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Prediction of deflection of high strength steel fiber reinforced concrete beams and columns

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Abstract. This paper presents an analytical procedure for the analysis of high strength steel fiber reinforced concrete members considering the cracking effect in the serviceability loading range. Modifications to a previously proposed formula for the effective moment of inertia are presented. Shear deformation effect is also taken into account in the analysis, and the variation of shear stiffness in the cracked regions of members has been considered by reduced shear stiffness model. The effect of steel fibers on the behavior of reinforced concrete members have been investigated by the developed computer program based on the aforementioned procedure. The inclusion of steel fibers into high strength concrete beams and columns enhances the effective moment of inertia and consequently reduces the deflection reinforced concrete members. The contribution of the shear deformation to the total vertical deflection of the beams is found to be lower for beams with fibers than that of beams with no fibers. Verification of the proposed procedure has been confirmed from series of reinforced concrete beam and column tests available in the literature. The analytical procedure can provide an accurate and efficient prediction of deflections of high strength steel fiber reinforced concrete members due to cracking under service loads. This procedure also forms the basis for the three dimensional analysis of frames with steel fiber reinforced concrete members.

Keywords: reinforced concrete; steel fibers; effective moment of inertia; effective shear modulus; cracking; deflections; high strength concrete.

1. Introduction

The engineering characteristics and economic advantages of high-strength concrete (HSC) are distinct from conventional concrete, thereby popularizing the HSC concrete in a large variety of applications in the construction industry. Used for long span bridges, HSC reduces the dead load of bridge girders for fewer and lighter bridge piers and thus enables greater underpass clearance widths. Used for high-rise buildings, HSC avoids the unacceptable oversized columns on the lower floors, allowing large column spacing and usable floor space, or increasing the number of possible stories without detracting from lower floors (Swamy 1987). The comparatively higher compressive strength of HSC is an attractive profit; whereas, the strength behaves against the ductility of the concrete by welcoming brittleness pronouncedly (Tasdemir *et al.* 1996). The HSC always possesses a steeper descending stress-strain curve in compression than does the normal strength concrete. The rapid decrease in compressive strength in the post-peak load region brings about a pronouncedly brittle mode of failure. To foster the compressive strength without sacrificing the ductility, a strategy

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is to add discrete steel fibers in HSC (Hsu and Hsu 1994, Brandt 2008). Addition of fibers to high-strength concrete converts its brittleness into a more ductile behavior. When concrete cracks, the randomly oriented fibers arrest both microcracking and its propagation, thus improving strength and ductility. A marginal improvement in tensile strength also results and fibre reinforced concrete has higher ultimate strain than plain concrete.

Many studies have been carried out to investigate the flexural behaviour of steel fiber high strength concrete members (Ezeldin and Balaguru 1992, Maalej and Lok 1999, Lima Junior and Giongo 2004, Thomas and Ramaswamy 2007, Ramadoss and Nagamani 2009, Chaekuk and Hyo-Gyoung 2011). A number of experimental and theoretical studies have also been conducted on concentrically or eccentrically loaded steel fiber high strength reinforced concrete columns in the past years. Tokgoz (2009) has investigated the effect of steel fibers on the strength and ductility of reinforced concrete columns. It has been concluded that the addition of steel fibers into high strength concrete significantly improves confinement, ductility and deformability of high strength concrete columns. Ganesan and Ramana Murthy (1990) have tested plain and steel fiber high strength concrete columns to examine the effects of steel fibers on strength and ductility. An analytical model has also been developed to describe the stress–strain behavior of steel fiber reinforced concrete column in the study. They have reported that the use of steel fibers improve strength and ductility of high strength columns.

The effect of steel fibers on the behavior of reinforced concrete beams have also been investigated experimentally and theoretically by several researches. As shown in Refs. (Lim and Nawy 2005, Craig and Decker 1987, Chunxiang and Patnaikuni 1999), the addition of steel fibers can reduce the deflection of high strength concrete beams and improve post cracking behavior significantly. However researches conducted on the flexural behavior of fiber reinforced concrete (FRC) members have been concentrated on the prediction of the ultimate flexural strength and the load deformation behavior in terms of various material parameters. Less attention was given to the flexural rigidity of FRC members in the serviceability loading range. Several methods have been proposed for computing the deflections of reinforced concrete flexural members subjected to short and long-term loadings (Al-Zaid *et al.* 1991, Corley and Sozen 1966, Cosenza 1990), however, those methods deal mainly with nonfiber concrete, and differences may exist for FRC members.

Deflections of reinforced concrete flexural members without steel fibers have been the focus of several research activities in the past decades, and as a consequent, various methods have been developed for predicting them under short-term deflection. For calculating the deflection of reinforced concrete members without steel fibers under working load, an empirical expression for the effective moment of inertia developed by Branson (1963) is perhaps the most widely used, and various form of Branson formula can be found in the ACI (1966) and AS 3600-1994 (Standarts Association of Australia) (1994). The inclusion of steel fibers into high-strength concrete members also enhances the arresting mechanism of crack propagation and consequently enhances the effective moment of inertia to be used in the deflection calculation. For high strength steel fiber reinforced concrete members, only limited research has been carried out to predict the flexural stiffness and the displacement in the serviceability loading range. However, in reinforced concrete construction, a designer must satisfy not only the strength requirements but also the serviceability requirements and therefore the control of the deformation is important to ensure serviceability criterion. For accurate determination of the deflections, the prediction of flexural and shear stiffness of members after cracking becomes important. Therefore, an analytical model which can include the effects of nonlinearity due to the concrete cracking on the flexural and shear stiffness of high strength steel fiber reinforced concrete members and accurately assess the deflections would be very useful.

A computer program was recently developed on the basis of iterative analytical method for the analysis of reinforced concrete frames (Dundar and Kara 2007). In the present study, this computer program has been modified to include the effect of steel fiber in the concrete members. This paper provides the analytical method to analyze high strength steel fiber reinforced concrete members considering the cracking effect in the serviceability loading range. Modifications to proposed formula for the effective moment of inertia are presented. The effect of steel fibers on the behavior of reinforced concrete beams and columns have been investigated by developed procedure. Shear deformation effect is also taken into account in the analysis, and the variation of shear stiffness in the cracked regions of members has been considered by reduced shear stiffness model. In the analysis, stiffness matrix method is applied to obtain the numerical solutions, and the cracked member stiffness equation is evaluated by including the uniformly distributed and point loads on the member. In obtaining the flexibility influence coefficients, a cantilever beam model is used which greatly simplifies the integral equations for the case of point load. Verification of the analytical procedure has been presented by the experimental results of beams and columns available in the literature.

