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Thermoelastic beam in modified couple stress thermoelasticity induced by laser pulse

Rajneesh Kumar[1] and Shaloo Devi[*2]

[1]*Department of Mathematics, Kurukshetra University, Kurukshetra, India*
[2]*Department of Mathematics & Statistics, Himachal Pradesh University, Shimla, India*

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Abstract. In this study, the thermoelastic beam in modified couple stress theory due to laser source and heat flux is investigated. The beam are heated by a non-Guassian laser pulse and heat flux. The Euler Bernoulli beam theory and the Laplace transform technique are applied to solve the basic equations for coupled thermoelasticity. The simply-supported and isothermal boundary conditions are assumed for both ends of the beam. A general algorithm of the inverse Laplace transform is developed. The analytical results have been numerically analyzed with the help of MATLAB software. The numerically computed results for lateral deflection, thermal moment and axial stress due to laser source and heat flux have been presented graphically. Some comparisons have been shown in figures to estimate the effects of couple stress on the physical quantities. A particular case of interest is also derived. The study of laser-pulse find many applications in the field of biomedical, imaging processing, material processing and medicine with regard to diagnostics and therapy.

Keywords: thermoelastic beam; modified couple stress theory; laser-pulse; classical coupled theory; heat flux

1. Introduction

Classical first gradient approaches in continuum mechanics do not address the size dependency that is observed in smaller scales. Consequently, a number of theories that include higher gradients of deformation have been proposed to capture, at least partially, size-effects at the nano-scale. Additionally, consideration of the second gradient of deformation leads naturally to the introduction of the concept of couple-stresses. Thus, in the current form of these theories, the material continuum may respond to body and surface couples, as well as spin inertia for dynamical problems.

Voigt (1887) was the first to introduce the concept of couple stresses in the material, by assuming that the interaction between two parts of the continuum not only depends upon force stress vector but also on the couple stress vector. This leads to the description of stress by means of two skew symmetric tensors namely force stress tensor and couple stress tensor. However, Cosserat and Cosserat (1909) gave the mathematical formulation to analyze materials with couple stresses, by considering that the deformation of the medium is described by displacement vector an independent rotation vector. Displacements and rotations are associated with skew symmetric stresses and couple stresses through constitutive relations. This was contrary to the classical theory which described stress as symmetric tensor. In spite of its novelty, this theory was not recognized at that time, later on many alternative theories on the same idea were given by Toupin (1962), Mindlin and Tiersten (1962), Mindlin (1964) and Koiter (1964).

When a solid is illuminated with a laser pulse, absorption of the laser pulse results in a localized temperature increase, which in turn causes thermal expansion and generates a thermoelastic waves in the solid. Most laser applications in material processing and medicine involve using of a continuous wave (CW) laser, short pulsed lasers are being used in a variety of applications such as remote sensing, optical tomography, laser surgery and ablation processes. The thermoelastic waves induced by a pulsed laser in solid is of great interest to many researchers due to extensive applications of pulsed laser technologies in biomedical, material processing, non-destructive detecting, manufacturing and characterization.

Pulsed lasers have the additional ability to control the width and depth of heating as well as induce high heating or cooling rates because of higher peak powers and shorter time duration. The uses of short-pulsed lasers in medicine, with regard to diagnostics and therapy, has gained attention in the last decade.

The advantages of using short-pulsed lasers rather than more traditional methods for surgical treatment include the precise control of the output energy of the device and the ability to control energy dissipation and the heat-affected zone. Thus, pulsed laser is used in a number of high-precision medical procedures like neurosurgery, ophthalmology, corneal surgery, and angioplasty.

Yang *et al.* (2002) modified the classical couple stress theory and proposed a modified couple-stress model, in which the couple stress tensor is symmetrical and only one material length parameter is needed to capture the size effect which is caused by micro-structure. Bernoulli-Euler

∗Corresponding author
E-mail: shaloosharma2673@gmail.com

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beam model based on a modified couple stress theory studied by Park and Gao (2006). Thermoelastic wave induced by pulsed laser heating discussed by Wang and Xu (2001). Banerjee *et al.* (2005) proposed the problem of temperature distribution in different materials due to short pulse laser. Ai Kah Soh *et al.* (2008) studied the vibration of micro/nano scale beam resonators induced by ultra short pulsed laser by considering the thermoelastic coupling term. Sun *et al.* (2008) explained the vibration phenomenon during pulsed laser heating of a micro beam using the Laplace transform technique.

Ghader *et al.* (2012) discussed problem of thermoelastic damping in a micro-beam resonator using modified couple stress theory. An eigenvalue formulation and Galerkin finite element method used to evaluate the problem of thermoelastic damping in vented microelectromechanical systems (MEMS) beam resonators presented by Guo *et al.* (2013). Sirafy *et al.* (2014) investigated the problem of thermoelastic beams in the context of dual-phase-lag model. The thermoelastic interaction in a gold nano beam resonator induced by ramp type heating in the context of three-phase-lag model of thermoelasticity theory studied by Sur and Kanoria (2014). A problem of two-temperature generalized thermoelasticity induced by pulsed laser heating is discussed in the context of coupled thermoelasticity and L-S theories. Mohammad-Abadi and Daneshmehr (2014) studied the size dependent buckling analysis of microbeams based on modified couple stress theory with high order theories and general boundary conditions.

