

## Study on fracture characteristics of reinforced concrete wedge splitting tests

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**Abstract.** To study the influence on fracture properties of reinforced concrete wedge splitting test specimens by the addition of reinforcement, and the restriction of steel bars on crack propagation, 7 groups reinforced concrete specimens of different reinforcement position and 1 group plain concrete specimens with the same size factors were designed and constructed for the tests. Based on the double-K fracture criterion and tests, fracture toughness calculation model which was suitable for reinforced concrete wedge splitting tensile specimens has been obtained. The results show that: the value of initial cracking load  $P_{ini}$  and unstable fracture load  $P_{un}$  decreases gradually with the distance of reinforcement away from specimens' top. Compared with plain concrete specimens, addition of steel bar can reduce the value of initial fracture toughness  $K_{Iini}$ , but significantly increase the value of the critical effective crack length  $a_c$  and unstable fracture toughness  $K_{Iun}$ . For tensional concrete member, the effect of anti-cracking by reinforcement was mainly acted after cracking, the best function of preventing fracture initiation was when the steel bar was placed in the middle of the crack, and when the reinforcement was across the crack and located away from crack tip, it plays the best role in inhibiting the extension of crack.

**Keywords:** reinforced concrete; reinforcement position; fracture parameters; crack propagation

### 1. Introduction

Concrete is a kind of multiple, heterogeneous, inhomogeneous cement-based composite materials, changes of pouring, curing conditions and environment can cause inevitable internal defects such as interspace, microcracks, etc. Some defects can gradually tend closed with environment and load conditions changes, but some defects get propagating to endanger the safety of structures. The reinforced concrete, a type of complex material using concrete as the matrix and composed of tensile reinforcement, combines the advantages of both, can effectively improve the structural soundness. In order to reduce the harm caused by concrete crack, controlling the concrete's crack and further extend, domestic and foreign scholars do many researches on

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reinforced concrete fracture and crack resistance of various materials, formed many mature theories. Vecchio, F.J. (2000) presented a conceptual model for describing the behavior of cracked reinforced concrete elements; Sener, S. etc. (2004) presented the test results of size effect on axially loaded reinforced concrete columns and the results showed that larger size columns exhibited larger imperfections; Englekirk, R.E. (2010) presented a discussion on the concept of effective stiffness of reinforced concrete columns on buildings; Sagaseta J etc. (2011) carried out two sets of push-off tests to investigate the effect of aggregate fracture on shear transfer through cracks in reinforced concrete structures; Carmona, J.R. (2013) presented a fracture mechanics model to describe the buckling behavior of lightly reinforced concrete columns; Lu Wenyao etc. (2013) reported the test results of reinforced concrete deep beams and proposed an analytical method for predicting the shear strength of deep beams. In recent years, some scholars do researches on different materials' anti-cracking role in different cases by adding the steel bar or other composite materials in concrete structure, provided an important theoretical basis for preventing and controlling the expansion of concrete crack. Hu Shaowei etc. (2012 and 2013) had a experimental research on adding reinforcement in three-point bending concrete beams, studied the limitation on crack propagation of concrete bending members by reinforcement, and created a suitable calculation model for reinforced concrete three-point bending beam fracture toughness; Maji A etc. (2001) analyzed the ductility and failure mode of FRP reinforced beams; Bencardino F etc. (2010) used two types of fibers established the fracture properties and fracture behavior of concrete; Kang S T etc. (2010) presented a study of the tensile fracture properties of Ultra High Performance Fiber Reinforced Concrete (UHPFRC) considering the effects of the fiber content; Zhang Xiaoxin etc. (2014) conducted three-point bending tests on notched beams of steel fibre-reinforced concrete (SFRC) by using both a servo-hydraulic machine and a drop-weight impact device.

Along with the deepening of research, achievements about adding steel bars into concrete bending members to restrict the cracking and destruction increasingly be rich. But for tensional concrete members, researches about its prevent crevice theory are relatively few. In order to study the restriction of steel bars on crack propagation for tensional concrete members, in this paper, different reinforced location as argument and reinforced concrete wedge splitting tensile test as basis, fracture toughness calculation model which was suitable for reinforced concrete wedge splitting tensile specimens was established and anti-crack function of steel bars in different location was obtained.

## 2. Calculation of fracture parameters

Through the analysis on crack propagation process during the experiments of concrete wedge splitting tensile specimens with steel bars, we can know the reinforced concrete specimens' fracture propagation also undergo 3 periods of fracture initiation, steady propagation and unstable failure. So, the fracture process of reinforced concrete wedge splitting tests can describe by double-K fracture criterion. Refer to double-K fracture criterion of concrete, the initial fracture toughness  $K_I^{ini}$  and unstable fracture toughness  $K_I^{un}$  suitable for reinforced concrete wedge splitting tensile test are lead up. However, because of the existence of reinforcement, the formula of fracture toughness in <Norm for fracture test of hydraulic concrete> (2005) are no longer applicable. According to specimen's shape and stress distribution during test, the superposition principle is used to calculate the double-K fracture parameters of reinforced concrete wedge

splitting specimens.

First of all, there are four basic assumptions: (1) In the process of specimens' cracking, there is no slip between steel bar and concrete; (2) During the test, the restriction of concrete fracture by reinforcement can equivalent to a unit counter force on the fracture surface of concrete; (3) There is a nonlinear softening constitutive relation between the cohesive force on the fictitious crack surface of concrete and the crack mouth opening displacement; (4) Steel bars meet the ideal elastoplastic damage constitutive relationship.

Based on the superposition principle, the stress intensity factor  $K_I$  on crack tip caused by the interaction of horizontal splitting tensile load, cohesive force and equivalent restraint force on reinforced concrete wedge splitting specimens is

$$K_I = K_{IP} - K_{I\sigma} - K_{IS} = K_{IF} - K_{IS} \quad (1)$$

In the above formula:  $K_{IP}$ ,  $K_{I\sigma}$ ,  $K_{IS}$  is the stress intensity factor on crack tip caused by horizontal splitting tensile load, cohesive force and equivalent restraint force. And  $K_{IF} = K_{IP} - K_{I\sigma}$  is the stress intensity factor of concrete wedge splitting specimens.

