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# A correction method for objective seismic damage index of reinforced concrete columns

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**Abstract.** In this paper, the sensitivity of a plastic-damage-based structural damage index on mesh density is studied. Multiple finite element meshes with increasing density are used to investigate their effect on the damage index values calculated from nonlinear finite element simulations for a reinforced concrete column subjected to cyclic loading. With the simulation results, this paper suggests a correction method for the objective damage index based on nonlinear regression of volumetric tensile damage ratio data. The modified damage index values are presented in the quasi-static cyclic simulation to show the efficacy of the suggested correction method.

**Keywords:** mesh-dependency; seismic damage index; reinforced concrete column; finite element analysis; plastic-damage model

## 1. Introduction

Seismic structural damage indices are used to evaluate the current damage states of structures due to cyclic and dynamic loads such as earthquakes. It is particularly important in the application to post-earthquake damage assessment, seismic fragility evaluation, loss estimation and retrofitting of structures. There are various structural damage indices proposed in terms of strains, curvature, rotation, lateral displacement, dissipated energy, etc. to capture structural damages due to excessive displacement and repetitive deformation of members (Park and Ang 1985, Teran-Gilmore and Jirsa 2005, Vielma *et al*. 2008, Jiang *et al*. 2011, Liang *et al*. 2011, Sinha and Shiradhonkar 2012, Hadzima-Nyarko *et al*. 2014, Azhdary and Shabakhty 2014). Those response-based damage indices, however, have objectivity problem because of ambiguity in determining their basis parameter values such as ultimate displacement and yield strength (Kang and Lee 2016).

To overcome the shortcomings, a plastic-damage-based structural damage index has been suggested for reinforced concrete columns subject to cyclic loading (Kang and Lee 2016). The damage index is formulated as a single monotonically-increasing function of the volume weighted average of local tensile damage variable over a reinforced concrete column, which is determined by the Lee and Fenves plastic-damage model. Despite the advantages of the plastic-damage-based structural damage index to reflect local stiffness degradation of structural members, the damage index value is expected to be sensitive to finite element mesh density because its evaluation is based on the result of nonlinear finite element analysis.

In this paper, the sensitivity of the plastic-damage-based structural damage index on mesh density is studied. Multiple finite element meshes with increasing density are used to investigate their effect on the damage index values calculated from nonlinear finite element simulations for a reinforced concrete column subjected to cyclic loading. With the simulation results, this paper suggests a correction method for the objective damage index based on nonlinear regression of volumetric tensile damage ratio data. Efficacy of the correction is discussed by comparing the modified damage index values with those obtained using individual meshes. In Section 2, the Lee and Fenves plastic-damage model and the reinforced concrete composite model in the context of finite element analysis are outlined. Then, in Section 3, the plastic-damage-based structural damage index is discussed. In Section 4, finite element modeling and damage analysis of the reinforced concrete column is presented. Finally, in Section 5, the damage index correction method and the results are presented.

## 2. Plastic-damage reinforced concrete model

The plastic-damage model (Lee and Fenves 1998a, 1998b, ABAQUS 2015) has been used to determine the local damage state on a cyclically loaded concrete structure. The local tensile damage variable from the plastic-damage model becomes a key variable of the structural damage index suggested by Kang and Lee (2016). In this section the plastic-damage model and the composite finite element model for damaged reinforced concrete structures are outlined (Lee and Fenves 1998b, Lee 2001).

In the Lee and Fenves plastic-damage model, the stress

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$\boldsymbol{\sigma}$ is decomposed into the degradation damage, $(1-D)$, and the effective stress, $\bar{\boldsymbol{\sigma}}$:

$$\boldsymbol{\sigma} = (1-D)\bar{\boldsymbol{\sigma}} \tag{1}$$

The scalar variable $D$ is assumed to represent the state of degradation damage on the stiffness: $\mathbf{E} = (1-D)\mathbf{E}_0$ where $\mathbf{E}_0$ is the initial elastic stiffness tensor. Using the yield surface function $F$, the effective stress and its admissibility is defined by the following inequality equation

$$\bar{\boldsymbol{\sigma}} = \mathbf{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \in \{\bar{\boldsymbol{\sigma}} | F(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \le 0\} \tag{2}$$

where $\boldsymbol{\varepsilon}^p$ is the plastic strain and $\boldsymbol{\kappa} = [\kappa_t \quad \kappa_c]^{\mathrm{T}}$ is a damage variable vector consisting of two monotonically-increasing scalars: the tensile damage variable $\kappa_t$ and the compressive damage variable $\kappa_c$. The factorization of the strength function into two functional forms, one for the effective stress and the other for the degradation damage variable, leads to the following damage evolution equation described with $\mathbf{H}$, a vector function of the effective stress and damage variable vector

$$\dot{\boldsymbol{\kappa}} = \dot{\lambda}\mathbf{H}(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) \tag{3}$$

where $\dot{\lambda}$ is a non-negative function referred to as the plastic consistency parameter.

