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Development of optimum design curves for reinforced concrete beams based on the INBR9

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Abstract. Structural optimization is one of the most important topics in structural engineering and has a wide range of applicability. Therefore, the main objective of the present study is to apply the Lagrange Multiplier Method (LMM) for minimum cost design of singly and doubly reinforced rectangular concrete beams. Concrete and steel material costs are used as objective cost function to be minimized in this study, and ultimate flexural strength of the beam is considered to be as the main constraint. The ultimate limit state method with partial material strength factors and equivalent concrete stress block is used to derive general relations for flexural strength of RC beam and empirical coefficients are taken from topic 9 of the Iranian National Building Regulation (INBR9). Optimum designs are obtained by using the LMM and are presented in closed form solutions. Graphical representation of solutions are presented and it is shown that proposed design curves can be used for minimum cost design of the beams without prior knowledge of optimization and without the need for iterative trials. The applicability of the proposed relations and curves are demonstrated through two real life examples of SRB and DRB design situations and it is shown that the minimum cost design is actually reached using proposed method.

Keywords: reinforced concrete beam; structural optimization; lagrange multiplier method; design curves; INBR9

1. Introduction

The objective of structural optimization is to find design parameters for the structure that usually minimize cost and satisfy various design requirements. Minimum cost design was initially developed during World War II to seek the optimum design of aircraft structural components subject to strength constraints. By 1960 it was known that many structural optimization problems could be defined as mathematical programming problems.

With proper attention to detailing and quality control, concrete structures can be comparable in weight, strength, and ductility to metal-framed structures at generally much lower cost (Ozbay *et al* 2010). Optimum design of Reinforced Concrete (RC) elements plays an important role in economic design of RC structures. Due to the high cost of the reinforcement, the optimal sections

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economic design of RC structures. Due to the high cost of the reinforcement, the optimal sections tended to present low rates of reinforcement; in most sections (Bordignon and Kripka 2012). Beam elements are usually the major components in RC skeletal structures, and hence their economical design is important to reduce total cost of the structure (Ceranic and Fryer 2000). The material costs of RC beams are dependent on their dimensions, reinforcement ratios and the unit costs of concrete and steel reinforcement (Saini *et al.* 2006). As pointed out by Karihaloo and Kanagasundaram (1993) labor cost may be included in each ingredient. The objective of optimization (e.g. minimum cost or weight), the design variables and the constraints considered by different studies vary widely and therefore, different optimization methods have been employed to provide the optimal design of RC beams (Rahmanian *et al.* 2014) .

Structural design requires judgment, intuition and experience, besides the ability to design structures to be safe, serviceable and economical. The design codes do not directly give a design satisfying all of the above conditions. Thus, a designer has to execute a number of design analyze cycles before converging on the best solution. The minimum cost design of RC beams is rather difficult using conventional office design methods as there are a large number of design solutions that can yield equal bending moment capacity (Fedghouche and Tiliouine 2012). In this case, recourse to a numerical optimization technique becomes necessary to develop a cost effective design approach (Al-Salloum and Siddiqi 1994; Fedghouche and Tiliouine 2010).

The optimization involves choosing the design variables in such a way that the cost of the beam is minimum, subject to behavioral and geometrical constraints as per recommended method of design codes. Doubly Reinforced Beams (DRB) are required to be designed when the depth of the beam is restricted by architectural considerations and the beam has to take moment greater than limiting moment of resistance of the corresponding Singly Reinforced Beam (SRB) (Saini *et al* 2007). The compression steel increases ductility and reduces long term deflections significantly. Some structural optimization works deal with minimization of weight of the structure (Atabay 2009), whereas most of the researchers have worked on cost optimization of the structure (Ö ztürk *et al.* 2016). Though, weight of a structure may be proportional to its cost, minimization of the cost should be the actual objective in economic design of RC structure elements. Most of the researchers have used ultimate load method for design of beams (Chakrabarty 1992a, b; Mukherjee and Deshpande 1995a, b), whereas a few have used limit state method (Al-Salloum and Saddiqi 1994; Ceranic and Fryer 2000).

In satisfaction of constraints, some researchers have given designs satisfying only moment capacity constraint (Al-Salloum and Saddiqi 1994; Ceranic and Fryer 2000), others included self-weight of structure in their analysis (Chakrabarty 1992a, b; Fedghouche and Tiliouine 2012; Mukherjee and Deshpande 1995a, b) and also few researchers have given designs satisfying equivalent allowable deflection corresponding to factored loads, which is a better approach (Adamu and Karihaloo 1994; Adamu *et al.* 1994).

Optimization techniques can be divided into three main categories: mathematical programming techniques, methods based on optimality criteria and heuristic search algorithms (Fedghouche and Tiliouine 2012). Some researchers have applied heuristic search algorithms such as artificial bee colony; simulated annealing and artificial neural networks for optimum design of a reinforced concrete beams (Ö ztürk *et al* 2012; Medeiros and Kripka 2013; and Kao & Yeh 2014). The LMMs have been successfully applied in engineering optimization when constrained problems are considered (Arora *et al.* 1994). The LMMs perform a direct transformation of a constrained problem to an unconstrained one, achieving a final solution through a series of successive unconstrained optimization sub-problems. This approach has been successfully employed for the minimum cost design of singly and doubly reinforced concrete rectangular beams to resist the action of flexural bending based on the British standard (Ceranic and Fryer 2000).

Some researchers have combined the LMM with other optimization approaches. For example, an application of the Continuum-type Optimality Criteria (COC) method to the design of RC beams where the conditions of minimality are derived using the augmented Lagrangian method has been proposed (Adamu *et al.* 1994). The cost that is minimized consists of concrete, reinforcement and formwork costs with active constraints on maximum deflection, bending and shear strength.