To calculate the stress intensity factor  $K_{IS}$  caused by equivalent restraint force of reinforcement on crack tip, according to the references (Sih 1973), there is a semi-infinite plate with a crack which length is  $a$  showing in Fig. 1, and the semi-infinite plate subjected to a unit counter force  $P$  across the crack away from boundary for  $b$ . If the plate thickness is  $B$ , for this cause, the stress intensity factor on crack tip is

$$K_{IS} = \frac{2}{\pi} \frac{1 + F\left(\frac{b}{a}\right)}{B\sqrt{a^2 - b^2}} P\sqrt{\pi a} \quad (2)$$

Above formula

$$F\left(\frac{b}{a}\right) = \left(1 - \frac{b}{a}\right) \left[ 0.2945 - 0.3912\left(\frac{b}{a}\right)^2 + 0.7685\left(\frac{b}{a}\right)^4 - 0.9942\left(\frac{b}{a}\right)^6 + 0.5094\left(\frac{b}{a}\right)^8 \right]$$

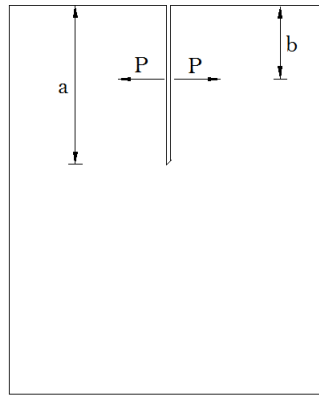


Fig. 1 Semi-infinite plate subjected to a unit counter force across the crack

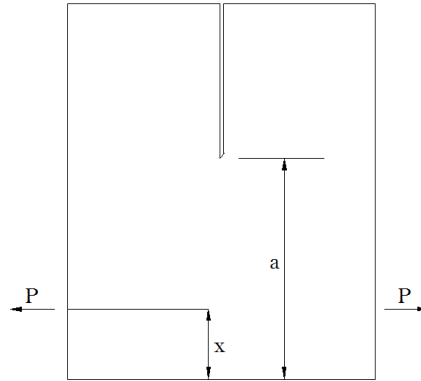


Fig. 2 Semi-infinite plate subjected to a unit counter force under the crack tip

When the equivalent restraint force of reinforcement subject under the crack tip, according to the references (Tada 2000), for the semi-infinite plate showing in Fig. 2, the crack tip away from the bottom edge for  $a$ , and there is a unit counter force  $P$  away from the bottom edge for  $x$  subject on the plate. If the plate thickness is  $B$ , for this cause, the stress intensity factor on crack tip is

$$K_{IS} = 3.522 \left( \frac{x}{a} - 0.368 \right) \frac{2P}{B\sqrt{\pi a}} \quad (3)$$

The <Norm for fracture test of hydraulic concrete 2005> put forward the viewpoint that when use concrete wedge splitting test to determine the fracture toughness, the effect of thickness on its fracture parameters is not considered, the thickness adopt 200mm that recommend by <Norm for fracture test of hydraulic concrete 2005>; when the height to width aspect ratio is less than or equal to 1, the width has no effect on specimen' failure (Hu and Xu 2015); for these specimens in this paper, when the reinforcement is located above the crack tip, it was restricted by top edge of the specimen, and when the reinforcement is located under the crack tip, it was restricted by bottom edge of the specimen, which are all correspond to the semi-infinite plate, so the formula is appropriate.

For standard concrete wedge splitting specimens, the formula of fracture toughness  $K_{IC}$  is

$$K_{IC} = \frac{P}{B\sqrt{H}} f(\alpha) \quad (4)$$

$$f(\alpha) = \frac{3.675[1 - 0.12(\alpha - 0.45)]}{(1 - \alpha)^{3/2}}, \alpha = \frac{a}{H}$$

But because of the specimens' size are not same as the standard specimens, this formula is not suitable anymore.

An improved testing device was used in testing, the loading location is located in the quartile of the upper part of specimen, bearings are located in the quartile of the bottom of specimen, this can ensure that the vertical component of external load, deadweight and end reaction are collinear, the

