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# Reliability analysis of concrete bridges designed with material and member resistance factors

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**Abstract.** Reliability analysis for a proposed limit state bridge design code is performed. In order to introduce reliability concept to design code, the proposed live load model is based on truck weight survey. Test data of domestic material strengths are collected to model statistical properties of member strengths. Sample RC and PSC girder sections are designed following the safety factor format of the proposed code and compared with the current design practice. Reliability indexes are calculated and examined for material and member resistance factor formats and sample calibrations of safety factors are presented. It is concluded that the proposed code provides reasonable level of reliability compared to the international design standards.

Keywords: structural reliability; truck live load; statistical data; resistance factor format; code calibration.

# 1. Introduction

Reliability analysis is conducted for a proposed limit state bridge design code. The proposed code is the first version of reliability based bridge design code introduced in Korea. There has been growing demand for globally standardized design code after experiencing rapid globalization of construction technique and market. Currently, most of the leading international codes accept limit state design format as well as reliability concepts.

The reliability analysis for the codes should be performed with sufficient and reliable statistical data for the loads, material strengths, member dimensions, and other necessary information. To achieve such a goal systematically in designing a structure, some international model codes including ISO (1998) and JCSS (2001) have provided the basis of required levels of reliability. As a

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common feature in the structural design codes developed in recent years, the limit state design concept has been widely introduced and thus the determination of partial safety factors in the design codes based on the reliability analysis has been a more critical problem.

Development of a live load model is essential for a probabilistic design code. Crespo-Minguillón and Casas (1997) modeled traffic actions on roadway bridges including algorithms for the simulation of continuous flow traffic and the maximum results in terms of return periods. Through truck surveys and simulation studies, Nowak (1999) developed the live load model for AASHTO LRFD design and determined the statistical parameters of the live load. Moses (2001) proposed statistical parameters for truck load for AASHTO LRFR based on weigh-in-motion data. The statistical properties for material and dimension are summarized in detail in Ellingwood et al. (1980). Based on these statistical data, Nowak (1999) developed a probabilistic model for various types of bridge girders. Nowak and Szerszen (2003) analyzed the material properties and presented that the quality of materials was improved over the last 30 years. Szerszen and Nowak (2003) presented the calibration of ACI 318 for RC and PSC elements with both new and old material data and recommended possible values of the resistance factors. Nowak et al. (2002) carried out the reliability analysis of PSC girders designed by Eurocode, Spanish Code and LRFD Code and compared the relative reliability levels of the codes. Du and Au (2004) also compared the prestressing requirements and reliability indices of PSC girders designed by Chinese Code, Hong Kong Code, and LRFD Code according to service limit state and strength limit state.

A future oriented performance-based model code was recommended to follow a probability-based method including reliability index which may be taken an advantage of as a measure of adequate performance of structure (Aktan *et al.* 2007; Walraven and Bigaj-van Vliet 2008). And Enevoldsen (2008) presented methods of practically implementing probability based assessment procedures to bridges demonstrating higher load carrying capacities than those evaluated by traditional deterministic analysis. Applications of reliability analysis to special structures such as cable bridges and offshore structures to investigate dominant design variables through sensitivity analysis can be found in many literatures including Cheng and Xiao (2005) and Islam and Ahmad (2007).

Depending on the codes from different continent, the format of defining the load and resistance models varies. The load factors appeared in current various design codes are much alike in the format. Depending on the target application (i.e. to member strength or to material strength), however, the type of resistance factors can be divided into three groups. Eurocode (2004) and CHBDC (2000) adopt the format of the partial safety factor of materials for concrete structures. On the other hand, the resistance factors of ACI (2005) and AASHTO LRFD (2007) are in the form of member strength reduction factor. Meanwhile, JSCE (2002) adopts the combination type of resistance factors with both material and member factors. Another example of combination type format is found in the assessment code where the condition factor or the system factor is applied as the member factor and the design strength is calculated based on material factor format.

One of the objectives of the first edition of the reliability based bridge design code is to assure that the designs based on the proposed code satisfy a reasonable level of reliability recommended by the international model code such as ISO (1998) and JCSS (2001). In addition, the designs based on the proposed reliability code need to minimize abrupt changes in the designed sections compared to those following the current domestic design practice.

In order to introduce reliability concept to design code, the proposed code adopts live load model based on truck weight field survey to update fast growing truck traffic volume. Also, test data of domestic material strengths are collected to model statistical properties of member strengths. The proposed code adopts a combined format of material and member resistance factors. Design for concrete structures applies mainly material resistance factor system similar to Eurocode (2004) and CHBDC (2000) format in order to take into account the characteristics of concrete and reinforcing steel more fundamentally, while member resistance factor is set to unity for the proposed code until further research requires certain values. Design for steel structures uses member resistance factor system with unit material factor similar to AASHTO LRFD (2007) format.

