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Statistical methods of investigation on the compressive strength of high–performance steel fiber reinforced concrete

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Abstract. The contribution of steel fibers on the 28-day compressive strength of high-performance steel fiber reinforced concrete was investigated, is presented. An extensive experimentation was carried out over water-cementitious materials (w/cm) ratios ranging from 0.25 to 0.40, with silica fume- cementitious materials ratios from 0.05 to 0.15, and fiber volume fractions (V_f = 0.0, 0.5, 1.0 and 1.5%) with the aspect ratios of 80 and 53. Based on the test results of 44 concrete mixes, mathematical model was developed using statistical methods to quantify the effect of fiber content on compressive strength of HPSFRC in terms of fiber reinforcing index. The expression, being developed with strength ratios and not with absolute values of strengths, is independent of specimen parameters and is applicable to wide range of w/cm ratios, and used in the mix design of steel fiber reinforced concrete. The estimated strengths are within ±3.2% of the actual values. The model was tested for the strength results of 14 mixes having fiber aspect ratio of 53. On examining the validity of the proposed model, there exists a good correlation between the predicted values and the experimental values of different researchers. Equation is also proposed for the size effect of the concrete specimens.

Keywords: silica fume; crimped steel fiber; fiber reinforcing index; high-performance fiber reinforced concrete; compressive strength; modeling; prediction.

1. Introduction

Steel fiber reinforced concrete (SFRC) is increasingly used as a structural material due to the enhanced mechanical properties. Balaguru and Shah (1992) and ACI Committee 544-1R-96 (1996) have reported that the addition of steel fibers in concrete matrix improves all mechanical properties, ductility and energy absorption capacity. ACI Committee 544 (1996) states that FRC is usually specified by strength and fiber content. An understanding of the compressive strength properties of SFRC and its variation with fiber amount is an important aspect of successful design. In this article, an investigation is carried out to relate the strength ratios of concrete mixtures with steel fiber reinforcing index to quantify the effect of fiber content on compressive strength of high-performance steel fiber reinforced concrete (HPSFRC) in terms of fiber reinforcing parameter. It is well documented that the use of silica fume in partial replacement of cement in combination with

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HRWRA, results in significant improvement in mechanical properties and durability of concrete (Hooton 1993, Iravani 1993, Duval and Kadri 1998, Banja and Sengupta 2002, Ramadoss 2008), and reduction of pore structures in the transition zone (Attcin 1998). Banja and Sengupta (2002) have observed that silica fume incorporation results in the improvement of compressive strength in the concrete. They developed a statistical model for the compressive strength of SF concrete with w/cm ratios ranging from 0.3 to 0.42 and SF replacement ranging from 0 to 30%, involving non-dimensional variables, is independent of the specimen parameters.

Wafa and Ashour (1992) have developed models for predicting the influence of fiber contents on strength properties (modulus of rupture, splitting tensile strength and compressive strength) of HSFRC with w/c ratio of 0.25, for fiber volume fraction ranging from 0 to 1.5%. Ezeldin *et al.* (1992) have developed a relationship between compressive strength of HSFRC and steel fiber reinforcing index (RI) with w/cm ratio of 0.35, for fiber content ranging from 0 to 59 kg/m^3. Nataraja *et al.* (1999) have proposed an equation to quantify the effect of fiber on compressive strength of concrete in terms of fiber reinforcing parameter. In their model the compressive strength ranging from 30 to 50 MPa, with fiber volume fraction of 0 to 1.0% and aspect ratio of 55 and 82 were used. Song and Hwang (2004) have developed an empirical expression for compressive strength of SFRC as a function of volume fraction of fiber. In all the models only a particular w/cm ratio with varying fiber content and particular type of fiber was used. The absolute strength values have been dealt with in all the models and thus those models are valid for a particular w/cm ratio and particular specimen size and shape. Zang *et al.* (2009) have studied the reinforced high strength slender columns under cyclic loading. Mansur *et al.* (1999) have studied the effect of fiber volume fraction (V_f= 0 to 1.5%) on compressive strength (from 70 to 120 MPa), and ductility of HSFC for w/cm ratios ranging from 0.2 to 0.4 and also developed an empirical expression for predicting 28-day compressive strength of fibrous concrete as a function of strength of plain concrete.

In spite of the wealth of information available in the existing literature on steel fiber reinforced concrete (SFRC), the relationship between compressive strength and fiber reinforcing index that can be used for predicting 28-day compressive strength at any fiber content in terms of fiber reinforcing parameter and at any water-cementitious materials ratio, is quite limited. Compressive strength is the most important property of hardened concrete. For the concrete mix design/proportioning, the prime property considered is the specified compressive strength. HPSFRC is a new construction material, which requires a large number of trial mixes to be tested for the desired compressive strength. Therefore, for economical mix proportioning, taking in to account the other influencing variables, mathematical modeling of strength is an essential one.

The objectives of the present investigation are: (1) to overcome this inherent weakness and to develop the statistical model involving non-dimensional parameters so that the effect of specimen size and shape (size effect), and wide range of w/cm ratios can be eliminated, and (2) to verify whether the proposed model performs well for various types of fibers and aspect ratios. Ramadoss and Nagamani (2006) have proposed the empirical expressions for the tensile strengths of HPFRC in terms of fiber reinforcing index involving non-dimension parameter, using crimped steel fibers of volume fraction, V_f= 0 to 1.5% with w/cm ratios ranging from 0.25 to 0.40.

Forty four series of concrete mixtures were used in this experimental investigation. Based on the test results of the specimens (containing fibers of aspect ratio = 80), mathematical model was developed to predict 28-day compressive strengths of HPFRC for a wide range of w/cm ratios irrespective of the size and shape of specimens, which may serve as a useful tool to quantify the effect of fiber reinforcement in terms of fiber reinforcing index for assessing the compressive strengths of concrete. Analyzing experimental data, using non-linear regression analysis, has derived the relationship between 28-day strength ratios of FRC to plain concrete and fiber reinforcing index. Test results of specimens (containing fibers of aspect ratio = 53) were tested using the mathematical model. The key feature of the model is its simplicity; since other factors relating to w/cm ratios, cement content, and pozzolanic material can be disregarded because both the fiber reinforced and pozzolanic mixtures have the similar material proportions and are assumed to have undergone the same curing history.