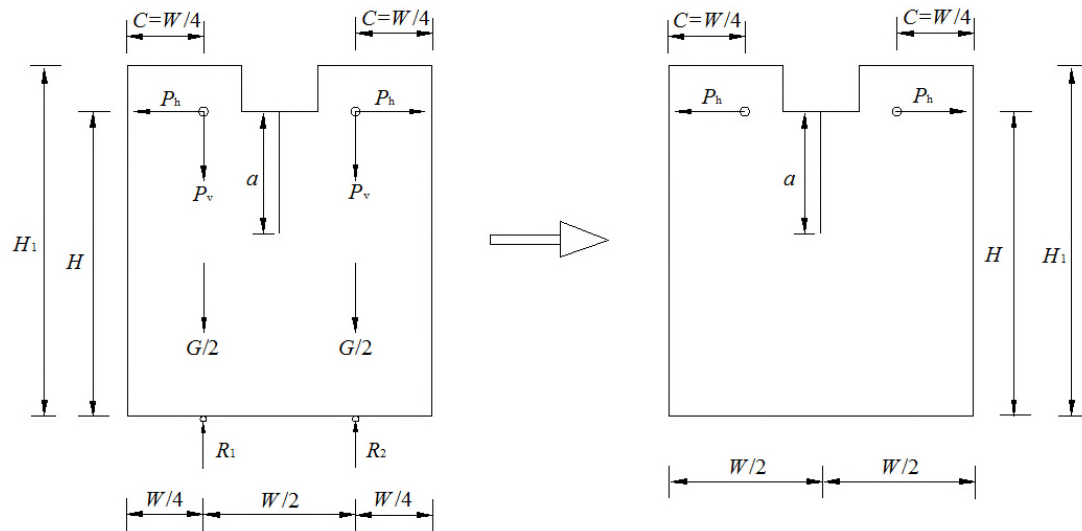


Fig. 3 The schematic diagram of wedge splitting test specimen

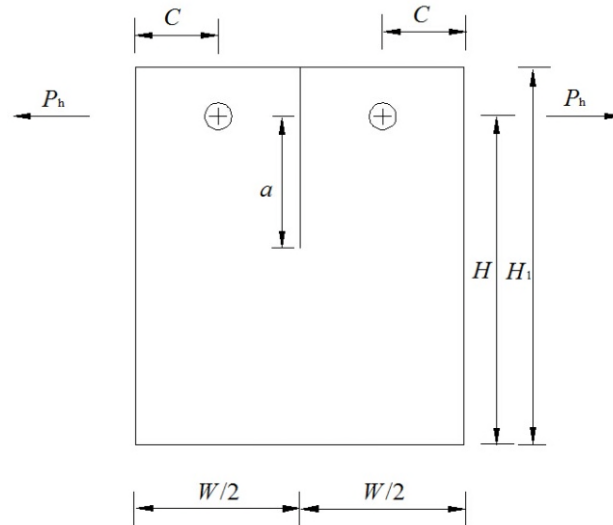


Fig. 4 The schematic diagram of compact tension specimen

schematic diagram is shown in Fig. 3. As shown in Fig. 3, the end reaction will balance out the vertical component of external load and deadweight, the force condition can be simplified to a pair of reverse tensile force on the specimen. The force condition of the compact tension specimen is shown in Fig. 4, we can see that the improved testing device can ensure the stress mode identical to compact tension specimens. Because the geometry characters and stress modes of concrete wedge splitting specimens are conform to compact tension specimens, so the formula of stress intensity factor for compact tension specimens can be used to wedge splitting specimens.

American Society for Testing and Materials (ASTM) obtained the formula of stress intensity factor for standard compact tension specimens ( $2C/W=0.54$ ,  $W/H=0.6$ ) is

$$K_{IF} = \frac{P}{B\sqrt{H}} f(\alpha) \quad (5)$$

$$f(\alpha) = 29.6\alpha^{1/2} - 185.5\alpha^{3/2} + 655.7\alpha^{5/2} - 1017.0\alpha^{7/2} + 638.9\alpha^{9/2}, \alpha = \frac{a}{H}$$

When the size factors is  $2C/W=0.5, W/H=0.6$ , reference (Srawley 1972) gives the  $f(\alpha)$  along with the change of initial seam height ratios  $\alpha$  as shown in Table 1. ( $C$  is the distance that loading location away from the edge of the specimen,  $W$  is half of the specimen width,  $H$  is specimen height.)

Table 1  $f(\alpha)$  along with the change of initial seam height ratios  $\alpha$

$\alpha$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$f(\alpha)$	4.50	5.77	7.38	9.65	13.63	21.56	41.04

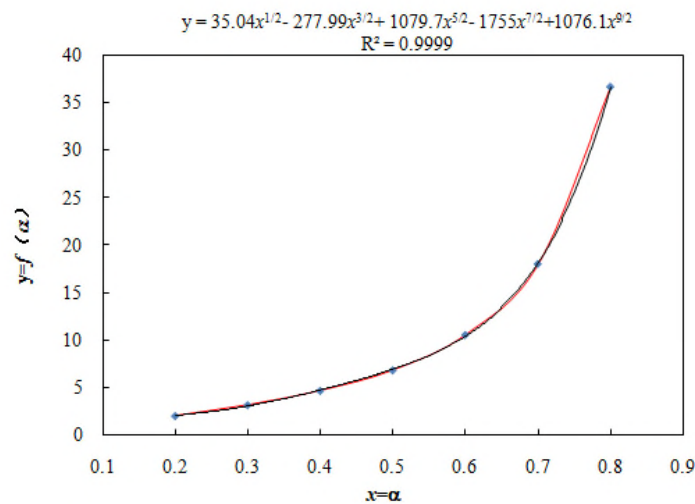


Fig. 5 Curve of  $f(\alpha)$  along with the change of initial seam height ratios  $\alpha$

A mathematical relational expression with fitting polynomial by using the method of minimum square is obtained

On the basis of the above mathematical relational expression, the stress intensity factor  $K_{IF}$  of nonstandard concrete wedge splitting specimens can be given

$$K_{IF} = \frac{P}{B\sqrt{H}} f(\alpha) \quad (6)$$

$$f(\alpha) = 35.04\alpha^{1/2} - 277.99\alpha^{3/2} + 1079.7\alpha^{5/2} - 1755.0\alpha^{7/2} + 1076.1\alpha^{9/2}, \alpha = \frac{a}{H}$$

When calculating, the  $a$  for initial fracture toughness  $K_{IF}^{ini}$  is specimen's initial fracture length  $a_0$ ; and the  $P$  is the horizontal splitting tensile load  $P_{ini}$  when concrete initial cracking. The  $a$  for unstable fracture toughness  $K_{IF}^{um}$  is specimen's critical effective crack length  $a_c$ ; and the  $P$  is the horizontal splitting tensile load  $P_{um}$  when the specimen is unstability.  $a_c$  can calculate by the following formula (Xu and Reinhardt 1999)

$$a_c = (H + h_0) \left[ 1 - \left( \frac{11.56}{\frac{CMOD_c EB}{P_{um}} + 9.397} \right)^{1/2} \right] - h_0 \quad (7)$$

Above formula:  $H$  is specimen height;  $h_0$  is thickness of steel disc pasted between the both side of crack mouth;  $CMOD_c$  is the critical value of crack mouth opening displacement;  $B$  is specimen thickness; elasticity modulus  $E$  can calculate by the following formula (Xu and Reinhardt 1999)

$$E = \frac{1}{Bc_i} \left[ 11.56 \times \left( 1 - \frac{a_0 + h_0}{H + h_0} \right)^{-2} - 9.397 \right] \quad (8)$$

$c_i$  is the initial compliance.

With this, on the basis of different steel location, put the test parameters of initial cracking and unstability into formula (2), (3) and (6) to calculate the stress intensity factor  $K_{IS}$  caused by equivalent restraint force of reinforcement and the stress intensity factor  $K_{IF}$  of concrete without regard to reinforcement. Then put  $K_{IS}$  and  $K_{IF}$  into formula (1) to work out initial fracture toughness  $K_I^{ini}$  and unstable fracture toughness  $K_I^{um}$  for reinforced concrete wedge splitting tensile test.