In the paper, typical RC and PSC girder sections are designed based on the safety factors systems of the proposed code and the reliabilities of the designed sections are compared with those designed by the current code. Reliability indexes are calculated for flexural and shear strength and the sensitivity on the reliability index of the design variables are examined. The effect of the material and member resistance factors on the reliability index is also examined through parameter study. In addition, sample calibrations of safety factors over different target reliability indexes are presented. By implementing the reliability concept, future changes in the construction environment could be taken into account more rationally by calibration of safety factors judged not only by experience but also by probabilistic information.

# 2. Live load model

## 2.1. Development of load modeling

For the reliability analysis of bridges, live load model and its statistics should be determined. The current live load model in Korea Bridge Design Code (MLTM 2005) needs to be updated to consider the fast growing truck traffic and weight. In this paper, a new live load model and its statistics are determined based on actual truck weight collected on various sites in the areas using BWIM (Bridge Weigh-In-Motion) system and portable WIM (Weigh-In-Motion) system. Table 1 shows locations and the number of truck weight data used in this study. From the collected data, maximum weights are estimated for each truck type and sites using the extreme analysis. In this study, it is assumed that only upper portion of truck weight data is relevant to the heavy truck event. Nowak (1999) used upper 20% of data for estimation. This study uses 10% and 20% of data and assumes that their distributions are Extreme Type I (Gumbel). Fig. 1 shows upper 10% of data of various sites plotted on Gumbel probability papers. Code 70 and 91 trucks represents 5 axle

Location	Road Type	No. of Data	System
Songpo	Highway	94,236	BWIM
Eunhyun	Provincial road	3,540	BWIM
Dongchun	National road	4,962	BWIM
Dogok	Highway	23,208	BWIM
Maebong	National road	17,200	BWIM
Pohang	National road	29,238	WIM
Munmak	National road	10,663	WIM
Bibong	National road	17,693	WIM
Total		200,740	

Table 1 Locations and number of trucks weight data



Fig. 3 Proposed live load model

single truck and 5 axle semi tractor and trailer, respectively, which are known as the heaviest trucks on the road. Maximum truck weights are calculated using linear extrapolation of data corresponding to the number of trucks during the bridge lifetime, which is assumed as 100 years in this study. There are not much difference in the estimation results between using 10% and 20% (KBDRC 2008).

Multiple presence of trucks in one lane (series of trucks) and two or more lanes (side-by-side trucks) are considered. The probability of multiple presences of trucks is determined from the video recording and other studies (Nowak 1999; KBDRC 2008). The probability of two uncorrelated trucks and fully correlated trucks in one lane are assumed as 1/70 and 1/350, respectively, which is similar to the value used in Nowak's study (1999). Probabilities of three or more trucks are based on two truck probabilities. The results are shown in Fig. 2.

Based on the envelop curve in Fig. 2, new live load models are proposed as shown in Fig. 3.

Loading Lane(s)	1	2	3	4	5 or more
Factor	1.0	0.9	0.8	0.7	0.6

Table 2 Proposed values for multiple presence factors

Probabilities of truck multiple presences in two or more lanes are also based on observations at different locations. The total truck weights of side-by-side trucks are loaded on two-lane and five-lane prestressed concrete girder bridges and the multiple presence factor is calculated as the load effect of side-by-side trucks divided by the load effect of single truck at each lane. Proposed factors are shown in Table 2.

The statistics of live load model are also based on the collected data and proposed live load model. The bias factors (or mean-to-nominal ratio) vary from about 0.95 to 1.05 depending on the span length but are generally assumed as 1.0. The coefficient of variation includes variations from truck weight probability function, site effects, structural analysis effects, dynamic load effects, and so on. Considering results of other studies and engineering decision, the coefficient of variation of live load is assumed to be about 20%.

## 2.2. Required strength

The effect of the current and the proposed live load models are compared with the revised load factors. The current load combination is as follows.

$$U = 1.3D + 2.15L \tag{1}$$

The proposed load combination is as follows.

$$U = 1.25D_1 + 1.50D_2 + 1.75L \tag{2}$$

Asphalt weight  $D_2$  is separated from total dead load  $D_1$  except asphalt in the proposed design. Structural analysis is performed by using finite element program MIDAS/CIVIL (2006) as shown in Fig. 4. The proposed live load model is heavier than the current live load model by the addition of distributed load component of 9.6 kN/m, however, the proposed load factors are decreased. Thus, the net effect of the required strength of the proposed model slightly decreases in RC girders, but remains the same in PSC girders as shown in Fig. 5(a) and (b), respectively.



