region method for damaged bar elements 1, 3, 4 and 8 is shown in Fig. 4. The results show that the SoD for each element can converges to the actual value by the proposed optimization algorithm. The convergence is achieved with less computational effort by the proposed method than the trust region method. The CPU run time for the proposed method is about 18.406 seconds, and the CPU run time for the trust region method is about 27.388 seconds. It is also seen that the SoDs can be predicted very accurately in the case that the uncertainties in modal parameters are ignored. The results show that the FE updating method with the proposed method is reliable in detecting damage in the deterministic case.

It is worthy to point out that the convergence of the proposed method can also be guaranteed even when the actual value is far away from the initial value in the case of extremely severe damage, although the work in this paper focuses on the relatively small damage detection considering that the uncertainties in modal parameters have more unfavorable effect on the



(a) Element 1 (b) Element 3 (c) Element 4 (d) Element 8

Fig. 4 Iterative process of SoDs for two methods (A: Trust region method, B: The proposed method)



(a) Element 1 (b) Element 3 (c) Element 4 (d) Element 8

Fig. 5 Iterative process of SoDs in case of severe damage in bar element 1

detection confidence of small damage. Fig. 5 shows the comparison of the iterative process and convergence of SoDs between two methods in the case that the elastic modulus of bar element 1 is reduced by 75%. It can be seen that the SoDs also converge to the actual values. However, the SoDs will not converge to the actual values in the case of more than 75% reduction in the elastic modulus of bar element 1. In fact, the convergence depends on the number of updating parameters as well as how far the actual values are from the initial values.

5.2 Uncertain analysis

As mentioned in Section 4, there are always substantial uncertainties in the modal parameters despite the significant efforts devoted to development and enhancement of the measurement instrumentation and modal identification techniques. In order to investigate the reliability of the proposed method in predicting structural damage in the case of uncertain modal parameters, four combinations of levels of uncertainty are used in consideration that modal frequencies are more accurately measured and identified than mode shapes. These combinations are: (1) Level 1: 0.5% COV for modal frequencies and 5% COV for modes shapes; (2) Level 2: 1% COV for modal frequencies and 5% COV for modes shapes; (3) Level 3:0.5% COV for modal frequencies and 10% COV for modes shapes; (4) level 4: 1% COV or modal frequencies and 10% COV for mode shapes. The uncertain analysis of SoD for every element in damaged truss can be implemented by using Monte-Carlo simulation. Firstly, 5000 samples of the modal parameters for the damaged structure, assumed normally distributed with the given COV, are generated and used as virtually measured data. Then, 5000 samples of the updating parameters can be obtained by conducting the

Table 2 Comparison of damaged and updated modal parameters after updating (Level 1)

Mode No.	Modal frequency (Hz)			MAC %
	Damaged truss	updated model	Relative error %	
1	9.9863	9.9883	0.02002	100
2	22.4781	22.4814	0.01468	99.9996
3	30.3913	30.4119	0.06778	99.9997
4	60.2776	60.2897	0.02007	99.9999
5	70.4548	70.4587	0.00553	99.9988
6	90.8862	90.8799	-0.00693	99.9996
7	106.9459	106.9577	0.01103	99.9989
8	115.6163	115.6426	0.02274	99.9660
9	117.1497	117.1892	0.03371	99.9679
10	123.4195	123.4209	0.00113	99.9994

Table 3 Comparison of damaged and updated modal parameters after updating (Level 2)

Mode No.	Modal frequency (Hz)			MAC %
	Damaged truss	updated model	Relative error %	
1	9.9863	9.9736	-0.12717	99.9998
2	22.4781	22.4355	-0.18951	99.9988
3	30.3913	30.3248	-0.21881	99.9984
4	60.2776	60.2793	0.00282	99.9998
5	70.4548	70.3726	-0.11667	99.9985
6	90.8862	90.8696	-0.01826	99.9999
7	106.9459	106.9634	0.01636	99.9983
8	115.6163	115.5831	-0.02871	99.9507
9	117.1497	117.2	0.04293	99.9487
10	123.4195	123.4287	0.00745	99.9964

FE model updating of 5000 times. Lastly, the means and variances of the updating parameters can be estimated by sample means and variances, and the COV values can be calculated.