### 3. Test

#### 3.1 Preparation

According to Table 2, 7 groups altogether 28 reinforced concrete wedge splitting specimens with different reinforced location are prepared for the test, and meanwhile 1 group altogether 4 plain concrete wedge splitting specimens with the same size factors are prepared for comparison. The concrete strength grade is C35, when pouring, the concrete mix design is cement: sand: stone: water=1.00 : 1.22 : 2.36 : 0.46, the cement is ordinary portland cement with strength grade 42.5, the stone is macadam with particle size of 5~20 mm, the sand is natural medium sand, fineness modulus is 2.6 and sand percentage is 34%. Pouring specimens in the wood pattern, sprinkling water curing indoors at atmospheric temperature for shaping in 28 days. Each specimen configure two plain hot rolled HPB235 reinforcement with specification  $\Phi 8$ , the distance between reinforcement and specimen's top boundary  $c$  is shown in Table 2, thickness of concrete cover is 25mm, pasting resistance strain gauge on the surface of reinforcement beforehand for testing the steel strain during test. Obtained by testing, the reinforcement yield strength is 240MPa and elasticity modulus is  $2.1 \times 10^5$ MPa.

Table 2 Design parameter value of specimen

Specimen serial number	$H \times W \times B/\text{mm}$	$a_0/H$	$a_0/\text{mm}$	$c/\text{mm}$	Reinforcement number	$\Phi/\text{mm}$	Quantity
RCWS-050	500×600×200	0.4	200	50	2	8	4
RCWS-095	500×600×200	0.4	200	95	2	8	4
RCWS-140	500×600×200	0.4	200	140	2	8	4
RCWS-185	500×600×200	0.4	200	185	2	8	4
RCWS-215	500×600×200	0.4	200	215	2	8	4
RCWS-260	500×600×200	0.4	200	260	2	8	4
RCWS-305	500×600×200	0.4	200	305	2	8	4

Notes :  $H$  is specimen height,  $W$  is specimen width,  $B$  is specimen thickness,  $a_0$  is initial crack length,  $a_0/H$  is initial crack-depth ratio,  $c$  is reinforced location,  $\Phi$  is diameter of rebar.



Fig. 6 Test loading device

### 3.2 Testing

Loading and testing device is shown in Fig. 6. In the process of test, control loading rate stable at 10N/s and test duration for about 30 minutes. An improved testing device is used in testing to ensure the stress mode identical to compact tension specimen. Use 0~50KN tension and compression load sensor to measure the value of external load  $P$ ; use fracture mechanics clip-on gages at steel disc pasted between the both side of crack mouth to measure crack mouth opening displacement, the measurement range of fracture mechanics clip-on gages is -1~4 mm; use resistance strain gauge with specification BF120-40AA (standard distance 4cm) to measure concrete strain, to monitor the initial cracking, extension and instability. Steel strain measured by strain gauges pasted beforehand. External load  $P$ , crack mouth opening displacement  $CMOD$ , strain of concrete and steel bars are collected by DH-3817 dynamic strain testing system, and connect it to computer to record in time.



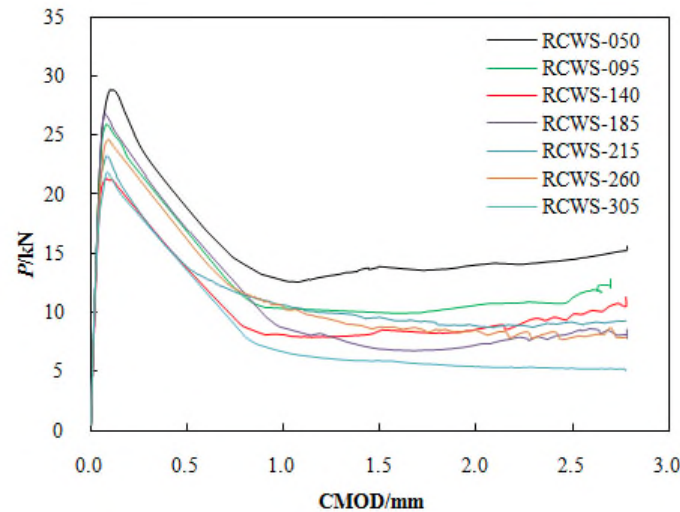


Fig. 7 Curves of P-CMOD

#### 4. Consequence and analysis

##### 4.1 P-CMOD curves and failure modes

The typical *P-CMOD* curves of each group by testing are shown in Fig. 7. As can be seen from the figure, whatever the reinforcement location is, the *P-CMOD* curves of each group showing the same trend. During the initiation of loading, the crack mouth opening displacement has a linear relation with load increasing, and the growth rate of load is significantly larger than the growth of crack mouth opening displacement. As load increases to initial cracking, the *P-CMOD* and load is no longer be the linear growth, the growth rate of load is decreasing, the growth rate of *CMOD* is accelerating, and concrete enters the fictitious crack propagation phase. Gradually with the increasing of load, the fictitious crack gradually propagating, the internal tiny cracks develop to macroscopic fracture, the constraining force by steel bars is gradually increased. When external load *P*, equivalent restraint force by reinforcement and cohesive force on fictitious crack surface reach a relative balance, cracks continue to extend stably. When the load increases to the unstable failure of concrete, concrete will no longer bearing load, along with the suddenly decrease of load and sharply increase of crack mouth opening displacement, a sudden drop process is shown in *P-CMOD* curves. At this time, all external load is undertook by steel, *P-CMOD* curves begin to flatten, and the expansion of crack is well restricted.