2. Experimental program

Four basic mixes for plain concrete, designated as FC1-0.0, FC2-0.0, FC3-0.0 and FC4-0.0 according to the respective water to cementitious materials (w/cm) ratios of 0.4, 0.35, 0.30 and 0.25 were selected. Tests were performed following the ASTM standards, BIS and ACI 544-1988 recommendations.

2.1 Materials and mixture proportions

Ordinary Portland Cement (OPC)- 53 grade, having 28-day compressive strength of 54.5 MPa, satisfying the requirements of IS: 12269-1987, and silica fume (SF) supplied by Elkhem India Ltd.,

Table 1 Physical properties of OPC

Parameter	Value
Specific gravity, S_g	3.16
B.E.T. Fineness (m^2/Kg)	245
Initial /Final setting time (minutes)	43/465
Soundness by Le-Chatelier expansion (mm)	5
28-day compressive strength (MPa)	54.50

Table 2 Physical properties and chemical analysis of silica fume (micro silica grade 920-D)

Physical properties

Parameter	Value
Appearance	Light gray powder
Specific gravity, S_g	2.25
Bulk density, D_{bulk}	670 kg/m^3
B.E.T Fineness	23000 m^2/kg
Greater than 45 micron	2.0 percent

Chemical analysis (analyzed for mandatory parameters of ASTM C1240-1999

Component	Result
Silicon dioxide, SiO_2	88.7%
Moisture content	0.7%
Loss of Ignition @ 975ºC	1.8%
Carbon	0.9%

Table 3 Physical properties of crimped steel fiber

Geometry and properties	Value
Fiber diameter, d (mm)	0.45
Fiber length, l (mm)	36 and 24
Aspect ratio, l/d	80 and 53
Ultimate tensile strength, f_u (MPa)	910
Elastic modulus, E_f (GPa)	200

Mumbai, complying the requirements of ACI 234R-1996 were used in the ratios of 9.5:0.5, 9:1 and 8.5:1.5 by weight of cementitious material in all the mixes. Fine aggregate of locally available river sand with fineness modulus of 2.65, a specific gravity of 2.63 and water absorption of 0.98% at 24 hrs., and coarse aggregate of crushed granite stones with 12.5 mm maximum size with a specific gravity of 2.70 and water absorption of 0.65% at 24 hrs were used. Crimped steel fibers (conforming to ASTM A820-2001) of diameter = 0.45 mm and lengths = 36 and 24 mm, giving the aspect ratios of 80 and 53, respectively, were used. Physical properties of OPC and SF, chemical composition of SF and physical properties of crimped steel fibers are given in Tables 1, 2 and 3, respectively.

Mixtures were proportioned in accordance with the recommended guidelines of the ACI 211.4R-93 (1999-Part1) and ACI Committee 544 (1993). Mixture proportions used in the test program are summarized in Table 4. For each water to cemetitious materials ratio, four plain concrete mixes with SF/Cm ratios of 0.05, 0.1, 0.15, and nine fibrous concrete mixes with fiber volume fractions, V_f = 0.5 1.0 and 1.5% (39, 78 and 117.5 kg/m^3) were prepared. This aspect of work has been published elsewhere (Ramadoss and Nagamani 2006). Due to the inclusion of the fibers to concrete mixes some minor adjustments in terms of different ingredients had to be made as shown in Table 4. Potable water was employed for mixing. Mixing water was adjusted to correct for aggregate absorption. Sulphonated naphthalene formaldehyde as HRWR admixture (superplasticizer) conforming to ASTM-C494 type F and IS: 9103-1999 with dosage range of 1.75 to 2.75% by weight of cementitious materials has been used to obtain the adequate workability of concrete mixes. Slump value obtained was 75±25 mm for plain concrete and VeBe value of 10±2 sec. for fiber concrete mixes. Degree of compaction obtained for different mixes was in between 0.95 and 0.80.

2.2 Mixing and curing

Due to high cement and micro silica contents, the small size coarse aggregate content and steel fiber content, the efficient mixing of the HPSFRC is more difficult than conventional concrete. The main requirement in SFRC mix is to have uniform dispersion of fibers and to prevent the balling of fibers during mixing. For these reasons, super-plasticizer was used to produce uniform concrete without any segregation. Concrete was mixed using a tilting type mixer and specimens were cast using steel moulds, compacted in two uniform layers by table vibrator. For each mix at least three 150 mm cubes, three 100 mm cubes, three 150 × 300 mm cylinders and three 100 × 200 mm cylinders were produced. After casting, specimens were covered with wet burlap to prevent moisture loss. Specimens were demoulded 24 hours after casting and water cured at 27±2°C until the age of testing at 28 days.

Table 4 Mix proportioning of HPSFRC (data for 1 m^3)

