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Forced vibration of an embedded single-walled carbon nanotube traversed by a moving load using nonlocal Timoshenko beam theory

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Abstract. Dynamic analysis of an embedded single-walled carbon nanotube (SWCNT) traversed by a moving nanoparticle, which is modeled as a moving load, is investigated in this study based on the nonlocal Timoshenko beam theory, including transverse shear deformation and rotary inertia. The governing equations and boundary conditions are derived by using the principle of virtual displacement. The Galerkin method and the direct integration method of Newmark are employed to find the dynamic response of the SWCNT. A detailed parametric study is conducted to study the influences of the nonlocal parameter, aspect ratio of the SWCNT, elastic medium constant and the moving load velocity on the dynamic responses of SWCNT. For comparison purpose, free vibration frequencies of the SWCNT are obtained and compared with a previously published study. Good agreement is observed. The results show that the above mentioned effects play an important role on the dynamic behaviour of the SWCNT.

Keywords: vibration; nonlocal Timoshenko beam theory; carbon nanotubes; moving loads.

1. Introduction

The physics of carbon nanotubes (CNTs) has rapidly evolved into a research field since their discovery by Iijima (1991) in multiwall form and as single-walled tubes two years later. Since then, theoretical and experimental studies in different fields, such as mechanics, optics, and electronics have focused on both the fundamental physical properties and on the potential applications of nanotubes. The modeling for the analysis of CNTs is generally classified into two categories. The first one is the atomistic modeling which is computationally expensive and is not suitable for analyzing large scale systems. Thus, the continuum mechanics models, such as beam and shell models, have been applied to bending, buckling and vibration of CNTs by several researchers. Although there are many studies related to vibration of nanotubes (Yoon *et al*. 2003, Yoon *et al*. 2004, Zhang *et al*. 2005, Wang *et al*. 2006, Aydogdu and Ece 2007, Aydogdu 2008, Mir *et al*. 2008) based on the classical continuum theory, the small-size scale and nanoscale surface effect associated with nanotechnology become significant and consequently the classical or local continuum theory can not predict the behavior of the nanoscale structures. The theory of nonlocal continuum mechanics was initiated by the studies of Eringen (1972,

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1983) and Eringen and Edelen (1972). In the classical or local continuum theory, the stress state at a given point is dependent uniquely on the strain state at the same point whereas the nonlocal continuum theory assumes that the stress state is a function of the strain states of all points in the body. Thus, the nonlocal continuum theory contains information about long range forces between atoms, and the internal length scale is introduced into the constitutive equations simply as material parameter. In this context, application of nonlocal continuum theory to nanotechnology problems was initially addressed by Peddieson *et al.* (2003), in which the static deformation analysis of Euler-Bernoulli beams are analyzed. The nonlocal continuum theory has been further extended to static (Wang and Liew 2007, Lim and Wang 2007, Reddy 2007, Civalek *et al.* 2009), buckling (Zhang *et al.* 2004, Zhang *et al.* 2006, Wang *et al.* 2006, Adali 2008, Kumar *et al.* 2008, Murmu and Pradhan 2009, Sato and Shima 2009), free vibration (Zhang *et al.* 2005, Ece and Aydogdu 2007, Li and Wang 2009, Lim *et al.* 2009, Civalek *et al.* 2010, Demir *et al.* 2010, Civalek and Akgöz 2010), and wave propagation analysis (Wang and Hu 2005, Lu *et al.* 2006, 2007, Lu 2007, Tounsi *et al.* 2008, Heireche *et al.* 2008, Hu *et al.* 2009, Narendar and Gopalakrishnan 2009) of carbon nanotubes. In addition to these studies, Reddy and Pang (2008) developed the nonlocal theories of Euler-Bernoulli and Timoshenko beams and derived analytical solutions for various boundary conditions for static analysis, buckling and free vibration of straight nanobeams. Aydogdu (2009) developed a nonlocal elastic rod model to investigate the small-scale effect on axial vibration of nanorods. Aydogdu (2009) proposed a generalized nonlocal beam theory based on Euler-Bernoulli, Timoshenko, parabolic shear deformation and general exponential shear deformation theory (Aydogdu 2009) to study bending, buckling and free vibration of nanobeams. Murmu and Pradhan (2009) investigated the effect of the small-scale parameter on free vibration of nonuniform nanocantilever by using differential quadrature method. Murmu and Pradhan (2009) studied the thermal vibration of SWCNTs based on the thermal elasticity mechanics and nonlocal Euler-Bernoulli beam theory by differential quadrature method. Wang (2009) developed a nonlocal Euler-Bernoulli beam theory for vibration and instability of tubular micro- and nanobeams conveying fluid by using differential quadrature method. Wang (2009) presented a nonlocal double-elastic beam model based on Euler-Bernoulli beam theory for the vibration analysis of DWCNTs conveying fluids. Lee and Chang (2009) analyzed the influences of nonlocal effect, viscosity effect, aspect ratio and elastic medium constant on the fundamental frequency of a viscous-fluid conveying SWCNT embedded in an elastic medium. Ke *et al.* (2009) studied nonlinear free vibration of embedded DWCNTs based on the nonlocal Timoshenko beam theory and von-Kármán geometric nonlinearity. XiaoDong and Lim (2009) investigated nonlinear free vibrations of a nanobeam due to finite stretching of the beam by using multiple scales method. Pradhan and Sarkar (2009) carried out bending, buckling and vibration analyses of functionally graded tapered beam using Eringen's nonlocal elasticity theory and Rayleigh-Ritz method. Pradhan and Phadikar (2009) studied the static, buckling and vibration analyses of nonhomogeneous nanotubes having various boundary conditions by general differential quadrature (GDQ) method. Murmu and Pradhan (2010) applied the nonlocal Euler-Bernoulli beam model to buckling analysis of a simply-supported SWCNT subjected to an axial compressive load and with the effect of temperature change and surrounding elastic medium. More recently, Kiani and Mehri (2010) have carried out the dynamic analysis of single-walled carbon nanotubes under excitation of a moving nanoparticle modeled by a moving constant load based on the nonlocal Euler–Bernoulli, Timoshenko and higher order beam theories. Şimşek (2010) has recently studied the dynamic behavior of a single-walled carbon nanotube subjected to a moving harmonic load based on Eringen's nonlocal elasticity theory.

The motion of neutral atoms and nanoparticles in nanotubes has been of considerable interest in view of the rapid progress of nanotechnology (Dedkov and Kyasov 2007), and carbon nanotubes are used as molecular channels for the transportation of nanoparticles, such as water and protons (Hummer *et al.* 2001). During these applications, nanotubes may be subjected to moving loads, and this leads to transverse vibration of nanotubes. Due to this fact, it is very important to understand the dynamical behavior of nanotubes under moving loads. However, the above review clearly shows that nearly all investigators have so far investigated static analysis, buckling or free vibration of nanotubes or nanobeams based on the local and the nonlocal elasticity theory. In this context, although there are many studies related to beams under moving loads using classical continuum models at macro-scale (i.e., Timoshenko and Young 1955, Fryba 1972, Lee 1994, Zheng *et al.* 1998, Wang 1997, Wang and Lin 1998, Wang and Sang 1999, Zhu and Law 1999, Abu-Hilal and Mohsen 2000, Klasztorny 2001, Kocatürk and Şimşek 2006a,b, Şimşek and Kocatürk 2007, Ling *et al.* 2008, Sniady 2008, Şimşek and Kocatürk 2009a, b, Şimşek 2010a, b), to the best knowledge of the author, there is no reported work on the dynamic analysis of embedded carbon nanotubes subjected to moving loads based on the nonlocal Timoshenko beam theory. Therefore, this study is the first attempt on the vibration of embedded carbon nanotubes under action of a moving load.