Abouelregal and Zenkour (2014) discussed the problem of an axially moving microbeam subjected to sinusoidal pulse heating and an external transverse excitation with one relaxation time by using Laplace transform and also studied the effects of the pulse-width of thermal vibration, moving speed and the transverse excitation. Sharma and Kaur (2014) studied transverse vibrations in thermoelastic-diffusive thin micro beam based on Euler-Bernoulli theory under clamped-clamped boundary conditions. Zenkour and Abouelregal (2015) studied the problem of thermoelastic vibration of an axially moving microbeam subjected to sinusoidal pulse heating. Yaghoub *et al.* (2015) studied the size-dependent equations of motion for functionally graded cylindrical shell on the basis of modified couple stress theory. Dehrouyeh-Semnani *et al.* (2015) studied the problems of microbeams based on modified couple stress theory.

Keeping in view the applications of sensors in detecting Infrared imaging, chemical and biological agents sensing, design and construction of precision thermometers as well as the use of beam type components and devices, in addition to mechanics and civil structures, the present study is devoted to investigate the behavior of lateral deflection, thermal moment and axial stress average of thermoelastic beam in the modified couple stress theory. The Laplace transform technique has been used to find the analytic solution of the model. The effect of couple stress on lateral deflection, thermal moment and axial stress average for Coupled thermoelastic (CT) theory under the influence of laser source and heat flux are computed numerically and shown graphically. A particular case of interest is also deduced from the present investigation.

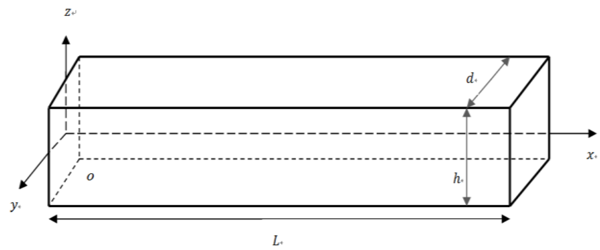

Fig. 1 Problem description

2. Basic equations

Following Yang *et al.* (2002) and Nowacki (1976) the equations governing the modified couple stress generalized thermoelastic medium in absence of body forces are:

(i) Constitutive relations

$$t_{ij} = \lambda e \delta_{ij} + 2\mu e_{ij} - \frac{1}{2}\, e_{kij} m_{lk,l} - \beta T \delta_{ij}, \tag{1}$$

$$m_{ij} = 2\alpha \chi_{ij}, \tag{2}$$

$$\chi_{ij} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right), \tag{3}$$

$$\omega_i = \frac{1}{2} e_{ipq} u_{q,p}, \tag{4}$$

i,j,k=1,2,3.

where t_{ij} are the components of stress tensor, λ and μ are material constants, δ_{ij} is Kronecker's delta, e_{ij} are the components of strain tensor, e_{ijk} is alternate tensor, m_{ij} are the components of couple-stress, $\beta=(3\lambda+2\mu)\alpha_T$, α_T is the coefficient of linear thermal expansion, T is the temperature change, α is the couple stress parameter, χ_{ij} is symmetric curvature, ω_i is the rotational vector.

(ii) Equations of motion

$$\left(\lambda + \mu + \frac{\alpha}{4}\Delta\right)\nabla\left(\nabla.\boldsymbol{u}\right) + \left(\mu - \frac{\alpha}{4}\Delta\right)\nabla^2 \boldsymbol{u} - \beta \nabla T = \rho \ddot{\boldsymbol{u}}, \tag{5}$$

where $\boldsymbol{u}=(u,v,w)$ is the components of displacement, ρ is the density, Δ is the Laplacian operator, ∇ is del operator.

(iii) Equation of heat conduction are given by Soh *et al.* (2008)

$$K\Delta T - \rho c_e \frac{\partial T}{\partial t} = T_0 \beta \frac{\partial}{\partial t}\left(\nabla.\boldsymbol{u}\right) - S_i, \tag{6}$$

where K is the coefficient of the thermal conductivity, c_e is the specific heat at constant strain, T_0 is the reference temperature assumed to be such that T/T_0<<1, S_i is the initial heat source.

3. Formulation of the problem

Let us consider an homogeneous isotropic, rectangular modified couple stress thermoelastic beam of length ($0\leq x\leq L$), width ($-d/2\leq y\leq d/2$) and thickness ($-h/2\leq z\leq h/2$), where x, y and z are Cartesian axes lying, respectively, along the length, width and thickness of the beam. We define the x-axis along the axis of the beam and the y and z axes correspond to the width and thickness, respectively.

According to the fundamental Euler-Bernoulli theory for small deflection of a simple bending problem, the displacement components are given by

$$u(x,y,z,t) = -z\frac{\partial w}{\partial x},\ v(x,y,z,t) = 0,\ w(x,y,z,t) = w(x,t), \tag{7}$$

where $w(x,t)$ is the lateral deflection of the beam and t is the time. The constitutive relation Eq. (1) in one-dimension along the axis and with the help of Eq. (7), we obtain

$$t_x = -(\lambda + 2\mu) z\frac{\partial^2 w}{\partial x^2} - \beta T, \tag{8}$$

Then the flexural moment of the cross-section of the beam is given by

$$M = M_\sigma + M_m = d\left(\int_{\frac{-h}{2}}^{\frac{h}{2}} t_x z\ dz + \int_{\frac{-h}{2}}^{\frac{h}{2}} m_{xy} dz\right) \tag{9}$$

where M_σ and M_m are the components of the bending moment due to the classic stress and couple stress tensors respectively.