Fig. 4 Proposed design truck model and its loading



Fig. 5 Required strength for the current and the proposed factored load (a) RC span (b) PSC span

# 3. Resistance model

# 3.1. Statistical properties of materials

In order to obtain statistical properties for material strengths, domestic construction field test data are collected. The results of the statistical analysis are shown in Table 3 and 4. In these tables the bias factor  $\lambda$  and the coefficient of variation (COV) V of material strength are defined as  $\lambda = \mu/x_n$  and  $V = \sigma/\mu$ , where  $\mu$  is mean,  $x_n$  is specified nominal value and  $\sigma$  is standard deviation.

Specified comp. strength $x_n$ [MPa]	Number of data $N$	Mean µ [MPa]	Standard deviation $\sigma$ [MPa]	Bias factor $\lambda$	COV V
18	1928	21.90	1.57	1.22	0.07
21	2932	25.05	1.62	1.19	0.07
24	7395	28.89	1.86	1.20	0.07
27	895	30.81	1.77	1.14	0.06
30	270	35.45	2.28	1.18	0.06
40	196	45.24	2.64	1.13	0.06
45	16	51.07	3.52	1.14	0.07

Table 3 Statistical properties for concrete compressive strength

Table 4 Statistical properties for yield strength of reinforcing bar and tensile strength of prestressing tendon

	Specified strength	Number of data	Average strength	Standard deviation	Bias factor	COV
	$x_n$ [MPa]	Ν	$\mu$ [MPa]	$\sigma$ [MPa]	λ	V
Re-bar	300	696	371.2	32.48	1.24	0.09
	400	925	478.8	47.77	1.20	0.10
	500	47	542.8	10.93	1.09	0.02
Tendon	1860	96	1939.8	34.00	1.04	0.02



Fig. 6 Statistical distribution (a) CDF of concrete strength and (b) PDF of PS tendon

Concrete data are collected from field reports of Korea Highway Corporation (KHC) from its 7 different construction sites during 1997 and 2004. The field reports includes compressive strength tested after 7 and 28 days and slump test values for concretes used for highway bridges, tunnels, walls and surrounding facilities. Concrete compressive strengths of 28 days are used for statistical analysis from the reports. In Table 3, the COV for concrete strength is 6 or 7% which seems low. It can be understood that the quality control of KHC has been very strict after the nation experiences the collapses of a major bridge and a department store building in Seoul in 1994 and 1995, respectively. These data are comparable to those of the ordinary ready-mix concrete in Nowak and Szerszen (2003) where the COV's for 21, 24, 28, 31, 34 and 41 MPa are 10.2, 7.9, 14.5, 4.2, 5.8, 4.2%, respectively. Fig. 6(a) shows cumulative distribution function (CDF) in normal probability paper for the collected concrete data.

Also, prestressing tendon data are obtained from manufacturer test report in 2005 and 2006. In Table 4, the standard deviation of PS tendon is 34.0 MPa which is similar to the corresponding values of the reinforcing bars in the same table. However, as the average value of the strength of PS tendon is 1939.8 MPa which is very large compared to that of re-bar, the COV of PS tendon is 0.02 which is very low compared to that of re-bar. The bias factor of 1.04 and the COV of 0.02 for PS tendon in this study are very similar to the corresponding values of 1.045 and 0.025 in Nowak and Szerszen (2003). Fig. 6(b) shows probability density function of the prestressing tendon.

Statistical data for reinforcing steel are obtained from the report to KHC of construction material properties. These data are based on the test report in 2005 and 2006 for the reinforcing steel made by the domestic manufacturers. In table 4,  $\lambda$ =1.20 and COV=0.10 for SD 400 MPa rebar in this study. The corresponding values of Grade 420 MPa in Nowak and Szerszen (2003) are  $\lambda$ =1.145 and COV=0.05 which are smaller than those in this study. As the current Korean Standard (KATS 2007) for manufacturing the rebars specifies only the lower limit of the yield strength, the domestic manufacturers tend to set the target strength high to satisfy this specification instead of minimizing the variation. It results in the higher values of  $\lambda$  and COV in the domestic rebar data. Fig. 7(a) shows the frequency distribution of the data in a bar graph and the PDF of the normal distribution with the same mean and standard deviation. As the frequency distribution for SD 400 has two peaks and the shape is different from the normal distribution curve, further examination is carried out. Total of 925 data are sorted with respect to manufacturer. In terms of the similarity in the mean