In the present study, the governing equations and boundary conditions are derived by using the principle of virtual displacement. The Galerkin method and the direct integration method of Newmark (1959) are employed to find the dynamic response of the embedded SWCNT. A detailed parametric study is conducted to study the influences of the nonlocal parameter, aspect ratio of the SWCNT, elastic medium constant and moving load velocity on the dynamic responses of SWCNT. For comparison purpose, free vibration frequencies of the SWCNT are obtained and compared with a previously published study. Good agreement is observed. The results show that the above mentioned effects play an important role on the dynamic behaviour of the SWCNT.

2. Nonlocal elasticity theory

According to Eringen (1983), the stress field at a point $\mathbf{x}$ in an elastic continuum not only depends on the strain field at the same point but also on strains at all other points of the body. Therefore, the nonlocal stress tensor σ at point $\mathbf{x}$ is defined by

$$\sigma(x) = \int_V K(|\mathbf{x}' - \mathbf{x}|, \tau) T(\mathbf{x}') \mathrm{d}V(\mathbf{x}') \tag{1a}$$

$$T(\mathbf{x}) = C(\mathbf{x}) : \varepsilon(\mathbf{x}) \tag{1b}$$

where $T(\mathbf{x}')$ is the classical, macroscopic stress tensor at point $\mathbf{x}$, $K(|\mathbf{x}' - \mathbf{x}|, \tau)$ is the nonlocal modulus or attenuation function incorporating into constitutive equations the nonlocal effects at the reference point $\mathbf{x}$ produced by local strain at the source $\mathbf{x}'$, C($\mathbf{x}$) is the fourth-order elasticity tensor, $\varepsilon(\mathbf{x})$ is the strain tensor, τ is the material constant which is defined as $\tau = e_0 a/l$ where e_0 is as constant to adjust the model to match the reliable results by experiments or other models, a and l are the internal and external characteristics lengths (such as the lattice spacing and wavelength). The parameter e_0 was estimated that $e_0 = 0.39$ by Eringen (1983) by comparing the results of lattice dynamics with nonlocal theory. According to Sudak (2003), values of e_0 need to be determined from experimental results, and in the results of Sudak (2003), it was concluded that L / a and e_0 should be the same order or one order less to have any significant nonlocal effect. Zhang *et al.* (2005) approximated that $e_0 \approx 0.82$ by matching the theoretical buckling strain obtained by the nonlocal elastic cylindrical shell model using Donell theory to those from molecular mechanics simulations given by Sears and Batra (2004). By using the strain gradient approach, the parameter e_0 was proposed that $e_0 \approx 0.288$ by Wang and Hu (2005). The above-mentioned studies clearly indicate that reasonable choice of the value of the parameter e_0a is crucial to ensure the validity of the nonlocal models, and therefore more works are required to determine the value of e_0a more accurately for CNTs. A conservative estimate of the nonlocal parameter $0 \le e_0a \le 2\text{nm}$ for an SWCNT is proposed by Wang (2005). Therefore, in this study, the nonlocal parameter is taken as $e_0a = 0$, 0.5, 1, 1.5, 2 nm to investigate nonlocal effects on the dynamic responses. In addition, the volume integral in Eq. (1a) is over the region V occupied by the body. However, it is difficult to solve the elasticity problems by using the integral constitutive relation in Eq. (1). Therefore, a simplified constitutive relation in a differential form is given by Eringen (1983) as follows

$$(1 - \tau^2 l^2 \nabla^2)\sigma = T,\ \tau = \frac{e_a a}{l} \tag{2}$$

where ∇ is the Laplacian operator. For a beam type structure, the nonlocal behavior can be neglected in the thickness direction. Thus, for a homogeneous isotropic Timoshenko beam, the nonlocal constitutive relations take the following form

$$\sigma_{xx} - (e_0a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\varepsilon_{xx} \tag{3a}$$

$$\sigma_{xz} - (e_0a)^2 \frac{\partial^2 \sigma_{xz}}{\partial x^2} = E\gamma_{xz} \tag{3b}$$

where E is the elasticity modulus, $G = 0.5E/(1 + \mathrm{v})$ is the shear modulus (where v is the Poisson's ratio), σ_{xx} is the axial normal stress, σ_{xz} is the shear stress, ε_{xx} is the axial strain and γ_{xz} is the shear strain. When the nonlocal parameter is taken as $e_0a = 0$, the constitutive relation of the local theory is obtained.

3. Nonlocal Timoshenko beam model

A simply-supported SWCNT having the length L, the diameter d and the thickness t_b is shown in Fig. 1. The SWCNT is subjected to a moving load $P(t)$, which moves in the axial direction of the nanotube with constant velocity, v_P. The SWCNT is embedded in an elastic medium which is modeled as Winkler foundation with spring constant k_w. It should be noted that the following assumptions are made in this study: (i) The nanotube is initially at rest, namely the initial conditions of the nanotube are zero. (ii) The moving particle is modeled as a moving load and the inertial effects of the moving load are negligible. (iii) The velocity of the moving load is constant and the moving load is in contact with the nanotube during the excitation.

Based on the Timoshenko beam theory, the displacement field at any point in the beam along x, y and z axes can be given as

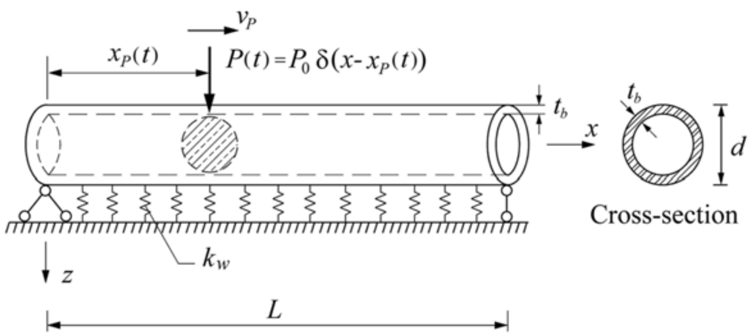

Fig. 1 A simply supported embedded SWCNT traversed by a moving load.

$$u_x(x,z,t) = z\phi(x,t) \tag{4a}$$

$$u_y(x,z,t) = 0 \tag{4b}$$

$$u_z(x,z,t) = w_0(x,t)\,, \tag{4c}$$

where ϕ is the rotation of the cross-sections, w_0 is the transverse displacements of any point on the neutral axis and t denotes time. The nonzero strains of the Timoshenko beam theory are

$$\varepsilon_{xx} = z\kappa_x,\ \kappa_x = \frac{\partial\phi(x,t)}{\partial x} \tag{5a}$$

$$\gamma_{xz} = \frac{\partial w_0(x,t)}{\partial x} + \phi(x,t) \tag{5b}$$

where κ_x is the curvature of the beam. The transverse shear force Q and the resulting the bending moment M can be obtained as

$$Q = \int_A \sigma_{xz}\mathrm{d}A,\ M = \int_A z\sigma_{xx}dA \tag{6}$$

where A is the area of the cross-section. The principle of virtual displacement for the Timoshenko beam is given by

$$\int_0^t\int_0^L\left(\rho A\frac{\partial w_0}{\partial t}\frac{\partial\delta w_0}{\partial t}+\rho I\frac{\partial\phi}{\partial t}\frac{\partial\delta\phi}{\partial t}-Q\delta\gamma_{xz}-M\delta\kappa_x-kw_0\delta w_0+q\delta w_0\right)\mathrm{d}x\mathrm{d}t = 0 \tag{7}$$

where ρ is the mass density of the beam, I is the second moment of area of the beam, q is the distributed load. Integrating Eq. (7) by parts and setting the coefficients of δw_0 and $\delta\phi$ to zero lead to the two variationally consistent governing equations of the nanotube

$$\frac{\partial Q}{\partial x} + q - k_w w_0 = \rho A \frac{\partial^2 w_0}{\partial t^2} \tag{8a}$$

$$\frac{\partial M}{\partial x} - Q = \rho I \frac{\partial^2 \phi}{\partial t^2} \tag{8b}$$