The main objective of this study is the application of the LMM for optimum design of both singly and doubly reinforced concrete rectangular beams based on the INBR9 criteria. The optimum design curves achieved in this study can be used for minimum cost design of the beams without prior knowledge of optimization and without the need for performing design process.

2. Optimum design

As mentioned in the previous section, in this study, the LMM method is employed for optimum design of RC beam element. An optimization problem's goal, in general, is to minimize the objective function of the following form

$$z = f(x_1, x_2, x_3, ..., x_n) \tag{1}$$

Subjected to constraints

$$h_i(x_1, x_2, x_3, ..., x_n) = 0 \qquad i = 1, 2, ..., p \tag{2}$$

Where n is the number of independent variables x_i and p is the number of constraints. To solve the optimization problem according to Eqs. (1) and (2), the unconstrained Lagrangian function L is constructed as follows

$$L(x_1, x_2, x_3, ..., x_n, \lambda_1, \lambda_2, ..., \lambda_p) = f(x_1, x_2, x_3, ..., x_n) + \sum_{i=1}^{p} \lambda_i h_i(x_1, x_2, x_3, ..., x_n) \tag{3}$$

Where parameters λ_i are Lagrange multipliers. The necessary conditions for the Lagrangian function to obtain an extreme design are:

$$\frac{\partial L}{\partial x_k} = \frac{\partial f}{\partial x_k} + \sum_{i=1}^{p} \lambda_i \frac{\partial h_i}{\partial x_k} = 0 \qquad k = 1, 2, ..., n \tag{4}$$

$$\frac{\partial L}{\partial \lambda_i} = h_i = 0 \qquad i = 1, 2, ..., p \tag{5}$$

Eqs. (4) and (5) form a system of $n+p$ equalities with $n+p$ unknowns and their solution will yield stationary values for $x_1, x_2, ..., x_n$ and $\lambda_1, \lambda_2, ..., \lambda_p$ from which an optimum design can be achieved.

For the purpose of this study i.e. optimum design of RC beam, we minimize cost of the beam subjected to ultimate internal bending moment M_u. In other words, cost of the beam is considered as the objective function and ultimate flexural strength as the active constraint. Therefore, the solution consists of three steps. In the first step, a cost function needs to be formulated as objective function to be minimized. In the second step, the ultimate flexural strength of a RC beam is determined and used as constraint function. In the final step, the Lagrangian function is formed according to Eq. (3) and the minimum cost solution is found through differentiation as in Eqs. (4) and (5). The above steps are accomplished by procedures that are explained in the following.

In the first step, we formulate the cost function. Total cost of unit length of a beam depends on material cost, geometry of the beam and reinforcement area. By introducing the material cost ratio $q=C_s/C_c$, where C_c and C_s are concrete and steel cost per unit volume; respectively, the cost objective function per unit length of a SRB can be defined as

$$C = C_c b\left[\rho q d + (1+r)d\right] \tag{6}$$

Where ρ is the reinforcement ratio A_s/bd, A_s is the tensile rebar area, b and d are beam width and effective depth; respectively and r is concrete cover ratio with respect to effective depth of the beam d. In this study, it is assumed that b is constant and concrete cover ratio r does not change during design process. Accordingly Eq. (6) can be re-written in the following simplified form:

$$C' = \rho q d + (1+r)d \tag{7}$$

Where $C_c b$ has been considered to be a constant (one). Similar equation can be defined for total cost of a DRB per unit length as follows

$$C' = q(\rho + \rho')d + (1+r)d \tag{8}$$

Where ρ' is the compressive rebar ratio. It is assumed that the compressive reinforcement is reached to the yield state and its value is computed by the relation $\rho' = \rho - \rho_{\max}$ where ρ_{max} is the maximum tensile rebar ratio according to the INBR9. Based on the last assumption, the final form of the objective cost function for a DRB will be as follows

$$C' = q(2\rho + \rho_{\max})d + (1+r)d \tag{9}$$

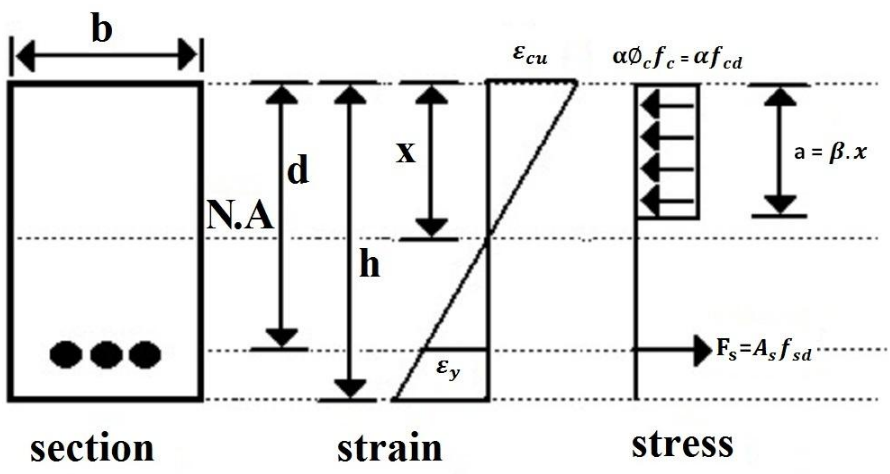

Fig. 1 Singly reinforced section with rectangular stress block based on the INBR9