Compared to the actual modal parameters of damaged structure, the means of modal parameters obtained by carrying out modal analysis of updated FE model with different levels of uncertainty are respectively shown in Tables 2-5. The results show that the relative differences in the modal frequencies and the MAC values are considerably improved after the FE model updating.

Using the proposed updating method, the iterative process and convergence of mean values of SoDs obtained by using random modal parameters coming from the Monte-Carlo simulations with different levels of uncertainty are shown in Fig. 6. The results show that the SoD for each element converges to a certain value near the actual SoD. This demonstrates a good performance of the proposed optimization algorithm. It is also seen that the uncertainty in modal parameters introduces the uncertainty in mean value of SoD for each element.

Table 4 Comparison of damaged and updated modal parameters after updating (Level 3)

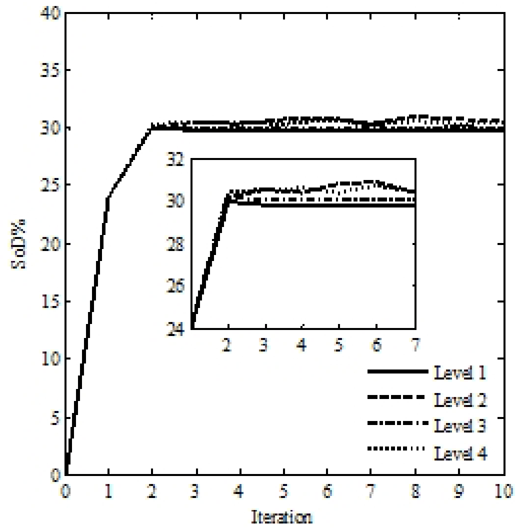
Mode No.	Modal frequency (Hz)			MAC %
	Damaged truss	updated model	Relative error %	
1	9.9863	9.9867	0.00401	100
2	22.4781	22.4746	-0.01557	99.9999
3	30.3913	30.3937	0.00789	100
4	60.2776	60.287	0.01559	100
5	70.4548	70.4871	0.04584	99.9999
6	90.8862	90.8904	0.00462	100
7	106.9459	106.9524	0.00608	99.9998
8	115.6163	115.6224	0.00527	99.9929
9	117.1497	117.1661	0.01399	99.9931
10	123.4195	123.4246	0.00413	99.9997

Table 5 Comparison of damaged and updated modal parameters after updating (Level 4)

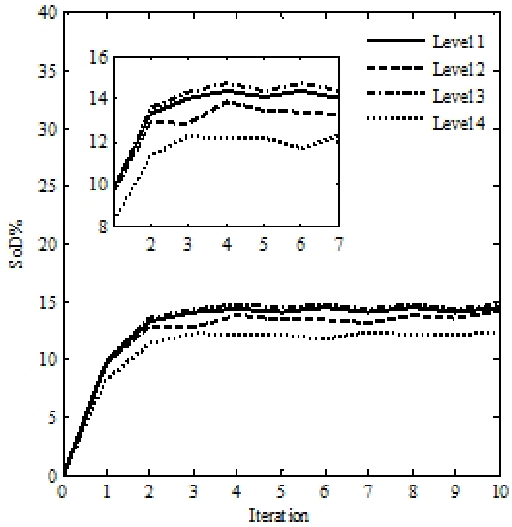
Mode No.	Modal frequency (Hz)			MAC %
	Damaged truss	updated model	Relative error %	
1	9.9863	9.9862	-0.00100	99.9999
2	22.4781	22.476	-0.00934	99.9994
3	30.3913	30.3519	-0.12964	99.9988
4	60.2776	60.2903	0.02106	99.9996
5	70.4548	70.3703	-0.11993	99.9951
6	90.8862	90.8575	-0.03157	99.9993
7	106.9459	106.9807	0.03253	99.9926
8	115.6163	115.6356	0.01669	99.8533
9	117.1497	117.2724	0.10473	99.8502
10	123.4195	123.4315	0.00972	99.9906

Fig. 7 shows the mean value and standard deviation of SoD for each damaged element compared to the corresponding actual value. For each element, the result shows that the higher level of uncertainty in the modal frequencies introduces more uncertainty in both mean and standard deviation of predicted SoD (seen from comparison between Level 1 and Level 2, or between level 3 and level 4), while the uncertainty in the mode shapes contributes less to the uncertainty in mean value and standard deviation of predicted SoD (seen from comparison between level 1 and level 3, or between Level 2 and level 4).