Furthermore, the following boundary conditions at the edges of the beam (at $x = 0$ and $x = L$) are obtained by application of the virtual displacement principle

$$\begin{aligned} &\text{either } w_0 = 0 \text{ or } Q = 0 \\ &\text{either } \phi = 0 \text{ or } M = 0 \end{aligned} \tag{9}$$

By using Eqs. (3), (5) and (6), the force-strain and the moment-strain relations of the nonlocal Timoshenko beam theory can be obtained as follows

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = EI \frac{\partial \phi}{\partial x} \tag{10a}$$

$$Q - (e_0 a)^2 \frac{\partial^2 Q}{\partial x^2} = k_s GA \left(\frac{\partial w_0}{\partial x} + \phi \right) \tag{10b}$$

where k_s is the shear correction factor that accounts for non-uniform shear stress distribution through the thickness of the beam. In order to obtain the governing equations in terms of the displacements (w_0, ϕ), first, eliminating the shear force Q between Eqs. (8a) and (8b) leads to the following equation

$$\frac{\partial^2 M}{\partial x^2} + q - k_w w_0 = \rho A \frac{\partial^2 w_0}{\partial t^2} + \rho I \frac{\partial^3 \phi}{\partial x \partial t^2} \tag{11}$$

The explicit expression of the nonlocal bending moment can be obtained by substituting for the second derivative of M from Eq. (11) into Eq. (10a) as follows

$$M = EI \frac{\partial \phi}{\partial x} + (e_0 a)^2 \left(\rho A \frac{\partial^2 w_0}{\partial t^2} + \rho I \frac{\partial^3 \phi}{\partial x \partial t^2} - q + k_w w_0 \right) \tag{12}$$

By substituting for the second derivative of Q from Eq. (8a) into Eq. (10b), the following relation for the nonlocal shear force is obtained

$$Q = k_s GA \left(\frac{\partial w_0}{\partial x} + \phi \right) + (e_0 a)^2 \left(\rho A \frac{\partial^3 w_0}{\partial x \partial t^2} - \frac{\partial q}{\partial x} + k_w \frac{\partial w_0}{\partial x} \right) \tag{13}$$

Finally, the following governing equations in terms of the displacements can be obtained by substituting for M and Q from Eqs. (12) and (13), respectively, into Eqs. (8a) and (8b) as

$$k_s GA\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial \phi}{\partial x}\right)-k_w w_0+(e_0 a)^2\left(k_w\frac{\partial^2 w_0}{\partial x^2}+\rho A\frac{\partial^4 w_0}{\partial x^2\partial t^2}\right)-\rho A\frac{\partial^2 w_0}{\partial t^2} = (e_0 a)^2\frac{\partial^2 q}{\partial x^2}-q \tag{14a}$$

$$EI\frac{\partial^2 \phi}{\partial x^2}-k_s GA\left(\frac{\partial w_0}{\partial x}+\phi\right)+(e_0 a)^2\rho I\frac{\partial^4 \phi}{\partial x^2\partial t^2}-\rho I\frac{\partial^2 \phi}{\partial t^2} = 0 \tag{14b}$$

It should be noted that when the nonlocal parameter $e_0 a$ is taken as zero in Eqs. (14a) and (14b), the local governing equations of Timoshenko beam theory are obtained. On the other hand, substituting $\phi = -\partial w_0/\partial x$ into Eqs. (14a) and (14b) yields the governing equation of the nonlocal Euler-Bernoulli beam theory. In order to investigate the dynamic behavior of the SWCNT, the responses of the SWCNT $w_0(x, t)$ and $\phi(x, t)$ are assumed to be in the following form of series

$$w_0(x,t) = \sum_{m=1}^{\infty} X_m(x)A_m(t) \tag{15a}$$

$$\phi(x,t) = \sum_{m=1}^{\infty} Y_m(x)B_m(t) \tag{15b}$$

where $A_m(t)$ and $B_m(t)$ are the unknown time-dependent generalized coordinates, and $X_m(x)$ and $Y_m(x)$ are the space-dependent coordinates which can be expressed for boundary conditions in Eq. (9) as follows

$$X_m(x) = \sin\left(\frac{m\pi x}{L}\right), m = 1,2,3,\ldots \tag{16a}$$

$$Y_m(x) = \cos\left(\frac{m\pi x}{L}\right), m = 1,2,3,\ldots \tag{16b}$$

Substituting Eqs. (15a) and (15b) into Eqs. (14a) and (14b), multiplying both sides of the first and the second resulting equations with $X_n(x)$ and $Y_n(x)$ respectively, and integrating them over the domain (0, L) yields the following coupled equations of motion in terms of the generalized displacements

$$\sum_{m=1}^{\infty} A_m\int_0^L k_s GAX_m''\,X_n\mathrm{d}x-\sum_{m=1}^{\infty} A_m\int_0^L k_w X_m X_n\mathrm{d}x+\sum_{m=1}^{\infty} A_m\int_0^L k_w(e_0 a)^2 X_m''\,X_n\mathrm{d}x+\sum_{m=1}^{\infty} B_m\int_0^L k_s GAY_m'\,X_n\mathrm{d}x$$

$$-\sum_{m=1}^{\infty}\ddot{A}_m\int_0^L\rho AX_m X_n\mathrm{d}x+\sum_{m=1}^{\infty}\ddot{A}_m\int_0^L\rho A(e_0 a)^2X_m''X_n\mathrm{d}x=\int_0^L\left((e_0 a)^2\frac{\partial^2 q}{\partial x^2}-q\right)X_n\mathrm{d}x \tag{17a}$$

$$-\sum_{m=1}^{\infty} A_m \int_0^L k_s GAX_m' \, Y_n \mathrm{d}x + \sum_{m=1}^{\infty} B_m \int_0^L EIY_m'' \, Y_n \mathrm{d}x - \sum_{m=1}^{\infty} B_m \int_0^L k_s GAY_m Y_n \mathrm{d}x - \sum_{m=1}^{\infty} \ddot{B}_m \int_0^L \rho I Y_m Y_n \mathrm{d}x$$

$$+ \sum_{m=1}^{\infty} \ddot{B}_m \int_0^L \rho I (e_0 a)^2 \, Y_m'' Y_n \mathrm{d}x = 0 \qquad m, n = 1, 2, 3, \ldots \tag{17b}$$

where prime and overdot denote the derivatives with respect to x and t, respectively. It is also to note that in Eqs. (17a) and (17b), $\int_L X_m X_n \mathrm{d}x = \int_L Y_m Y_n \mathrm{d}x = 0$ for $m \neq n$. After some arrangements, Eqs. (17a) and (17b) can be written in a matrix form as

$$\begin{bmatrix} [\mathbf{K}_{11}] & [\mathbf{K}_{12}] \\ [\mathbf{K}_{21}] & [\mathbf{K}_{22}] \end{bmatrix} \begin{Bmatrix} \mathbf{A}(t) \\ \mathbf{B}(t) \end{Bmatrix} + \begin{bmatrix} [\mathbf{M}_{11}] & [0] \\ [0] & [\mathbf{M}_{22}] \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{A}}(t) \\ \ddot{\mathbf{B}}(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}(t) \\ \mathbf{0} \end{Bmatrix} \tag{18}$$