Fig. 7 Frequency histogram and PDF for reinforcing steel (a) whole data (b) manufacturer group A (SD 400) and (c) manufacturer group B (SD 400)

value of the yield strength, the manufacturers are grouped in A and B. The frequency diagram and PDF are plotted in Fig. 7(b) and (c) for Group A and B separately, and it is much closer to normal distribution compared to the original diagram in Fig. 7(a). It can be concluded that the distribution for the reinforcing bar is close to normal distribution when the data are analyzed for each manufacturer. Therefore, the values shown in Table 4 with normal distribution are decided to be used.

## 3.2. Design strength

Design strength of a concrete section can be obtained by multiplying the appropriate resistance factors to nominal strength of the section. In this study, in order to examine both the member force resistance factor (MEMF) format and the material resistance factor (MATF) format, a combination type of resistance factor (COMF) format is applied. Design strength  $R_d$  of flexural moment of a PSC section can be written as in Table 5 for each resistance factor format.

In Table 5,  $\phi_m$  and  $\phi_i$  stands for the member resistance factor and the material resistance factor,

Table 5 Formul	lation for	design	strength	for (	each	resistance	factor	format
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Format	Design strength	Design strength for flexure	
Member factor format	$R_d = \phi_m R(X_{k,i})$	$\phi_f \left\{ A_{ps} f_{ps} \left( d_p - \frac{A_{ps} f_{ps}}{b f_{ck}} \right) \right\}$	(3)
Material factor format	$R_d = R(\phi_i X_{k,i})$	$A_{ps}\phi_s f_{ps} \left( d_p - 0.59 \frac{A_{ps}}{b} \frac{\phi_s f_{ps}}{\phi_c f_{ck}} \right)$	(4)
Combination factor format	$R_d = \phi_m R(\phi_i X_{k,i})$	$\phi_f \left\{ A_{ps} \phi_s f_{ps} \left( d_p - 0.59 \frac{A_{ps}}{b} \frac{\phi_s}{\phi_c} \frac{f_{ps}}{f_{ck}} \right) \right\}$	(5)

respectively, and  $X_{k,i}$  stands for the characteristic strength of material. Also in the table,  $\phi_f$  is the member resistance factor for flexure,  $\phi_s$  and  $\phi_c$  are the material resistance factor for prestressing tendon and concrete, respectively,  $A_{ps}$  is tendon area,  $f_{ps}$  is tendon stress at nominal strength,  $d_p$  is effective depth to tendon,  $\rho_p$  is reinforcement ratio of tendon and  $f_{ck}$  is the specified compressive strength of concrete. The effect of the resistance factor format on the reliability of design is further examined in section 4 of this paper. When MATF is applied, member factors are set to 1 and when MEMF is applied, material factors are set to 1 in Eq. (5) in the table. Nominal strength is obtained when all the resistance factors are equal to 1. Formula for RC member is basically the same as in Table 5 for PSC if the corresponding values of reinforcing steel are inserted to those of prestressing tendon in the equation and is not shown for brevity.

Design shear strength  $V_d$  is calculated using the truss model with variable angles as given in EN1992 (2004) as follows.

$$V_d = \frac{A_v \phi_s f_y z}{s} \operatorname{cot} \theta \le V_{dmax} = \frac{v \phi_c f_{ck} b_w z}{\cot \theta + \tan \theta}$$
(6)

in which  $A_v$  is the area of transverse reinforcement, z is the inner lever arm, s is the stirrup spacing,  $\theta$  is angle between concrete compression struts and the main tension chord, v is given as  $0.6(1-f_{ck}/250)$  and  $b_w$  is web width. Resistance factor system for shear is not examined separately in this paper as the formulation for shear strength in Eq. (6) has only one term regarding shear reinforced beam. The effect of the material resistance factor  $\phi_s$  is the same as that of the member resistance factor  $\phi_v$  for shear for the example sections considered in this paper. Formulas to calculate shear strength is the same for PSC and RC members except that in PSC case the amount of the load balancing effect of the vertical component of prestressing tendon is subtracted from shear force caused by external load.