For modeling the cyclic behavior of concrete, which has very different tensile and compressive yield strengths, it is necessary to use two cohesion variables in the yield function: $c_t$, a tensile cohesion variable, and, $c_c$, a compressive cohesion variable. The yield function in Lubliner *et al*. (1989), which only models isotropic hardening behavior in the classical plasticity sense, is modified to include two cohesion variables as follows

$$F(\bar{\boldsymbol{\sigma}}, \boldsymbol{\kappa}) = \frac{1}{1-\alpha}\left[\alpha I_1 + \sqrt{3J_2} + \beta(\boldsymbol{\kappa})\langle\hat{\sigma}_{max}\rangle - c_c(\boldsymbol{\kappa})\right] \tag{4}$$

where $\hat{\sigma}_{\max}$ denotes the algebraically maximum principal stress, and $\alpha$ is a parameter which is evaluated by the initial shape of the yield function. The evolution of the yield function is determined by defining $\beta$, which is, in contrast, a constant in the original model

$$\beta = \frac{c_c(\boldsymbol{\kappa})}{c_t(\boldsymbol{\kappa})}(1-\alpha) - (1+\alpha) \tag{5}$$

A non-associative flow rule derived from the Drucker-Prager-type plastic potential function is used to generate the dilatancy exhibited by frictional materials

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\left(\frac{\mathbf{s}}{\|\mathbf{s}\|} + \alpha_p \mathbf{I}\right) \tag{6}$$

where $\|\mathbf{s}\| = \sqrt{\mathbf{s}:\mathbf{s}}$ denotes the norm of the deviatoric effective stress $\mathbf{s}$, and the parameter $\alpha_p$ is chosen to give the proper dilatancy for concrete.

The experimental cyclic tests of concrete demonstrate that the degradation of stiffness from microcracking in tension and compression becomes more significant as the strain increases. The mechanism of stiffness degradation under cyclic loading is complicated because of the opening and closing of microcracks. The crack opening/closing behavior can be modeled as elastic stiffness recovery during elastic unloading from a tensile state to a compressive state. Using a multiplicative parameter $0 \le s \le 1$ on the tensile degradation variable $D_t$, the degradation damage variable is defined: $D = 1 - (1 - D_c(\boldsymbol{\kappa}))(1 - sD_t(\boldsymbol{\kappa}))$, where $D_c$ is the compressive degradation variable. Accordingly, the total stress $\boldsymbol{\sigma}$ is written as

$$\boldsymbol{\sigma} = (1 - D_c(\boldsymbol{\kappa}))(1 - sD_t(\boldsymbol{\kappa}))\mathbf{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \tag{7}$$

The parameter is chosen to represent the stiffness recovery as follows

$$s(\bar{\boldsymbol{\sigma}}) = \frac{\sum_{i=1}^{3}\langle\hat{\bar{\sigma}}_i\rangle}{\sum_{i=1}^{3}|\hat{\bar{\sigma}}_i|} \tag{8}$$

To avoid the ill-posedness in representing the softening behavior with a model based on rate-independent plasticity, the regularization scheme based on the Duvaut-Lions viscoplastic model is applied to the rate-independent plastic strain and degradation damage variable (Lee and Fenves 1998b, Lee 2001)

$$\dot{\boldsymbol{\varepsilon}}^i = \frac{1}{\mu}(\boldsymbol{\varepsilon}^p - \boldsymbol{\varepsilon}^i) \tag{9}$$

$$\dot{\bar{D}} = \frac{1}{\mu}(D - \bar{D}) \tag{10}$$

where $\mu$ is the viscosity parameter, $\boldsymbol{\varepsilon}^i$ is the viscoplastic strain, and $\bar{D}$ is a viscously regularized degradation variable. Accordingly, the stress-strain relation in Eq. (7) is restated using the new rate-dependent variables in Eqs. (9)-(10) as

$$\boldsymbol{\sigma} = (1 - \bar{D})\mathbf{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^i) \tag{11}$$

For a finite element reinforced concrete model, the composite of the plastic-damage concrete, reinforcing steel bar and bond-slip link elements is used (Lee 2001). To reproduce the realistic reinforced concrete column behavior under cyclic and dynamic loading, a concrete model must represent initiation and localization of tensile cracking and compressive crushing damage, as well as stiffness degradation and stiffness recovery on crack closing, which is successfully implemented by the Lee and Fenves plastic-damage model (Lee and Fenves 1998a).