Mix designation	W/Cm	C kg	FA kg	CA kg	SF kg	W kg	SP (%)	Steel fiber V_f(%)
FC1-0	0.4	416	691	1088	22	175	1.75	0
FC1-0.5	0.4	416	687	1079	22	175	1.75	0.5
FC1-1	0.4	416	682	1071	22	175	1.75	1
FC1-1.5	0.4	416	678	1062	22	175	1.75	1.5
FC1*-0	0.4	394.2	691	1088	43.8	175	1.75	0
FC1*-0.5	0.4	394.2	687	1079	43.8	175	1.75	0.5
FC1*-1	0.4	394.2	682	1071	43.8	175	1.75	1
FC1*-1.5	0.4	394.2	678	1062	43.8	175	1.75	1.5
FC1**-0	0.4	372.2	691	1088	65.8	175	1.75	0
FC1**-1	0.4	372.2	682	1071	65.8	175	1.75	1
FC1**-1.5	0.4	372.2	678	1062	65.8	175	1.75	1.5
FC2-0	0.35	461.7	664	1088	24.3	170	2	0
FC2-0.5	0.35	461.7	660	1079	24.3	170	2	0.5
FC2-1	0.35	461.7	655	1071	24.3	170	2	1
FC2-1.5	0.35	461.7	651	1062	24.3	170	2	1.5
FC2*-0	0.35	437.4	664	1088	48.6	170	2	0
FC2*-0.5	0.35	437.4	660	1079	48.6	170	2	0.5
FC2*-1	0.35	437.4	655	1071	48.6	170	2	1
FC2*-1.5	0.35	437.4	651	1062	48.6	170	2	1.5
FC2**-0	0.35	413.1	664	1088	72.9	170	2	0
FC2**-1	0.35	413.1	655	1071	72.9	170	2	1
FC2**-1.5	0.35	413.1	651	1062	72.9	170	2	1.5
FC3-0	0.3	522.5	624	1088	27.5	165	2.5	0
FC3-.5	0.3	522.5	620	1079	27.5	165	2.5	0.5
FC3-1	0.3	522.5	615	1071	27.5	165	2.5	1
FC3-1.5	0.3	522.5	611	1062	27.5	165	2.5	1.5
FC3*-0	0.3	495	624	1088	55	165	2.5	0
RC3*-0.5	0.3	495	620	1079	55	165	2.5	0.5
FC3*-1	0.3	495	615	1071	55	165	2.5	1
FC3*-1.5	0.3	495	611	1062	55	165	2.5	1.5
FC3**-0	0.3	467.5	624	1088	82.5	165	2.5	0
FC3**-1	0.3	467.5	615	1071	82.5	165	2.5	1
FC3**-1.5	0.3	467.5	611	1062	82.5	165	2.5	1.5
FC4-0	0.25	608	562	1088	32	160	2.75	0
FC4-0.5	0.25	608	558	1079	32	160	2.75	0.5
FC4-1	0.25	608	553	1071	32	160	2.75	1
FC4-1.5	0.25	608	549	1062	32	160	2.75	1.5
FC4*-0	0.25	576	562	1088	64	160	2.75	0
FC4*-0.5	0.25	576	558	1079	64	160	2.75	0.5
FC4*-1	0.25	576	553	1071	64	160	2.75	1
FC4*-1.5	0.25	576	549	1062	64	160	2.75	1.5
FC4**-0	0.25	544	562	1088	96	160	2.75	0
FC4**-1	0.25	544	553	1071	96	160	2.75	1
FC4**-1.5	0.25	544	549	1062	96	160	2.75	1.5

In mix designation FC1 to FC4, FC1* to FC4*, and FC1** to FC4**, silica fume replacement is 5, 10 and 15 percent respectively by weight of cementitious materials, after hyphen denotes fiber volume fraction in percent.
SP (%) - Super plasticizer in percent by weight of cementitious material
Water present in Super plasticizer is excluded in calculating the water to cementitious material ratio.
V_f(%) denotes Steel fiber volume fraction in percent in volume of concrete.

2.3 Strength testing

Compressive strength tests were performed according to IS: 516-1979, and ASTM C39-1992 standards using 150 mm cubes, 100 mm cubes, 100 × 200 mm cylinders and 150 × 300 mm cylinders loaded uniaxially. The tests were done in a servo-controlled compression testing machine by applying load at the rate of 14 MPa/min. Compressive strengths were measured on four different types of specimens-150 mm cubes, 100 mm cubes, 150 × 300 mm cylinders and 100 × 200 mm cylinders. Each strength value was average of three specimens and in few cases more samples (4 or 5) were considered. For same concrete four different 28-day compressive strength values were determined and database consisting of results on 296 concrete specimens (aspect ratio of fiber = 80), was analyzed for statistical modeling and results of 42 numbers of 150 mm Ø concrete specimens (aspect ratio of fiber = 53), were tested by using the mathematical model developed.

3. Analysis of results and statistical modeling

3.1 Analysis of data

The average coefficient of variation for all strength results was found to be approximately 1.5%. This low variation indicates reliability of the results and adherence to standard concreting and testing procedure. The coefficient of correlation, R for non-linear model (Fig. 1) was found to be close to unity and the reliability coefficient found to be 0.94, indicating that the quadratic model is a good description for the relationship between the two variables. The coefficient of determination, $R^2 = 0.878$, which indicates that 87.8% of the variation in strength is explained by the reinforcing parameter, taking in to account the sample size and number of independent variable. Another statistical tool to evaluate the suitability of the non-linear model is by observing residuals plotted as a function of the reinforcing index. If the relationship between X (independent variable) and Y

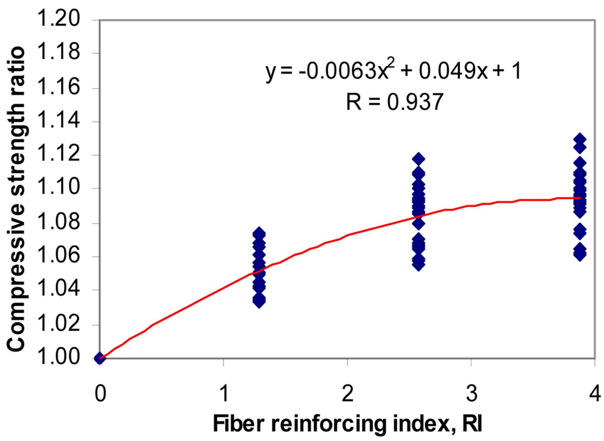

Fig. 1 Relationship between compressive strength ratio (f_{cf}/f_c) and fiber reinforcing index (RI)