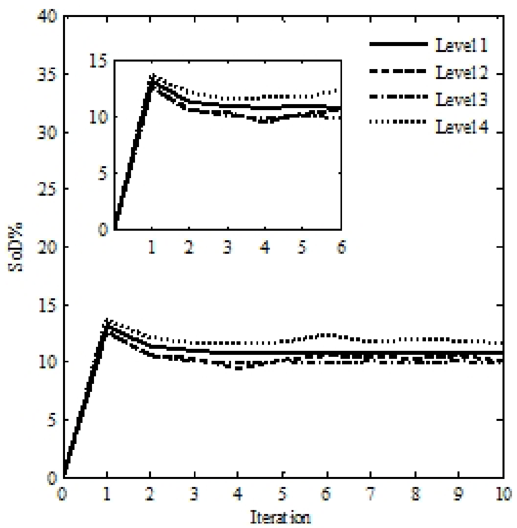
The COVs of SoDs with different levels of uncertainty is given in Table 6. The results show that the COVs of SoDs for different elements are significantly different in magnitude for different levels of uncertainty, ranging from 4.54% to 13.13% for element 1 (30% SoD), 29.76% to 65.31% for element 3(15% SoD), 28.93% to 63.13% for element 4 (10% SoD) and 57.32% to 143.16% for element 8 (5% SoD) when the level of uncertainty varies from Level 1 to Level 4. It is also seen



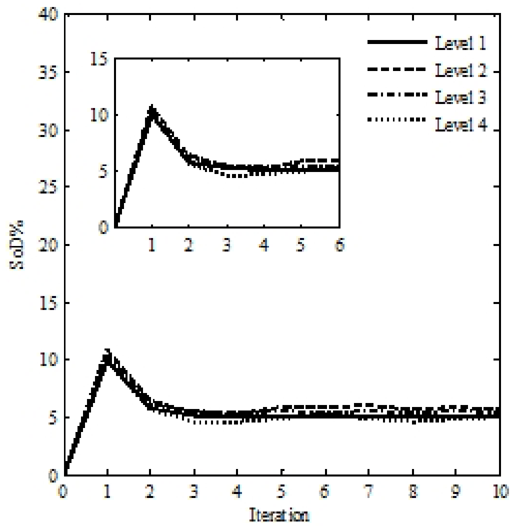
(a) Element 1



(b) Element 3



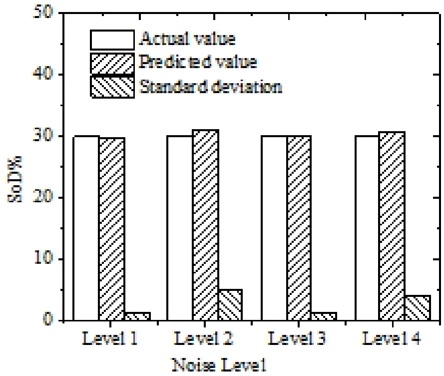
(a) Element 4



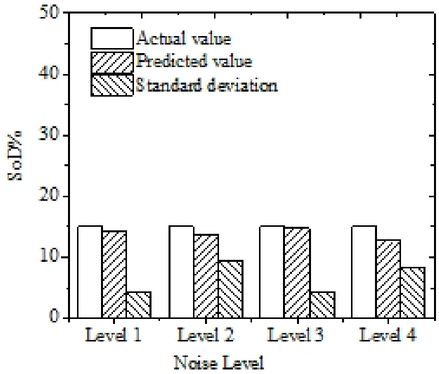
(b) Element 8

Fig. 6 Iterative process of SoDs (mean values) for damaged elements

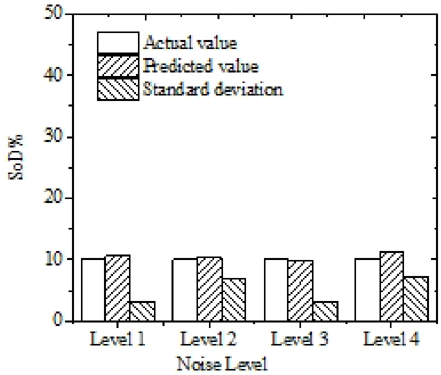
that both the less severe damage and the higher level of uncertainty can lead to the higher COV values, which means the larger dispersion of damage prediction from the mean value. It is also seen that the dispersion of damage detection data is sensitive to the uncertainty in modal frequencies and insensitive to the uncertainty in mode shape. In fact, the uncertainty in modal frequencies contributes much more to the changes in COV values than that in mode shape. A small COV of predicted SoD assures that the damage can be detected with good accuracy in the presence of uncertainty of measured modal parameters, while a large one indicates that the damage detection result could be totally annihilated by measurement uncertainties.