In this composite model approach, the modified uniaxial model proposed by Filippou *et al.* (1983) is used for the cyclic constitutive relation of longitudinal and transverse steel reinforcement. The constitutive model for the steel reinforcement is implemented in truss elements to represent reinforcing bars separately from concrete. This separated modeling provides better representation for the cracked reinforced concrete body than the so-called embedded model, because this modeling approach can simulate more damaged states (Lee 2001). In order to model the cyclic bond-slip behavior, reinforcing bars are assumed to be indirectly connected to surrounding concrete through imaginary bond-slip material, which is modeled by the discrete link described in Eligehausen *et al.* (1983).

## 3. Plastic-damage-based structural damage index

The new structural damage index for cyclically-loaded reinforced concrete columns based on a local tensile

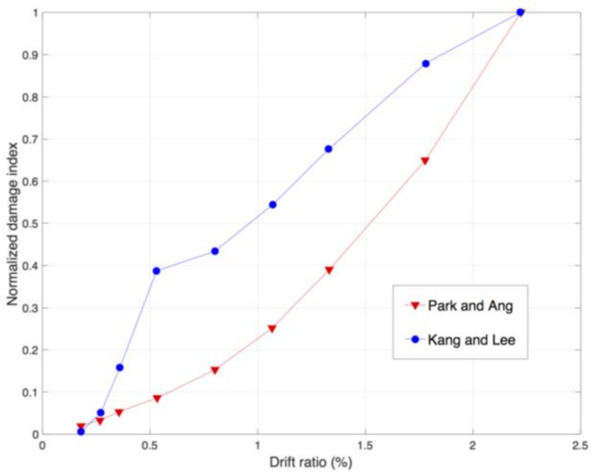



Fig. 1 Comparison of normalized damage indices versus drift ratio curves for a reinforced concrete column

damage variable of the Lee and Fenves plastic-damage model has been suggested by Kang and Lee (2016). In contrast to the response-based damage index such as Park and Ang's one (1985), the suggested global damage index is well-defined in the form of a single monotonically-increasing function of the volume weighted average of local damage distribution, and provides the necessary computability and objectivity because it does not require to compute the ultimate displacement and yield strength.

Since the damage variable of the Lee and Fenves plastic-damage model gives the fundamental information for measuring a damage level of the entire structure subject to cyclic and dynamic loading, it can be used to generate a major damage index parameter, the volumetric damage ratio $K_\upsilon$, which is defined as

$$K_\upsilon = \frac{\upsilon \int_\Omega \kappa_t d\Omega}{V} \tag{12}$$

where $\kappa_t$ is the local tensile damage variable in Eq. (3), $\upsilon$ is the modification factor, $\Omega$ is the volumetric domain, and $V$ is the total volume of a column. In Eq. (12), the modification factor $\upsilon$ is introduced to account for the slenderness effect on the damage distribution over a column structure

$$\upsilon = \min(H, nD)/H \tag{13}$$

where $H$ is the height and $D$ is the width (or diameter) of the column. It is noted that the range of the volumetric damage ratio becomes: $0 \le K_\upsilon \le 1$. It is also noted that $K_\upsilon$ is a monotonically-increasing function of the field local damage variable $\kappa_t$, which is also monotonically-increasing at all the points in $\Omega$.

From the nonlinear regression of experimental column test data, the global damage index, $\chi$, is defined in the form as

$$\chi = cQ(\rho_w)K_\upsilon^{1.5} \tag{14}$$

where $c$ is a coefficient, and $Q(\rho_w) = 0.8^{\rho_w}$ is a confinement factor function of the lateral confinement reinforcement ratio, $\rho_w$, in percentage. After the calibration to determine the coefficient $c$ in Eq. (14), the plastic-damage-based structural damage index by Kang and Lee (2016) becomes the final formulation written as

$$\chi = 27(0.8^{\rho_w})K_\upsilon^{1.5} \tag{15}$$

As shown in Fig. 1, regardless of the difference in the curve shape, both Kang and Lee's (2016) and Park and Ang's (1985) damage indices show the monotonically increase function for the peak drift ratio value for a reinforced concrete column specimen, which implies that the plastic-damage-based structural damage index is also appropriate as a damage index function.