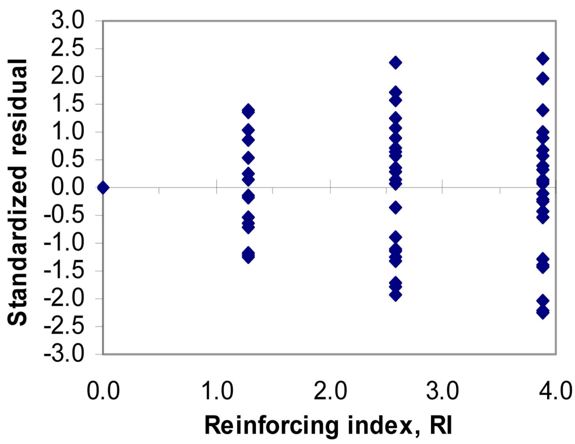

Fig. 2 Standardized residuals for compressive strength ratios

Table 5 28-day compressive strength of fiber reinforced concrete and plain concrete, and their corresponding ratios and strength variations in percent

Mix designation	w/cm	RI l/d = 80	150 mm Cube f_c, MPa	150 mm Cube f_{cf}/f_c	150×300 mm. Cylinder f_c, MPa	150×300 mm. Cylinder f_{cf}/f_c	Predict. by model Cube % error	Predict. by model Cylind. % error
FC1-0	0.4	0	55.62	1.000	46.85	1.000	0.00	0.00
FC1-0.5	0.4	1.29	59.28	1.066	48.94	1.045	1.23	-0.78
FC1-1	0.4	2.58	62.2	1.118	52.00	1.110	3.02	2.29
FC1-1.5	0.4	3.88	62.85	1.130	52.68	1.124	3.07	2.59
FC1*-0	0.4	0	61.03	1.000	52.56	1.000	0.00	0.00
FC1*-0.5	0.4	1.29	64.75	1.061	54.77	1.042	0.78	-1.02
FC1*-1	0.4	2.58	66.85	1.095	56.01	1.066	0.99	-1.77
FC1*-1.5	0.4	3.88	67.38	1.104	57.40	1.092	0.79	-0.29
FC1**-0	0.4	0	65.73	1.000	55.70	1.000	0.00	0.00
FC1**-1	0.4	2.58	71.58	1.089	60.21	1.094	0.41	0.88
FC1**-1.5	0.4	3.88	72.15	1.098	61.17	1.110	0.22	1.36
FC2-0	0.35	0	62.32	1.000	52.69	1.000	0.00	0.00
FC2-0.5	0.35	1.29	65.43	1.050	55.64	1.056	-0.27	0.33
FC2-1	0.35	2.58	67.72	1.087	57.85	1.098	0.20	1.19
FC2-1.5	0.35	3.88	68.36	1.097	58.23	1.105	0.15	0.91
FC2*-0	0.35	0	66.87	1.000	55.85	1.000	0.00	0.00
FC2*-0.5	0.35	1.29	69.23	1.035	59.65	1.068	-1.68	1.43
FC2*-1	0.35	2.58	71.4	1.068	61.05	1.093	-1.57	0.78
FC2*-1.5	0.35	3.88	71.96	1.076	61.44	1.100	-1.78	0.42
FC2**-0	0.35	0	71.15	1.000	59.42	1.000	0.00	0.00
FC2**-1	0.35	2.58	76.2	1.071	63.41	1.067	-1.26	-1.61
FC2**-1.5	0.35	3.88	77.48	1.089	64.59	1.087	-0.58	-0.75
FC3-0	0.3	0	65.8	1.000	60.10	1.000	0.00	0.00
FC3-0.5	0.3	1.29	70.6	1.073	62.81	1.045	1.88	-0.74
FC3-1	0.3	2.58	72.41	1.100	64.01	1.065	1.45	-1.84
FC3-1.5	0.3	3.88	72.94	1.109	64.56	1.074	1.19	-1.94
FC3*-0	0.3	0	72.75	1.000	63.86	1.000	0.00	0.00
FC3*-0.5	0.3	1.29	75.87	1.043	67.12	1.051	-0.94	-0.18
FC3*-1	0.3	2.58	76.96	1.058	68.91	1.079	-2.52	-0.47
FC3*-1.5	0.3	3.88	77.29	1.062	69.67	1.091	-3.09	-0.36
FC3**-0	0.3	0	77.8	1.000	62.28	1.000	0.00	0.00
FC3**-1	0.3	2.58	82.38	1.059	67.6	1.085	-2.42	0.09
FC3**-1.5	0.3	3.88	82.86	1.065	68.12	1.094	-2.84	-0.14
FC4-0	0.25	0	75.21	1.000	71.64	1.000	0.00	0.00

In mix designation FC1 to FC4 and FC1* to FC4* and FC1** to FC4**, silica fume replacement is 5, 10 and 15 percent respectively, after hyphen denotes fiber volume fraction, percent

f_{cf} represents compressive strength of SFRC, f_c refers to the strength of plain concrete

Fiber reinforcing index (RI) = w_f *(l/d) and average unit weight of HPFRC = 2415 kg/m^3

Weight fraction (w_f) = (density of fiber/density of fibrous concrete)*V_f

Aspect ratio (l/d) = length of fiber/diameter of fiber.

Table 5 Continued

Mix designation	w/cm	RI l/d = 80	150 mm Cube f_c, MPa	150 mm Cube f_{cf}/f_c	150×300 mm. Cylinder f_c, MPa	150×300 mm. Cylinder f_{cf}/f_c	Predict. by model: Cube % error	Predict. by model: Cylind. % error
FC4-0.5	0.25	1.29	80.74	1.074	74.15	1.035	1.94	-1.76
FC4-1	0.25	2.58	82.97	1.103	75.65	1.056	1.69	-2.74
FC4-1.5	0.25	3.88	83.03	1.104	76.09	1.062	0.79	-3.16
FC4*-0	0.25	0	78.54	1.000	74.87	1.000	0.00	0.00
FC4*-0.5	0.25	1.29	82.83	1.055	77.42	1.034	0.18	-1.82
FC4*-1	0.25	2.58	85.91	1.094	79.96	1.068	0.86	-1.57
FC4*-1.5	0.25	3.88	86.47	1.101	80.41	1.074	0.52	-2.00
FC4**-0	0.25	0	83.26	1.000	77.95	1.000	0.00	0.00
FC4**-1	0.25	2.58	90.74	1.090	85.98	1.108	0.49	2.14
FC4**-1.5	0.25	3.88	91.29	1.096	86.99	1.116	0.11	1.85