The current resistance factors are defined in MEMF format and the values are as follows.

$$\phi_f = 0.85 \text{ and } \phi_v = 0.80$$
 (7)

The proposed resistance factors are defined in MATF format and the values are as follows.

$$\phi_s = 0.90 \text{ and } \phi_c = 0.65$$
 (8)

Sample girder sections of RC and PSC are given in Fig. 8 and Table 6. Girder spans investigated



Fig. 8 Concrete girder section (a) RC (b) PSC



Table 6 Dimensions of RC and PSC beam sections, mm

Fig. 9 Required area of steel reinforcement for flexure (a) RC re-bar (b) PSC tendon

in this study are 9, 12, 15 and 18 meters for RC and 20, 25, 30, 35 and 40 meters for PSC. PSC sections of 25, 30 and 35 meters are the standard sections recommended by KHC and those of 20 and 40 meters are decided by extrapolation.

The areas of reinforcing bars and prestressing tendons are calculated to provide the required strength given in Fig. 5 and the results are shown in Fig. 9. In this figure, the required area of steel reinforcement decreases on average by 13.6% and 6.1% for RC rebar and PSC tendon, respectively.

#### 3.3. Statistical properties of members

The statistical properties of the member strength are obtained by simulation. Basic statistical properties for material strengths shown in Table 3 and 4 of this study are used in the analysis. The statistical properties for dimensions and professional factors are found from Nowak *et al.* (1994).

Moment-curvature diagram for RC girder and PSC girder are presented in Fig. 10 for spans of 12 meters and 25 meters, respectively. Mean and standard deviation of flexural moment of girders at each curvature point are obtained and mean and mean plus and minus one standard deviation values are plotted. Flexural strength of concrete section can be estimated by nonlinear analysis using stress-strain relation of materials. Short-term stress-strain relation given in Eurocode (2004) is used for concrete. Idealized elastic and perfectly plastic stress-strain model is used for reinforcing steel. Stress-strain diagram for prestressing tendon is applied by modifying the equation given in Nawy (1999). After finding out the nominal value for tendon stress  $f_{ps}$  at failure from the result of



Fig. 10 Moment curvature diagram of mean and mean plus and minus one standard deviation (a) RC (b) PSC



Fig. 11 Member statistical properties (a) Bias factor and (b) coefficient of variation

		$\lambda_{ m MF}$	$V_{\text{MF}}$	$\lambda_{ m P}$	V <sub>P</sub>	$\lambda_{ m R}$	V <sub>R</sub>
RC	Flexure	1.205	0.115	1.020	0.06	1.229	0.130
	Shear	1.199	0.104	1.075	0.10	1.289	0.144
PSC	Flexure	1.045	0.041	1.010	0.06	1.056	0.073
	Shear	1.185	0.097	1.075	0.10	1.274	0.139

Table 7 Bias factor and coefficient of variation for member strength

nonlinear analysis for section strength, a simulation to get statistical properties of section strength can be carried out using rectangular stress block analysis which yields almost the same results for low reinforced sections as in this study and is simpler to be included in the reliability analysis computer program.

Bias factors and coefficients of variation resulted from 10,000 simulations are shown in Fig. 11 and Table 7. In the table, MF accounts for uncertainties in material and dimensional properties, and P accounts for uncertainties in the analysis model.  $\lambda_{MF}$  and  $V_{MF}$  are obtained after material and dimensional variables are randomly generated and simulated. After professional factors are taken into account, the bias factor and the coefficient of variation of member strength  $\lambda_R$  and  $V_R$  are calculated using the relation  $\lambda_R = \lambda_{FM} \lambda_P$  and  $V_R = (V_{FM}^2 + V_P^2)^{1/2}$ .

## 4. Reliability analysis

# 4.1. Reliability index and sensitivity factor

For calculating reliability index, Rackwitz-Fiessler procedure described in Nowak and Collins (2000) is used to take into account the non-normal distribution of the variables in the limit state function. In this procedure, non-normal variables are transformed into equivalent normal variables at design point to possess the same values of CDF and PDF as shown in Eq. (9), respectively,

$$F_x(x^*) = \varPhi\left(\frac{x^* - \mu_x^e}{\sigma_x^e}\right); \quad f_x(x^*) = \frac{1}{\sigma_x^e} \oint\left(\frac{x^* - \mu_x^e}{\sigma_x^e}\right) \tag{9}$$

where F and f are non-normal CDF and PDF of a design random variable x, respectively,  $\Phi$  and  $\phi$  are CDF and PDF for the standard normal distribution, respectively, asterisk \* denotes design point and e denotes equivalent normal value. The expression for the equivalent mean and standard deviation can be obtained as follows.

$$\mu_x^e = x^* - \sigma_x^e [\Phi^{-1}(F_x(x^*))]; \quad \sigma_x^e = \frac{1}{f_x(x^*)} \phi [\Phi^{-1}(F_x(x^*))]$$
(10)