2. Models and method

2.1 The effective flexural and shear stiffness models for the member in cracked state

The concrete members that contribute flexural stiffness have varying degrees of cracking ranging from the uncracked regions to the fully cracked regions. ACI (1966) model which considers the contribution of tensile resistance of concrete to flexural rigidities by moment-curvature relationships have been proposed to define effective flexural behavior of reinforced concrete cracked sections. In the ACI model the effective moment of inertia is given in the following form

$$I_{eff}=\left(\frac{M_{cr}}{M}\right)^m I_1+\left[1-\left(\frac{M_{cr}}{M}\right)^m\right]I_2, \quad \text{for} \quad M \geq M_{cr} \tag{1a}$$

$$I_{eff}= I_1, \qquad \text{for} \qquad M < M_{cr} \tag{1b}$$

where $m = 3$. This equation was first presented by Bronson (1963) with $m = 4$ when I_{eff} is required for the calculation of curvature in an individual section. In Eq. (1), I_1 and I_2 are the moments of inertia of the gross uncracked section which accounts for the reinforcing steel to the stiffness, and the cracked transformed section, respectively, M is the bending moment, M_{cr} is the moment corresponding to flexural cracking considered.

The inclusion of steel fibers into high strength concrete members enhances the arresting mechanism of crack propagation and consequently enhances the effective moment of inertia to be used in the deflection calculation. In the present study, the following equation have been proposed to include the effect of steel fiber in high strength concrete beams and columns by modifying the exponent, m, in ACI code equation

$$m = \frac{4}{1+0.12V_f} \tag{2}$$

in which V_f is the steel fiber content by volume. In the literature (Cosenza 1990, Sakai and Kakuta 1980), the effective moment of inertia given by ACI is perhaps the most widely used and the best among the commonly accepted simplified method for the estimation of instantaneous deflection of reinforced concrete beams without steel fibers. Although ACI model is usually considered for beams, in the present study this model is modified to include effect of steel fiber both in high strength concrete beams and columns considering the axial force in the determination of the cracking moment. The cracking moment, M_{cr} is also calculated by the program using the following equation

$$M_{cr} = \frac{(f_r + \sigma_v)I_1}{y_t} \tag{3}$$

where σ_v is the axial compressive stress, f_r is the flexural tensile strength of steel fiber concrete, and y_t is the distance from centroid of gross section to extreme fiber in tension.

In the present study, shear deformation effect is also taken into account in the analysis. The shear modulus of the cracked concrete $\overline{G}_c$ proposed by Al-Mahaidi (1978) for plain concrete and used later in references (Abdul-Razzak 1996, Al-Hasan 2004) for fibrous concrete is used. Al-Mahaidi (1978) recommended the following hyperbolic expression for the reduced shear stiffness $\overline{G}_c$ to be employed in the constitutive relation of cracked concrete

$$\overline{G}_c = \frac{0.4G_c}{\varepsilon_1 / \varepsilon_{cr}}, \quad \text{for} \quad \varepsilon_1 \geq \varepsilon_{cr} \tag{4}$$

where G_c is the elastic shear modulus of uncracked concrete, ε_1 is the principal tensile strain normal to the crack and ε_{cr} is the cracking tensile strain considering the steel fiber effect.

In this study, since three dimensional analysis is considered in the analysis, I_{eff}, M_{cr}, M, I_1, I_2, ε_1 and ε_{cr} are the values related to the flexure in local y and z directions.

2.2 Formulation of the analytical procedure

In the present study, the stiffness matrix method has been employed to obtain the numerical solutions of the analytical procedure. This analytical method was firstly developed by Dundar and Kara (2007) for the analysis of reinforced concrete frames, and have been modified in the present study to include the effect of steel fiber in the concrete members. This procedure does not increase

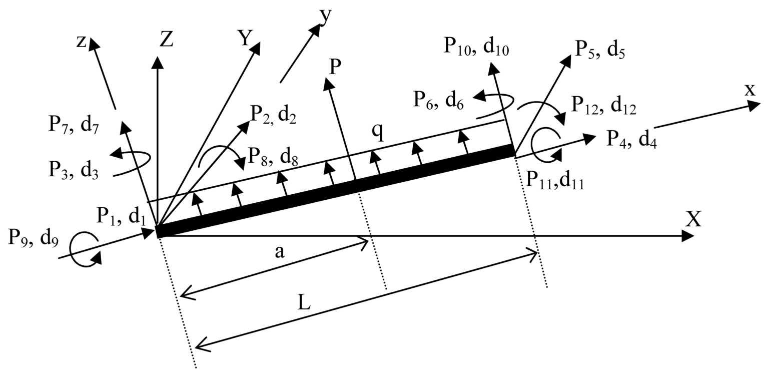


Fig. 1 A typical three dimensional member subjected to a point and a uniformly distributed loads

Fig. 2 A cantilever model for calculating the relations between the nodal actions and basic deformation parameters

the numbers of degrees of freedom with respect to common procedure and, at the same time, is particularly useful in the case of highly statically indeterminate structures. The formulation of the analytical procedure is obtained for the three dimensional analysis of frame with steel fiber reinforced concrete members to include general case.

In this part the flexibility influence coefficients of a member will first be evaluated and then the stiffness matrix and the load vector of a member with some region in the cracked state will be obtained by using compatibility conditions and equilibrium equations. The basic formulations are given in the next paragraph for better understanding the fundamental of the proposed analytical procedure.

Fig. 1 shows a typical member, and positive end forces with corresponding displacements. For calculating the relations between nodal actions and basic deformation parameters of a general space element, a cantilever model is used (Fig. 2). The basic deformation parameters of a general space element may be established by applying unit loads in turn in the directions of 1-3 and 7-9. Then, the compatibility conditions give the following equation in matrix form

$$\begin{bmatrix} f_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & f_{22} & f_{23} & 0 & 0 & 0 \\ 0 & f_{32} & f_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & f_{77} & f_{78} & 0 \\ 0 & 0 & 0 & f_{87} & f_{88} & 0 \\ 0 & 0 & 0 & 0 & 0 & f_{99} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_7 \\ P_8 \\ P_9 \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_7 \\ d_8 \\ d_9 \end{bmatrix} \tag{5}$$

in which, f_{ij} is the displacement in i-th direction due to the application of unit loads in j-th direction, and can be obtained by means of the principal of virtual work as follows

$$f_{ij} = \int_0^L \left(\frac{M_{zi}M_{zj}}{E_c I_{effz}} + \frac{M_{yi}M_{yj}}{E_c I_{effy}} + \frac{V_{yi}V_{yj}}{\overline{G}_c A}s + \frac{V_{zi}V_{zj}}{\overline{G}_c A}s + \frac{M_{bi}M_{bj}}{G_c I_o} + \frac{N_i N_j}{E_c A} \right) dx \tag{6}$$

In Eq. (6), M_{zi}, M_{zj}, M_{yi}, M_{yj}, V_{zi}, V_{zj}, V_{yi}, V_{yj}, M_{bi}, M_{bj}, N_i and N_j are the bending moments, shear forces, torsional moments and axial forces due to the application of unit loads in i-th and j-th directions, respectively, E_c denotes the modulus of elasticity of concrete, s and A are the shape factor and the cross sectional area, respectively.