(dependent variable) is non- linear and if the various assumptions made in regression analysis are true, then a plot of residuals against the values of X will show no apparent trend or pattern with changes in X. Fig. 2 shows the plot of standardized residuals against RI, for all the mixtures. Standardized residual is a raw residual divided by the standard error of estimate, which is a measure of the actual variability about the regression plane of the underlying population. If the residuals are normally distributed about the regression line, 95% of the standardized residuals should lie between -2 and +2 and 99% between -2.5 and +2.5. Fig. 2 shows almost 95% of the standardized residuals are between -2 and +2 and 100% between -2.5 and +2.5 indicating that there are no outlier or extreme residual values (Cook 1982). Fig. 2 also shows a reasonably well-scattered plot.

Analysis of variance (ANOVA) was performed using spread sheet developed by the authors to ascertain the quadratic term has any significant influence on the quadratic model compared to linear model. The following null hypothesis for the quadratic model $y_{est} = \beta_0 + \beta_1 x + \beta_{11}x^2$, was tested: i) H0: $\beta_{11} = 0$ (coefficient of quadratic term is zero). ANOVA table and the analysis of the significance of the quadratic term are shown in Table 6(a) and 6(b). At 95% confidence level (the result is significant at 5% level. i.e. probability, $p = 0.05$), quadratic term shows effect and highly improves the model over linear regression alone and therefore the quadratic term contributes significantly to the model.

3.2 Strength estimation model

The variation of the compressive strength, f_{cf}, as obtained for cube and cylindrical concrete specimens on the effect of fiber content in terms of fiber reinforcing index, and the strength ratios between fiber reinforced and plain concrete (silica fume concrete) are presented in Table 5. Since the same concrete was used to prepare four different types of specimens, the results obtained on all the specimens exhibit a common trend as expected. If absolutely identical methods of preparation and testing conditions can be ensured, then the ratio of strength of two concretes measured on different specimens should be identical and independent of the specimen parameters such as shape and size. But identical conditions of specimen preparation and testing are virtually impossible to

Table 6(a) ANOVA-Test for significance of regression for the quadratic model

Source	DF	SS	MS	F value	P value
prob>F					
Model	2	0.14714	0.073570	334.4091	0
Residual	85	0.01880	0.000221		
c Total	n-1= 87	0.16594			
Root MSE		0.014832	R-square	0.88	
Dep. Mean		1.058100	adj.Rsquare	0.87	
C.V.		0.014018	Observation	88	

Quadratic regression model is significant at 95% confidence level

Table 6(b) Test for significance of increase of quadratic over linear model

Source	DF	SS	MS	F value	P value
Model	2	0.148944	0.074472	334.4091	0
Linear	1	0.119475	0.119475	371.4399	0
Quadratic	1	0.029469	0.029469	133.2352	0
Residual	85	0.01880	0.000221		
c Total	n-1=87	0.04827			

Quadratic term in regression is highly significance improvement over linear regression alone.

achieve in practice. Hence, the ratios of strengths for the same concretes are observed to be somewhat different for different specimens. These ratios can be utilized for the development of the generalized expression which, being free from the influence of varying w/cm ratios and specimen parameters, can be used for the prediction of 28-day compressive strength of any type of specimen. The absolute values of compressive strength depend on many parameters. It is very difficult to define a model that can cater to all these variables simultaneously and can make accurate predictions. But if the strength of SFRC is defined with respect to plain concrete and expressed in the form of ratios, then the uncertainties involved in predictions, are considerably reduced. A scatter diagram was prepared to get the information in the search of an appropriate mathematical model. The scatter diagram for both 150 mm cube and 150 mm dia. cylindrical specimens exhibited a non-linear relationship. Accordingly, regression analysis was performed for developing a model and the unknown parameters were determined. Fig. 1 shows compressive strength ratios, (f_{cf}/f_c) as a function of the fiber-reinforcing index, RI of the concrete. The validity of model was investigated by examining relevant statistical coefficients (Bhattacharya 1977, Spiegel 2000).

Based on the results of the present investigation, a second degree polynomial equation for predicting the compressive strength ratios of fiber reinforced to plain concrete as a function of the fiber reinforcing index, RI for w/cm ratios ranging from 0.25 to 0.40, using regression analysis by least-square method, has been obtained as

$$f_{cf}/f_c = 1 + 0.049\,(\mathrm{RI}) - 0.0063\,(\mathrm{RI})^2 \tag{1}$$

where f_c = compressive strength of plain concrete (HPC), MPa

f_{cf} = compressive strength of HPSFRC, MPa and

RI = steel fiber reinforcing index

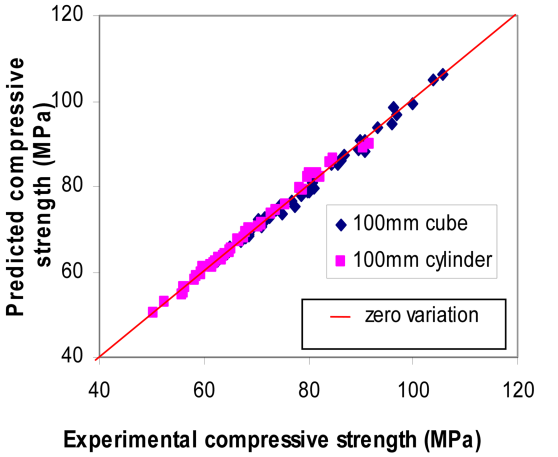

Fig. 3 Correlation of 100 mm cube and 100 mm dia. cylinder strengths (MPa) with the predicted values of compressive strengths (MPa)