where $\{\mathbf{A}(t)\} = \{A_1(t), A_2(t), \ldots A_m(t)\}^T$ and $\{\mathbf{B}(t)\} = \{B_1(t), B_2(t), \ldots B_m(t)\}^T$. For the moving load, the load $q(x, t)$ can thus be written as follows

$$q(x, t) = P_0 \delta(x - x_p) \tag{19}$$

where $\delta(\cdot)$ is the Dirac delta function, P_0 is the magnitude of the moving load, x_P ($0 \leq x_P = v_P t \leq L$) is the coordinate of the moving load. By using the following general property of Dirac delta function

$$\int_b^c g(x) \delta^{(n)}(x - x_0) \mathrm{d}x = \begin{cases} (-1)^n g^{(n)}(x_0) & \text{if } b < x_0 < c \\ 0 & \text{Otherwise} \end{cases} \tag{20}$$

where $\delta^{(n)}(\cdot)$ represents nth derivative of Dirac delta function, the generalized load vector $\mathbf{F}(t)$ can be expressed as

$$F_n(t) = P_0[(e_0 a)^2 X_n''(x_P) - X_n(x_P)] \text{ for } t_1 = 0 \leq t \leq t_2 = L/v_P \tag{21a}$$

$$F_n(t) = 0 \text{ for } t > t_2 = L/v_P \tag{21b}$$

The dynamic responses of the SWCNT are computed by solving Eq. (18) in the time domain with the aid of the average acceleration method of Newmark (Newmark 1959). In addition, SWCNT is at rest before the arrival of the moving load, namely initial displacements and velocities of the SWCNT are zero. For free vibration analysis, the time-dependent generalized displacement coordinates can be expressed as follows

$$A_m(t) = \overline{A}_m e^{i\omega t} \tag{22a}$$

$$B_m(t) = \overline{B}_m e^{i\omega t} \tag{22b}$$

where $i = \sqrt{-1}$ and ω stands for circular frequency. Substituting Eqs. (22a) and (22b) into Eq. (18) and setting the generalized load vector to zero yield the following simultaneous sets of linear algebraic equations (frequency equation) in the matrix form

$$\left(\begin{bmatrix} [\mathbf{K}_{11}] & [\mathbf{K}_{12}] \\ [\mathbf{K}_{21}] & [\mathbf{K}_{22}] \end{bmatrix} - \omega^2 \begin{bmatrix} [\mathbf{M}_{11}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{M}_{22}] \end{bmatrix}\right)\begin{Bmatrix} \bar{\mathbf{A}} \\ \bar{\mathbf{B}} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix} \tag{23}$$

The natural frequencies, ω_m, and corresponding mode shapes, are obtained by setting the determinant of the characteristic equation from (23) equal to zero.

4. Numerical results

In the numerical results, vibration of the simply supported embedded SWCNT due to the moving load is investigated. The following parameters are used in computing the numerical results: E = 1TPa, ρ = 2300 kg/m^3, ν = 0.2, d = 1 nm, t_b = 0.34 nm. The length of the nanotube is taken as variable for the various values of the aspect ratio L / d. The shear correction factor (k_s) for hollow circle cross-section is taken as follows (Cowper 1966)

$$k_s = \frac{6(1+\nu)(1+k^2)^2}{(7+6\nu)(1+k^2)^2 + (20+12\nu)k^2} \tag{24}$$

where k is the ratio of inner diameter to the outer diameter of the nanotube. The dynamic transverse displacements of the SWCNT are normalized by the static deflection D = $P_0L^3 / 48EI$, of a beam under a point load P_0 at the mid-span. The effect of the velocity of the moving load is represented by the dimensionless velocity parameter α, as follows

$$\alpha = \frac{v_p}{v_{cr}} \tag{25}$$

where v_{cr} is the critical velocity defined as (Fryba 1972)

$$v_{cr} = \frac{\omega_1 L}{\pi} \tag{26}$$

where ω_1 is the fundamental frequency of the SWCNT. The dimensionless time t^* is defined by

$$t^* = \frac{x_P}{L} = \frac{v_P t}{L} \tag{27}$$

Therefore, when $t^* = 0$ the moving load is at the left edge of the beam, i.e., $x_P = 0$, and when $t^* = 1$ the load is at the right edge of the beam, i.e., $x_P = L$.

Fig. 2 shows the convergence studies for the number of time steps in Newmark integration method,

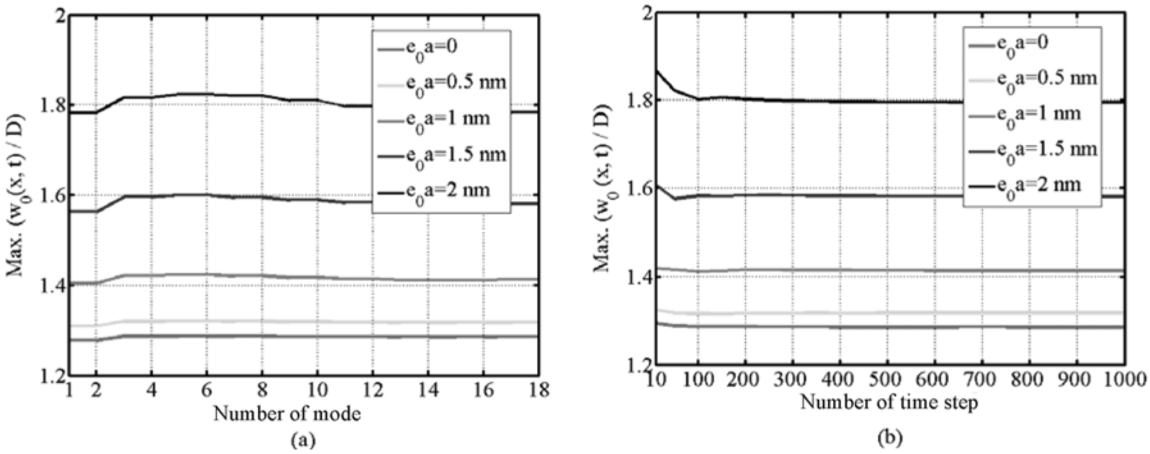

Fig. 2 The effects of the number of mode and of the time step on the non-dimensional dynamic deflections for various values of e_0a and $L / d = 10$, $\alpha = 0.25$, $k_w = 0$.

and the effect of the number of the modes on the maximum non-dimensional dynamic deflections of the SWCNT for various values of the nonlocal parameter. From these analysis conducted, the first mode is certainly the most significant, and the response contribution of higher modes can be neglected. On the other hand, the dynamic deflections are saturated when twelve natural modes are taken. Further, it is seen from Fig. 2(b) that when the number of time step is taken to be more than 100, the numerical accuracy of the responses improved only slightly. It should be noted that procedures that lead to bounded solutions if the time step is shorter than some stability limit are called conditionally stable procedures. Procedures that lead to bounded solutions regardless of the time step are called unconditionally stable procedures (Chopra 2001). The average acceleration method is stable if $\Delta t / T_i < \infty$ (where T_i is the natural period of vibration of system) (Chopra 2001). This implies that the average acceleration method is unconditionally stable. Consequently, in the subsequent numerical calculations, setting the number of the modes to 12 and the number of time steps to 500 is very satisfactory for the desired numerical precision.