Stiffness matrix of space frame members is obtained by inverting the flexibility matrix in Eq. (5) and using the equilibrium conditions.

The member fixed-end forces for the case of a point and a uniformly distributed load can be obtained by using the compatibility and equilibrium conditions as follows

$$P_{10} = P_{20} = P_{30} = P_{40} = P_{50} = P_{60} = P_{90} = P_{110} = 0 \tag{7a}$$

$$P_{70} = -(f_{88}\,f_{70} - f_{78}\,f_{80}) \,/\, (f_{77}\,f_{88} - f_{78}\,f_{87}) \tag{7b}$$

$$P_{80} = -(f_{77}\,f_{80} - f_{78}\,f_{70}) \,/\, (f_{77}\,f_{88} - f_{78}\,f_{87}) \tag{7c}$$

$$P_{100} = -(q\,L + P + P_{70}) \tag{7d}$$

$$P_{120} = -(q\,L^2/2 + P\,(L-a) + P_{70}\,L + P_{80}) \tag{7d}$$

where f_{i0} ($i = 7,\ 8$) is the displacement in i-th direction due to the application of span loads which can be evaluated by means of the principal of virtual work in the following form

$$f_{i0} = \int_0^L \left(\frac{M_{yi} M_0}{E_c I_{effy}} + \frac{V_{zi} V_0}{\overline{G}_c A} s \right) dx \tag{8}$$

where M_0 and V_0 are the bending moment in local y direction and shear force in local z direction due to the span loads. Finally, the member stiffness equation can be obtained as

$$\underline{k}\underline{d} + \underline{P}_0 = \underline{P} \tag{9}$$

where $\underline{k}$ (12×12) is the stiffness matrix, $\underline{d}$ (12×1) is the displacement vector, $\underline{P}_0$ (12×1) is the fixed end force vector and $\underline{P}$ (12×1) is the total end force vector of the member. Eq. (9) is given in the member coordinate system (x, y, z). Hence it should be transformed to the structure coordinate system (X, Y, Z).

The reinforced concrete members have varying degrees of cracking ranging from the uncracked region to the fully cracked region under lateral and vertical loads (Fig. 3). It should be noted that, since the member has cracked and uncracked regions, the integral operations in Eqs. (6) and (8) will be carried out in each region individually.

The flexibility influence coefficient can now be obtained by means of Eqs. (6) and (8), with the following procedure.

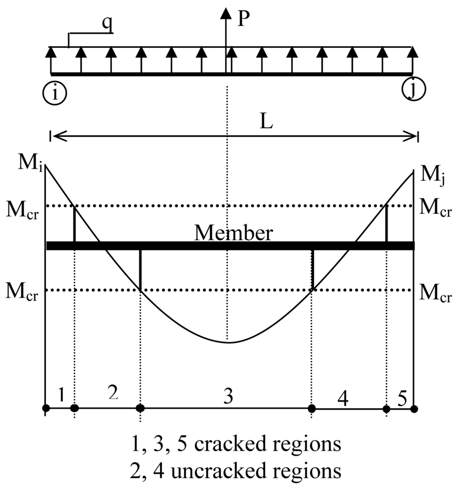


Fig. 3 Cracked and uncracked regions of the member

The moments and shear forces obtained from the application of unit and span loads are given in terms of x coordinate as follows

$$M_2(x) = x;\ V_2(x) = 1 \tag{10a}$$

$$M_3(x) = -1;\ V_3(x) = 0 \tag{10b}$$

$$M_7(\mathrm{x}) = -x;\ V_7(x) = 1 \tag{10c}$$

$$M_8(x) = -1;\ V_8(x) = 0 \tag{10d}$$

$$M_9(x) = -1;\ V_9(x) = 0 \tag{10e}$$

$$M_0(x) = \begin{cases} -\dfrac{qx^2}{2} & 0 \le x \le a \\ -\dfrac{qx^2}{2} - P(x-a), & a < x \le L \end{cases} \tag{10f}$$

$$V_0(x) = \begin{cases} qx, & 0 \le x \le a \\ qx + P, & a < x \le L \end{cases} \tag{10g}$$

The flexibility influence coefficient can now be evaluated using Eqs. (6), (8) and (10) as follows

$$f_{22} = \frac{1}{E_c}\int_0^L \frac{x^2}{I_{effz}}dx + \frac{s}{A}\int_0^L \frac{1}{G_c}dx \tag{11a}$$

$$f_{23} = \frac{1}{E_c}\int_0^L \frac{(-x)}{I_{effz}}dx \tag{11b}$$

$$f_{33} = \frac{1}{E_c}\int_0^L \frac{1}{I_{effz}}dx \tag{11c}$$

$$f_{77} = \frac{1}{E_c}\int_0^L \frac{x^2}{I_{effy}}dx + \frac{s}{A}\int_0^L \frac{1}{G_c}dx \tag{11d}$$

$$f_{78} = \frac{1}{E_c}\int_0^L \frac{x}{I_{effy}}dx \tag{11e}$$

$$f_{88} = \frac{1}{E_c}\int_0^L \frac{1}{I_{effy}}dx \tag{11f}$$

$$f_{70} = \frac{q}{2E_c}\int_0^L \frac{x^3}{I_{effy}}dx + \frac{qs}{A}\int_0^a \frac{x}{G_c}dx + \frac{P}{E_c}\int_a^L \frac{x(x-a)}{I_{effy}}dx + \frac{s}{A}\int_a^L \frac{P}{G_c}dx \tag{11g}$$

$$f_{80} = \frac{q}{2E_c}\int_0^L \frac{x^2}{I_{effy}}dx + \frac{P}{E_c}\int_a^L \frac{(x-a)}{I_{effy}}dx \tag{11h}$$

The stiffness of a cracked member varies according to the amount of crack formation occurring in the members. In the cracked regions where $M > M_{cr}$, I_{eff} and $\overline{G}_c$ vary with M along the region. Therefore, the integral values in these regions should be computed by a numerical integration technique. The variation of effective moment of inertia and effective shear modulus of concrete in the cracked regions necessitate the redistribution of the moments in the structure. Hence, iterative procedure should be applied to obtain the final deflections and internal forces of the structure (Kara and Dundar 2009).