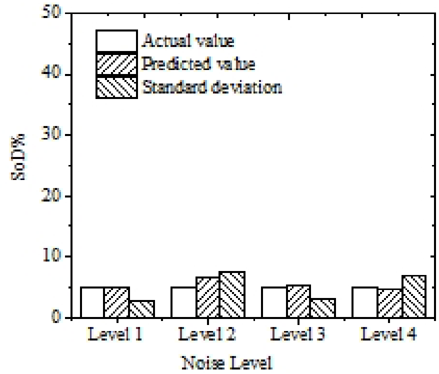
(a) Element 1



(b) Element 3



(a) Element 4



(b) Element 8

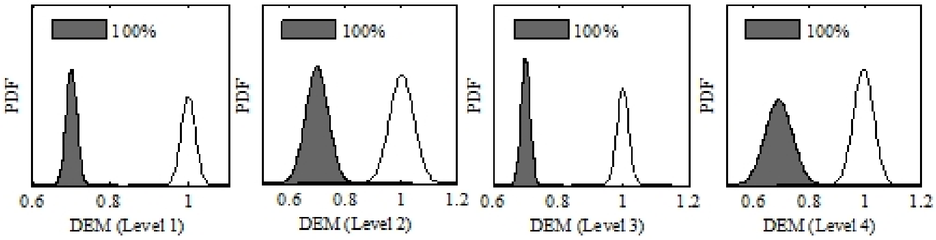
Fig. 7 R-square of mean values and standard deviation of SoD for damaged elements

Table 6 COVs of SoDs with different levels of uncertainty

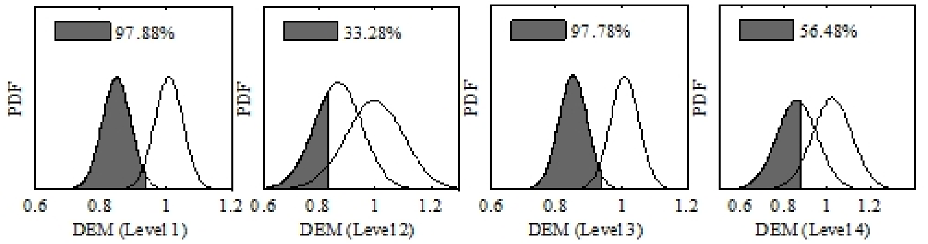
Element No.	Level of uncertainty			
	Level 1 (0.5%, 5%)	Level 2 (1%, 5%)	Level 3 (0.5%, 10%)	Level 4 (1%, 10%)
1	0.0454	0.1662	0.0459	0.1313
3	0.2976	0.6901	0.2922	0.6531
4	0.2893	0.6556	0.3277	0.6313
8	0.5732	1.1394	0.5638	1.4316

5.3 Probability of damage detection

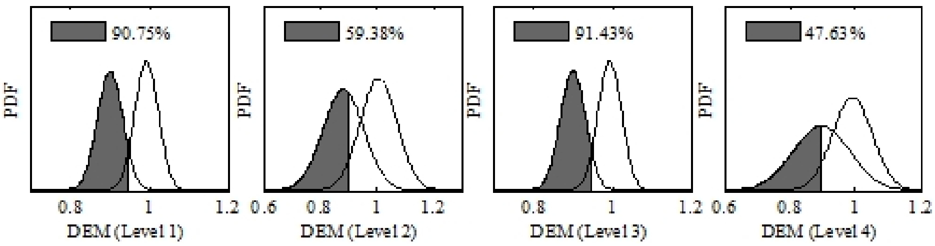
To further assess the reliability of the proposed method in damage detection at different levels of uncertainty, the PDD values are estimated comparing the statistics of the stiffness parameters (dimensionless elastic modulus, DEM) of the undamaged and damaged structures using Eq. (18) and Eq. (19). With 95% confidence level, $\phi(1-\alpha)$ in Eq. (18) will be 1.645, which is the 5% quantile of the standard normal distribution for a single-side confidence interval.