To validate the present formulation and the computer program developed by the author, a comparison study for free vibration analysis is made between the present results and the previously published results of Aydogdu (2009) by inserting the material and section properties used in this reference in the present formulation. The three different aspect ratios $L / h = 10$, 20 and 50 are selected, where h is the height of nanobeam, and the small scale effect is reflected by the nonlocal parameter $\mu = (e_0a)^2$ by Aydogdu (2009). The frequencies are normalized by $\lambda = \omega L^2\sqrt{\rho A/EI}$. Table 1 shows that the present results agree well with the results of Aydogdu (2009). It is seen that the frequencies decrease with increasing the nonlocal parameter, and the effect of the nonlocal parameter decreases with the increase of the aspect ratio L / h. The reduction may be explained as follows: The small scale effects make the SWCNT more flexible as the nonlocal model may be viewed as atoms linked by elastic springs (Wang and Hu 2005), while the local continuum model assumes the spring constant to take an infinite value (Tounsi *et al.* 2008). Therefore, the nonlocal beam models should be used to obtain accurate predictions of vibrational characteristics of nanotubes.

Fig. 3 shows the effects of the aspect ratio L / d and beam models on the non-dimensional dynamic deflections. It is observed from this figure that the difference between the deflections of Timoshenko

Table 1 Comparison of the non-dimensional fundamental frequencies $\lambda = \omega L^2 \sqrt{\rho A/EI}$ for the simply supported nanobeam

L / h	$\mu = (e_0 a)^2$	Non-dimensional fundamental frequency	
		Present study	Aydogdu (2009)
10	0	9.7074	9.7443
	1	9.2612	9.2931
	2	8.8713	8.8994
	3	8.5268	8.5517
	4	8.2196	8.2419
20	0	9.8281	9.8381
	1	9.7090	9.7187
	2	9.5942	9.6036
	3	9.4834	9.4924
	4	9.3763	9.3850
50	0	9.8629	9.8645
	1	9.8435	9.8451
	2	9.8242	9.8258
	3	9.8050	9.8066
	4	9.7859	9.7875

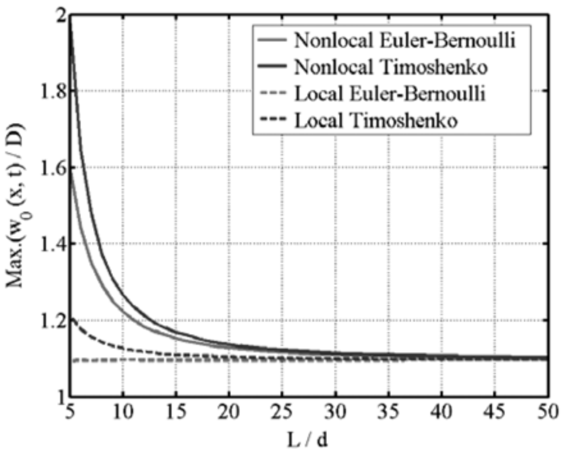

Fig. 3 The effects of aspect ratio of length to diameter L / d and beam models on the non-dimensional dynamic deflections for $\alpha = 0.1$, $k_w = 0$, $e_0 a = 1$nm.

and Euler-Bernoulli beam models is significant when the aspect ratio of the SWCNT is less than $L / d = 10$. The discrepancy between the deflections of the two beam models is due to the effect of the shear deformation. This means that the shear deformation and the rotary inertia effects gain importance as the aspect ratio is decreased. This means that shear deformable beam models should be used in the analysis of short nanotubes (i.e., $L / d \leq 10$). In addition, the dynamic deflections are saturated after the value of $L / d = 25$, and all the beam models considered here give almost the same results when the SWCNT has the aspect ratio which is larger than $L / d = 25$. Also, as seen from Fig. 3, the difference between the dynamic deflections of the two nonlocal beam models is greater than that of the two local beam models.

This situation, which is very explicit for $L / d = 5$, implies that the nonlocality effect plays an important role on the dynamic behavior of the short nanotubes. Note that the results of the local Euler-Bernoulli beam model are independent of the aspect ratio.

The effect of the nonlocal parameter on the dynamic deflections of the SWCNT is shown in Fig. 4 for various values of the aspect ratio. This figure shows that the effect of the nonlocal parameter is dependent on the aspect ratio, as mentioned in many studies in the literature. The effect of the nonlocal parameter is almost is insignificant for nanotubes with large aspect ratios (i.e., $L / d = 50$ or 100 as seen from Fig. 4). With an increase in the nonlocal parameter, the non-dimensional dynamic deflections are also increased. This is because increasing the nonlocal parameter decreases the stiffness of the SWCNT.

Fig. 5 studies the effect of the moving load velocity on the dynamic behavior of the SWCNT by

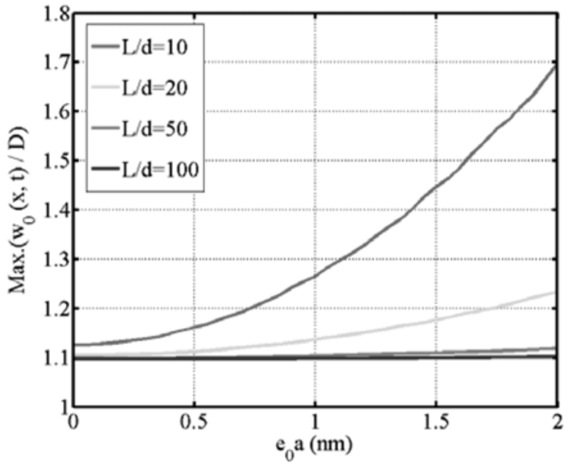

Fig. 4 The effect of the nonlocal parameter e_0a on the non-dimensional dynamic deflections for various values of L / d and $\alpha = 0.1$, $k_w = 0$.

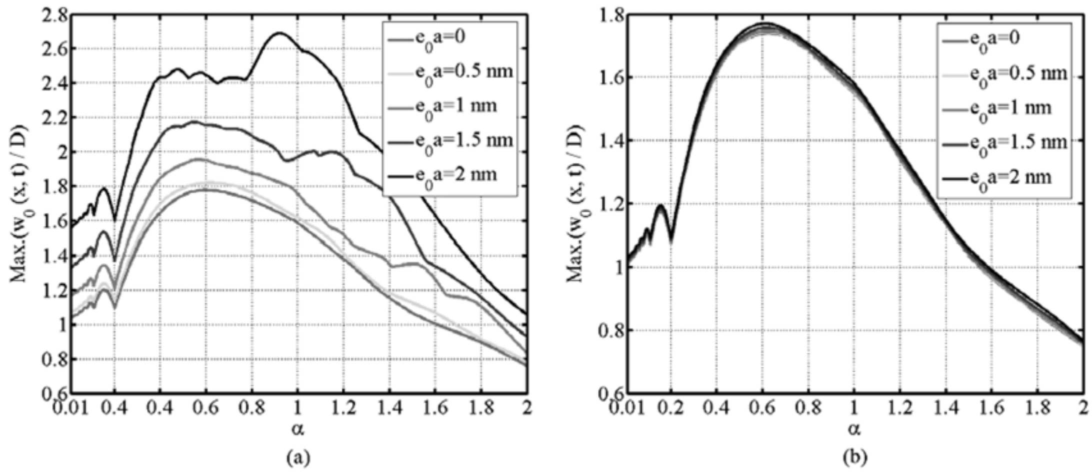

Fig. 5 The effect of the velocity parameter α on the non-dimensional dynamic deflections for various values of e_0a and $k_w = 0$, a) $L / d = 10$, b) $L / d = 50$.