In the present study, the total load is divided into (n1) load increments and each load increment (ΔQ) is applied step by step. Iterative procedure has also been adopted in each loading step. In the iterative procedure developed on the basis of stiffness matrix method, member equations are first

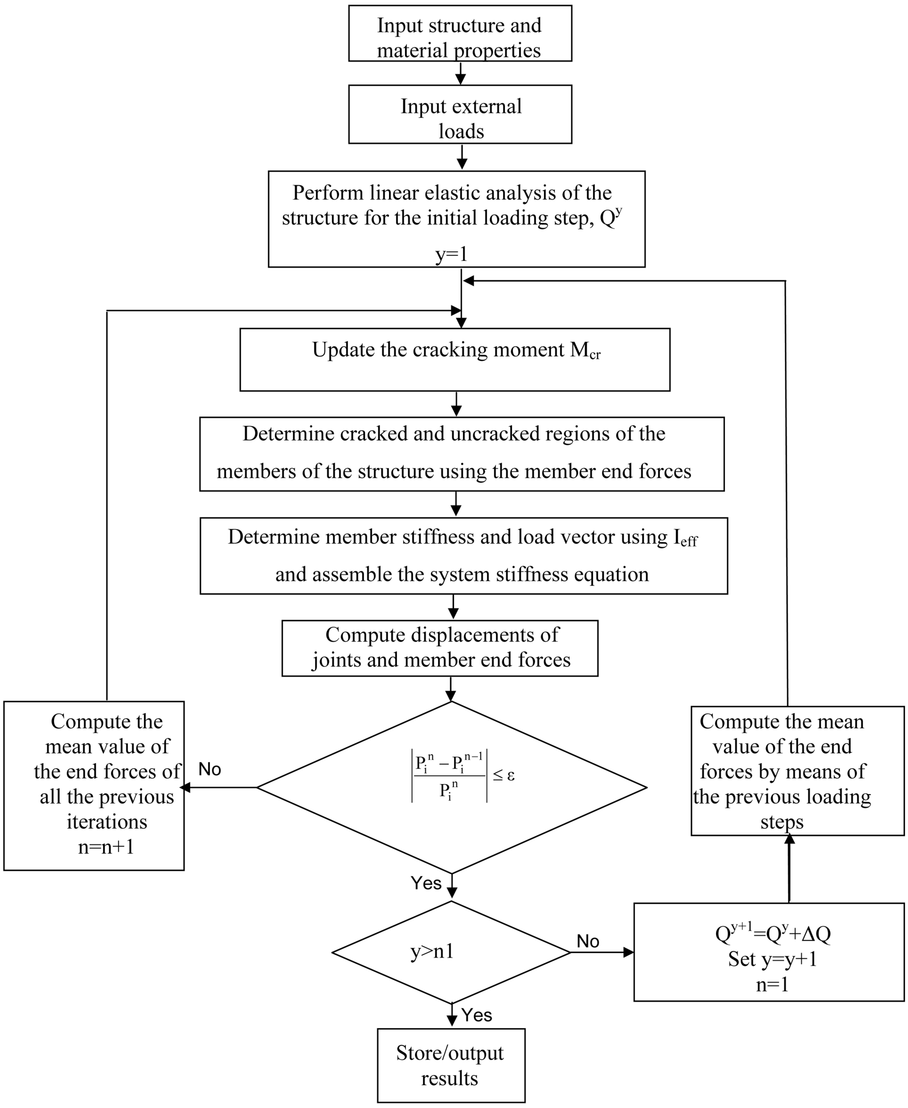


Fig. 4 Solution procedure of the program

obtained and then the system stiffness matrix and system load vector are assembled. Finally, the system displacements and member end forces are determined by solving the system equation. This procedure is repeated step by step in all iterations.

3. Computer program

A general purpose computer program developed on the basis of iterative procedure is coded in FORTRAN 77 language. The flow chart of the solution procedure of the program is given in Fig. 4. The proposed analytical procedure provides the history of nonlinear behavior of high strength steel fiber reinforced concrete beams and columns due to cracking effect by applying the external load in an incremental manner. In the solution procedure, the member end forces used at each iteration step are taken as the mean value of the end forces of all previous iterations. In the program

$$\left|\frac{P_i^n - P_i^{n-1}}{P_i^n}\right| \le \varepsilon \tag{12}$$

is used as convergence criterion. Here, n is the iteration number, ε is the convergence factor and P_i^n ($i = 1, 12$) is the end forces of each member of the structure for n-th iteration.

4. Applications, results and discussions

In order to verify the applicability and determine the limitation of the proposed analytical procedure, all the examples are taken from the literature. The first example is the high strength steel fiber reinforced concrete columns subjected to short term axial and biaxial bending. The last two examples are the simply

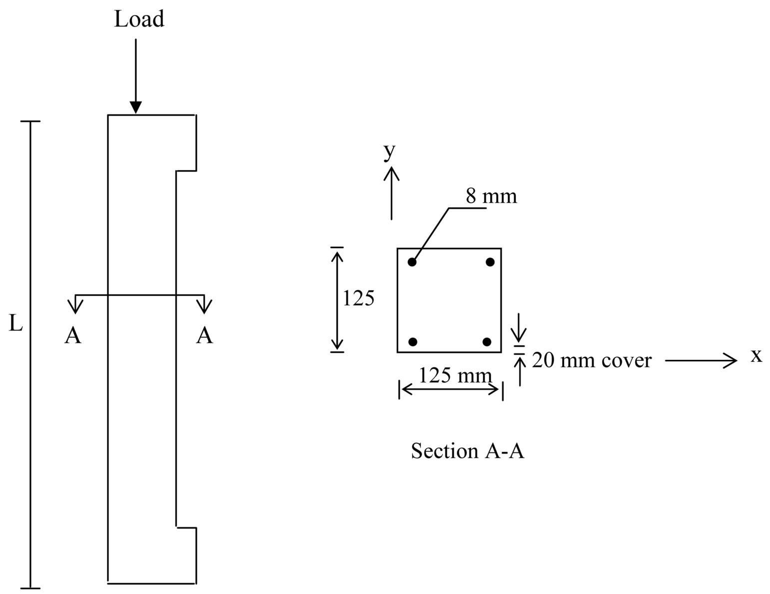


Fig. 5 The steel fiber and plain high strength columns subjected to short-term axial load and biaxial bending

Table 1 Details of the columns

Column no.	Column properties				
	L (mm)	e_x (mm)	e_y (mm)	f_c (MPa)	SF (kg/m^3)*
SF4C	1300	45	45	51.34	60
SF7C	1300	45	45	50.42	40
SF9C	1300	40	40	71.78	20
SF10C	1300	45	45	53.71	20
SF11C	1300	50	50	61.11	20
C02	1300	45	45	54.91	0

*SF: Steel Fiber

supported high strength concrete beams reinforced with steel bars with different fiber contents by volume.