The reduced variate z for the design variable x can be determined as follows.

$$z^* = \frac{x - \mu_x^e}{\sigma_x^e} \tag{11}$$

The  $\{G\}$  vector contains the partial derivatives of the limit state function with respect to z.

$$\{G\} = \{G_1 \ G_2 \ \dots \ G_n\}^T$$
 (12)

$$G_i = -\frac{\partial g}{\partial z_i}\Big|_{*}$$
(13)

In the present study, the limit state function g can be written as follows.

$$g = R(f_{ck}, f_y, A_s, d) - D_1 - D_2 - L$$
(14)

From Eq. (11) and (13),  $G_i$  can be expressed as follows.

$$G_{i} = -\frac{\partial g}{\partial x_{i}} \frac{\partial x_{i}}{\partial z_{i}} \Big|_{*} = -\frac{\partial g}{\partial x_{i}} \Big|_{*} \sigma_{x_{i}}^{e}$$
(15)

Then the reliability index can be estimated as

$$\beta = \frac{\{G\}^{T}\{z^*\}}{\sqrt{\{G\}^{T}\{G\}}}$$
(16)

where the  $\{z\}$  vector contains the values of the reduced variate  $z_i$  at the design point.

$$\{z^*\} = \{z_1^* \ z_n^* \ \dots \ z_n^*\}^T \tag{17}$$

Iterations are required until the value of  $\beta$  and the design point converge.

Reliability analysis is conducted to compare the reliability level of the proposed design and the current design. In Eq. (14), the statistical properties for R and L obtained in this study are used and



Fig. 12 Reliability of the proposed load and resistance vs. the current load and resistance (a) RC flexure (b) RC shear (c) PSC flexure (d) PSC shear

those for  $D_1$  and  $D_2$  are found from Nowak (1999). Fig. 12 presents the reliability level of RC and PSC for flexural moment and shear strength for different spans.

In this figure, CR and PR stands for the sectional resistance of the current design and the proposed design, respectively, and CQ and PQ stands for the effect of external load of the current load model and the proposed load model, respectively. The reliability index of the proposed design code (PRPQ) is lower than that of the current design code (CRCQ) by 0.86 and 1.12 for RC flexure and shear, respectively, and 1.07 and 1.24 for PSC flexure and shear, respectively, on average over the examined spans. Considering that the target reliability levels of the international bridge design codes are usually around 3.5 or 3.8 for strength limit state, the reliability level of the proposed design code is much closer to the international reliability level than the current code. For reference, the reliability level of the member strength which is designed based on the current code and subjected to the proposed load is examined as indicated by CRPQ in Fig.12.

Sensitivity analysis is conducted to investigate which design parameters affect much on the reliability index. Fig. 13 shows an example of sensitivity on the reliability index of possible scenarios of % deviation of design parameters for the PSC girder of 30m span. From Fig. 13(a) it can be seen that concrete compressive strength  $f_{ck}$  has much less effect on the reliability of flexural strength than the tensile strength of tendon,  $f_{ps}$ , and the tendon area  $A_{ps}$ . Also, the dead load D<sub>1</sub> of structure and the live load L affect more on the reliability index than the asphalt weight D<sub>2</sub>. Fig. 13(b) shows similar result that deviation of the partial safety factors of concrete  $\phi_c$  and asphalt



Fig. 13 Sensitivity of % deviation of design parameters on the reliability index (a) design variables of load and resistance (b) partial safety factors

weight  $\gamma_{d2}$  has much less effect on the reliability index than that of flexural member factor  $\phi_{f}$ , material factor for prestressing tendon  $\phi_s$ , dead load factor  $\gamma_{d1}$  and live load factor  $\gamma_{f}$ .

## 4.2. Parameter study on resistance factor system

Reliability analysis is performed to examine the effects of material resistance factors and member resistance factors. Table 8 shows the reliability index of RC and PSC girders designed in MATF and MEMF format with different span lengths. Average value of the reliability index of RC flexure is 3.89 for MATF with  $\phi_s = 0.90$  and  $\phi_c = 0.65$ . It is slightly larger than 3.86 of the reliability index for MEMF with  $\phi_f = 0.90$  which is set to be the same value as  $\phi_s$  for comparison purposes. From this result and by examining Eq. (3) and (4), it can be said that the material factors for steel reinforcement controls the overall reliability of the flexural member strength and the effect of the