Table 2 Peak values of the maximum non-dimensional dynamic deflections and the corresponding velocity parameters for Fig. 5

L / h	e_0a (nm)	Max.$(w_0(x,t) / D)$	α
10	0	1.78011	0.60
	0.5	1.82106	0.61
	1	1.95674	0.57
	1.5	2.17472	0.55
	2	2.69108	0.92
50	0	1.74067	0.63
	0.5	1.74268	0.62
	1	1.74833	0.62
	1.5	1.75837	0.62
	2	1.77152	0.61

presenting the maximum normalized dynamic deflection as a function of the velocity parameter, which ranges from $0.01 \leq \alpha \leq 2$ with an increment $\Delta\alpha = 0.01$, for various values of e_0a and $L / d = 10, 50$. Table 2 presents the values of the maximum non-dimensional dynamic deflections and the corresponding velocity parameter values. It is observed from these figures that the normalized dynamic deflection is greatly influenced by the moving load velocity. As seen from Fig. 5 and Table 1, the peak values of the displacements are obtained at different velocity parameters depending on the nonlocal parameter, and in general, increase in the nonlocal parameter leads to a decrease in the velocity parameter which corresponds to peak values. It is interesting to note that for $L / d = 10$ in Fig. 5(a), behavior of the deflection curves are almost the same until $\alpha = 0.4$ for all the values of e_0a considered in the study, but after this value of the velocity parameter, increasing in the nonlocal parameter causes a fluctuation in the displacement curves. As an expected result, the maximum dynamic deflections and the corresponding velocity parameters are insensitive to the change in the nonlocal parameter for the SWCNT with $L / d = 50$. Also, it should be mentioned that the dynamic deflections tend to zero for very large values of the moving load velocity.

Figs. 6,7 and Table 3 show the effect of the elastic medium constant on the dynamic deflections of the

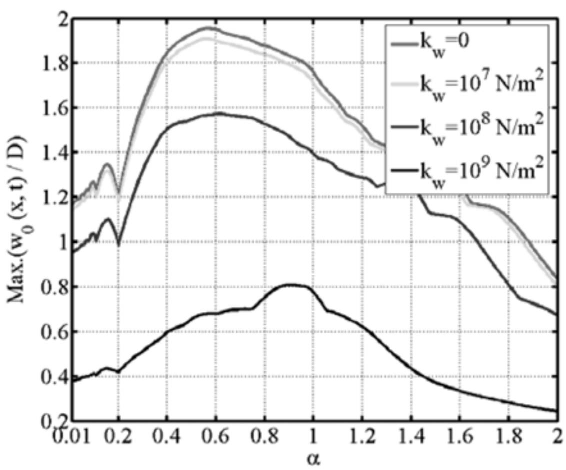

Fig. 6 The effect of the velocity parameter α on the non-dimensional dynamic deflections for various values of k_w and $L / d = 10$, $e_0a = 1$ nm.

Table 3 Peak values of the maximum non-dimensional dynamic deflections and the corresponding velocity parameters for Fig. 6.

k_w (N/m^2)	Max.($w_0(x, t) / D$)	α
0	1.95674	0.57
10^7	1.90864	0.56
10^8	1.57232	0.62
10^9	0.80990	0.91

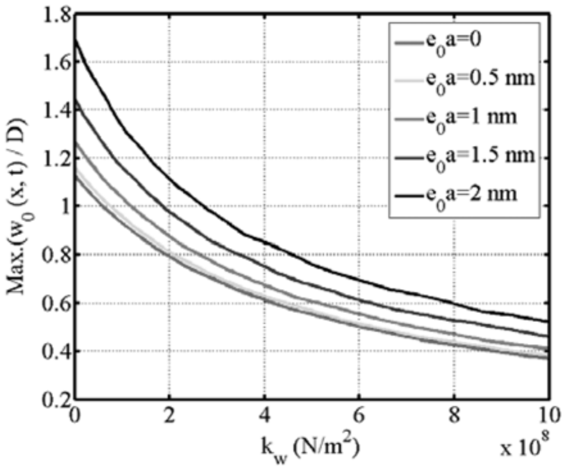

Fig. 7 The effect of the elastic medium constant k_w on the non-dimensional dynamic deflections for various values of e_0a and $\alpha = 0.1$, $L / d = 1.0$.

SWCNT with $L / d = 10$. It is clearly shown that the normalized dynamic deflections are decreased as the elastic medium constant k_w is increased. This is because the nanotube becomes stiffer with an increase in the value of the elastic medium constant. The effect of the spring constant on the dynamic deflections can be neglected for relatively small value of the spring constant (i.e., $k_w = 10^7$ N/m^2). Furthermore, as seen from Table 3, as the elastic medium constant is increased, the velocity parameter which corresponds to peak value of the displacement is increased.

Fig. 8 depicts the time history of the midspan deflections of the SWCNT with $L / d = 10$, 20 for various values of the elastic medium constant, and $e_0a = 1$ nm at the constant moving load velocity ($\alpha = 0.1$). It is found that for a fixed value of k_w, the effect of the elastic medium constant on the dynamic deflections is increased as the aspect ratio of the nanotube is increased. Also, from this figure, the SWCNT oscillates freely after time $t^* = 1$ at which the moving load leaves the nanotube.

5. Conclusions

In this study, the effects of the nonlocal parameter, aspect ratio, elastic medium constant and the moving load velocity on the dynamic behavior of the nanotube are investigated based on the nonlocal Timoshenko beam theory. Numerical results show that the nonlocal parameter and the shear

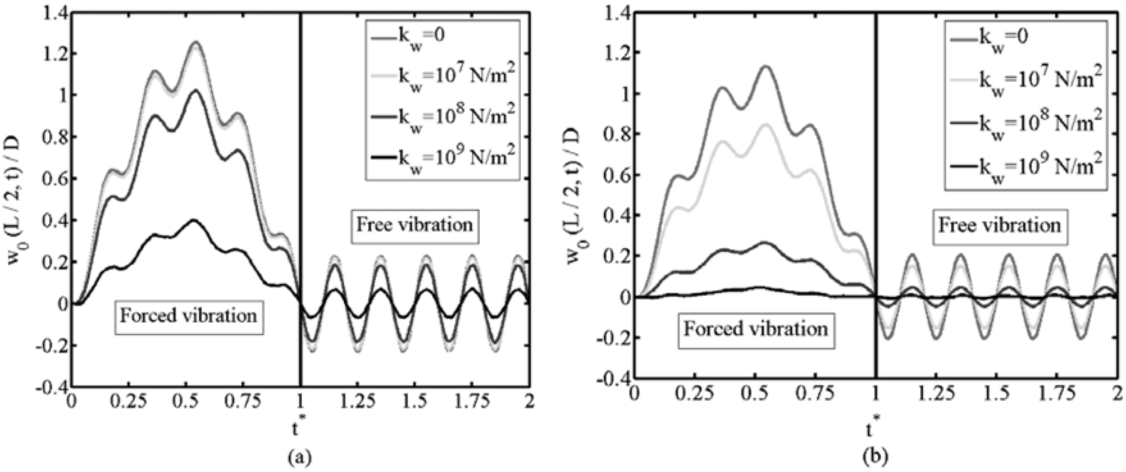

Fig. 8 Time history of the midspan displacements for various values of the elastic medium constant k_w, $\alpha = 0.1$, $e_0a = 1$ nm and a) $L / d = 10$, b) $L / d = 20$.

deformation greatly affect the dynamic behavior of the nanotube with lower aspect ratios (i.e., $L / d \leq 10$), and cannot be neglected in the analysis of short nanotubes. As the nonlocal parameter is increased, dynamic deflections are increased which means that local beam models underestimate the dynamic responses. Increasing in elastic medium constant causes a decrease in the dynamic deflections and this decreasing in the dynamic responses with elastic medium constant is influenced by the aspect ratio of the nanotube. The nonlocal beam models should be used to obtain accurate predictions of vibrational characteristics of nanotubes.

6. References

Abu-Hilal, M. and Mohsen, M. (2000), "Vibration of beams with general boundary conditions due to moving harmonic load", *J. Sound Vib.*, **232**(4), 703-717.

Adali, S. (2008), "Variational principles for multi-walled carbon nanotubes undergoing buckling based on nonlocal elasticity theory", *Phys. Lett. A*, **372**(35), 5701-5705.