Through the observation of *P-CMOD* curves and test process, reinforced concrete wedge splitting tensile specimens' fracture overall process shows the following characteristics: During the initiation of loading, the specimen is in elastic state, the deformation of reinforcement is very small or almost no deformation, which can be thought the constraint effect of reinforcement to concrete can be neglected at this time. When the stress intensity factor on crack tip caused by the interaction of external load *P*, equivalent restraint force and cohesive force reaches to the concrete initial fracture toughness, concrete begin to cracking. After concrete cracking, with the steady expansion of concrete, the constraint function of reinforcement gradually be strength, cracks extend slowly



Fig. 8 State chart diagram of typical specimen failure

and steady by its limitation. When the stress intensity factor on crack tip caused by the interaction of external load  $P$ , equivalent restraint force and cohesive force reaches to the concrete unstable fracture toughness, concrete be unstable and destructive, concrete is no longer bear load and reinforcing steel bars bear the main load. Fig. 8 is specimen's typical failure mode, through the observation it could be found that during the test reinforced steels are all not yield and there are some tiny cracks occur around the reinforcement. It shows that in the process of test the emergence and extension of tiny cracks need to consume energy, the consumption of energy could improve the bearing capacity of reinforced concrete. In addition, when each specimen is damaged, there is no slip happened between steel bar and concrete, which verified the basic assumption (1) in section 1 is correct.

#### 4.2 Calculating and analyzing on fracture parameters

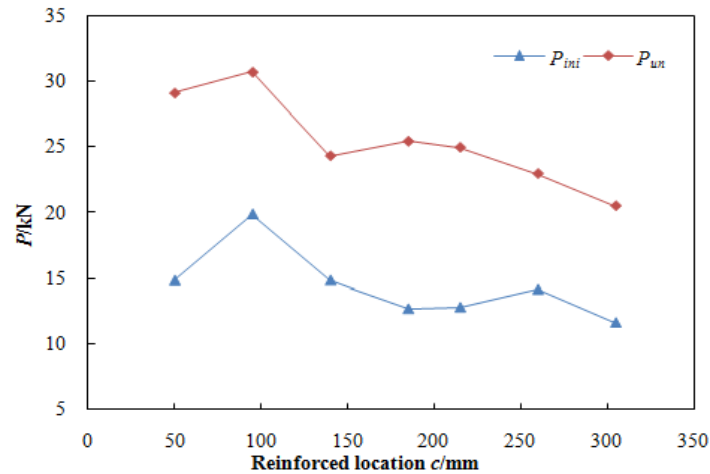
Corresponding fracture parameters of each specimen is shown in Table 3. The initial craking load  $P_{ini}$  is measured by means of resistance strain gauge, the unstable fracture load  $P_{un}$  is the loading maximum value by test. Because during the test reinforced steels are all not yield, so the value of equivalent restraint force caused by reinforcement at the moment of initial cracking and unstability can use this formula  $P_s = E_s A_s \varepsilon_s$ , above the formula:  $E_s$  is the elasticity modulus of reinforced steels,  $A_s$  is steel area,  $\varepsilon_s$  is the value of steel strain and when calculating it valued by the moment of initial cracking and unstability. In addition, the fracture parameters of plain concrete wedge splitting specimens for comparison are as followed: The initial craking load  $P_{ini}$  is 16.86kN and the unstable fracture load  $P_{un}$  is 24.20kN, the critical effective crack length  $a_c$  is 241.65mm, above date are not shown in Table 3.

Table 3 Calculated results on fracture parameters of each specimen

Specimen serial number	$P_{ini}$ /kN	$P_{un}$ /kN	$P_s^{ini}$ /kN	$P_s^{un}$ /kN	$a_c$ /mm	$\Delta a_c$ /mm	$P_{ini} / P_{un}$
RCWS-050-1	12.62	30.32	1.84	5.62	255.58	55.58	0.416
RCWS-050-2	16.87	29.96	2.43	5.29	256.00	56.00	0.563