Making use of Euler-Bernoulli assumption Eq. (7) and with the aid of Eq. (8) in Eq. (9), we obtain

$$M = -(\lambda + 2\mu) I\frac{\partial^2 w}{\partial x^2} - M_T - \alpha h\frac{\partial^2 w}{\partial x^2}, \tag{10}$$

where I is the second moment of the cross-section area of the beam and M_T is the thermal moment and I and M_T are given by

$$I = \int_{\frac{-h}{2}}^{\frac{h}{2}} dz^2 dz = \frac{dh^3}{12},\ M_T = \beta d\int_{\frac{-h}{2}}^{\frac{h}{2}} Tz\ dz. \tag{11}$$

The equation of transverse motion of the beam is given by (Rao 2007).

$$\frac{\partial^2 M}{\partial x^2} - \rho A\frac{\partial^2 w}{\partial t^2} = 0, \tag{12}$$

where ρ denotes the beam density and $A=dh$ is the cross-sectional area of the beam.

Now from Eqs. (10) and (12), we obtain

$$\left[(\lambda + 2\mu) I + \alpha h\right]\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 M_T}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} = 0, \tag{13}$$

and the heat conduction equation can be written as

$$\nabla^2 T - \frac{\rho c_e}{K}\left(\frac{\partial T}{\partial t}\right) + \frac{\beta T_0}{K}\frac{\partial}{\partial t}\left(z\frac{\partial^2 w}{\partial x^2}\right) + \frac{S_i}{K} = 0, \tag{14}$$

Multiplying Eq. (14) by $z\ dz$ and integrate from interval $(-h/2, h/2)$, we obtain

$$\begin{aligned}&\left(\frac{\partial^2 M_T}{\partial x^2} - \frac{1}{h^2}M_T\right)(x,t) + \frac{h\beta d}{2}\left\{\frac{\partial T}{\partial z}\left(x,\frac{h}{2},t\right) + \frac{\partial T}{\partial z}\left(x,-\frac{h}{2},t\right)\right\}\\ &-\frac{\rho c_e}{K}\left(\frac{\partial M_T}{\partial t}\right) + \frac{\beta^2 T_0 I}{K}\left(\frac{\partial^3 w}{\partial x^2 \partial t}\right) - S_1(t) = 0,\end{aligned} \tag{15}$$

where M_T is mathematically approximated as the difference between the temperatures at the upper and bottom surfaces of the beam, the temperature is assumed to vary linearly through the thickness of the beam, thus, we have

$$T\left(x,\frac{h}{2},t\right) + T\left(x,-\frac{h}{2},t\right) = \frac{1}{\beta dh^2}M_T(x,t) \tag{16}$$

And

$$S_1(t) = \frac{\beta d}{K}\int_{-\frac{h}{2}}^{\frac{h}{2}} S_i z dz, \tag{17}$$

The following dimensionless quantities are introduced

$$\begin{aligned}&x' = \frac{x}{L},\ z' = \frac{z}{L}, w' = \frac{w}{L},\ t' = \frac{vt}{L},\ T' = \frac{\beta T}{E},\\ &M' = \frac{M}{dEh^2},\ M_T' = \frac{M_T}{dEh^2}, t_x' = \frac{t_x}{E},\ v^2 = \frac{E}{\rho}.\end{aligned} \tag{18}$$

Using Eq. (18) in Eqs. (13) and (15), after dropping the dashes for convenience, we obtain

$$\frac{\partial^4 w}{\partial x^4} + a_1\frac{\partial^2 M_T}{\partial x^2} + a_2\frac{\partial^2 w}{\partial t^2} = 0, \tag{19}$$

$$\begin{aligned}&\left(\frac{\partial^2 M_T}{\partial x^2} - a_3^2 M_T\right)(x,t) + \frac{a_3}{2}\left\{\frac{\partial T}{\partial z}\left(x,\frac{h}{2},t\right) + \frac{\partial T}{\partial z}\left(x,-\frac{h}{2},t\right)\right\}\\ &-a_4\left(\frac{\partial M_T}{\partial t}\right) + a_5\frac{\partial^3 w}{\partial x^2 \partial t} - S_2(t) = 0,\end{aligned} \tag{20}$$

Where

$$\begin{aligned}&a_1 = \frac{dELh^2}{(EI + \alpha h)},\ a_2 = \frac{\rho A v^2 L^2}{(EI + \alpha h)},\ a_3 = \frac{L}{h},\\ &a_4 = \frac{\rho c_e v L}{K},\ a_5 = \frac{T_0 \beta^2 I v}{KdEh^2},\ S_2(t) = \frac{L^2}{dEh^2}S_1(t).\end{aligned}$$

4. Method of solution

We define the Laplace transform as

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t) dt = \bar{f}(s). \tag{21}$$

where s is the Laplace transform parameter.