Aydogdu, M. and Ece, M.C. (2007), "Vibration and buckling of in-plane loaded double-walled carbon nanotubes", *Turkish J. Eng. Env. Sci.*, **31**, 305-310.

Aydogdu, M. (2008), "Vibration of multi-walled carbon nanotubes by generalized shear deformation theory", *Int. J. Mech. Sci.*, **50**(4), 837-844.

Aydogdu, M. (2009), "Axial vibration of the nanorods with the nonlocal continuum rod model", *Physica E*, **41**(5), 861-864.

Aydogdu, M. (2009), "A general nonlocal beam theory: Its application to nanobeam bending, buckling and vibration", *Physica E*, **41**(9), 1651-1655.

Aydogdu, M. (2009), "A new shear deformation theory for laminated composite plates", *Compos. Struct.*, **89**(1), 94-101.

Chopra, A.K. (2001), *Dynamics of structures*, Prentice Hall, Inc, New Jersey.

Civalek, Ö., and Akgöz, B. (2009), "Static analysis of single walled carbon nanotubes (SWCNT) based on Eringen's elasticity theory", *Int. J. Eng. Appl. Sci.*, **2**, 47-56.

Civalek, Ö., Demir, Ç. and Akgöz, B. (2010), "Free vibration and bending analyses of cantilever microtubules

based on nonlocal continuum model", *Math. Comput. Appl.*, **15**, 289-298.
Civalek, Ö., Akgöz, B. (2010), "Free vibration analysis of microtubules as cytoskeleton components: Nonlocal Euler-Bernoulli beam modeling", *Scientica Iranica, Transaction B- Mech. Eng.*, **17**, 367-375.
Cowper, G.R. (1966), "The shear coefficient in Timoshenko's beam theory", *ASME J. Appl. Mech.*, **33**, 335-340.
Dedkov, G.V. and Kyasov, A.A. (2007), "Fluctuational-electromagnetic interaction of a moving nanoparticle with walls of a flat dielectric gap", *Tech. Phys. Lett.*, **33**(1), 51-53.
Demir, Ç., Civalek, Ö. and Akgöz, B. (2010), "Free vibration analysis of carbon nanotubes Based on shear deformable beam theory by discrete singular convolution technique", *Math. Comput. Appl.*, **15**, 57-65.
Ece, M.C. and Aydogdu, M. (2007), "Nonlocal elasticity effect on vibration of in-plane loaded double-walled carbon nano-tubes", *Acta. Mech.*, **190**(1-4), 185-195.
Eringen, A.C. (1972), "Nonlocal polar elastic continua", *Int. J. Eng. Sci.*, **10**(1), 1-16.
Eringen, A.C. and Edelen, D.G.B. (1972), "On nonlocal elasticity", *Int. J. Eng. Sci.*, **10**(3), 233-248.
Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710.
Fryba, L. (1972), *Vibration of solids and structures under moving loads*, the netherlands: Noordhoff International, Groningen.
Garinei, A. (2006), "Vibrations of simple beam-like modeled bridge under harmonic moving loads", *Int. J. Eng. Sci.*, **44**(11-12), 778-787.
Hu, Y.G., Liew, K.M. and Wang, Q. (2009), "Nonlocal elastic beam models for flexural wave propagation in double-walled carbon nanotubes", *J. Appl. Phys.*, **106**(4), 044301.
Heireche, H., Tounsi, A., Benzair, A., Maachou, M. and Adda, B.E.A. (2008), "Sound wave propagation in single-walled carbon nanotubes using nonlocal elasticity", *Physica E*, **40**(8), 2791-2799.
Hummer, G., Rasaiah. J.C. and Noworyta, J.P. (2001), "Water conduction through the hydrophobic channel of a carbon nanotube", *Nature*, **414**, 188-190.
Iijima, S. (1991), "Helical microtubes of graphitic carbon", Nature, **354**, 56-58.
Ke, L.L., Xiang, Y., Yang, J. and Kitipornchai, S. (2009), "Nonlinear free vibration of embedded double-walled carbon nanotubes based on nonlocal Timoshenko beam theory", *Comput. Mater. Sci.*, **47**(2), 409-417.
Kiani K. and Mehri, B., (2010), "Assessment of nanotube structures under a moving nanoparticle using nonlocal beam theories", *J. Sound Vib*. **329**(11), 2241–2264.
Klasztorny, M. (2001), "Vertical vibrations of a multi-span beam steel bridge induced by a superfast passenger train" *Struct. Eng. Mech.*, **12**(3), 267-281.
Kocatürk, T. and Şimşek, M. (2006a), "Vibration of viscoelastic beams subjected to an eccentric compressive force and a concentrated moving harmonic force", *J. Sound Vib.*, **291**(1-2), 302-322.
Kocatürk, T. and Şimşek, M. (2006b), "Dynamic analysis of eccentrically prestressed viscoelastic Timoshenko beams under a moving harmonic load", *Comput. Struct.*, **84**(31-32), 2113-2127.
Kumar, D., Heinrich, C. and Waas, A.M. (2008), "Buckling analysis of carbon nanotubes modeled using nonlocal continuum theories", *J. Appl. Phys.*, **103**(7), 073521.
Lee, H.P. (1994), "Dynamic response of a beam with intermediate point constraints subject to a moving load", *J. Sound Vib.*, **171**(3), 361-368.
Lee, H.W. and Chang, W.J. (2009), "Vibration analysis of a viscous-fluid-conveying single-walled carbon nanotube embedded in an elastic medium", *Physica E*, **41**(4), 529-532.
Li, X.F. and Wang, B.L. (2009), "Vibrational modes of Timoshenko beams at small scales", *Appl. Phys. Lett.*, **94**(10), 101903.
Lim, C.W. and Wang, C.M. (2007), "Exact variational nonlocal stress modeling with asymptotic higher-order strain gradients for nanobeams", *J. Appl. Phys.*, **101**(5), 054312.
Lim, C.W., Li, C. and Yu, J.L. (2009), "The effects of stiffness strengthening nonlocal stress and axial tension on free vibration of cantilever nanobeams", *Interaction and Multiscale Mechanics*, **2**, 223-233.
Lu, P., Lee, H.P., Lu, C. and Zhang, P.Q. (2006), "Dynamics properties of flexural beams using a nonlocal elasticity model", *J. Appl. Phys.*, **99**(7), 073510.
Lu, P., Lee, H.P., Lu, C. and Zhang, P.Q. (2007), "Application of nonlocal beam models for carbon nanotubes", *Int. J. Solids. Struct.*, **44**(16), 5289-5300.
Lu, P. (2007), "Dynamic analysis of axially prestressed micro/nanobeam structures based on nonlocal beam