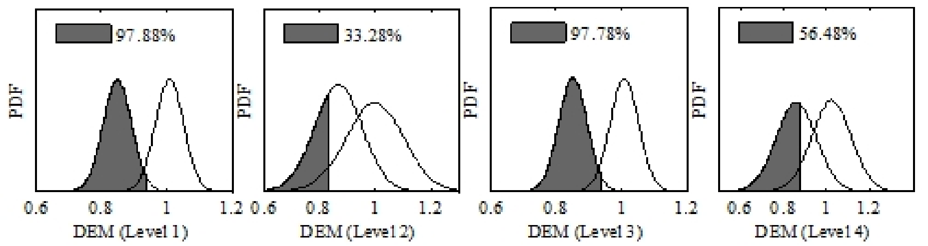
(a) Element 1



(b) Element 3



(c) Element 4



(d) Element 8

Fig. 8 PDD values with damaged and undamaged PDFs

The PDD of every element with different levels of uncertainty is shown in Fig. 8. It is observed that the highest PDE occurs at the element 1 (30% SoD) and the lowest PDE is with element 8 (5% SoD) in the same level of uncertainty. In the case of Level 1, for example, the PDD is 100% for element 1 while the PDD is 54.33% for element 8. The results indicate that the damage can be confidently identified when it is severer, and the damage cannot be confidently detected when it is not significant. This means that the modal uncertainties will have a significant influence on the results of damage detection, when the damage levels are small and the changes of modal parameters due to damage are not apparent. The results also show that the PDDs decrease apparently when a significantly high level of uncertainty is introduced in modal frequencies, and the change of PDDs is relatively small when a significantly high level of uncertainty is introduced in mode shapes. For example, the PDD in element 3 decreases from 97.88% to 33.28% when the uncertainty in frequencies increases from 0.5% to 1% with the uncertainty in mode shapes unchanged (5%), while the change of PDD is from 97.88% to 97.78% when the uncertainties in mode shapes increases from 5% to 10% with the uncertainty in frequencies unchanged (0.5%). The same situation occurs to other damaged elements. This indicates that the PDD is more sensitive to the uncertainty in frequencies while the uncertainty in mode shapes contributes less to the uncertainties in damage detection.

6. Conclusions

Based on FE updating method, a novel simple but computationally efficient optimization algorithm is proposed and applied to structural damage detection. The proposed method integrates the Gauss-Newton method with the region truncation at each iteration step. Through the proposed method, not only the constraints are introduced instead of the penalty function, but also the design variables are restricted in the searching region. The accuracy of the proposed method is verified by the trust region method. Compared to the trust region method, the convergence can be achieved with less computational effort by the proposed method.

The reliability of the proposed method in damage detection is also investigated by considering different levels of uncertainties in modal parameters. Using the Monte-Carlo simulation, mean values and standard deviations of damages are obtained and the results show that both the severer damage and the higher level of uncertainty lead to the large dispersion of damage detection data from the corresponding mean value. Further, the probability of damage detection (PDD) is estimated by using the probabilistic method, and it is found that the PDD is much more sensitive to the uncertainty in frequencies than that in mode shapes. The conclusion is made that the proposed method can be reliably applied to damage detection with relatively less computational efforts in comparison with trust region method.

References

Bakir, P.G., Reynders, E. and Roeck, G.D. (2007), "Sensitivity-based finite element model updating using constrained optimization with a trust region algorithm", *J. Sound Vib.*, **305**(1-2), 211-225.

Carden, E. and Fanning, P. (2004), "Vibration based condition monitoring: a review", *Struct. Health Monit.*, **3**(4), 355-377.

Cawley, P. and Adams, R.D. (1979), "The location of defects in structures from measurements of natural frequencies", *J. Strain Anal. Eng. Des.*, **14**(2), 49-57.

Chakraborty, S. and Sen, A. (2014), "Adaptive response surface based efficient finite element model updating", *Finite Elem. Anal. Des.*, **80**, 33-40.

Chellini, G., Roeck, G.D., Nardini, L. and Salvatore, W. (2010), "Damage analysis of a steel concrete composite frame by finite element model updating", *J. Constr. Steel Res.*, **66**(3), 398-411.