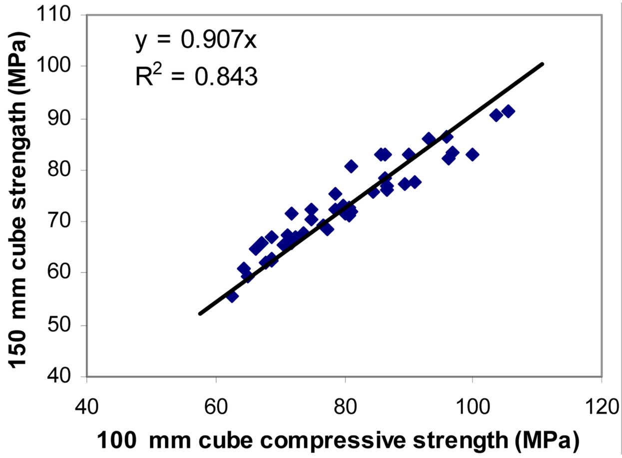

Fig. 4 Relationship between Compressive strengths of 150 mm cube and 100 mm cube of fiber reinforced concretes

The values of the correlation coefficient (r) and the standard error of estimate (s) have been obtained as 0.94 and 0.015 respectively. The maximum percent variation in absolute has been obtained as 3.34. Eq. (1), if expanded for f_{cf} (the compressive strength of HPSFRC), the second term with coefficient (= $0.049 \times f_c \times$ RI) represents the contribution of matrix strength-fiber interaction explicitly, which depends on the pullout characteristics of fibers in matrix.

The computed equation (Eq. (1)) was then used to estimate the compressive strength of 100 mm cubes and 100 mm diameter cylinders over different w/cm ratios and the results of which are presented in Table 7. Overall, it is observed that 100% of the estimated values lie within ±3.3% of the actual values. The test results of 150 mm Ø specimens prepared with fiber aspect ratio = 53 and RI values ranging from 0 to 2.57, are also used to verify the mathematical model developed. Fig. 3 shows the correlation of 100 mm cube and 100 mm diameter cylinder compressive strengths with the strengths estimated by the model (Eq. (1)). The average absolute percent variation obtained was 0.867 and the correlation coefficient, $r = 0.99$.

Fig. 4 shows the correlation between the compressive strength values measured using the 150 mm cubes and that using the 100 mm cubes for various fiber content and w/cm ratios. The following relationship is provided by using the regression analysis.

$$f_{cf}(150) = 0.907 f_{cf}(100) \tag{2}$$

3.3 Stress-strain curves

The stress-strain response of HPC (SF concrete) and SFRC with compressive strength ranging from 52.5-57.4 MPa with w/cm ratio = 0.40 and SF replacement = 10% for varying fiber volume fractions has also been investigated. The stress-strain relationship of concrete essentially consists of two distinct branches- an ascending branch up to peak stress followed by descending branch until the concrete fails. Pull out effect of fibers has significant role in the post-peak stress-strain behavior of concrete in compression and this effect is improved due to the addition of SF in concrete as it

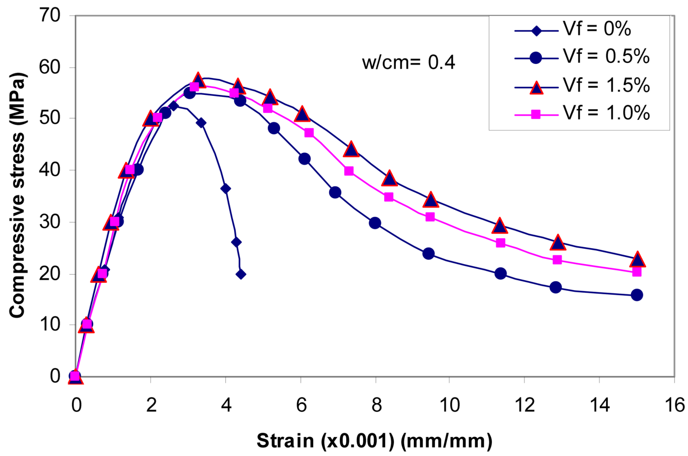

Fig. 5 Typical stress-strain curve for SF concrete and steel fiber reinforced concrete (w/cm = 0.40, SF content = 10%)

has enhanced the strength and filler effect of concrete. Typical stress-strain curve for SF concrete and SFRC is shown in Fig. 5. From the stress-strain curves generated in this study, it can be observed that an increase in concrete strength increases the extent of curved portion in ascending branch, and renders the drop in the descending part steeper for non-fibrous concrete and gradually flatter for SFRC in post-peak region with a residual stress even at a strain of 0.015 mm/mm. The post-peak strain values also show that ductility gains can be achieved through the introduction of fibers into the silica fume concrete mix. Previous researchers noticed that crimped and hook end fibers are effective in improving the mechanical properties, energy absorption at post-peak load capacity and ductility.

Fanella and Naaman (1985), Ezheldin and Balaguru (1992) have also used an ultimate strain of 0.015 for computing the toughness as it is sufficient to represent the trend of post peak behavior of SFRC. This toughness is compared to the toughness of a rigid plastic material in the form of toughness ratio (TR). It is observed in the present study that the increase in RI yields a large area under stress-strain curve making a flatter descending part and a higher toughness ratio as is evidenced by the post-peak stress-strain behavior of SFRC (refer Fig. 5). From the post-peak stress-strain relation, it is seen that descending portion of the curve changes remarkably and toughness of SFRC is comparatively increased as the fibers are more effective in providing post-peak toughness. The toughness ratios of HPSFRC obtained are 0.2038, 0.6276, 0.6574, and 0.6715%, at V_f of 0, 0.5, 1.0 and 1.5%, respectively. The maximum ductility factor achieved at fiber volume fraction of 1.5% is 3.9 and it is postulated that advantage can be obtained using steel fibers.