4.1 Example1

In this example, the applicability of the proposed analytical procedure to high strength reinforced concrete columns with different fiber contents by volume have been tested by means of experimental results available in the literature (Tokgoz 2009). The test columns are square 125 mm cross-section and all the column specimens consisted of four 8 mm in diameter longitudinal deformed bars located at each corner of the section. The end hooked RC 65/35 type steel fibers with a length of 35 mm, diameter of 0.55 mm, aspect ratio of 64 were used. The reinforcing steel in the column, the span and the load are shown in Fig. 5. The geometric properties of the columns are also listed in Table 1. The high strength columns were tested in the vertical position with pinned conditions at both ends subjected to short-term axial load and biaxial bending. The load was also applied with different eccentiriticies. The flexural tensile strength of steel fiber concrete members is computed using the following equation, firstly recommended by Naaman and Reinhardt (2003) without considering the effect of fiber shape and concrete type (β), and then redefined (Choi *et al.* 2007) as

$$f_f = 2.5[f_t(1-V_f)+\alpha_1\alpha_2\tau V_f(L/d)\beta] \quad \text{(Mpa)} \tag{13}$$

where L = length of fiber, d = diameter of fiber, τ = average interfacial bond stress of fiber matrix, f_t = tensile strength of concrete in pure tension, α_1 = coefficient representing the fraction of bond mobilized at first matrix cracking, a_2 = efficiency factor of fiber orientation in the uncracked state of the composite, β = factor for fiber shape and concrete type. Based on the results of the previous study by Naaman and Reinhardt (2003), α_1 = 0.5, α_2 = 0.1. In the present study, τ = 4.15 proposed by Swamy and Bahia (1985) and Ashour *et al.* (1984) is used. It is concluded from the experimental verifications that the addition of steel fibers into high strength concrete has no significantly effects on modulus of elasticity (Tokgoz 2009). Therefore, in the proposed procedure, the modulus of elasticity for steel fiber and plain high strength concrete is assumed as follows

$$E_c = 4730\sqrt{f_c} \text{ (N/mm}^2\text{)} \tag{14}$$

in which, f_c is the compressive strength of concrete.

The comparison between experimental and theoretical results for the maximum deflection of the

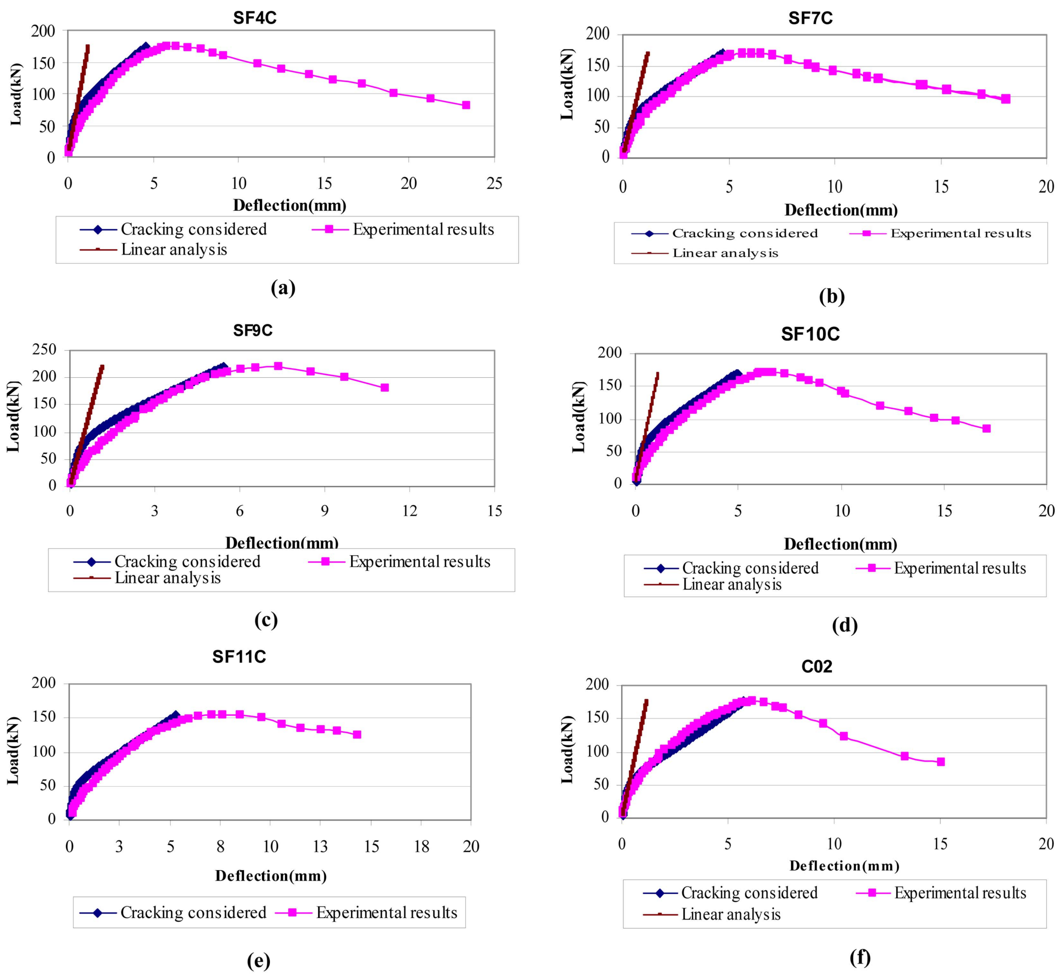


Fig. 6 (a)-(f) Load-deflection diagrams of the plain and steel fiber high strength columns

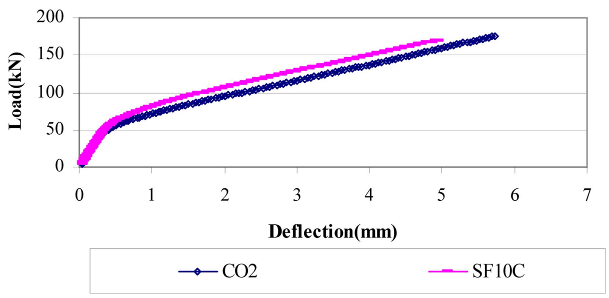


Fig. 7 Comparison of steel fiber and plain concrete columns

steel fiber and plain high strength concrete columns are presented in Fig. 6(a) through (f). The numerical results obtained from the present computer program are in good agreement with the test results for applied loads up to approximately 90% of the maximum load. The proposed analytical method also predicts the deflection in the serviceability loading range with good accuracy for all columns with different fiber contents. When the load is beyond serviceability loading range the differences between the experimental and theoretical results increase with an increase in the loads. Such differences are primarily due to the fact that steel and concrete material becomes nonlinear beyond the serviceability loading level. The slenderness effect of the columns may also become significant with increasing loads. It can be seen from the figures that the differences between the deflections obtained from the linear and cracking analysis increase with increasing the loads. The difference also becomes significant at higher load.

Fig. 7 shows the comparison of steel fiber and plain concrete columns. This figure indicates that with the addition of steel fibers, the column shows a steeper slope in the load displacement curve, which means the columns posses higher flexural rigidity.

4.2 Example 2

In this example, the simply supported high strength concrete beams with different reinforcement ratio and steel fiber contents by volume are considered (Ashour *et al.* 2000). The test beams subjected to two point loads had the same dimensions of 200×250 mm cross-section. The reinforcing steel in the beams, the span and the load are shown in Fig. 8. The geometric properties of the test beams are also listed in Table 2. Hooked-ends mild carbon steel fibers with average length of 60 mm, nominal diameter of 0.8 mm, aspect ratio of 75 were used. The effect of steel fiber on the behavior of high strength concrete beams has been investigated by developed computer program.