Doebling, S.W., Farrar, C.R. and Prime, M.B. (1998), "A summary review of vibration-based damage identification methods", *Shock Vib. Digest*, **30**(2), 91-105.

Fang, S.E., Perera, R. and Roeck, G.D. (2008), "Damage identification of a reinforced concrete frame by finite element model updating using damage parameterization", *J. Sound Vib.*, **313**(3-5), 544-559.

Friswell, M.I. and Mottershead, J.E. (1995), *Finite Element Model Updating in Structural Dynamics*, Kluwer Academic, The Netherlands.

Gola, M.M., Somae, A. and Botto, D. (2001), "On theoretical limits of dynamic model updating using a sensitivity-based approach", *J. Sound Vib.*, **244**(4), 583-595.

Jaishi, B. and Ren, W.X. (2005), "Structural finite element model updating using ambient vibration test results", *J. Struct. Eng.*, **131**(4), 617-628.

Jaishi, B. and Ren, W.X. (2006), "Damage detection by finite element model updating using modal flexibility residual", *J. Sound Vib.*, **290**(1-2), 369-387.

Jaishi, B. and Ren, W.X. (2007), "Finite element model updating based on eigenvalue and strain energy residuals using multiobjective optimization technique", *Mech. Syst. Signal Pr.*, **21**(5), 2295-2317.

Kosmatka, R. and Ricles, J. (1999), "Damage detection in structures by modal vibration characterization", *J. Struct. Eng.*, **125**(12), 1384- 1392.

Link, M. (1999), "Updating of analytical models—review of numerical procedures and application aspects", P*roceedings of the Structural Dynamics Forum SD2000*, Los Alamos, NM, USA.

Meruane, V. and Heylen, W. (2011), "An hybrid real genetic algorithm to detect structural damage using modal properties", *Mech. Syst. Signal Pr.*, **25**(5), 1559-1573.

Mottershead, J.E., Link, M. and Friswell, M.I. (2011), "The sensitivity method infinite element model updating: A tutorial", *Mech. Syst. Signal Pr.*, **25**, 2275-2296.

Natke, H. (1998), "Problems of model updating procedures: a perspective resumption", *Mech. Syst. Signal Pr.*, **12**(1), 65-74.

Qiu, S.Q. and Chen, Z.W. (2012), "A new penalty-free-type algorithm based on trust region techniques", *Appl. Math. Comput.*, **218**(22), 11089-11099.

Ren, W.X. and Chen, H.B. (2010), "Finite element model updating in structural dynamics by using the response surface method", *Eng. Struct.*, **32**(8), 2455-2465.

Reynders, E., Teughels, A. and Roeck, G.D. (2010), "Finite element model updating and structural damage identification using OMAX data", *Mech. Syst. Signal Pr.*, **24**(5), 1306-1323.

Ribeiro, D., Calcada, R. and Delgado, R. (2012), "Finite element model updating of a bowstring-arch railway bridge based on experimental modal parameters", *Eng. Struct.*, **40**, 413-435.

Shi, Z., Law, S. and Zhang, L. (2000), "Damage localization by directly using incomplete mode shapes", *J. Eng. Mech.*, **126**(6), 656-660.

Shi, Z.J. and Xu, Z.W. (2009), "The convergence of subspace trust region methods", *J. Comput. Appl. Math.*, **231**(1), 365-377.

Sipple, J.D. and Sanayei, M. (2014), "Finite element model updating of the UCF grid benchmark using measured frequency response functions", *Mech. Syst. Signal Pr.*, **46**(1), 179-190.

Shiradhonkar, S.R. and Shrikhande, M. (2011), "Seismic damage detection in a building frame via finite element model updating", *Comput. Struct.*, **89**(23-24), 2425-2438.

Tu, Z.G. and Lu, Y. (2008), "FE model updating using artificial boundary conditions with genetic algorithms", *Comput. Struct.*, **86**(7-8), 714-727.

Yang, Y.B. and Chen, Y.J. (2009), "A new direct method for updating structural models based on measured modal data", *Eng. Struct.*, **31**(1), 32-42.

Zhang, Q., Chang, C. and Chang, T. (2000), "Finite element model updating for structures with parametric constraints", *Earthquake Eng. Struct. Dyn.*, **29** (7), 927-944.

CC