3.4 Validation of the model

In order to testify whether the proposed model is independent of the specimen parameters, strength results of steel fiber reinforced concrete at different w/cm ratios obtained by different researchers on different size and shape of specimens have been considered. The model performance was assessed by using 88 data of 100 mm cube and 100 mm diameter cylinder compressive strength results obtained from experimental investigation (refer Table 7) and the maximum absolute

Table 7 28-day compressive strength of high-performance steel fiber reinforced concrete for 100 mm cube and 100 mm dia. cylinder and their percentage variation

Mix	w/cm	RI	100 mm Eq. (1) Cube (model)		100 × 200 mm. Eq. (1) Cylinder (model)	
designation		l/d = 80	f_{cf}, MPa	% error	f_{cf}, MPa	% error
FC1-0	0.4	0	62.51	0.00	50.41	0.00
FC1-0.5	0.4	1.29	64.95	-1.32	52.55	-0.99
FC1-1	0.4	2.58	67.89	0.15	55.67	1.80
FC1-1.5	0.4	3.88	68.65	0.27	56.06	1.51
FC1*-0	0.4	0	64.3	0.00	56.13	0.00
FC1*-0.5	0.4	1.29	66.35	-2.02	58.49	-1.02
FC1*-1	0.4	2.58	68.87	-1.25	59.76	-1.86
FC1*-1.5	0.4	3.88	71.06	0.89	61.18	-0.49
FC1**-0	0.4	0	67.13	0.00	58.15	0.00
FC1**-1	0.4	2.58	71.71	-1.53	62.84	-0.35
FC1**-1.5	0.4	3.88	74.95	1.90	63.86	0.27
FC2-0	0.35	0	68.62	0.00	56.37	0.00
FC2-0.5	0.35	1.29	70.51	-2.45	59.38	0.06
FC2-1	0.35	2.58	73.64	-1.06	61.64	0.82
FC2-1.5	0.35	3.88	77.35	2.83	62.03	0.47
FC2*-0	0.35	0	72.56	0.00	59.6	0.00
FC2*-0.5	0.35	1.29	76.74	0.46	63.47	1.15
FC2*-1	0.35	2.58	80.12	1.78	64.9	0.41
FC2*-1.5	0.35	3.88	80.98	1.86	65.3	0.03
FC2**-0	0.35	0	80.68	0.00	62.23	0.00
FC2**-1	0.35	2.58	86.73	-0.88	66.41	-1.62
FC2**-1.5	0.35	3.88	90.92	2.81	67.65	-0.75
FC3-0	0.3	0	71.75	0.00	63.93	0.00
FC3-0.5	0.3	1.29	74.79	-0.99	66.7	-0.90
FC3-1	0.3	2.58	78.65	1.07	67.92	-2.08
FC3-1.5	0.3	3.88	79.98	1.74	68.48	-2.25
FC3*-0	0.3	0	80.7	0.00	67.75	0.00
FC3*-0.5	0.3	1.29	84.34	-0.73	71.1	-0.31
FC3*-1	0.3	2.58	86.78	-0.85	72.92	-0.76
FC3*-1.5	0.3	3.88	89.48	1.22	73.7	-0.69
FC3**-0	0.3	0	90.84	0.00	64.89	0.00
FC3**-1	0.3	2.58	96.37	-2.23	70.43	0.08
FC3**-1.5	0.3	3.88	99.85	0.36	70.97	-0.14
FC4-0	0.25	0	78.59	0.00	75.71	0.00
FC4-0.5	0.25	1.29	81.05	-2.08	78.28	-1.82
FC4-1	0.25	2.58	85.75	0.61	79.8	-2.89
FC4-1.5	0.25	3.88	86.36	0.33	80.27	-3.31
FC4*-0	0.25	0	86.87	0.00	79.01	0.00
FC4*-0.5	0.25	1.29	89.97	-1.06	81.61	-1.92
FC4*-1	0.25	2.58	93.24	-0.46	84.2	-1.76
FC4*-1.5	0.25	3.88	95.83	1.28	84.77	-2.09
FC4**-0	0.25	0	96.86	0.00	82.14	0.00
FC4**-1	0.25	2.58	103.74	-1.19	90.62	1.70
FC4**-1.5	0.25	3.88	105.62	-0.38	91.67	1.86

Table 8 Comparison between the results of previous researchers and the predicted values of the present model

Researcher/type of specimen	Reinforcing index, RI	Actual strength f_c, MPa	Predicted strength f_c, MPa	Error (%)
Wafa and Ashour	0	93.49	93.49	0.00
	0.609	95.1	96.06	-1.10
150 × 300 mm cylinder	1.209	97.32	98.17	-0.87
Hooked end steel fiber	1.819	96.47	99.87	-3.53
(*l/d*) = 60/0.8 = 75	2.419	97.14	101.12	-4.10
	3.637	97.83	102.36	-4.63
Ezeldin *et al.*	0	75.9	75.90	0.00
100 × 200 mm cylinder	1.25	73.83	79.80	-8.09
Hooked end steel fiber	2.50	73.83	82.21	-11.35
	0.75	75.9	78.42	-3.32
	1.88	81.42	81.20	0.27
Trottier and Banthia				
100 × 200 mm cylinder	0	52	52.00	0.00
Hooked end *l/d* = 75	1.242	56	54.66	2.39
Crimped-circular *l/d* = 60	0.994	52	54.21	-4.25
Crimped-flat *l/d* = 41	0.679	53	54.21	-1.09
Twin cone *l/d* = 62	1.027	53	54.27	-2.40
	0	85	85.00	0.00
Hooked end *l/d* = 75	1.242	90	89.37	0.73
Crimped- circular *l/d* = 60	0.994	84	88.61	-5.49
Crimped-flat *l/d* = 41	0.679	83	88.61	-5.52
Twin cone *l/d* = 62	1.027	86	88.71	-3.15
Barros and Figueiras Drawmix steel fiber	0	41.5	41.50	0.00
(hooked end) *l/d* = 75	0.937	43.4	43.17	0.52
	1.367	47.4	43.79	7.61

error obtained was 3.30 percent. A comparison between the experimental results obtained by Wafa and Ashour 1992, Ezeldin and Balaguru 1992, Trottier and Banthia 1994, Barros and Figueiras 1999, Mansure *et al*. 1999 and those predicted by the present model (Eq. 1) is presented in Table 8 and 9 respectively. The proposed model was also tested with the test results of 150 mm Ø specimens with aspect ratio of fiber = 53 (RI = 0–2.57), with average absolute percent variation as 1.19 and standard error of the estimate (s) as 1.05, and the predicted values are presented in Table 10. It was observed that the proposed model performs very well for different types of fibers and aspect ratios.