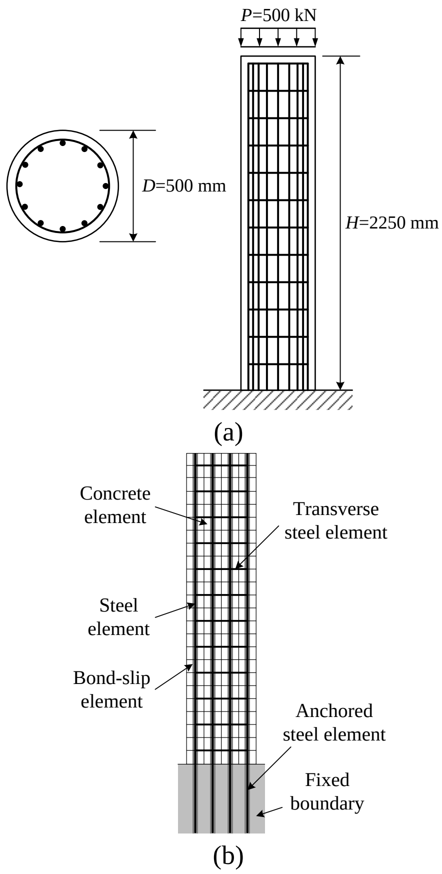
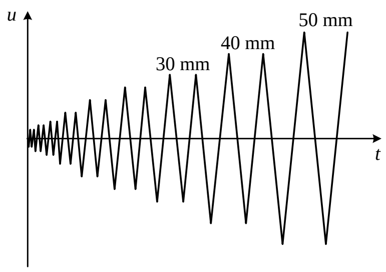



Fig. 2 Finite element modeling of the reinforced concrete column; (a) column configuration; (b) composition of reinforced concrete structural elements

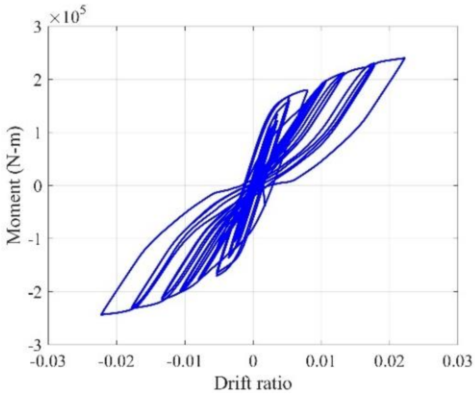
## 4. Mesh sensitivity analysis of the seismic damage index

### *4.1 Numerical modeling of a reinforced concrete column*

To apply the damage index $\chi$ in seismic damage quantification, a reinforced concrete column subjected to a lateral cyclic loading at the top is considered. Fig. 2(a) shows the configuration of the column, of which the diameter and height are 500 mm and 2250 mm, respectively. The column is fixed at the bottom end while supporting a vertical static load of 500 kN. The compressive strength of concrete is 28 MPa. Longitudinal and transverse




(a)




(b)

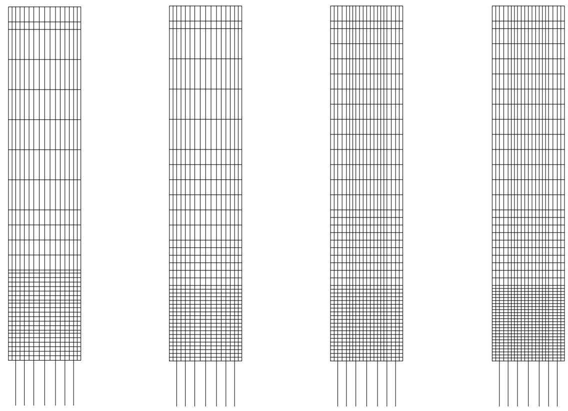
Fig. 3 Moment vs. drift ratio hysteresis curves of the column subjected to a lateral displacement cyclic loading; (a) cyclic loading; (b) hysteresis curve