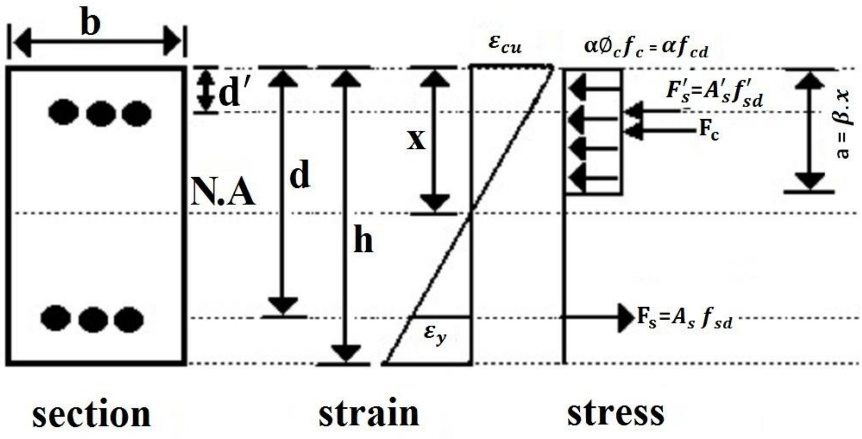

Fig. 2 Doubly reinforced section with simplified rectangular stress block based on the INBR9

In the second step, we derive the ultimate flexural strength constraint function. Rectangular RC beam sections are primarily designed to resist internal bending moments. These beams are generally classified as "singly reinforced" or "doubly reinforced". In singly reinforced beams, rebars are designed to resist tensile forces only, while in doubly reinforced beams, rebars are designed to resist both tensile and compressive forces. To develop the flexural constraint, we used the Ultimate Limit State (ULS) method with equivalent concrete stress block and partial concrete and steel strength reduction factors which is commonly used by current codes of practice. The empirical coefficients are taken directly from the INBR9. This, however, does not limit the generality of the method and empirical coefficients of other codes can be also used in exactly the same manner.

Geometry of a SRB and DRB with the rectangular stress block is shown in Figs. 1 and 2. If the ultimate design moment exceeds the maximum balance moment of a singly reinforced concrete beam, utilization of compressive reinforcement will be necessary.

Taking moments about the centre of the tensile rebar, the flexural moment equilibrium constraints are obtained according to Eqs. (10a) and (10b) for a SRB and DRB; respectively

$$\frac{M_u}{bd^2} = \rho\, f_{yd}\left(1 - \frac{\rho\, f_{yd}}{2\alpha\, f_{cd}}\right) \tag{10a}$$

$$\frac{M_u}{bd^2} = \left[f_{yd}(\rho - \rho_{\max})(1 - r) + f_{yd}\rho_{\max}\left(1 - \frac{350\beta}{700 + f_y}\right)\right] \tag{10b}$$

Where M_u is the ultimate factored design moment, f_{yd} is the reduced yield strength of steel, $\emptyset_y f_y$, and f_{cd} is the reduced compressive cylinder strength of concrete, $\emptyset_c f_c$, where $\emptyset_y = 0.85$ and $\emptyset_c = 0.65$ are the strength reduction factors of steel and concrete; respectively. α and β are coefficients to define dimensions of equivalent concrete stress block. Values of these coefficients can be determined according to the INBR9 by the relations: $\alpha = 0.85 - 0.0015 f_c$ and $\beta = 0.97 - 0.0025 f_c$; where f_c is the concrete characteristic strength (compressive cylinder strength of concrete).

The last step consists of forming the Lagrangian function and solving the problem to obtain the minimum cost design. According to the proposed method, the unconstrained problem is defined using the Lagrangian function L by using Eq. (3) as follows:

$$L = \rho q d + (1+r)d + \lambda\left[f_{yd} b \rho q^2 \left(1 - \frac{f_{yd}}{2\alpha f_{cd}}\rho\right) - M_u \right] \tag{11a}$$

$$L = q(2\rho - \rho_{\max}) + a_3 d + \lambda\left[f_{yd} b d^2 (\rho - \rho_{\max})(1-r) + \rho_{\max}\left(1 - \frac{350\beta}{700 + f_y}\right) - M_u \right] \tag{11b}$$

Eqs. (11a) and (11b) are the Lagrangian functions for a SRB and DRB; respectively.

Taking partial derivatives of the Lagrangian functions according to Eqs. (11a) and (11b) and equating them to zero and solving the respective equation system, the optimum tensile reinforcement ratio is determined:

$$\rho_{s\,opt} = \frac{1}{\dfrac{q}{1+r} + \dfrac{f_{yd}}{\alpha f_{cd}}} \tag{12a}$$

$$\rho_{d\,opt} = \frac{1+r}{2q} + 1.5\rho_{\max} - 2\rho_{\max}\left(1 - \frac{350\beta}{700 + f_y}\right)\left(\frac{1}{1-r}\right) \tag{12b}$$

Eqs. (12a) and (12b) are used to obtain the optimum tensile reinforcement ratio for a SRB and DRB; respectively. The optimum compressive reinforcement ratio for a DRB is calculated by the following equation

$$\rho'_{opt} = \rho_{d\,opt} - \rho_{\max} \tag{12c}$$

The respective optimal effective depth is then derived by using Eqs. (10) and (12)

$$d_{s\,opt} = \sqrt{\frac{M_u}{\rho_{s\,opt} f_{yd} d \left(1 - \dfrac{\rho_{s\,opt} f_{yd}}{2\alpha f_{cd}}\right)}} \tag{13a}$$

$$d_{d\,opt} = \sqrt{\frac{M_u}{\left[f_{yd}\rho_{\max}\left(1 - \dfrac{350\beta}{700 + f_y}\right) + f_{yd}(\rho_{d\,opt} - \rho_{\max})(1-r)\right] b}} \tag{13b}$$

Eqs. (13a) & (13b) are used to determine the optimum effective depth for a SRB and DRB; respectively.