The comparison between the test and theoretical results for the maximum vertical deflection of beams is presented in Fig. 9(a) through (l). The numerical results obtained from the present computer program are in good agreement with the test results for applied loads up to approximately 94% of the maximum load. This analytical method predicts the deflection with a high degree of accuracy in the serviceability loading range for all beams with different reinforcement ratio and

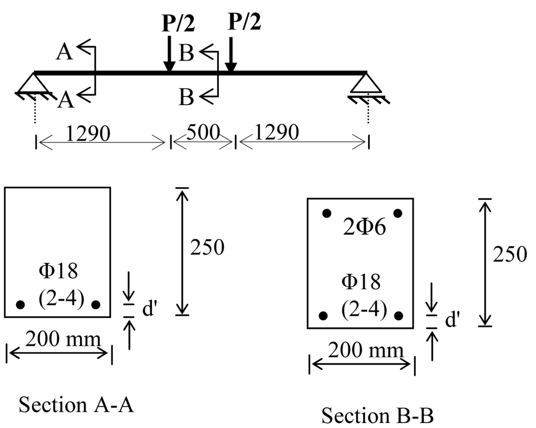


Fig. 8 The simply supported steel fiber high strength concrete beams subjected to two point loads

Table 2 Details of the beam specimens

Beam type	Beam properties		
	A_s	f_c (Mpa)	V_f(%)
B0.5M2	2Φ18	81.99	0.5
B0.5M3	3Φ18	81.99	0.5
B0.5M4	4Φ18	81.99	0.5
B1M2	2Φ18	87.37	1
B1M3	3Φ18	87.37	1
B1M4	4Φ18	87.37	1
B1N2	2Φ18	65.16	1
B1N3	3Φ18	65.16	1
B1N4	4Φ18	65.16	1
B0N2	2Φ18	48.61	0
B0N3	3Φ18	48.61	0
B0N4	4Φ18	48.61	0

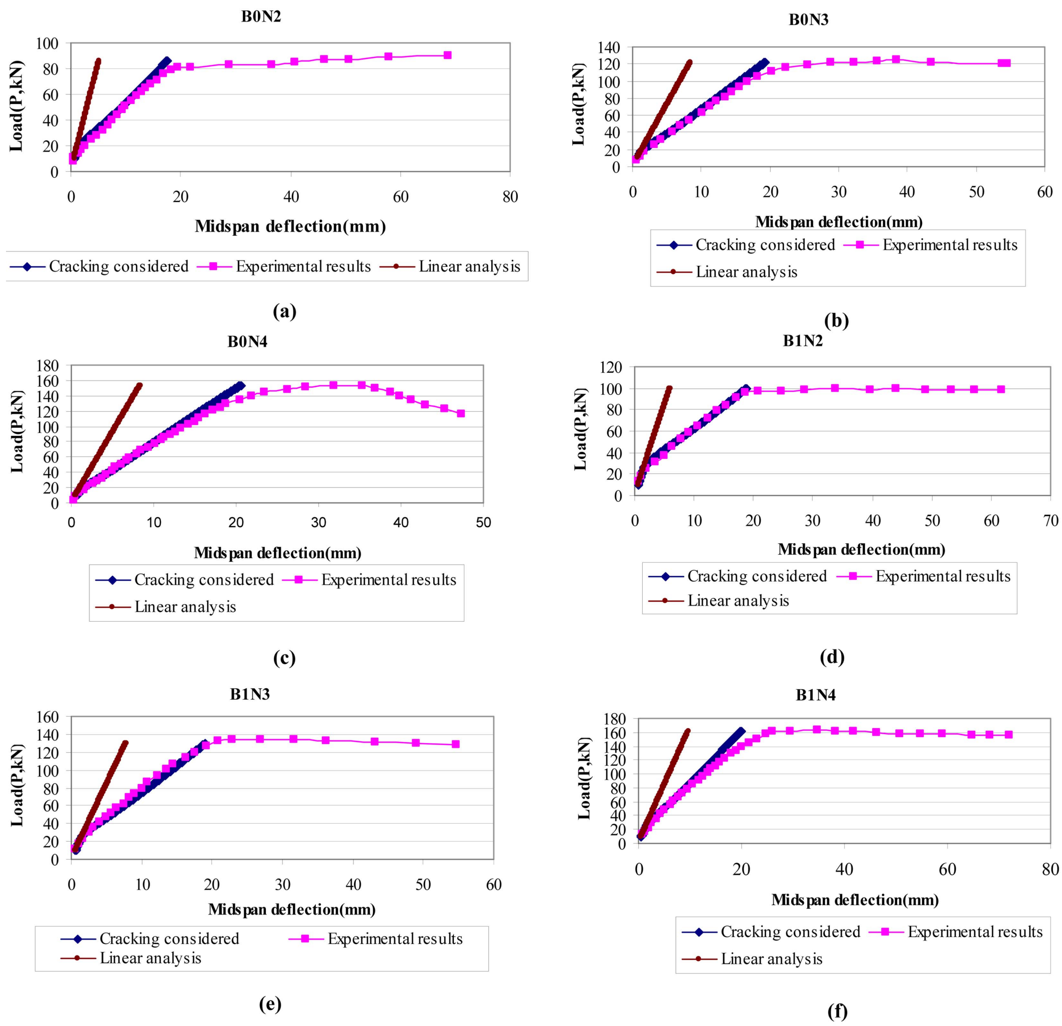


Fig. 9 (a)-(l) Comparison between experimental and analytical results of the deflection of beams

steel fiber contents by volume.

Fig. 10(a)-(b) show the comparisons of steel fiber and plain high strength concrete beams. These figures indicate that with the addition of steel fibers, concrete beams show a steeper slope in the load displacement curve, which means the beams posses higher flexural rigidity. It can also be seen from the figure that the presence of steel fibers reduces the deflection of concrete beams in the serviceability loading range.

Fig. 11(a)-(d) also show the influence of shear deformation on the total vertical deflection of the plain and steel fiber reinforced concrete members. It is seen that the contribution of the shear deformation to the total vertical deflection is lower for beams with fibers than that of beams with no fibers.