3.5 Application

The strength of an OPC concrete can generally be estimated with good accuracy based on the strength charts or by experience if mixture proportion, age and curing conditions are known. With

Table 9 Comparison of experimental data of Mansur *et al.* with the compressive strength values (MPa) predicted by the present model

V_f (%)	100 mm cube (Hooked end fiber, $l/d = 60$)			100 × 200 mm cylinder (Hooked end fiber, $l/d = 60$)			100 × 100 × 200 mm prism (Hooked end fiber, $l/d = 60$)		
	Exper. value	Predicted strength	Error (%)	Exper. value	Predicted strength	Error (%)	Exper. value	Predicted strength	Error (%)
0	74.7	74.70	0.00	70.2	70.20	0.00	64.1	64.10	0.00
1	80.5	81.01	-0.63	79.9	76.13	4.72	74.6	69.52	6.82
0	86.4	86.40	0.00	79.7	79.70	0.00	71.4	71.40	0.00
1	88.6	93.70	-5.76	85.9	86.43	-0.62	76.7	77.43	-0.95
0	88.3	88.30	0.00	85.9	85.90	0.00	85.3	85.30	0.00
1	102.4	95.76	6.48	93.4	93.16	0.26	83.9*	92.51	-10.3*
0	108.5	108.50	0.00	103.6	103.60	0.00	94.2	94.20	0.00
0.5	110.8	114.22	-3.09	103.5	109.06	-5.37	101.7	99.17	2.49
1	114.4	117.67	-2.86	106.6	112.35	-5.40	103.5	102.16	1.30
1.5	111	118.84	-7.06	104	113.47	-9.11	96.6	103.18	-6.81
0	111.3	111.30	0.00	119.9	119.90	0.00	113.5	113.5	0.00
1	123	120.70	1.87	115	-	-	112.6*	123.09	-9.32*

*The experimental value shows negative variation for the increased fiber content, may be discarded

Table 10 28-day compressive (150 mm dia. cylinder) strength of high-performance steel fiber reinforced concrete and their percentage variations-aspect ratio = 53

Mix designation	w/cm	RI $l/d = 53$	150 dia. × 300 mm cylinder Experimental value f_c, MPa	Predicted by eq. (1) (model) f_c, MPa	Error (%)
FC1*-0	0.4	0	52.56	52.56	0.00
FC1*-0.5	0.4	0.86	55.21	54.52	1.25
FC1*-1	0.4	1.71	55.75	55.99	-0.44
FC1*-1.5	0.4	2.57	58.46	56.99	2.51
FC1**-0	0.4	0	55.7	55.70	0.00
FC1**-1	0.4	1.71	60.83	59.34	2.45
FC1**-1.5	0.4	2.57	61.85	60.40	2.35
FC3*-0	0.3	0	63.86	63.86	0.00
FC3*-0.5	0.3	0.86	67.45	66.24	1.79
FC3*-1	0.3	1.71	68.99	68.03	1.39
FC3*-1.5	0.3	2.57	70.78	69.25	2.17
FC3**-0	0.3	0	64.28	64.28	0.00
FC3**-1	0.3	1.71	67.95	68.48	-0.78
FC3**-1.5	0.3	2.57	70.83	69.70	1.59

this model, strength of a blended mixture at particular steel fiber content in terms of fiber reinforcing index, RI can be estimated based on a prior knowledge of its equivalent plain concrete strength. The proposed model will be of value in the design process of FRC mixtures that avoids large number of trial mixes, where specific target strength needs to be achieved at 28-days. Since the model drastically reduces the number of trial mixes, economical mixes can be produced and time also be saved. In civil engineering optimization problems, objective function is the cost minimization. In optimization of concrete mixtures, constraints are strength and workability. In such problems, strength prediction model is required. Therefore, the model developed, can also be used in the optimization problems.

4. Conclusions

Extensive experimentation was carried out to quantify the effect of steel fiber content in terms of fiber reinforcing index on compressive strength of concrete at water-cementitious materials ratios ranging from 0.25 to 0.40

1. On the basis of regression analysis of a large number of experimental results, the statistical model has been developed, which can serve as the useful tools for predicting and optimizing the compressive strength of high-performance fiber reinforced concrete over a wide range of w/cm ratios and fiber reinforcing index ranging from 0 to 3.88 (fiber volume fraction 0 to 1.5 percent). The model involves the non- dimensional variables, is independent of specimen parameters and is suitable for wide range of w/cm ratios and all types of specimens.
2. The proposed model was found to have good accuracy in estimating the 28-day compressive strength of fiber reinforced concrete with different fiber aspect ratios, where 100% of the estimated values are within ± 3.3% of the actual values.
3. The ratio of compressive strength of 150 mm to 100 mm cube specimens was about 0.91, regardless of the steel fiber content.
4. The validity of the model has been tested, and verified with results of different researchers and the variation in percent obtained was given in Table 7 and 8.
5. It was observed that the proposed model performs very well for different types of fibers and aspect ratios.

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Notation

The following symbols are used in this paper

f_c = compressive strength of plain concrete, MPa

f_{cf} = compressive strength of HPSFRC, MPa

HPC = high performance concrete

HPSFRC = high-performance steel fiber reinforced concrete

SFRC = steel fiber reinforced concrete

V_f = fiber volume fraction in percent

w_f = weight fraction

RI = fiber reinforcing index.