Since Eqs. (12a) and (13a) are valid only for SRB, it is necessary to calculate an upper bound to $\rho_{s\,opt}$ beyond which the optimum solution will yield a doubly reinforced beam. According to the

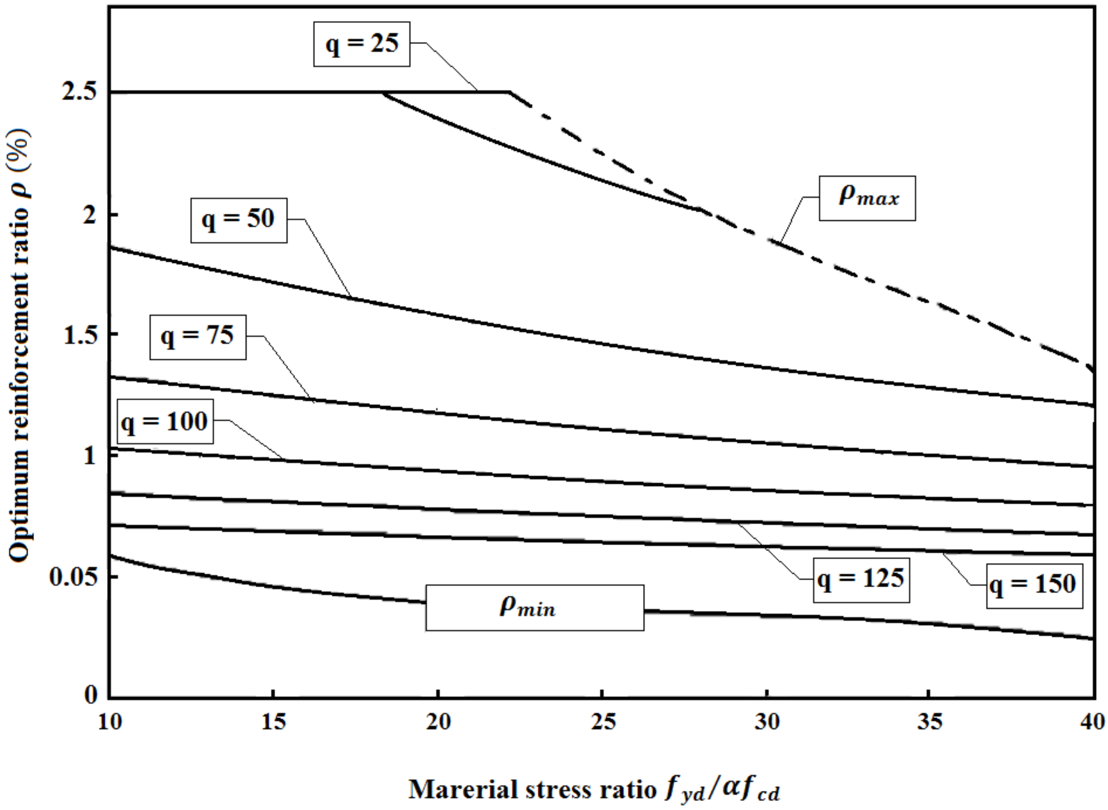

Fig. 3 Optimum tensile reinforcement ratio for singly reinforced concrete beams

INBR9, tensile reinforcement ratio must be bounded by the allowable maximum reinforcement ratio $\rho_{\max} = \min\{0.025, \rho_b\}$, where ρ_b is the limiting reinforcement ratio for which both steel and concrete simultaneously reach their ultimate strength assuming Bernoulli's principle and linear strains. This is a boundary value which controls whether the fracture is brittle or ductile. The balance reinforcement ratio is obtained by Eq. (14a). The compressive reinforcement ratio must be bounded by the allowable minimum reinforcement ratio as per INBR9 regulations (Eq. (14b)).

$$\rho_b = \frac{\alpha\, f_{cd}}{f_{yd}} \frac{700\beta}{700 + f_y} \tag{14a}$$

$$\rho_{\min} = \max\left\{\frac{1.4}{f_y}, \frac{0.25\sqrt{f_c}}{f_y}\right\} \tag{14b}$$

Fig. 3 is a graphical representation of the optimum tensile reinforcement ratio given by Eq. (12a). A family of curves have been drawn for different material cost ratio q for a fixed value of