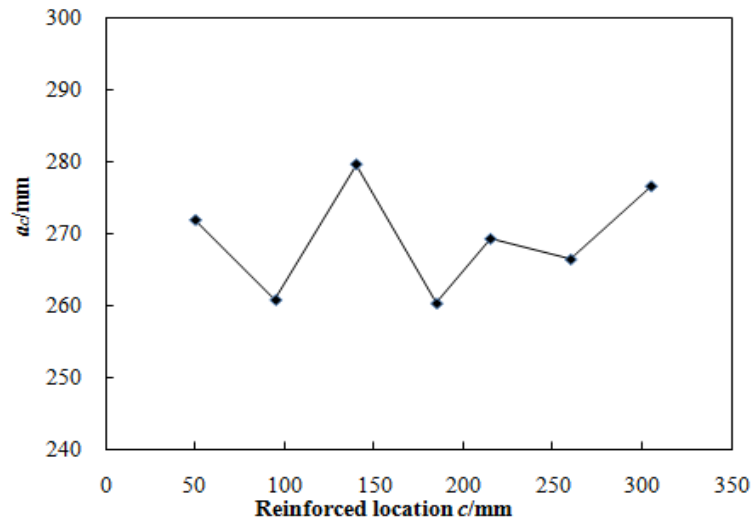
Table 3 Continued

RCWS-050-3	15.94	28.89	2.84	7.66	274.11	74.11	0.552
RCWS-050-4	13.91	27.27	1.78	7.71	301.74	101.74	0.510
Mean	14.84	29.11	2.23	6.57	271.86	71.86	0.510
Standard Deviation	1.67	1.19	0.44	1.12	18.80	18.80	0.058
RCWS-095-1	22.89	32.38	3.45	8.76	285.02	85.02	0.707
RCWS-095-2	19.97	31.90	2.09	5.23	251.07	51.07	0.626
RCWS-095-3	21.63	32.59	3.32	5.96	252.82	52.82	0.664
RCWS-095-4	15.02	25.94	2.21	6.66	254.33	54.33	0.579
Mean	19.88	30.70	2.77	6.65	260.81	60.81	0.647
Standard Deviation	2.99	2.76	0.62	1.32	14.03	14.03	0.047
RCWS-140-1	18.05	26.57	2.50	6.89	321.50	121.50	0.679
RCWS-140-2	16.28	26.76	1.67	3.32	241.90	41.90	0.608
RCWS-140-3	11.25	21.28	2.24	5.31	273.45	73.45	0.529
RCWS-140-4	13.81	22.64	1.99	6.34	281.44	81.44	0.610
Mean	14.85	24.31	2.10	5.46	279.57	79.57	0.611
Standard Deviation	2.57	2.40	0.31	1.36	28.36	28.36	0.053
RCWS-185-1	9.80	26.24	1.04	5.41	271.94	71.94	0.373
RCWS-185-2	13.26	23.41	1.00	3.24	266.41	66.41	0.566
RCWS-185-3	17.36	25.27	1.61	3.47	258.98	58.98	0.687
RCWS-185-4	10.19	26.76	0.47	2.91	243.96	43.96	0.381
Mean	12.65	25.42	1.03	3.76	260.32	60.32	0.498
Standard Deviation	3.03	1.28	0.40	0.97	10.51	10.51	0.132
RCWS-215-1	12.92	28.14	0.58	5.01	264.88	64.88	0.459
RCWS-215-2	14.81	25.32	1.02	3.03	274.06	74.06	0.585
RCWS-215-3	11.28	23.20	0.61	3.28	271.13	71.13	0.486
RCWS-215-4	12.07	23.19	0.55	3.07	267.00	67.00	0.520
Mean	12.77	24.96	0.69	3.60	269.27	69.27	0.512
Standard Deviation	1.31	2.03	0.19	0.82	3.56	3.56	0.047
RCWS-260-1	12.81	23.62	0.06	2.17	270.19	70.19	0.542
RCWS-260-2	15.48	24.65	0.18	2.97	264.34	64.34	0.628
RCWS-260-3	11.68	21.49	0.15	1.84	252.53	52.53	0.544
RCWS-260-4	16.59	22.06	0.29	3.55	278.57	78.57	0.752
Mean	14.14	22.96	0.17	2.63	266.41	66.41	0.616
Standard Deviation	1.98	1.25	0.08	0.67	9.47	9.47	0.086
RCWS-305-1	11.85	18.83	0.10	1.25	297.89	97.89	0.629
RCWS-305-2	14.92	21.83	0.14	1.78	285.46	85.46	0.683
RCWS-305-3	9.81	18.57	0.35	0.52	258.82	58.82	0.528
RCWS-305-4	9.83	22.76	0.08	1.30	263.97	63.97	0.432
Mean	11.60	20.50	0.17	1.21	276.53	76.53	0.566
Standard Deviation	2.09	1.83	0.11	0.45	15.87	15.87	0.096

Fig. 9 Curves of  $P$  and  $c$ 

Curves of initial cracking load  $P_{ini}$  and unstable fracture load  $P_{un}$  along with the change of reinforced location is shown in Fig. 9. As can be seen from Fig. 9, along with the reinforced location goes down, the value of initial cracking load  $P_{ini}$  and unstable fracture load  $P_{un}$  all present a downward trend, what shows that along with the down of reinforced location, the external load bore by testing specimen is gradually decrease. When the distance between reinforced location and top boundary is 95mm, the initial cracking load and unstable fracture load of reinforced concrete wedge splitting tensile specimens are maximal, the restriction by reinforcement for testing specimen is largest in this case. From Table 3, when the distance between reinforced location and top boundary is 95mm, the value of equivalent restraint force at the moment of initial cracking and instability are all maximal, it presents that at this moment the external load bore by steel bars is largest and the restrictive role by reinforcement for testing specimen is most significant.

Compared with the initial cracking load value 16.86kN of plain concrete wedge splitting specimens, the date of reinforced concrete specimens shown in Table 3 except the group RCWS-095 are all less than 16.86kN, what shows that the value of initial cracking load for each specimen are all reduced except the RCWS-095 group which restrictive role by reinforcement is most significant. For reinforced concrete specimens, before initial cracking, reinforcement and concrete work together and have the same deformation, but because the concrete area is much larger than the steel area, so the external load bore by concrete is much larger than steel. When concrete achieve the ultimate tensile strain and begin cracking, the tension bore by steel is very small and the restrictive role for concrete cracking is very small even can be ignored. And with the addition of steel, it destroys the integrity of concrete, moreover, during the test there many tiny cracks occurred by extrusion of steel around the reinforcement in concrete, these above function led to the acceleration of initial cracking for reinforced concrete specimens and this situation is most obviously for the two groups that reinforcement is mostly near the crack tip. Compared with the unstable fracture load, except the two groups RCWS-260 and RCWS-305 which restrictive role is the least by reinforcement, the value of unstable fracture load are all improved, it presents that there is an obvious effect for steel to restrict the expansion of crack, and the effect of reinforcement for reinforced concrete specimen is mainly manifested after initial cracking which

Fig. 10 Curve of  $a_c$  and  $c$ 

conform to the reinforced concrete tension member in practical engineering.

Curve of critical effective crack length  $a_c$  along with the change of reinforced location is shown in Fig. 10. As can be seen from Fig. 10, when reinforced position is changed, the value of critical effective crack length changed accordingly, and the varying value changed between 260.32mm to 279.51mm, which has a small changing scale and no remarkably trend. What shows that the critical effective crack length of reinforced concrete wedge splitting tensile specimens has nothing to do with the location of the bar placed. Compared with the critical effective crack length value 241.65mm of plain concrete wedge splitting specimens, the value of each reinforced concrete specimen are all have a greater degree of improve. It presents that the existence of reinforcement has a significant restrictions on crack propagation and it can effectively improve the ductility of concrete structures.

The value of initial fracture toughness  $K_I^{ini}$  and unstable fracture toughness  $K_I^{un}$  for each specimen can be obtained by putting the fracture parameters as shown in Table 3 into the formula in section 1. The result of initial fracture toughness is shown in Table 4 and the result of unstable fracture toughness is shown in Table 5. The  $K_{IC}^{ini}$  and  $K_{IC}^{un}$  is the value of initial fracture toughness and unstable fracture toughness of comparative plain concrete specimens in each table. In the table, the formula for increment is  $\frac{K_I - K_{IC}}{K_{IC}} \times 100\%$  and the minus means the value of  $K_I$  is less than  $K_{IC}$ .