	RC						PSC	2	
		Flexure		Shear			Flexure		Shear
Length	MATF	ME	CMF	MATF	Length	MATF	ME	MF	MATF
(m)	$\phi_s = 0.90$	<i>∳</i> <sub>f</sub> =0.85	φ <sub>f</sub> =0.90	$\phi_s = 0.90$	(m)	$\phi_s = 0.90$	<i>¢</i> <sub>f</sub> =0.85	φ <sub>f</sub> =0.90	$\phi_s = 0.90$
9	3.82	4.04	3.81	3.80	20	3.84	4.17	3.81	3.86
12	3.90	4.12	3.88	3.86	25	3.98	4.32	3.94	3.90
15	3.94	4.14	3.89	3.90	30	4.14	4.50	4.08	3.94
18	3.91	4.10	3.86	3.87	35	4.30	4.70	4.23	3.99
Average	3.89	4.10	3.86	3.86	40	4.44	4.87	4.36	4.05
					Average	4.14	4.51	4.08	3.95

Table 8 Reliability of RC and PSC for material and member resistance factor system



Fig. 14 Comparison of reliability of MEMF, MATF and COMF for flexure (a) RC (b) PSC

concrete material factor is not significant for the example sections in this study.

Several values of resistance factors in different format are applied to RC and PSC flexural design and the reliability indexes,  $\beta$ 's, are plotted in Fig. 14. When  $\beta$ 's with the same values of  $\phi_s$  in MATF format and  $\phi_f$  in MEMF format are compared, MATF format yields almost the same but slightly smaller values of  $\beta$ 's than those of MEMF format as was shown before in Table 8. For example,  $\beta$ 's for  $\phi_s = \phi_f = 0.90$  or 0.95 are similar as can be seen from the lower two groups of lines in Fig. 14.

Also, when both MATF and MEMF are used to form COMF format, the effect of multiplying MEMF is the same as decreasing MATF by the amount of the decrease in the MEMF from the value 1.0. For example, when MEMF  $\phi_f = 0.95$  is multiplied to MATF format with  $\phi_s = 0.90$  to form COMF format, the values of  $\beta$ 's are almost the same as those of MATF format with  $\phi_s = 0.85$  as can be found from the second groups of lines from the top of Fig. 14.

Next, the effect of the reinforcement ratio on the safety of the girder section is examined. The equivalent member resistance factor,  $\phi_{equi}$ , is introduced for MATF format as defined in Eq. (18) for flexural moment,

$$\phi_{equi} = \frac{R_d}{R_n} = \frac{R(\phi_i X_{k,i})}{R_n} \tag{18}$$

where  $R_n$  denotes the nominal flexural strength.

Fig. 15 shows  $\phi_{equi}$  for two MATF sets of  $\phi_s = 0.90$  and 0.95 with  $\phi_c = 0.65$  fixed. As the reinforcement ratio increases,  $\phi_{equi}$  decreases for MATF system. For the reference purpose, MEMF sets of  $\phi_s = 0.85$  and 0.90 are also plotted in the figure. If MATF of  $\phi_s = 0.90$  is compared with MEMF of  $\phi_f = 0.90$ ,  $\phi_{equi}$  is similar at low reinforcement ratio but the difference increases as  $\rho$  increases until the MEMF line bent down and cross the MATF line at near  $0.7\rho_b$ . The MEMF line is bent down because the section reaches the transition section as defined in ACI (2005) and KCI (2007). When the reinforcement ratio is large and the net tensile strain of the reinforcement is smaller than 0.004, it is defined as the transition section which is located between the tension-controlled section and the compression-controlled section. The MEMF for the compression-controlled section.

However, in practical design for T-sections, due to the serviceability categories such as allowable



Fig. 15 Equivalent member resistance factor over reinforcement ratio

tensile stress in concrete, crack and deflection, etc., and also due to very large flange width, the reinforcement ratio is usually kept very low. Therefore, in the low  $\rho$  region, the equivalent member resistance factor,  $\phi_{equi}$ , is close to 0.9 for MATF with  $\phi_s = 0.90$  as was the case in Table 8 and Fig. 14. For the standard sections of both of PSC girders and the RC T-sections used in this study, it is calculated as  $\phi_{equi}=0.89$ .

## 4.3. Calibration of safety factors

By applying the reliability analysis procedure in this study, calibration of the safety factors with respect to the target reliability index  $\beta_T$  could be performed. Reference for a code calibration and reliability analysis program can be found from Faber *et al.* (2003). In this study, a computer program is written in order to include both material and member resistance factors for the reliability analysis as well as the load factors. Results of sample calculation are shown in Fig. 16 and Table 9.