Making use of Eq. (21) in Eqs. (19) and (20), we obtain

$$\left(D^4 + a_2 s^2\right)\bar{w} + a_1 D^2 \bar{M}_T = 0, \tag{22}$$

$$\left(D^2 - b_1\right)\bar{M}_T(x,s) + b_2 D^2 \bar{w}(x,s) + \bar{Q}(s) = 0, \tag{23}$$

The differential equation of the lateral deflection $\bar{w}$

and the thermal moment $\bar{M}_T$ are

$$\left\{D^6 - pD^4 + qD^2 - r\right\}\begin{bmatrix}\bar{w}\\ \bar{M}_T\end{bmatrix} = \begin{bmatrix}0\\ -a_2 s^2 \bar{Q}\end{bmatrix}, \tag{24}$$

Where

$$D = \frac{d}{dx},\ b_1 = \left(a_3^2 + a_4 s\right),\ b_2 = a_5 s,$$
$$\bar{Q}(s) = \frac{a_3}{2}\left\{\frac{d\bar{T}}{dz}\left(x, \frac{h}{2}, s\right) + \frac{d\bar{T}}{dz}\left(x, -\frac{h}{2}, s\right)\right\} \text{-} S_3(s), \tag{25}$$
$$p = \left(b_1 + a_1 b_2\right),\ q = a_2 s^2,\ r = q b_1,\ S_3(s) = S_2(t).$$

The differential equation governing the lateral deflection $\bar{w}$ can take the form

$$\left(D^2 - \lambda_1^2\right)\left(D^2 - \lambda_2^2\right)\left(D^2 - \lambda_3^2\right)\bar{w} = 0, \tag{26}$$

where $\pm\lambda_1$, $\pm\lambda_2$ and $\pm\lambda_3$ are the characteristic roots of the equation

$$\lambda^6 - p\lambda^4 + q\lambda^2 - r = 0. \tag{27}$$

and satisfy the well-known relations

$$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = p,\ \lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 = q,\ \lambda_1^2\lambda_2^2\lambda_3^2 = r. \tag{28}$$

Then the lateral deflection is given by

$$\bar{w}(x, s) = \sum_{i=1}^{3}\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right), \tag{29}$$

where A_i and B_i, i=1,2,3, are constant coefficients and are depending on the Laplace variable s.

The thermal moment is given by

$$\bar{M}_T(x, s) = \sum_{i=1}^{3}\left(A_i^{'} e^{\lambda_i x} + B_i^{'} e^{-\lambda_i x}\right) + \frac{\bar{Q}}{b_1}, \tag{30}$$

Where A_i, A'_i $A_i, A_i^{'}$ and B_i, B'_i i=1,2,3, are constant coefficients and are depending on the Laplace variable s. Substituting Eqs. (29) and (30) in Eq. (23), yields

$$\begin{pmatrix}A_i^{'}\\ B_i^{'}\end{pmatrix} = \frac{b_2\lambda_i^2}{\left(b_1 - \lambda_i^2\right)}\begin{pmatrix}A_i\\ B_i\end{pmatrix}, \qquad i = 1,2,3. \tag{31}$$

Substituting Eq. (31) in Eq. (30), we obtain

$$\bar{M}_T(x, s) = \sum_{i=1}^{3}\frac{b_2\lambda_i^2}{\left(b_1 - \lambda_i^2\right)}\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right) + \frac{\bar{Q}}{b_1}. \tag{32}$$

Making use of Eqs. (8), (16), (17), (18) and Eq. (21), with the aid of Eqs. (29) and (32), the axial stresses can be written as

$$\bar{T}_x(x, s) = \sum_{i=1}^{3}\frac{-\lambda_i^2}{L}\left(\frac{h}{12} + \frac{b_2}{Ld\left(b_1 - \lambda_i^2\right)}\right)\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right) - \frac{\bar{Q}}{L^2 d b_1}, \tag{33}$$

where

$$\bar{T}_x(x, s) = \bar{t}_{x_{avg}}(x, s) = \bar{t}_x\left(x, \frac{h}{2}, s\right) - \bar{t}_x\left(x, -\frac{h}{2}, t\right) \tag{34}$$

5. Boundary conditions

We consider a beam with both ends simply supported and isothermal. The boundary conditions are as follows

$$w(0, t) = 0,\quad \frac{\partial^2 w(0, t)}{\partial x^2} = 0,\quad M_T(0, t) = 0 \tag{35}$$

$$w(L,t) = 0,\ \frac{\partial^2 w(L,t)}{\partial x^2} = 0,\ M_T(L,t) = 0. \tag{36}$$

Using Eqs. (18) and (21) in the boundary conditions Eqs. (35) and (36), yield

$$\bar{w}(0,s) = 0,\quad \frac{\partial^2 \bar{w}(0,s)}{\partial x^2} = 0,\quad \bar{M}_T(0,s) = 0 \tag{37}$$

$$\bar{w}(1, s) = 0,\quad \frac{\partial^2 \bar{w}(1, s)}{\partial x^2} = 0,\quad \bar{M}_T(1, s) = 0. \tag{38}$$

6. Application

(i) Laser source with non-Gaussian form temporal profile set along the upper surface$\left(z = \frac{h}{2}\right)$, following Tang and Araki (2000)

$$I(t) = \frac{I_0 t}{t_p^2} exp\left(\frac{-t}{t_p}\right) \tag{39}$$

where t_p is the laser pulse duration, I_0 is the laser intensity which is defined as the total energy carried by a laser pulse per unit cross-section of the laser beam. The thermal conduction in the beam can be modeled as a one-dimensional problem with an energy source S_i near the surface, i.e.,

$$S_i(z, t) = \frac{(1-R)}{\delta} exp\left(\frac{z - h/2}{\delta}\right) I(t). \tag{40}$$

where δ is the penetration depth of heating energy and R is the surface reflectivity.