reinforcement ratios of the column are 1.89% and 0.39%, respectively. Fig. 2(b) shows a finite element model of the reinforced concrete column. The model consists of a plane stress isoparametric quadrilateral elements with the Lee and Fenves plastic damage model for concrete, nonlinear truss element for rebar, and nonlinear bond-slip connection element with zero length. The concrete, reinforcing bar, and bond-slip models are implemented within FEAP (Taylor 2008), a nonlinear finite element analysis program. The column is subjected to a lateral displacement cyclic loading at the top as prescribed in Fig. 3(a). By using the described column model, nonlinear finite element analysis was performed to calculate the response of the column under the cyclic loading. Fig. 3(b) represents moment versus drift ratio hysteretic curves obtained by the quasi-static numerical simulation. During the finite element simulation, initiation and localization of concrete tensile cracking were quantified, from which the structural damage index $\chi$ could be evaluated using Eq. (15).

### *4.2 Mesh sensitivity of damage index*

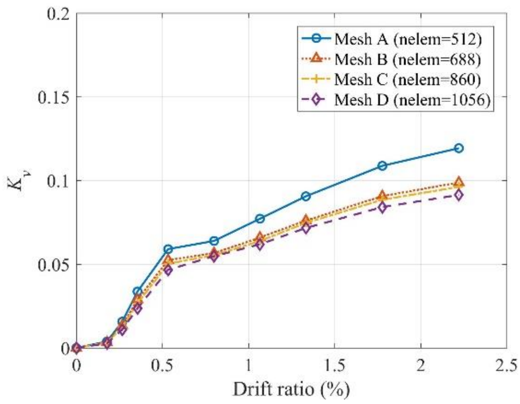
To investigate the mesh sensitivity of the seismic damage index, four different meshes, as shown in Fig. 4, are applied in the finite element analysis of the reinforced concrete column. The meshes have 512, 688, 860, and 1056 concrete elements, respectively. If the number of concrete elements is greater, the number of rebar and bond-slip elements is also greater. Therefore, the column model has increasingly fine mesh from mesh A to mesh D.

Fig. 5(a) shows the change of volumetric damage ratio

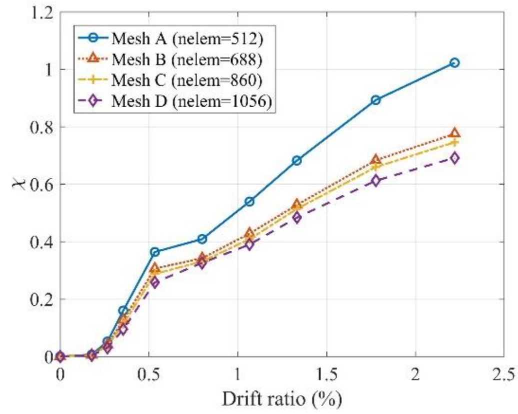


(a) Mesh A (b) Mesh B (c) Mesh C (d) Mesh D

Fig. 4 Four meshes used for modeling of reinforced concrete column; the number of concrete elements is (a) 512, (b) 588, (c) 860, and (d) 1056




(a)




(b)

Fig. 5 Change of $K_v$ and $\chi$ with peak drift ratio of the reinforced concrete column subjected to a cyclic loading; (a) volumetric damage ratio, $K_v$; (b) seismic damage index, $\chi$

$K_v$ with the peak drift ratio of cyclic loading for the four mesh cases. The graphs indicate that volumetric tensile damage over the entire column domain increases monotonically as the peak drift ratio is increased. In Fig. 5(b), the seismic damage index curves are plotted from the finite element simulation results for each of the mesh cases. As the peak drift ratio in the cyclic loading is increased, the

Table 1 Coefficients $a_0$ and $a_1$ of power function $K_v = a_0 n^{a_1}$

| Drift ratio (%) | Power function coefficient | |
|---|---|---|
| | $a_0$ | $a_1$ |
| 0.18 | 0.0028 | -0.4473 |
| 0.27 | 0.0111 | -0.4749 |
| 0.36 | 0.0238 | -0.4762 |
| 0.53 | 0.0467 | -0.3152 |
| 0.80 | 0.0535 | -0.2178 |
| 1.07 | 0.0603 | -0.3119 |
| 1.33 | 0.0698 | -0.3260 |
| 1.78 | 0.0820 | -0.3584 |
| 2.22 | 0.0889 | -0.3723 |

damage index values are monotonically increased. It can be seen, from Fig. 5, that the values of $K_v$ and $\chi$ converge as the number of elements in the column mesh increases. When using the smaller number of elements, the damage index tends to overestimate damage state for each of the peak drift ratios considered. To overcome such mesh sensitivity problem of the global damage index $\chi$ and thus to improve the objectivity of the damage index, a correction method for reflecting mesh density is proposed for the damage index by using nonlinear data regression.