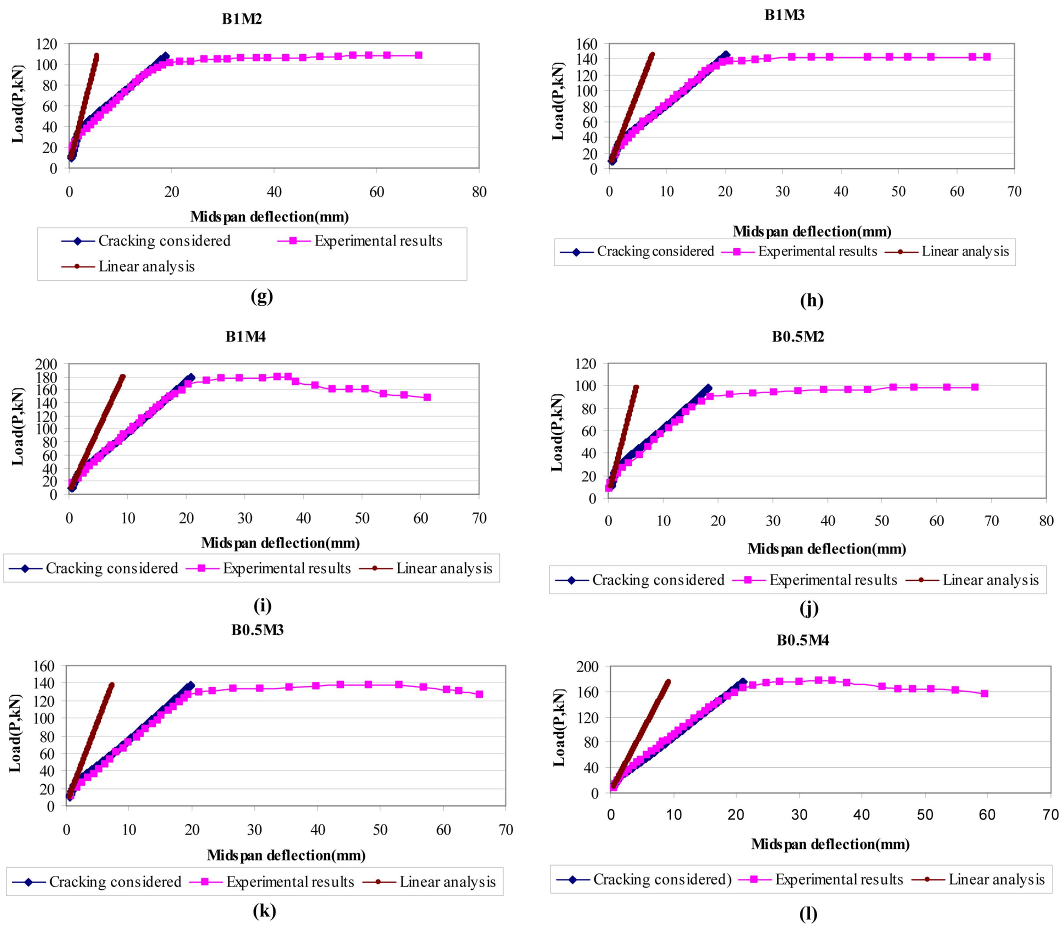


Fig. 9 Continued

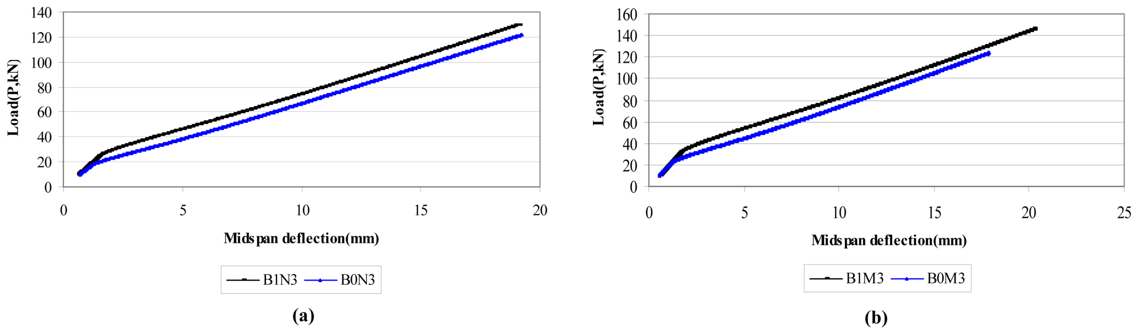


Fig. 10 (a)-(b) Comparisons of steel fiber and plain high strength concrete beams

4.3 Example 3

The accuracy of the proposed analytical procedure to the high strength steel fiber concrete beams has also been investigated using other test results available in the literature (Chunxiang and Patnaikuni

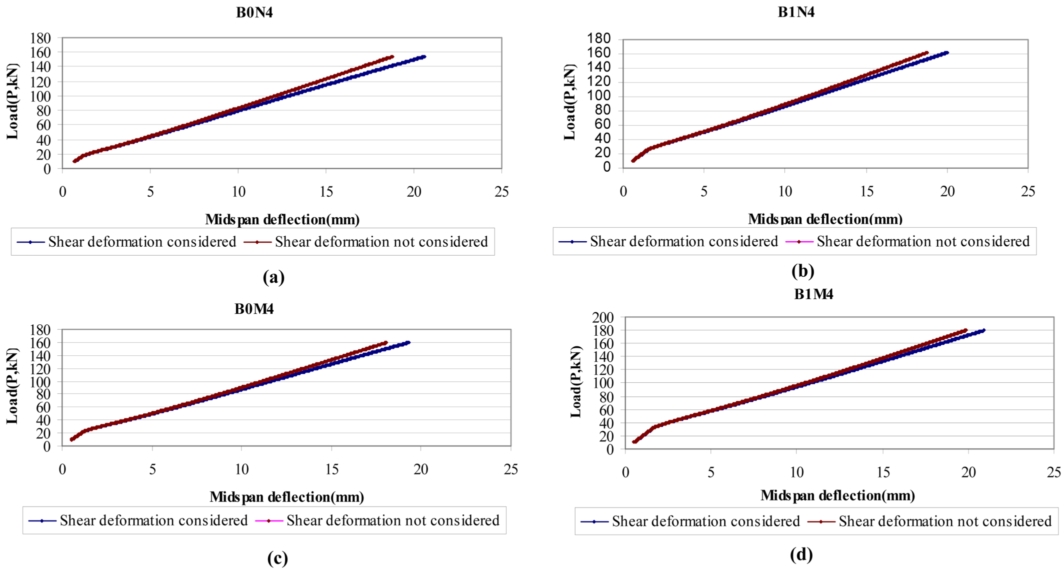
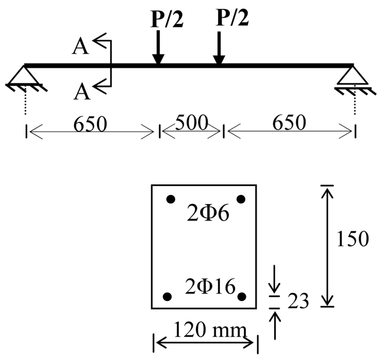


Fig. 11 (a)-(d) Theoretical influence of shear deformation on the vertical deflection of plain and high strength concrete beams



Section A-A

Fig. 12 Simply supported steel fiber reinforced concrete beam with two point loads tested by Chunxiang and Patnaikuni (1999)

1999). Comparisons have been made with the results reported of beams with rectangular sections under two loading points. The test beams had the same dimensions of 120×150 mm cross-section. The reinforcing steel in the beams, the span and the load are shown in Fig. 12. The cross-sections of the fibers were rectangular with length-nominal diameter (L/d) ratios of 46 (Type I), 38 (Type II).