There is no heat flow across the upper and lower surfaces of the beam, i.e.,

$$\frac{\partial T}{\partial z}\left(x, \frac{h}{2}, t\right) = \frac{\partial T}{\partial z}\left(x, -\frac{h}{2}, t\right) = 0. \tag{41}$$

Now, we can evaluate the thermal influence $\bar{Q}$ given by

$$\bar{Q} = \frac{(1-R) a_\delta a_3 a_6 a_7}{2\left(a_6 s + 1\right)^2}. \tag{42}$$

where

$$a_\delta = \left[\left(1 - \frac{2\delta}{h}\right) + \left(1 + \frac{2\delta}{h}\right)e^{-\frac{h}{\delta}}\right],\ a_6 = \frac{\nu t_p}{L},\ a_7 = \frac{L\beta I_0}{EKt_p}.$$

Substituting the values of $\bar{w}$ and $\bar{M}_T$ from Eqs. (29) and (32) in the boundary conditions Eqs. (37) and (38), with the aid of Eq. (42), after some simplification, we obtain the expressions of lateral deflection, thermal moment and axial stress average for laser source as

$$\bar{w}(x, s) = \sum_{i=1}^{3}\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right), \tag{43}$$

$$\bar{M}_T(x, s) = \sum_{i=1}^{3} M_i\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right) + \frac{\bar{Q}}{\Gamma_1}, \tag{44}$$

$$\bar{T}_x(x, s) = \sum_{i=1}^{3} N_i\left(A_i e^{\lambda_i x} + B_i e^{-\lambda_i x}\right) - \frac{\bar{Q}}{L^2 db_1}, \tag{45}$$

where A_1, A_2, A_3, B_1, B_2 and B_3, are given in the Appendix.

(ii) Constant heat flux normal to the upper surface $z = \frac{h}{2}$.

We replace the laser source term with a constant heat flux ($-q_0$) normal to the upper surface $\left(z = \frac{h}{2}\right)$ of the beam and keeping bottom surface $\left(z = -\frac{h}{2}\right)$ at zero temperature gradient. The boundary conditions on the upper and bottom surfaces of heat conduction equation as

$$q_0 = K\frac{\partial T}{\partial z}\left(x, \frac{h}{2}, t\right),\ \frac{\partial T}{\partial z}\left(x, -\frac{h}{2}, t\right) = 0. \tag{46}$$

Using Eqs. (18) and (21) on Eq. (46), we obtain

$$\frac{d\bar{T}}{dz}\left(x,\ \frac{h}{2}, s\right) = \frac{q_0}{K},\ \frac{d\bar{T}}{dz}\left(x, -\frac{h}{2}, s\right) = 0. \tag{47}$$

From Eqs. (25) and (47), the thermal influence is given by

$$\bar{Q} = \frac{a_3 q_0}{2K}. \tag{48}$$

Making use of the value of thermal influence $\bar{Q}$ from Eq. (48) in Eqs. (43)-(45), after some simplification, we yield the expressions of lateral deflection, thermal moment and axial stress.

6. Particular cases

(i) If α=0, in Eqs. (43)-(45), we obtain the results for lateral deflection, thermal moment and axial stress average in a thermoelastic beam and these results in a special case are similar as obtained by Sirafy *et al.* (2014) for thermoelastic beam theory without dual-phase-lag model.

7. Inversion of the laplace transform

To obtain the solution of the present application in the physical domain, we first apply the well-known formula

$$f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{f}(s)\, e^{-st} ds, \tag{49}$$

Secondly, we adopt a numerical inversion method on the fourier series expansion, by which the integral Eq. (40) can be approximated as a series theory in the absence and presence of couple stress for laser and heat flux applications are computed numerically and shown graphically in Figs. 2-7 respectively. In all these Figs., solid line for $0 \le t \le 2t_1$.

$$f(t) = \frac{e^{ct}}{t_1}\left[-\frac{1}{2}\operatorname{Re}\bar{f}(c) + \sum_{j=0}^{\infty}\operatorname{Re}\left(\bar{f}\left(c + \frac{ij\pi}{t_1}\right)\right)\cos\left(\frac{j\pi}{t_1}\right) - \sum_{j=0}^{\infty}\operatorname{Im}\left(\bar{f}\left(c + \frac{ij\pi}{t_1}\right)\right)\sin\left(\frac{j\pi}{t_1}\right)\right] - \sum_{j=1}^{\infty} \mathrm{e}^{-2cjt_1} f(2jt_1 + t). \tag{50}$$

The above series Eq. (50) is called the Durbin formula and the last term in which is called the discretization error. Honig (1984) developed a method for accelerating the convergence of the Fourier series and a procedure that computes that computes approximately the best choice of the free parameters.