## 5. Correction method for damage index objectivity

Since the seismic damage index $\chi$ is a function of $K_v$ as in Eq. (15), it can be modulated to reflect mesh density by considering the mesh dependency of $K_v$. To establish the relation of $K_v$ with mesh density, the values of $K_v$ at each drift ratio are optimized by regression analysis to the number of elements. Specifically, a nonlinear regression equation of power function form as in Eq. (16) is utilized to set up the relationship at each drift ratio

$$K_v = a_0 n^{a_1} \tag{16}$$

In Eq. (16), $a_0$ and $a_1$ are power function coefficients, and $n$ is the normalized mesh density, which is the number of elements normalized with the largest number of elements in a mesh set. The nonlinear regression analysis can be performed at each drift ratio to obtain the power function coefficients. The coefficients $a_0$ and $a_1$ obtained by the nonlinear regression analysis at each drift ratio are listed in Table 1. Fig. 6 shows the regression curves of $K_v$ for the normalized number of elements at each drift ratio.

To obtain the optimal relationship between the drift ratio and the power function coefficients, a linear regression analysis is performed for each of the coefficients $a_0$ and $a_1$. Fig. 7 shows the regression lines for the power function coefficients against drift ratio. From the regression line equations, one can get the coefficients $a_0$ and $a_1$ for specific drift ratio so as to calculate $K_v$ by Eq. (16). Then the calculated value of $K_v$ is used in Eq. (15) to evaluate the damage index $\chi$. By the described procedure, mesh density can be reflected in calculating the global damage index $\chi$ for structures subjected to cyclic or dynamic loadings. Fig. 8 shows a flowchart of the damage index

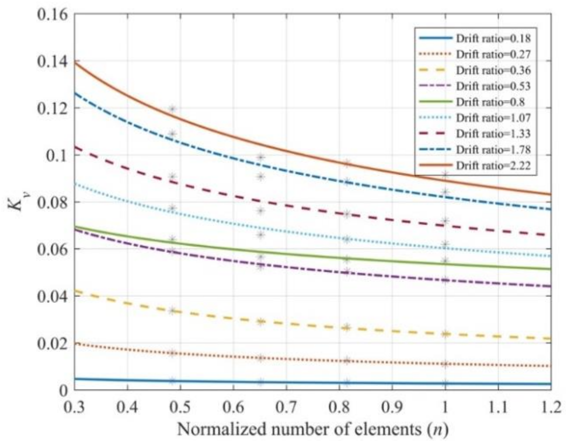
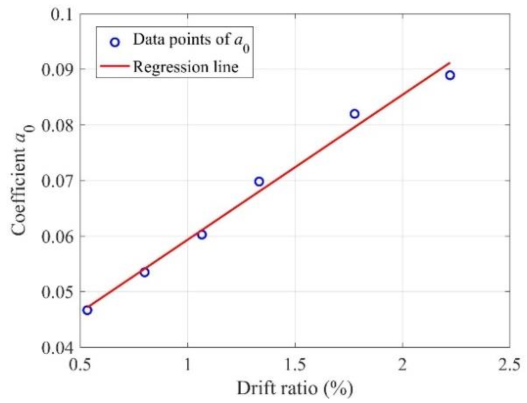
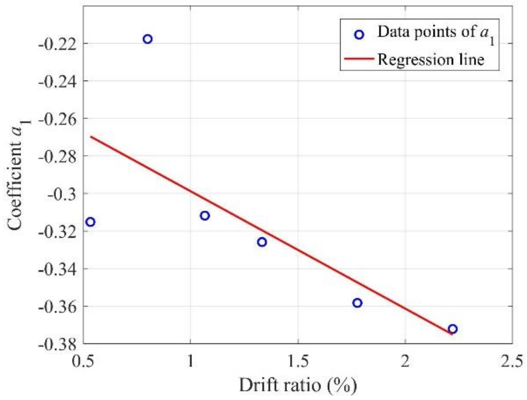



Fig. 6 Regression curves of $K_v$ for normalized number of elements in finite element mesh




(a)




(b)

Fig. 7 Regression lines for the power function coefficients (a) $a_0$ and (b) $a_1$

correction method to reflect mesh density. In Fig. 9, tensile damage contours obtained from the cyclic loading simulation of the reinforced concrete column are presented. It can be said that the column damage is overestimated when the column model has low mesh density.