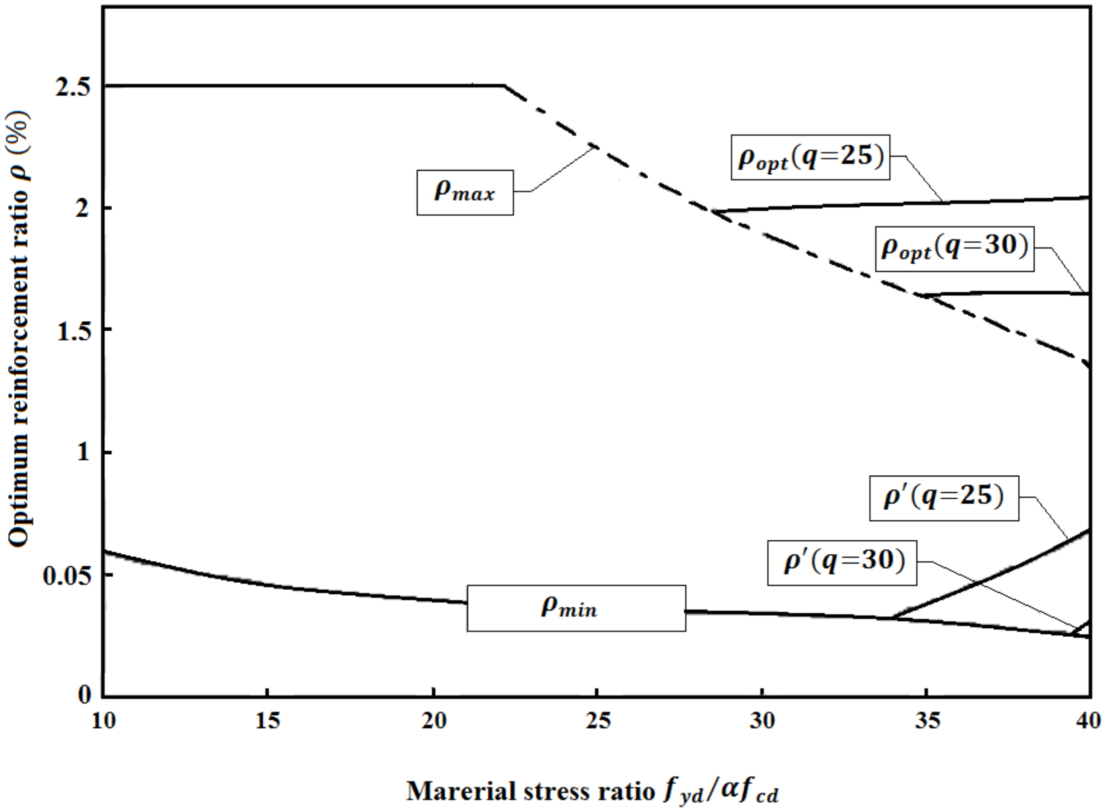

Fig. 4 Optimum tensile reinforcement ratio for doubly reinforced concrete beams

concrete cover ratio of r =0.15. The curves are bounded by the maximum and minimum limitations (ρ_{max} & ρ_{min}) on the reinforcement ratio given by the INBR9 code. It must be noticed that these limitations are considered as the side constraints in the optimization process and ultimate flexural strength of the beam is considered as the only main constraint which has been assumed to be active in the process. Similar graphs can be made for different values of r but the results show that since practical variations of r are very limited thus the objective cost function is not too sensitive to this parameter. It can be deducted from Fig. 3 that for an increase in material cost ratio, a similar decrease in reinforcement ratio $\rho_{s\ opt}$ is required. Under identical loading conditions this decrease is compensated by an increase in the optimal effective depth from Eq. (13a). The q curves are valid only until they intersect with the upper bound value ρ_{max}. The solution for the space above this bound will be a doubly reinforced beam section.

Fig. 4 is a graphical representation of the optimum tensile reinforcement ratio given by Eq. (12b) for a doubly reinforced concrete beam, showing a group of q curves for a fixed value of r equal to 0.15.The plotted values for $\rho_{d\ opt}$ are bounded by the ρ_{min} as the lower limit. Although a series of similar graphs for different values of r can be plotted, however the results show that the objective cost function is not particularly sensitive to changes of these parameters. Fig. 4 illustrates that for an increase in material cost ratio, a similar decrease in reinforcement ratio is required.

Under identical loading conditions, this is compensated for by an increase in the effective depth d. The q lines for $\rho_{d\ opt}$ are only valid until they intersect with the ρ_{max} curve which represents the singly reinforced beam solution. Therefore for values less than the bound value ρ_{max} Fig. 3 should be used.

3. Sensitivity analysis

In this section, the optimum solutions for singly and doubly reinforced sections for different values of material stress ratio $f_{yd}/(\alpha\, f_{cd})$ are compared and different regions of practical solution are identified.

For a singly reinforced beam, the reinforcement ratio $\rho_{s\ opt}$ must be limited to the bounding value ρ_b. Accordingly, the following criterion can be obtained:

$$\frac{f_{yd}}{\alpha\ f_{cd}} \le \frac{700\beta}{700(1-\beta)+f_y}\,\frac{q}{1+r} \tag{15}$$

Since for a doubly reinforced beam section, the tensile reinforcement ratio $\rho_{d\ opt}$ must be greater than the bounding value ρ_b , the following criterion can be obtained:

$$\frac{f_{yd}}{\alpha\ f_{cd}} \ge \left[4\left(1-\frac{350\beta}{700+f_y}\right)\left(\frac{1}{1-r}\right)-1\right]\left(\frac{700\beta}{700+f_y}\right)\left(\frac{q}{1+r}\right) \tag{16}$$

With reference to Eqs. (15) and (16), two distinct zones can be defined based on material stress ratio $f_{yd}/(\alpha\ f_{cd})$. The boundary of these zones depends on values of q and r. Fig. 5 shows a