8. Numerical results and discussion

For numerical computations, we take the magnesium material. The physical data chosen for magnesium are taken as Daliwal and Singh (1980) and Sirafy *et al.* (2014).

$$\lambda = 2.696\times10^{10}\,\mathrm{Kg\ m^{-1}s^{-2}},$$

$$\mu = 1.639\times10^{10}\ \mathrm{Kgm^{-1}s^{-2}},$$

$$\rho = 1.74\times10^{3}\,\mathrm{Kg\ m^{-3}},$$

$$T_0 = 0.293\times10^{3}\ \mathrm{K},$$

$$E = 45\times10^{3}\,\mathrm{GPa},$$

$$\alpha_t = 1.78\times10^{-5}\ \mathrm{K^{-1}},$$

$$\nu = 0.35,$$

$$c_e = 1.04\times10^{3}\,\mathrm{JKg^{-1}K^{-1}},$$

$$q_0 = 2.25\times10^{11}\,\mathrm{Wm^{-2}},$$

$$\alpha = 2.5\ \mathrm{Kg\ m\ s^{-2}},$$

$$K = 1.7\times10^{2}\ \mathrm{Wm^{-1}K^{-1}},$$

$$t = 1s,$$

$$R = 0.93,$$

$$L/h = 10,\ b/h = 0.5,\ h = 10\mu\mathrm{m},$$

$$I_0 = 1000J/m^2,\ t_p = 0.02s,\ \delta = 0.1h.$$

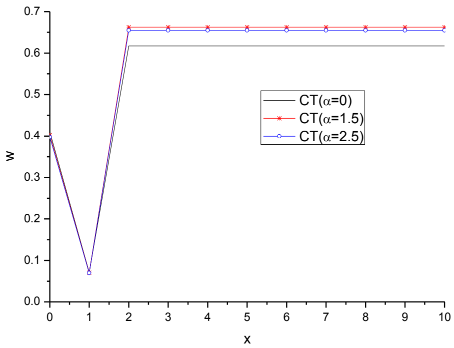

Fig. 2 Variation of laser induced lateral deflection with length

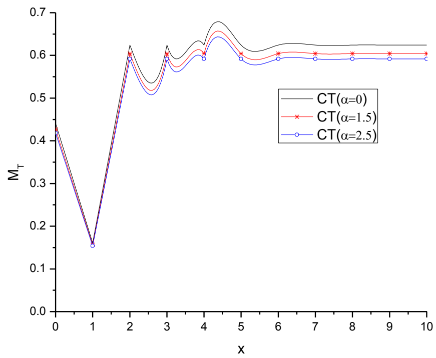

Fig. 3 Variation of laser induced thermal moment with length

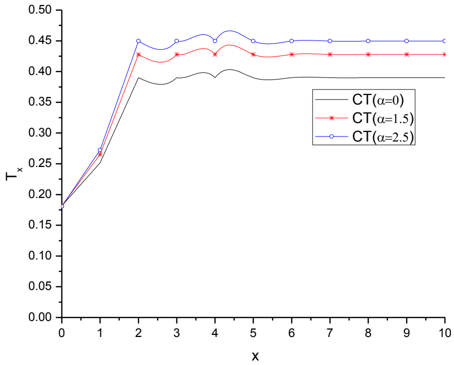

Fig. 4 Variation of laser induced axial stress average with length

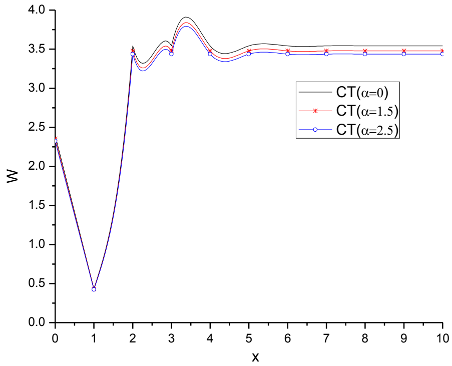

Fig. 5 Variation of heat flux induced lateral deflection with length

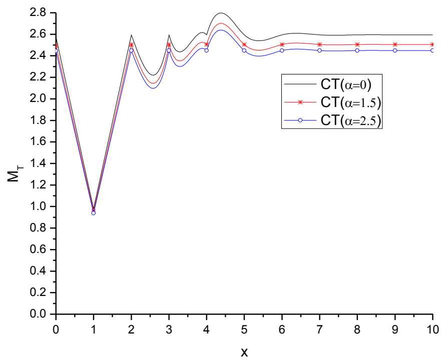

Fig. 6 Variation of heat flux induced thermal moment with length

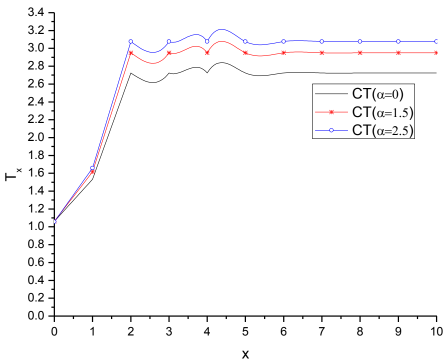

Fig. 7 Variation of heat flux induced axial stress average with length

The software MATLAB 7.10.4 have been used to determine the absence and presence of couple stress on lateral deflection w, thermal moment M_T and axial stress average T_x for coupled thermoelastic (CT) corresponds to CT(α=0), solid line with centre symbol (−*−) corresponds to CT(α=1.5), solid line with centre symbol (−o−) corresponds to CT(α=2.5).

8.1 Laser application

Fig. 2 shows the variation of laser induced lateral deflection w with respect to length x. It is observed that from the figure that the lateral deflection initially decreases and then increases in the range $0 \leq x < 3$ and then remain stationary in the considered range. The lateral deflection of the beam goes on increasing as the couple stress increases further with respect to length. Fig. 3 depicts the variation of laser induced thermal moment M_T with respect to length x. It is evident that the value of thermal moment initially decreases and then increases monotonically for the assumed region. On the other hand, the values of thermal moment is higher in the presence of couple stress for coupled thermoelasticity CT(α=1.5,2.5) and smaller in the absence of coupled stress for coupled thermoelasticity CT(α=0). Fig. 4 represents the variation of laser induced average of axial stress T_x with respect to length x. It is noticed that the behavior and variation of axial stress are similar for all cases of coupled thermoelastic model but difference in their values. It is also clear from the figure that the value of laser induced axial stress greater as the couple stress goes on increasing.