The comparison between experimental and theoretical results for the midspan deflection is presented in Fig. 13(a) through 13(d). The numerical results obtained from the present computer program agree well with the test results for applied loads up to approximately 96% of the maximum load. The analytical method predicts the deflection with a high degree of accuracy in the serviceability loading range for all beams with different types of fibers. These figures also indicate that the differences between the deflections obtained from the linear and cracking analysis increase

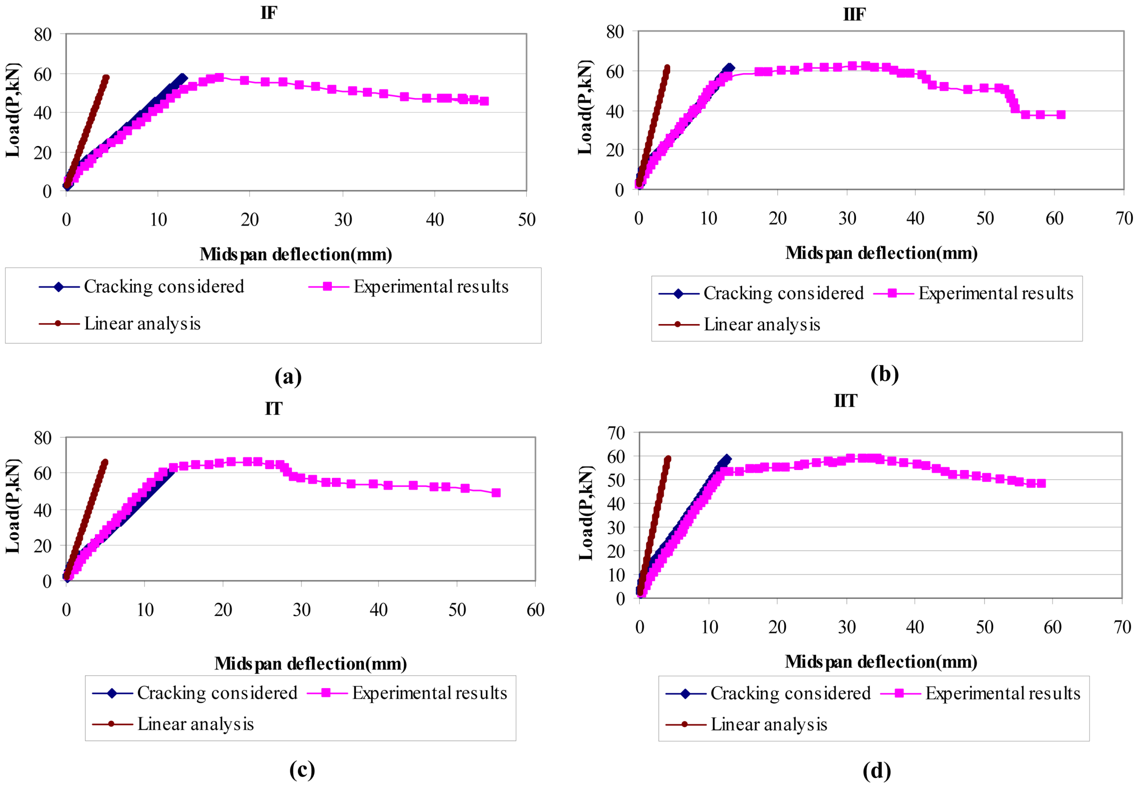


Fig. 13 (a)-(d) Comparison between experimental and predicted deflection of beams

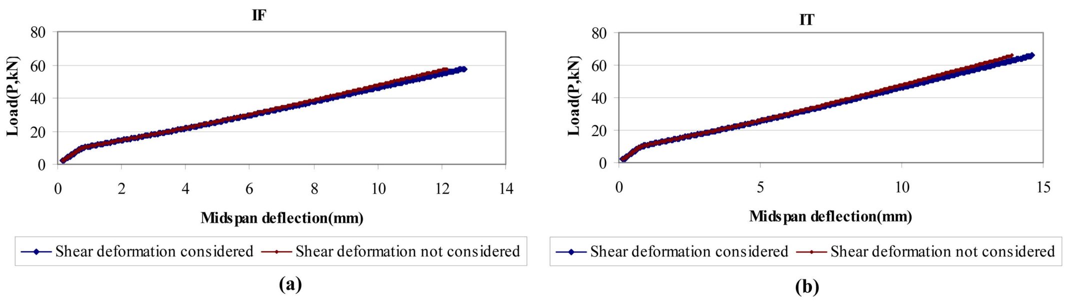


Fig. 14 (a)-(b) Theoretical influence of shear deformation on the vertical deflection of high strength concrete beams

Table 3 Simply supported beams tested by Chunxiang and Patnaikuni (1999)

Beam no.	Beam properties		
	Fiber Type	Compressive Strength (Mpa)	V_f(%)
IF	I	79.9	1
IIF	II	95.7	1
IT	I	81	1
IIT	II	92.2	1

with an increase in the loads. The difference also becomes significant at higher loads.

Fig. 14(a)-(b) show the contribution of the shear deformation effect in the overall vertical deflection. The results indicate that the percentage of shear deformation in the total deflection of beams is approximately 4%.

5. Conclusions

This paper presents an iterative procedure to analyze high strength steel fiber reinforced concrete members considering the cracking effect in the serviceability loading range. The proposed analytical procedure provides the nonlinear force-deformation relationships for the cracked members with steel fibers.

The variation of the flexural stiffness of a cracked member has been evaluated by modifying ACI model to include steel fiber effect. Shear deformations are also taken into account in the analysis. The variation of shear rigidity in the cracked regions of the member has been considered by reduced shear stiffness of non-fiber reinforced concrete members.

The feasibility for applying the proposed procedure to high strength steel fiber reinforced concrete beams and columns has been tested by a comparison between experimental and numerical results. The numerical results of the analytical procedure have been found to be in good agreement with the test results for applied loads up to approximately 90% of the maximum load of the members. The analytical procedure predicts the deflections with a high degree of accuracy in the serviceability loading range for beams and columns with different steel fiber content and concrete compressive strength.

The analytical results show that the presence of steel fibers reduces the deflection of concrete beams and columns in the serviceability loading range. These results also indicate that with the addition of steel fibers, reinforced concrete members show a steeper slope in the load displacement curve, which means the beams and columns posses higher flexural rigidity.

Stiffness matrix method has been employed to obtain the numerical solutions of the proposed procedure. This procedure is efficient from the viewpoints of computational effort and convergence rate. This iterative analytical procedure also forms the basis for the three dimensional analysis of structures with steel fiber reinforced concrete members.

The numerical results of the analytical procedure indicate that the contribution of the shear deformation to the total vertical deflection is lower for beams with fibers than that of beams with no fibers. The numerical results of the analytical procedure indicate that the effect of shear deformation on the total deflection has a significant factor on the beams with no fibers, while it has less significant effect on the beams with fibers.

The results obtained by the proposed procedure could be taken into consideration in design of flexural elements produced with steel fiber added reinforced concrete.

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