In Fig. 10, the corrected damage index curve using the mesh density of Mesh D, $n = 1$, is plotted with the originally simulated damage index curves against drift ratio with Meshes A to D. The damage index was corrected after the drift ratio of 0.53% because the damage index values at

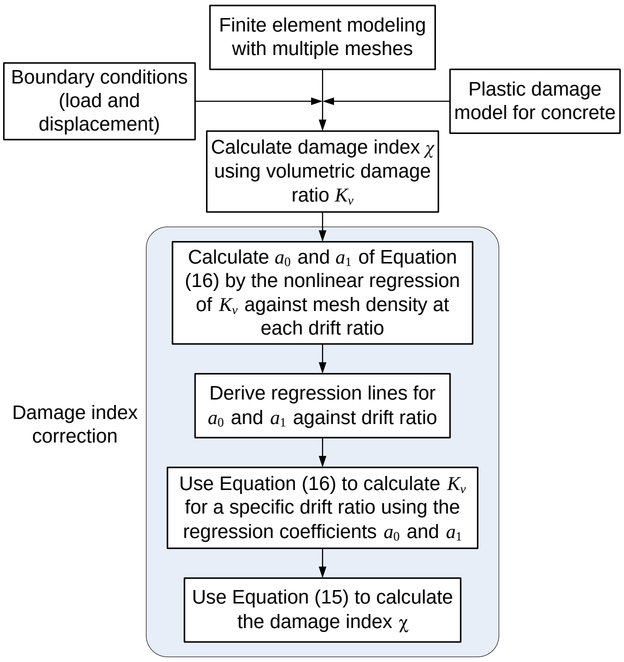



Fig. 8 Flowchart of the damage index correction method to reflect mesh density

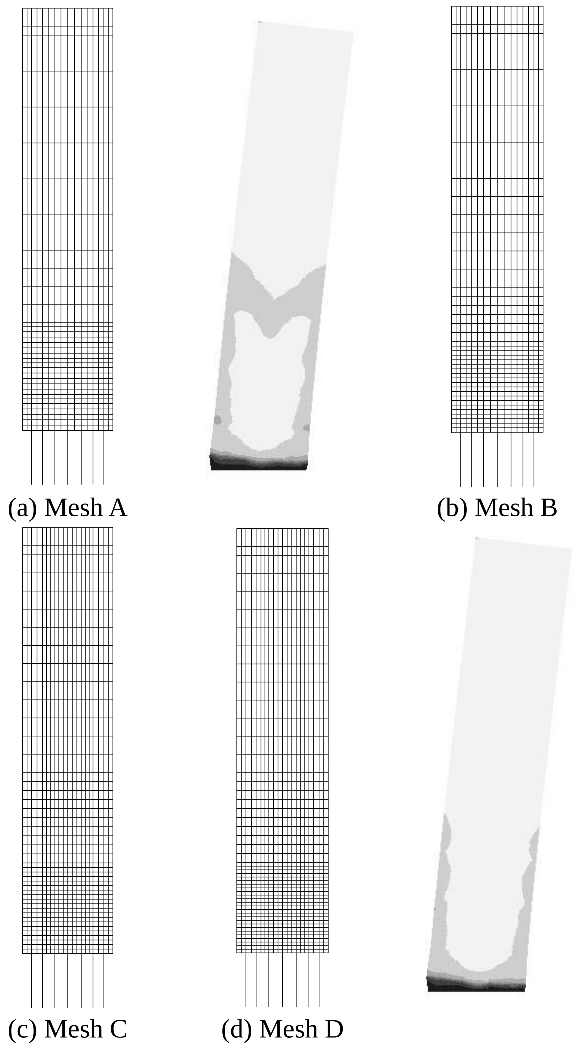



Fig. 9 Tensile damage contours obtained from the cyclic loading simulation of the reinforced concrete column