theory", *J. Appl. Phys*., **101**(7), 073504.
Mir, M., Hosseini, A., Majzoobi, G.H. (2008), "A numerical study of vibrational properties of single-walled carbon nanotubes", *Comput. Mater. Sci*., **43**(3), 540-548.
Murmu, T. and Pradhan, S.C. (2009), "Buckling analysis of a single-walled carbon nanotube embedded in an elastic medium based on nonlocal elasticity and Timoshenko beam theory and using DQM", *Physica E*, **41**(7), 1232-1239.
Murmu, T. and Pradhan, S.C. (2009), "Small-scale effect on the vibration of nonuniform nanocantilever based on nonlocal elasticity theory", *Physica E*, **41**(8), 1451-1456.
Murmu, T. and Pradhan, S.C. (2009), "Thermo-mechanical vibration of a single-walled carbon nanotube embedded in an elastic medium based on nonlocal elasticity theory", *Comput. Mater. Sci*., **46**(4), 854-859.
Murmu, T. and Pradhan, S.C. (2010), "Thermal effects on the stability of embedded carbon nanotubes", *Comput. Mater. Sci*., **47**(3), 721-726.
Narendar, S. and Gopalakrishnan, S. (2009), "Nonlocal scale effects on wave propagation in multi-walled carbon nanotubes", *Comput. Mater. Sci*., **47**(2), 526-538.
Newmark, N.M. (1959), "A method of computation for structural dynamics", *ASCE Eng. Mech. Div*., **85**, 67-94.
Peddieson, J., Buchanan, G.R. and McNitt, R.P. (2003), "Application of nonlocal continuum models to nanotechnology", *Int. J. Eng. Sci*., **41**(3-5), 305-312.
Pradhan, S.C., Sarkar, A. (2009), "Analyses of tapered fgm beams with nonlocal theory", *Struct. Eng. Mech*., **32**(6), 811-833.
Pradhan, S.C. and Phadikar, J.K. (2009), "Bending, buckling and vibration analyses of nonhomogeneous nanotubes using GDQ and nonlocal elasticity theory", *Struct. Eng. Mech.,* **33**, 193-213.
Reddy, J.N. (2007), "Nonlocal theories for bending, buckling and vibration of beams", *Int. J. Eng. Sci*., **45**(2-8), 288-307.
Reddy, J.N. and Pang, S.D. (2008), "Nonlocal continuum theories of beams for the analysis of carbon nanotubes", *J. Appl. Phys*., **103**(2), 023511.
Sato, M. and Shima, H. (2009), "Buckling characteristics of multiwalled carbon nanotubes under external pressure", *Interaction and Multiscale Mechanics*, **2**, 209-222.
Sears, A. and Batra, R.C. (2004), "Macroscopic properties of carbon nanotubes from molecular-mechanics simulations", *Phys. Rev. B*, **69**(23), 235406.
Sniady, P. (2008), "Dynamic response of a Timoshenko beam to a moving force", *ASME J. Appl. Mech.*, **75**(2), 0245031-0245034.
Sudak, L.J. (2003), "Column buckling of multiwalled carbon nanotubes using nonlocal continuum mechanics", *J. Appl. Phys*., **94**(11), 7281.
Şimşek, M. and Kocatürk, T. (2007), "Dynamic analysis of an eccentrically prestressed damped beam under a moving harmonic force using higher order shear deformation theory", *ASCE J. Struct. Eng*., **133**(12), 1733-1741.
Şimşek, M. and Kocatürk, T. (2009a), "Nonlinear dynamic analysis of an eccentrically prestressed damped beam under a concentrated moving harmonic load", *J. Sound Vib*., **320**(1-2), 235-253.
Şimşek, M. and Kocatürk, T. (2009b), "Free and forced vibration of a functionally graded beam subjected to a concentrated moving harmonic load", *Compos. Struct*., **90**(4), 465-473.
Şimşek. (2010), "Vibration analysis of a functionally graded beam under a moving mass by using different beam theories", *Compos. Struct*., **92**(4), 904-917.
Şimşek, M. (2010), "Non-linear vibration analysis of a functionally graded Timoshenko beam under action of a moving harmonic load", *Compos. Struct*., **92**(10), 2532-2546.
Şimşek, M. (2010), "Vibration analysis of a single-walled carbon nanotube under action of a moving harmonic load based on nonlocal elasticity theory", *Physica E*, **43**(1), 182-191.
Timoshenko, S. and Young, D.H. (1955), *Vibration Problems in Engineering*, Van Nostrand Company, New York.
Tounsi, A., Heireche, H., Berrabah, H.M., Benzair, A. and Boumia, L. (2008), "Effect of small size on wave propagation in double-walled carbon nanotubes under temperature field", *J. Appl. Phys*., **104**(10), 104301.
Wang, R.T. (1997), "Vibration of multi-span Timoshenko beams to a moving force", *J. Sound Vib*., **207**(5), 731-742.

Wang, R.T. and Lin, J.S. (1998), "Vibration of T-type Timoshenko frames subjected to moving loads", *Struct. Eng. Mech.,* **6**, 229-243.
Wang, R.T. and Sang, Y.L. (1999), "Out-of-plane vibration of multi-span curved beam due to moving loads", *Struct. Eng. Mech.,* **7**(4), 361-375.
Wang, C.M., Tan, V.B.C. and Zhang, Y.Y. (2006), "Timoshenko beam model for vibration analysis of multi-walled carbon nanotubes", *J. Sound. Vib*., **294**(4-5), 1060-1072.
Wang, Q. (2005), "Wave propagation in carbon nanotubes via nonlocal continuum mechanics", *J. Appl. Phys*., **98**(12), 124301.
Wang, L. and Hu, H. (2005), "Flexural wave propagation in single-walled carbon nanotubes", *Phys. Rev. B*, **71**(19), 195412.
Wang, Q., Varadan, V.K. and Quek, S.T. (2006), "Small scale effect on elastic buckling of carbon nanotubes with nonlocal continuum models", *Phys. Lett. A*, **357**(2), 130-135.
Wang, Q. and Liew, K.M. (2007), "Application of nonlocal continuum mechanics to static analysis of micro- and nano-structures", *Phys. Lett. A*, **363**(3), 236-242.
Wang, L. (2009), "Vibration and instability analysis of tubular nano- and micro-beams conveying fluid using nonlocal elastic theory", *Physica E*, **41**(10), 1835-1840.
Wang, L. (2009), "Dynamical behaviors of double-walled carbon nanotubes conveying fluid accounting for the role of small length scale", *Comput. Mater. Sci*., **45**(2), 584-588.
XiaoDong, Y. and Lim, C.W. (2009), "Nonlinear vibrations of nano-beams accounting for nonlocal effect using a multiple scale method", *Science in China Series E: Technological Sciences*, **52**(3), 617-621.
Yoon, J., Ru, C.Q. and Mioduchowski, A. (2003), "Vibration of an embedded multiwall carbon nanotube", *Compos. Sci. Techn*., **63**(11), 1533-1542.
Yoon, J., Ru, C.Q. and Mioduchowski, A. (2004), "Timoshenko-beam effects on transverse wave propagation in carbon nanotubes", *Compos. Part B: Eng*., **35**(2), 87-93.
Yu, L., Chan, T.H.T. and Zhu, J.H. (2008), "A MOM-based algorithm for moving force identification: Part II. Experiment and comparative studies", *Struct. Eng. Mech.,* **29**(2), 155-169.
Zhang, Y., Liu, G. and Han, X. (2005), "Transverse vibrations of double-walled carbon nanotubes under compressive axial load", *Phys. Lett. A*, **340**(1-4), 258-266.
Zhang, Y.Q., Liu, G.R and Wang, J.S. (2004), "Small-scale effects on buckling of multiwalled carbon nanotubes under axial compression", *Phys. Rev. B*, **70**(20), 205430.
Zhang, Y.Q., Liu, G.R. and Han, X. (2006), "Effect of small length scale on elastic buckling of multi-walled carbon nanotubes under radial pressure", *Phys. Lett. A*, **349**(5), 370-376.
Zhang, Y.Q., Liu, G.R. and Xie, X.Y. (2005), "Free transverse vibrations of double-walled carbon nanotubes using a theory of nonlocal elasticity", *Phys. Rev. B*, **71**(19), 195404.
Zheng, D.Y., Cheung, Y.K., Au, F.T.K. and Cheng, Y.S. (1998), "Vibration of multi-span non-uniform beams under moving loads by using modified beam vibration functions" *J. Sound Vib*., **212**(3), 455-467.
Zhu, X.Q. and Law, S.S. (1999), "Moving force identification on multi-span continuous bridge", *J. Sound Vib*., **228**(2), 377-396.

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