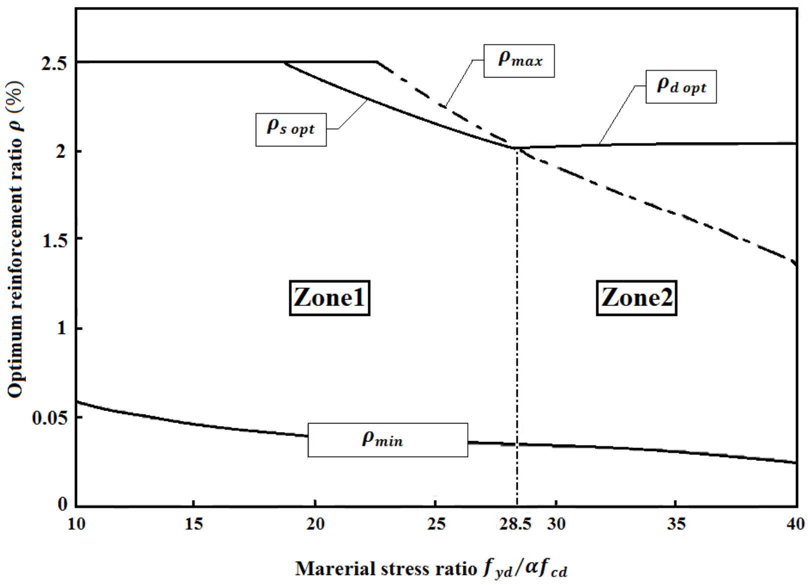

Fig. 5 Optimum reinforcement ratio of the two distinct zones

Table 1 Valid stress ratio ranges for r=0.15

Material cost ratio *(q)*	Single reinforcement optimum range $\left(f_{yd}/\alpha\, f_{cd}\right)$	Double reinforcement optimum range $\left(f_{yd}/\alpha\, f_{cd}\right)$
25	18.3 – 28.4	28.7 - 40
30	14 - 35	35 - 40
35	10 – 38.5	39.6 - 40
45	10 – 40	0.00
55	10 – 40	0.00
65	10 – 40	0.00
75	10 – 40	0.00
85	10 – 40	0.00
95	10 – 40	0.00
100	10 – 40	0.00

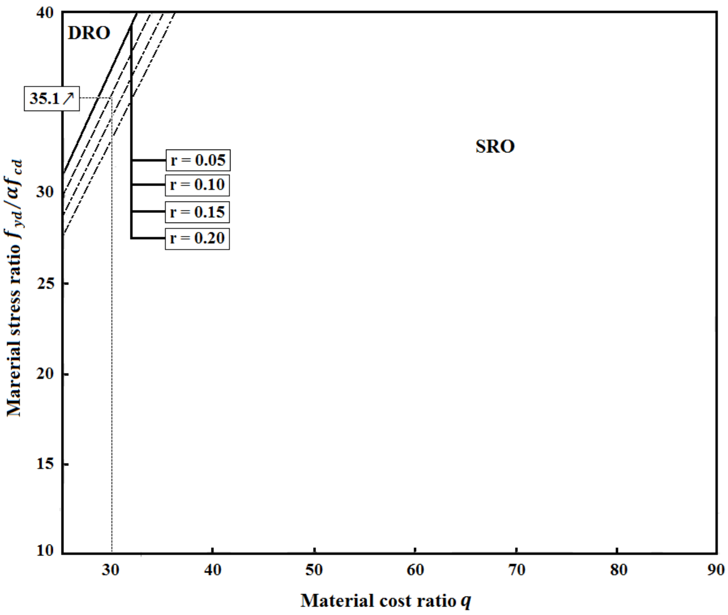

Fig. 6 Optimal curves for r ranged from 0.05 to 0.2

representation of the aforementioned zones for q=25 and r=0.15. Material stress ratio $f_{yd}/(\alpha\, f_{cd})$ is ranged from 10 to 40 which covers all the possible ranges of the INBR9.

Zone 1 represents an optimum singly reinforced concrete beam design with stress ratio $f_{yd}/(\alpha\, f_{cd})$ between 10 and 28.5. Zone 2 represents the optimum design for a doubly reinforced concrete beam whose stress ratio $f_{yd}/(\alpha\, f_{cd})$ ranges from its lower bound 28.5 to the upper bound of 40. For any other values of q, it is possible to mathematically derive the valid stress ratio range for each design type. For example Table 1 is derived using value 0.15 for r.

A series of similar tables with different values of r can be produced. Thus for a designed problem the optimum design can be reached without resorting to repetitive and cumbersome calculations. Hence the proposed approach provides an easy and simple method of selecting the optimum solution and formula.

In practice, material stress ratio $f_{yd}/(\alpha\ f_{cd})$ takes discrete values depending on f_{yd} and $\alpha\ f_{cd}$ predetermined by the INBR9 and market availability. To help the designer reach the optimal design, a series of graphs that show optimal regions for singly and doubly reinforced solutions are developed. A sample of these graphs for typical values of r ranged from 0.05 to 0.2 is presented in Fig. 6.

By selecting q and r values, the boundary value for material stress ratio $f_{yd}/(\alpha\ f_{cd})$ can be obtained from the graph. For example with q=30 and r=0.15, boundary value of $f_{yd}/(\alpha\ f_{cd})$ i.e. the upper bound for a singly reinforced design and the lower bound for a doubly reinforced design will be 35.1. If the values for f_{yd} and $\alpha\ f_{cd}$ are selected in such a way that their ratio is less than 35.1 then the optimum design will be a singly reinforced design. Otherwise the optimum solution will be a doubly reinforced design.

To compare individual material cost with the total cost, concrete and steel cost ratios C_{tc}/C_t and C_{ts}/C_t are defined where C_{tc} and C_{ts} are concrete and steel costs respectively and C_t is total cost of the beam.

For a singly reinforced beam section, concrete and steel cost ratios are obtained by the following relations:

$$\frac{C_{tc}}{C_t} = \frac{1}{1 + \rho_{s\,opt}\dfrac{q}{1+r}} \tag{17a}$$

$$\frac{C_{ts}}{C_t} = \frac{1}{1 + \dfrac{1+r}{\rho_{s\,opt}\,q}} \tag{17b}$$

For a doubly reinforced section, concrete and steel cost ratios can be calculated by the following relations

$$\frac{C_{tc}}{C_t} = \frac{1}{\left[\dfrac{q}{1+r}\left(2\rho_{d\,opt} - \rho_{\max}\right) + 1\right]} \tag{18a}$$

$$\frac{C_{ts}}{C_t} = \frac{2\rho_{d\,opt} - \rho_{\max}}{\left[\left(2\rho_{d\,opt} - \rho_{\max}\right) + \dfrac{1+r}{q}\right]} \tag{18b}$$

As can be seen in Eqs. (17) and (18), the material costs ratios are dependent on values of $f_{yd}/(\alpha\ f_{cd})$, r, and q. In Fig. 7, the material cost ratios have been compared for f_y=400 MPa, f_c=30 MPa and r=0.1.