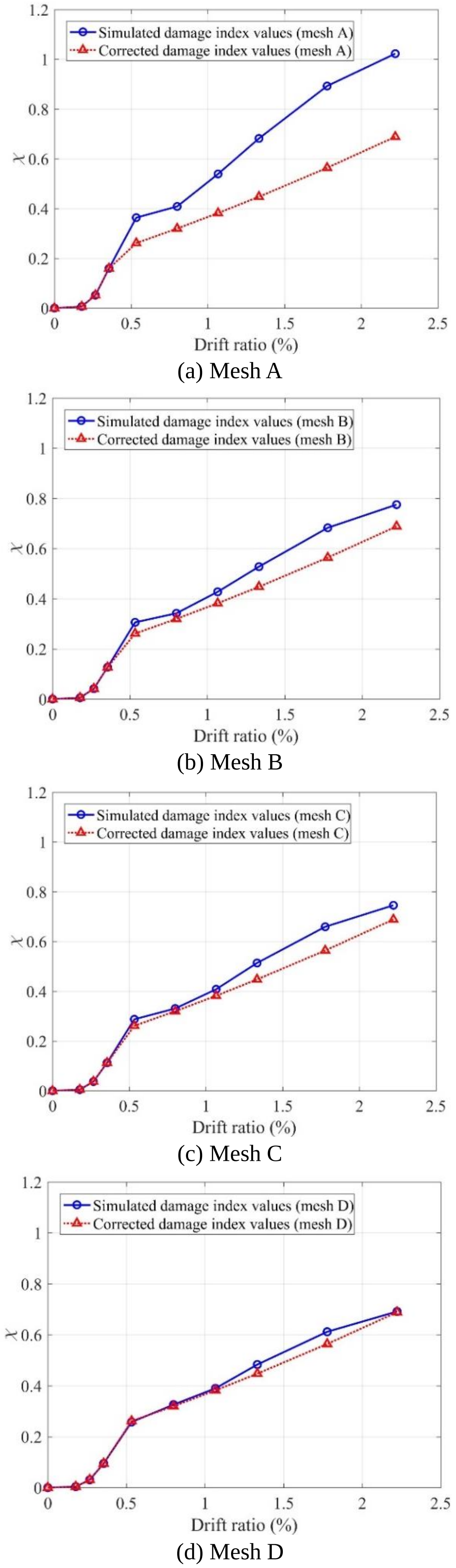



Fig. 10 Damage index $\chi$ corrected using normalized mesh density $n = 1$

smaller drift ratios show little difference for the Meshes A to D. It is noted, from Fig. 5(b), that the damage index curves for Meshes B, C, and D are close to each other, whereas the damage index curve for Mesh A is somewhat farther up. In Fig. 10, the overestimated structural damage index values computed by using Mash A, a coarse mesh, are most significantly corrected by the proposed damage index correction method. It is shown in Fig. 10 that the mesh becomes denser from Meshes B to D, the discrepancies between the original and corrected damage index values become smaller.

From the observation of Fig. 6, $K_v$ tends to converge as the mesh density in the normalized mesh density, $n$, approaches to the value of 1.2, which yields the following formulation for the objective structural damage index $\chi$ from Eqs. (15)-(16)

$$\chi = 27(0.8^{\rho_w})(0.0261d + 0.0333)(1.2)^{-1.5(0.0625d+0.2364)} \tag{17}$$

where the drift ratio, $d \geq 0.53$. The objective structural damage index in Eq. (17) can be efficiently used to evaluate damage index values in seismic fragility analysis of the reinforced concrete column. It is noted that the present procedure to implement the suggested correction method is applicable for other reinforced concrete columns subject to cyclic and earthquake loading.

## 6. Conclusions

An objectivity problem of the plastic-damage-based structural damage index in the result of nonlinear finite element simulation has been studied. Multiple finite element meshes for a reinforced concrete column were used to investigate the effect of mesh density on the structural damage index in a quasi-statically cyclic loading condition. From the simulation results, damage states of the test column tend to be overestimated when the mesh becomes coarse. A new correction method to alleviate the mesh dependency problem of the structural damage index is suggested based on nonlinear regression of volumetric tensile damage ratio data. A nonlinear regression equation of power function form is utilized to set up the relationship at each drift ratio. Then, linear regression analysis is performed for each of the power function coefficients to complete the correction formulation as a function of mesh density and drift ratios. By applying the suggested method, the structural damage index values computed with a coarse mesh were corrected appropriately to be comparable with those evaluated with a finer mesh. The present correction method provides an efficient procedure to improve the objectivity of the plastic-damage-based structural damage index of reinforced concrete columns subject to cyclic and earthquake loading.

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