Table 4 Calculated results of initial fracture toughness

Specimen serial number	$K_{IF}^{ini}/\text{MPa}\cdot\text{m}^{1/2}$	$K_{IS}^{ini}/\text{MPa}\cdot\text{m}^{1/2}$	$K_I^{ini}/\text{MPa}\cdot\text{m}^{1/2}$	$K_{IC}^{ini}/\text{MPa}\cdot\text{m}^{1/2}$	Increment ( % )
RCWS-050-1	0.667	0.024	0.643	0.891	-27.799
RCWS-050-2	0.892	0.032	0.860	0.891	-3.440

Table 4 Continued

RCWS-050-3	0.843	0.037	0.806	0.891	-9.558
RCWS-050-4	0.736	0.023	0.712	0.891	-20.053
Mean	0.784	0.029	0.755	0.891	-15.213
Standard Deviation	0.088	0.006	0.084	0	9.386
RCWS-095-1	1.210	0.049	1.161	0.891	30.304
RCWS-095-2	1.056	0.030	1.026	0.891	15.156
RCWS-095-3	1.144	0.048	1.096	0.891	23.034
RCWS-095-4	0.794	0.032	0.763	0.891	-14.421
Mean	1.051	0.040	1.011	0.891	13.518
Standard Deviation	0.158	0.009	0.151	0	16.997
RCWS-140-1	0.954	0.044	0.910	0.891	2.171
RCWS-140-2	0.861	0.029	0.831	0.891	-6.689
RCWS-140-3	0.595	0.039	0.555	0.891	-37.665
RCWS-140-4	0.730	0.035	0.695	0.891	-21.983
Mean	0.785	0.037	0.748	0.891	-16.042
Standard Deviation	0.136	0.005	0.135	0	15.183
RCWS-185-1	0.518	0.035	0.484	0.891	-45.716
RCWS-185-2	0.701	0.033	0.668	0.891	-25.016
RCWS-185-3	0.918	0.053	0.865	0.891	-2.967
RCWS-185-4	0.539	0.016	0.523	0.891	-41.285
Mean	0.669	0.034	0.635	0.891	-28.746
Standard Deviation	0.160	0.013	0.149	0	16.761
RCWS-215-1	0.683	0.012	0.671	0.891	-24.700
RCWS-215-2	0.783	0.022	0.762	0.891	-14.528
RCWS-215-3	0.596	0.013	0.584	0.891	-34.489
RCWS-215-4	0.638	0.012	0.627	0.891	-29.676
Mean	0.675	0.015	0.661	0.891	-25.848
Standard Deviation	0.070	0.004	0.066	0	7.396
RCWS-260-1	0.677	0.001	0.676	0.891	-24.084
RCWS-260-2	0.819	0.003	0.816	0.891	-8.440
RCWS-260-3	0.618	0.002	0.615	0.891	-30.949
RCWS-260-4	0.877	0.005	0.873	0.891	-2.051
Mean	0.748	0.003	0.745	0.891	-16.381
Standard Deviation	0.104	0.001	0.104	0	11.619
RCWS-305-1	0.627	0.001	0.626	0.891	-29.793
RCWS-305-2	0.789	0.001	0.788	0.891	-11.614
RCWS-305-3	0.519	0.004	0.515	0.891	-42.185
RCWS-305-4	0.520	0.001	0.519	0.891	-41.756
Mean	0.614	0.002	0.612	0.891	-31.337
Standard Deviation	0.110	0.001	0.111	0	12.426

Curves of fracture toughness along with the change of reinforced location are shown in Fig. 11. As can be seen from Fig. 11 and Table 4, except the group RCWS-095, the value of initial fracture toughness for each group is less than the value of plain concrete specimens, it shows that for concrete tensile members, the addition of reinforcement cannot effectively restrict the initial cracking of concrete, instead it will reduce the initial fracture toughness and speed up the concrete cracking. It also illustrates that the limited role of reinforcement for concrete tensile members cracks is mainly manifested after initial cracking. Meanwhile, as shown in Fig. 11 and Table 4, the value of initial fracture toughness for these groups has a close agreement and no obvious trend, it presents that the addition of reinforcement will reduce the initial fracture toughness but the value of initial fracture toughness has nothing to do with reinforced location. When the distance between reinforced location and top boundary is 95mm, the initial fracture toughness is maximum. And from the  $K_I^{ini}$  increment in Table 4, the value of initial fracture toughness for this group is larger than the plain concrete group, it shows that reinforcement can effectively limit the initial cracking of concrete at this moment, it also shows that place reinforcement in the middle of the fracture has a significant restriction for concrete initial cracking.

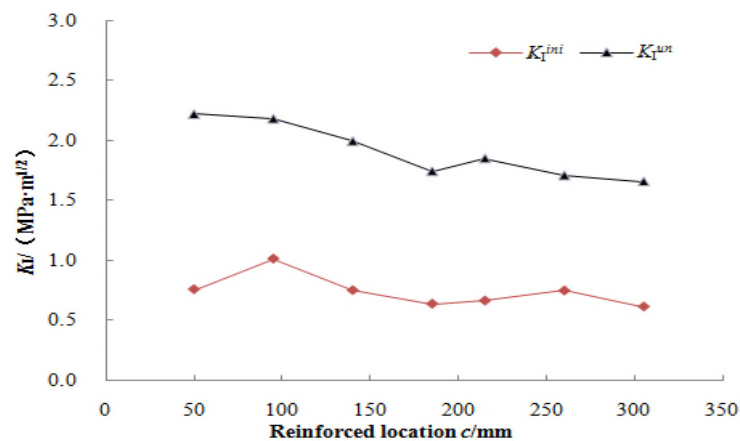
Fig. 11 Curves of  $K_I$  and  $c$ 

Table 5 Calculated results of unstable fracture toughness

Specimen serial number	$K_{IF}^{un}/MPa \cdot m^{1/2}$	$K_{IS}^{un}/MPa \cdot m^{1/2}$	$K_I^{un}/MPa \cdot m^{1/2}$	$K_{IC}^{un}/MPa \cdot m^{1/2}$	Increment ( % )
RCWS-050-1	2.165	0.073	2.091	1.601	30.636
RCWS-050-2	2.144	0.069	2.075	1.601	29.599
RCWS-050-3	2.296	0.100	2.196	1.601	37.162
RCWS-050-4	2.622	0.100	2.522	1.601	57.504
Mean	2.307	0.086	2.221	1.601	38.725
Standard Deviation	0.191	0.015	0.180	0	11.223
RCWS-095-1	2.759	0.126	2.634	1.601	64.497
RCWS-095-2	2.222	0.069	2.153	1.601	34.467
RCWS-095-3	2.292	0.085	2.206	1.601	37.813