Two distinct zones can be defined based on whether the optimal design is performed for a singly or doubly reinforced section. The boundary value of q for these two zones is almost 19. It can be noted that with an increase in cost ratio q, the concrete cost ratio decreases steadily. To further evaluate this matter, the concrete cost ratio for a singly reinforced section is re-written by substituting Eq. (12a) in Eq. (17a) as

$$\frac{C_{ts}}{C_t} = 1 - \frac{\dfrac{q}{1+r}}{\dfrac{2q}{1+r} + \dfrac{f_{yd}}{\alpha\, f_{cd}}} \tag{19}$$

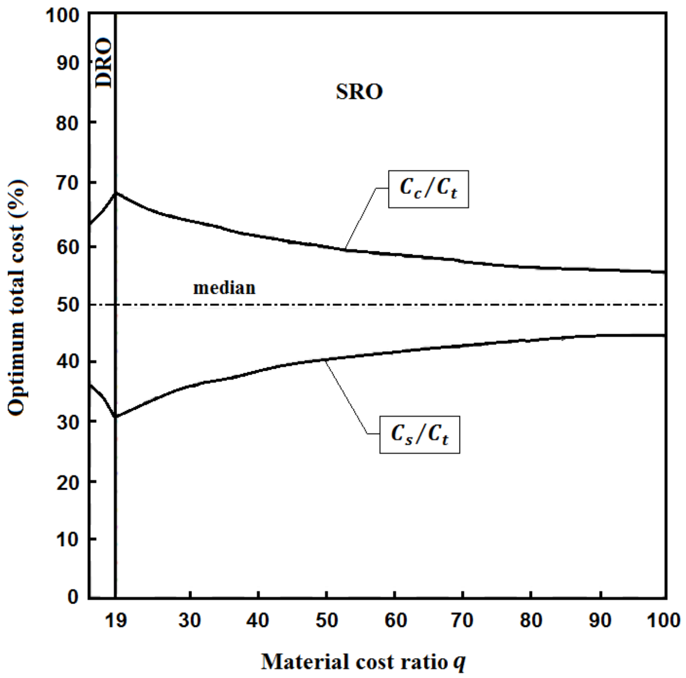

Fig. 7 Material cost ratios for f_y= 400 MPa, f_c= 30 MPa and r = 0.10

Table 2 Design results using proposed and conventional methods for example 1

Design number	Effective depth (mm)	Area of tension reinforcement (mm^2)	Total material costs (×C_c) (\$/m)
1	440	1396.673	0.361301
2	460	1319.498	0.356625
3	480	1251.341	0.353301
4	500	1190.574	0.351086
5	520	1135.962	0.349794
6	544.98	1074.96	0.349262
7	570	1020.555	0.349733
8	640	895.4067	0.355111
9	680	837.3996	0.36021
10	720	786.7532	0.366413

11	760	742.1027	0.373515

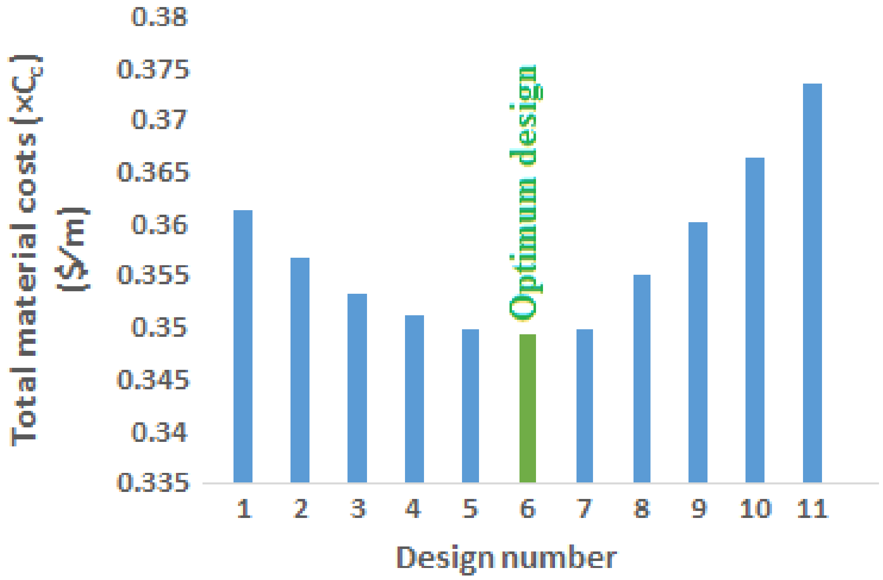

Fig. 8 Comparison between the optimum and conventional designs for example 1

4. Real life numerical examples

To illustrate efficiency and applicability of the proposed relations and graphs for optimal design of either singly or doubly reinforced section, two real life numerical examples are presented. The optimal solution is compared with the conventional methods as per INBR9.