8.2 Heat flux application

Fig. 5 depicts the variation of heat flux induced lateral deflection w with respect to length x. It is evident that the value of lateral deflection initially decreases and then increases smoothly for the considered region. As we seen from the figure that the value of lateral deflection is smaller as the couple stress parameter increases with respect to length but there is very small difference between them. Fig. 6 shows the variation of heat flux induced thermal moment M_T with respect to length x. Oscillatory behavior is shown for smaller value of length but remain stationary for higher value of length. As couple stress increases, the value of heat flux induced thermal moment decreases for coupled thermoelastic (CT) theory. Fig. 7 represents the variation of heat flux induced average of axial stress T_x with respect to length x. It is noticed from the figure that the value of axial stress increases monotonically for the whole range. The value of heat flux induced axial stress increases as couple stress increases further for all cases of coupled thermoelastic theory.

9. Conclusions

In this present study, the vibration of a simply supported Euler-Bernoulli thermoelastic rectangular microscale beam in the context of Coupled thermoelastic (CT) model of thermoelasticity have been studied during two different ways of heating: the laser pulse heating and the constant heat flux on the upper surface. The method of Laplace transform is used to solve the problem. A numerical technique has been adopted to recover he solutions in the physical domain. The expressions for lateral deflection, thermal moment and axial stress average have been derived successfully and shown graphically in the absence and presence of couple stress for Laser and heat flux applications. It is observed from the figures that as couple stress increases, the value of lateral deflection increases for laser application and decreases for heal flux application. It is clear from the figures that the value of thermal moment decreases and axial stress increases as couple stress increases for both laser and heat flux applications. It is also seen from the figures that the behavior and variation are approximately similar for both laser-pulse and heat flux applications. The results obtained in the study should be beneficial for people working in material processing, remote sensing, optical tomography, laser surgery, medical science, thermomechanical, engineering, accelerometers, sensors, resonators and also working in the field of thermoelastic beam in modified couple stress theory for Coupled thermoelastic model.

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CC

Appendix

$$A_1=\frac{\Delta_1}{\Delta},\ A_2=\frac{\Delta_2}{\Delta},\ A_3=\frac{\Delta_3}{\Delta},\ B_1=\frac{\Delta_4}{\Delta},\ B_2=\frac{\Delta_5}{\Delta},\ B_3=\frac{\Delta_6}{\Delta}.\ \text{And}$$

$$\Delta=\left(\lambda_1^2e^{-\lambda_1}-\lambda_2^2e^{-\lambda_2}\right)\begin{bmatrix}\left(\lambda_1^2-\lambda_2^2\right)R_1+\left(\lambda_3^2-\lambda_1^2\right)R_2\\+\left(\lambda_2^2-\lambda_3^2\right)R_3\end{bmatrix}-\left(\lambda_2^2e^{-\lambda_2}-\lambda_3^2e^{-\lambda_3}\right)\begin{bmatrix}\left(\lambda_1^2-\lambda_2^2\right)R_4-\left(\lambda_2^2-\lambda_3^2\right)R_5\\-\left(\lambda_3^2-\lambda_1^2\right)R_6\end{bmatrix}$$

$$-\left(\lambda_3^2e^{\lambda_3}-\lambda_1^2e^{-\lambda_1}\right)\left[\left(\lambda_1^2-\lambda_2^2\right)R_7-\left(\lambda_2^2-\lambda_3^2\right)R_8\right]+\left(\lambda_2^2e^{\lambda_2}-\lambda_3^2e^{\lambda_3}\right)\begin{bmatrix}\left(\lambda_1^2-\lambda_2^2\right)R_9+\left(\lambda_3^2-\lambda_1^2\right)R_{10}\\+\left(\lambda_2^2-\lambda_3^2\right)R_{11}\end{bmatrix}$$

$$+\left(\lambda_1^2e^{\lambda_1}-\lambda_2^2e^{\lambda_2}\right)\left[-\left(\lambda_2^2-\lambda_3^2\right)R_{12}+\left(\lambda_3^2-\lambda_1^2\right)R_{13}-\left(\lambda_1^2-\lambda_2^2\right)R_{14}\right].$$

$$R_1=\left\{\left(k_2-k_3\right)\left\{\left(2g_1-g_5\right)l_1-g_2l_2-g_3l_3\right\}+\left(k_3-k_1\right)\left\{g_4l_4\right\}\right\},$$

$$R_2=\left\{\left(k_1-k_2\right)\left\{g_5\left(l_3-l_5\right)-g_4\left(l_4-l_5\right)\right\}+\left(k_2-k_3\right)\left\{g_6\left(l_4-l_1\right)\right\}\right\}$$

$$R_3=\left\{\left(k_1-k_2\right)\left\{\left(g_2-g_5\right)l_2+g_1\left(l_4-l_1\right)\right\}+\left(k_2-k_3\right)\left\{g_3\left(l_2-l_5\right)\right\}+\left(k_3-k_1\right)\left\{g_5l_5+g_6\left(l_1-l_4\right)-g_4l_5\right\}\right\},$$