Fig. 16 Variation of reliability index for calibration of a safety factor with different weights as shown in Table 9

D/(D+L)	Wi	$eta_i$	$\beta_T$	$w_i (\Delta \beta_i)^2$
0.1	0.00	2.95	3.75	0
0.2	0.00	3.03	3.75	0
0.3	0.00	3.12	3.75	0
0.4	0.10	3.23	3.75	0.02684
0.5	0.50	3.37	3.75	0.07201
0.6	1.00	3.54	3.75	0.04225
0.7	1.00	3.76	3.75	0.00007
0.8	1.00	3.97	3.75	0.04690
0.9	0.50	3.83	3.75	0.00352
Result : $\phi_s = 0.963$			$\Sigma(w_i(\Delta\beta_i))$	$)^{2})=0.19159$

Table 9 Example calibration result of material resistance factor to target reliability index

For the entire range of the ratio of dead load to total load, 0 < D/(D+L) < 1, the reliability indexes are calculated and the differences are squared and summed with weights multiplied. The calculation is iterated until minimum value of the difference is reached. In this particular example of PSC flexure, as the usual D/(D+L) values range from 0.55 to 0.75, weights are allocated as shown in Table 9, for example. Optimum material resistance factor  $\phi_s$  of tendon for  $\beta_T = 3.75$  can be determined as 0.963 in this case.

 $\beta_T$  can be different in design and assessment. The values between 2.5 and 4.7 for the reference period of 1 year proposed in different countries and international bodies are presented in SB-LRA (2007). The variations of the calibrated safety factors according to the target reliability index  $\beta_T$  are shown in Fig. 17 for the resistance and load factors. In Fig. 17(a) the calibrated values for the resistance factors  $\phi_f$  and  $\phi_s$  for the MEMF format and the MATF format, respectively, are shown by solid lines and are similar to each other over the studied range of  $\beta_T$ . The slight difference is due to the effect of  $\phi_c$  which is set to 0.65 for MATF format in this study. The effect of introducing the combination type of resistance factors is examined by setting  $\phi_s = 0.95$  for MEMF and  $\phi_f = 0.95$  for MATF and the calibrated values for  $\phi_f$  and  $\phi_s$  are plotted in dotted line labeled as COMF. The factors calibrated in COMF format are increased by 0.05 from the corresponding values of MEMF



Fig. 17 Variation of safety factors with respect to  $\beta_T$  (a) resistance factors and (b) load factors

and MATF. An example of COMF format is found in the assessment code where the design strength is calculated based on MATF format.

Following the similar procedure, load factors of  $\gamma_l$  and  $\gamma_d$  for live load and dead load, respectively, could be calibrated according to the target reliability index and the results are shown in Fig. 17(b). The basic values for safety factors used for the calibration of each safety factor are  $\phi_s=0.90$ ,  $\phi_c=0.65$ ,  $\gamma=1.75$  and  $\gamma_d=1.25$  in this example.

## 5. Conclusions

Reliability analysis is performed for a proposed limit state bridge design code. As the load model and the safety factors of the current code have been based mainly on the experience in Korea, the proposed code is the first version to attempt to introduce the probabilistic concept to design. In order to introduce the reliability concept to the bridge design code, a live load model is proposed based on truck weight field survey to update fast growing volumes of truck traffic. Also, test data of domestic material strengths are collected to model statistical properties of member strengths. Reliability analysis demonstrates that the proposed design code provides reliability level between 3.86 to 4.10 for RC and PSC flexure and shear on the average. The reliability level of the proposed code seems to be reasonable considering that it is much closer to the target reliability levels of the international design codes, which are around 3.5 or 3.8. In addition, the proposed design can become more economical than the current design of which the estimated reliability level is around 4.74 to 5.34 on the average which is recognized relatively too conservative.

A combination type of resistance factor format has been examined to compare the effect of the material resistance factor format introduced in the proposed code and the member resistance factor format of the current code. For the standard T-sections examined in this study, the material resistance factor for the steel reinforcement dominates the flexural strength of the section, and has similar effect on the member strength as the member resistance factor does. Sensitivity analysis shows that the concrete compressive strength does not affect much on the flexural strength of the concrete girder compared to the area and strength of the tensile reinforcement, effective depth, dead load and live load.

As the proposed design yields reinforcing steel areas slightly less than those of the current code for the standard sections, the changes in design practice is not abrupt and the design gets more economical. In addition, by developing statistical load and resistance model and introducing reliability based analysis procedure of the safety factors, it can be concluded that future changes in the design and construction environment could be taken into account more rationally by calibration of safety factors based on not only experience but also the statistical information of loads and resistances.

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