4.1 Example 1

The first design example is given to obtain the optimum design for a singly reinforced beam. A beam of width 300 mm is subjected to an ultimate bending moment of 185 kN.m. The ratio r is taken as 0.10, material cost ratio q as 150, and the cost of concrete C_c as 100000 $/m^3. Characteristic strength of steel and concrete are considered to be 400 and 30 MPa, respectively. Based on the above strength defaults and according to the INBR9, the values of α and β are 0.805 and 0.895; respectively, and values of f_{cd} and f_{yd} are 19.5 and 340 MPa respectively. Therefore the material stress ratio $f_{yd}/(\alpha f_{cd})$ is achieved as 21.7. The lower and upper bounds for effective depths are assumed to be 300 mm and 800 mm, respectively.

Fig. 6 shows that the optimum solution is a singly reinforced section. By using Eq. (12a), the optimal tensile reinforcement ratio is 0.00658. The corresponding optimum effective depth obtained from Eq. (13a) is 544.98 mm and the required rebar area is calculated as 1074.96 mm^2. The minimum material cost of the beam per unit length is then calculated from Eq. (6) as 0.349262 C_c.

Table 2 shows the design results using proposed and conventional methods. It is obviously observed that the optimum design gives the minimum material cost. Also, comparison between the optimum and conventional designs for the example according to Fig. 8 shows that material costs of all the conventional designs (design 1-5 & 7-11) are more than cost of the optimum design (design 6) and the minimum material cost is related to the optimum design.

Table 3 Design results using proposed and conventional methods for example 2

Design number	Effective depth (mm)	Area of compression reinforcement (mm^2)	Area of tension reinforcement (mm^2)	Total material costs ($\times C_c$) ($\$/m^3$)
1	320	553.143	2276.985	0.181153
2	330	447.296	2225.008	0.180658
3	340	345.0149	2176.597	0.18034
4	355.3	194.7405	2108.744	0.180166
5	370	56.67525	2049.867	0.180314
6	380	0	2012.98	0.180572
7	390	0	1978.428	0.180948

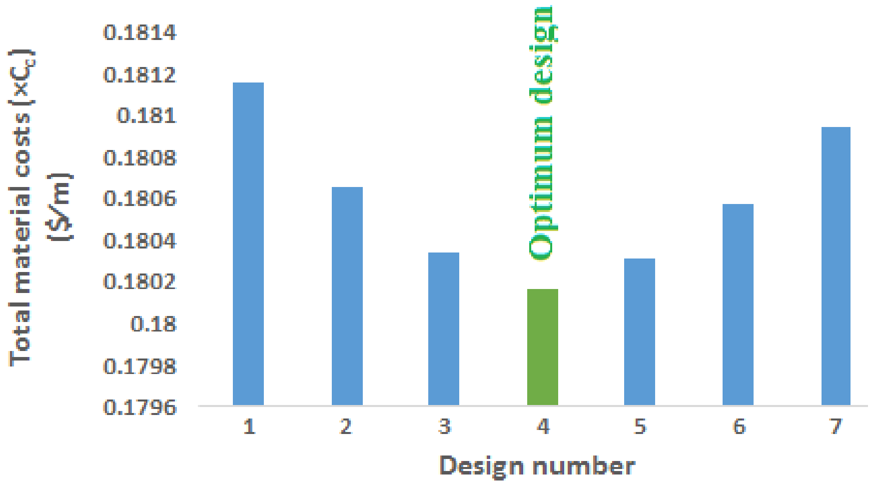

Fig. 9 Comparison between the optimum and conventional designs for example 2

4.2 Example 2

The second design example is given to obtain the optimum design for a doubly reinforced beam. The design parameter values are the same as assumed in example 1 with the exception that the material cost ratio is 25, the concrete strength is 20 MPa and the lower-bound effective depth is 240 mm. Based on the new assumption for the concrete strength in this example, the material stress ratio $f_{yd}/(\alpha f_{cd})$ is achieved as almost 32. Fig. 6 indicates that the optimum solution for this example is a doubly reinforced section.

Utilizing Eq. (12b) the optimal tensile reinforcement ratio is 0.019784 and from Eq. (13b) the corresponding optimal effective depth is 355.3 mm. Thus an area of 2108.74 mm^2 for the tensile reinforcement is required. The minimum cost per unit length of the beam is obtained as 0.180166 C_c. It can be observed that the optimum solution lies on the doubly reinforced stress constraint boundary with the objective function being tangential to the curve. The feasible region is bounded by the effective depth corresponding to a boundary reinforced section, its corresponding area of steel and the bending stress constraint for a doubly reinforced section. Table 3 and Fig. 9 show that the optimum solution (design 4) gives the minimum cost which is less than all the conventional ones (design 1-3 & 5-6).

5. Conclusions

This study dealt with the problem of minimum cost design of RC beams. The objective function was chosen as total cost of concrete and steel materials. The formwork and labor costs were not considered in this study. The beam's resistance against internal bending moment was considered as the constraint for this optimization problem. The bending capacity of the beam was derived using the ULS method. The minimum cost design was found using a LMM based method. The results were presented in closed form relations which can easily be used by practicing engineers. Also graphical representations of the results were provided for better comprehension of the optimization process. Analysis of the graphical representations revealed that a marginal value of material stress ratio can be found which divides the design space into to sub-region. Each of the mentioned regions conforms to either a SRB or DRB design. Relations were derived to easily find this marginal value and determine the optimum design type. A further sensitivity analysis of the total cost revealed that the total cost varies in accordance with steel to concrete cost ratio for both SRB and DRB designs with lower values of cost ratio corresponding to DRB design. Finally, to illustrate the applicability of the proposed method, two real life examples of RC beam designs were presented. It was shown that a practicing engineer can easily defines the optimum design type (SRB or DRB) by selecting the material cost and stress ratios and find the optimum design (rebar area and beam depth) using closed form solutions or respective design graphs.

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