$$R_4=\left\{\left(k_2-k_3\right)\left\{g_1\left(l_5+l_6\right)+\left(g_7-g_6\right)l_2\right\}+\left(k_3-k_1\right)\left\{g_5\left(l_6-l_5\right)+\left(g_6-g_7\right)l_4\right\}\right\},$$

$$R_5=\left\{\left(k_1-k_2\right)\left\{g_7l_2-g_1l_6\right\}+\left(k_3-k_1\right)\left\{g_6l_6-g_7l_5\right\}+\left(k_1-k_2\right)\left\{g_1l_5-g_6l_2\right\}\right\},$$

$$R_6=\left\{\left(k_1-k_2\right)\left\{-g_5l_6+\left(g_7-g_4-g_6\right)l_4+g_5l_5\right\}+\left(k_2-k_3\right)\left\{-g_6l_6+g_7l_5\right\}\right\},$$

$$R_7=\left\{\left(k_2-k_3\right)\left\{\left(g_7+g_6\right)\left(l_1+l_4\right)-\left(g_2+g_5\right)l_6+\left(g_2-g_5\right)l_5\right\}-2\left(k_1-k_2\right)\left\{g_4l_4\right\}\right\},$$

$$R_8=\left\{\left(k_1-k_2\right)\left\{\left(g_2+g_5\right)\left(l_6-l_5\right)+\left(g_6-g_7\right)\left(l_1+l_4\right)\right\}+2\left(k_2-k_3\right)\left(g_7l_5-g_6l_6\right)\right\},$$

$$R_9=\left\{\left(k_3-k_1\right)\left\{\left(g_7-g_6\right)l_1-g_2\left(l_6-l_5\right)\right\}+\left(k_2-k_3\right)\left\{g_1\left(l_6-l_5\right)-\left(g_6+g_7\right)l_2\right\}+2\left(k_1-k_2\right)\left\{g_4l_2-g_1l_1\right\}\right\},$$

$$R_{10}=\left\{\left(k_1-k_2\right)\left\{g_2l_6-g_6l_1\right\}-\left(k_2-k_3\right)\left\{g_6l_6-g_7l_5\right\}\right\},$$

$$R_{11}=\left\{\left(k_1-k_2\right)\left\{g_7l_2-g_3l_6+g_6l_4-g_5l_5\right\}+\left(k_3-k_1\right)\left\{g_6l_6-g_7l_5\right\}\right\},$$

$$R_{12}=\left\{\left(k_3-k_1\right)\left\{g_7l_4-g_5l_6\right\}+\left(k_1-k_2\right)\left\{g_5l_2-g_1l_4\right\}+2\left(k_2-k_3\right)\left\{g_5l_6-g_7l_2\right\}\right\},$$

$$R_{13}=\left\{\left(k_2-k_3\right)\left\{g_7l_1-g_4l_6+g_5l_6-g_7l_4\right\}+\left(k_1-k_2\right)\left\{g_4l_4-g_5l_1\right\}\right\},$$

$$R_{14}=\left\{\left(k_2-k_3\right)\left\{g_1l_1-g_2l_2\right\}+\left(k_3-k_1\right)\left\{g_2l_4-g_5l_1\right\}+\left(k_2-k_3\right)\left\{g_5l_2-g_1l_4\right\}\right\},$$

$$M_i=\frac{\Gamma_2\lambda_i^2}{\left(\Gamma_1-\lambda_i^2\right)}$$

$$N_i = \frac{-\lambda_i^2}{L}\left(\frac{h}{12} + \frac{\xi_1\Gamma_2}{Ld\left(\Gamma_1 - \lambda_i^2\right)}\right)$$

$$g_1 = \left(e^{\lambda_3} - e^{-\lambda_1}\right), g_2 = \left(e^{-\lambda_2} - e^{-\lambda_3}\right),$$

$$g_3 = \left(e^{\lambda_3} - e^{-\lambda_1}\right),$$

$$g_4 = \left(e^{-\lambda_2} - e^{-\lambda_3}\right), g_5 = \left(e^{\lambda_2} - e^{\lambda_3}\right), g_6 = \left(e^{\lambda_1} - e^{\lambda_2}\right), g_7 = \left(e^{-\lambda_1} - e^{-\lambda_2}\right),$$

$$l_1 = \left(k_2 e^{-\lambda_2} - k_3 e^{-\lambda_3}\right),$$

$$l_2 = \left(k_3 e^{\lambda_3} - k_1 e^{-\lambda_1}\right), l_3 = \left(k_2 e^{\lambda_2} - k_3 e^{-\lambda_3}\right), l_4 = \left(k_2 e^{\lambda_2} - k_3 e^{\lambda_3}\right), l_5 = \left(k_1 e^{\lambda_1} - k_2 e^{\lambda_2}\right), l_6 = \left(k_1 e^{-\lambda_1} - k_2 e^{-\lambda_2}\right).$$

Δ_i(i=1,……….,6) are obtain by replacing 1st, 2nd, 3rd, 4th, 5th and 6th column by

$$\left[0,0,\left(\frac{Q}{\Gamma_1\Gamma_2}\right),0,0,\left(\frac{Q}{\Gamma_1\Gamma_2}\right)\right]^T \text{ in } \Delta_i.$$