Table 5 Continued

RCWS-095-4	1.839	0.096	1.744	1.601	8.917
Mean	2.278	0.094	2.184	1.601	36.424
Standard Deviation	0.327	0.021	0.315	0	19.688
RCWS-140-1	3.025	0.122	2.903	1.601	81.339
RCWS-140-2	1.775	0.059	1.716	1.601	7.193
RCWS-140-3	1.684	0.094	1.590	1.601	-0.657
RCWS-140-4	1.884	0.112	1.772	1.601	10.697
Mean	2.092	0.097	1.996	1.601	24.643
Standard Deviation	0.543	0.024	0.528	0	32.991
RCWS-185-1	2.058	0.180	1.878	1.601	17.323
RCWS-185-2	1.777	0.108	1.669	1.601	4.250
RCWS-185-3	1.838	0.115	1.723	1.601	7.632
RCWS-185-4	1.794	0.097	1.698	1.601	6.049
Mean	1.867	0.125	1.742	1.601	8.814
Standard Deviation	0.113	0.032	0.081	0	5.057
RCWS-215-1	2.117	0.106	2.011	1.601	25.612
RCWS-215-2	2.011	0.064	1.947	1.601	21.638
RCWS-215-3	1.811	0.069	1.741	1.601	8.760
RCWS-215-4	1.766	0.065	1.701	1.601	6.267
Mean	1.926	0.076	1.850	1.601	15.569
Standard Deviation	0.144	0.017	0.132	0	8.225
RCWS-260-1	1.833	0.034	1.799	1.601	12.365
RCWS-260-2	1.849	0.046	1.802	1.601	12.561
RCWS-260-3	1.509	0.029	1.480	1.601	-7.561
RCWS-260-4	1.802	0.056	1.747	1.601	9.104
Mean	1.748	0.041	1.707	1.601	6.617
Standard Deviation	0.139	0.011	0.133	0	8.300
RCWS-305-1	1.758	0.013	1.745	1.601	9.005
RCWS-305-2	1.866	0.018	1.847	1.601	15.390
RCWS-305-3	1.350	0.005	1.345	1.601	-16.021
RCWS-305-4	1.703	0.013	1.690	1.601	5.555
Mean	1.669	0.012	1.657	1.601	3.483
Standard Deviation	0.193	0.005	0.189	0	11.800

For unstable fracture toughness, as shown in Fig. 11: When place the reinforcement through the crack, the value of unstable fracture toughness gradually decreases as reinforced location close up to the crack tip. It illustrates that when place the reinforcement through the crack, the more reinforcement away from crack tip, the stronger restricted function for reinforcement to limit the crack extension, the better for the performance of crack resistance, and the stronger bearing



capacity for specimen. When the location of reinforcement is not across the crack, the value of unstable fracture toughness gradually decreases as reinforced location far away from the crack tip. It presents that for the specimens that the location of reinforcement are not across the crack, the more reinforcement close to the crack tip, the greater function of restraint for concrete cracking of reinforcement. For the two groups that place reinforcement nearby the crack tip, the value of unstable fracture toughness for the group that place reinforcement not across the crack is slightly bigger than the group that place reinforcement across the crack. It presents that when place reinforcement nearby the crack tip, the performance of crack resistance for specimens that place reinforcement not across the crack is better than the group that place reinforcement across the crack. In addition, as can be seen from Fig. 11: When the specimens that place reinforcement across the crack and far away from the crack tip, the value of unstable fracture toughness is much larger than the specimens that place reinforcement not across the crack. It shows that: For tensional concrete members, when place reinforcement across the crack and far away from the crack tip, the restrictive role on crack extension is best and can mostly improve the bearing capacity of components, so in the actual project, to anchoring the tensional concrete members containing cracks should place the bolt across the crack and far away from the crack tip. Compared with the plain concrete specimens, as the increment of unstable fracture toughness  $K_I^{un}$  shown in Table 5, except the extremely specific specimens, the value of unstable fracture toughness for reinforced concrete specimens are all larger than the plain concrete specimens, it shows that the addition of reinforcement can effectively control the crack extension and the reinforcement has an effective role in restricting crack propagation. And from the date in Table 5, along with the reinforced location down gradually the increment value has also been gradually reduced, it also presents that when place reinforcement across the crack and far away from the crack tip, the performance of crack resistance is the best.

## 5. Conclusions

- Along with the increase of the distance between reinforced location and specimen's top boundary, the initial craking load  $P_{ini}$  and unstable fracture load  $P_{un}$  present a downward trend, and the bearing capacity of specimen gradually decrease. When the distance is 95mm, the bearing capacity of specimen is the largest.
- The critical effective crack length  $a_c$  has nothing to do with the reinforced location. But the addition of reinforcement can effectively improve the value of  $a_c$  and the ductility of concrete structures.
- Compared with the plain concrete specimens, addition of reinforcement reduces the initial fracture toughness  $K_I^{ini}$  of reinforced concrete specimens, but the value of  $K_I^{ini}$  has nothing to do with the reinforced location and tends to be stable; the unstable fracture toughness  $K_I^{un}$  improves significantly with the addition of reinforcement and its value gradually decreases along with the increase of the distance between reinforced location and specimen's top boundary.
- For tensional concrete members, addition of reinforcement has no obvious effect on restricting the initial cracking of concrete, on the contrary it would speed up the concrete initial cracking to some extent, the limited role of reinforcement for concrete is mainly manifested after initial cracking. When place reinforcement in the middle of the fracture has the best restriction for concrete initial cracking; when place reinforcement across the crack and far away from the crack tip, the restrictive role on crack extension is best and the crack resistance effect is obvious.

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