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RC beams retrofitted using external bars with additional anchorages–a finite element study

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Abstract. Study on flexural retrofitting of RC beams using external bars with additional intermediate anchorages at soffit is reported in this paper. Effects of varying number of anchorages in the external bars at soffit were studied by finite element analysis using ANSYS 12.0 software. The results were also compared with available experimental results for beam with only two end anchorages. Two sets of reference and retrofitted beam specimens with two, three, four and five anchorages were analysed and the results are reported. FE modeling and non-linear analysis was carried out by discrete reinforcement modeling using Solid65, Solid45 and Link8 elements. Combin39 spring elements were used for modeling the frictional contact between the soffit and the external bars. The beam specimens were subjected to four-point bending and incremental loading was applied till failure. The entire process of modeling, application of incremental loading and generation of output in text and graphical format were carried out using ANSYS Parametric Design Language.

Keywords: external bars; anchorages; soffit; ANSYS

1. Introduction

Flexural retrofitting of beams is one of the common requirements of RC buildings in order to accommodate increased seismic forces due to codal revisions; increased loading requirements; extend the life of buildings vulnerable to natural disasters. Though the objectives of retrofitting RC flexural members are wider, the ultimate aim is to minimize the use of raw materials. Though methods such as beam jacketing, bonded steel plating, external post-tensioning and FRP wrapping are widely adopted for the retrofitting of structural elements, they have limitations such as high cost, loss of aesthetics, increase in self-weight, need for careful surface preparation, unexpected de-lamination failure, etc. The external reinforcement technique, proposed by the authors (Kothandaraman and Vasudevan 2010, Vasudevan and Kothandaraman 2014) can be adopted as one of the easy-to-use and cost effective flexural retrofitting technique due reasons such as speed and simplicity of installation; minimal disruption during installation; use of cost effective materials;

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minimal surface preparation of concrete substrate; no possibility of de-lamination failure problems, as experienced in bonded plates and FRP laminates (Hassan and Rizkalla 2003, Thomsen *et al.* 2004). In this paper, results of the non-linear finite element analysis (NLFEA) of 2 sets of beam specimens with external bars at the soffit with varying numbers of anchorages were carried out and the results are compared. Due to of heterogenic and cracking behaviour of concrete beam with external bars provided at the soffit, the non-linear analysis is complicated. Also, the external bars at the soffit anchored at the ends behaves differently from the conventional reinforcing bars due to absence of proper bonding. The finite element modeling is carried using Solid65, Solid45, Link8 and Combin39 elements (ANSYS Commands Reference 2005).

2. Review of literature

Buckhouse (1997) carried out experimental testing on the flexural behaviour of RC beams and the critical results were compared with analytical values. Wolanski (2004) implemented finite element analysis of flexural behaviour RC using ANSYS on the experimental beams carried out by Buckhouse (1997) and the results were compared and validated. Wolanski (2004) has used Solid65, Solid45 and Link8 elements to model concrete, steel cushion at the supports and loading points by considering one quarter of the beam model. The steel reinforcements were incorporated in the concrete elements through the nodes created by the mesh of the concrete volume. Boundary conditions were applied at points of symmetry and at the supports. The performance of RC beams with externally bonded Carbon Fiber Reinforced Polymer fabric using ANSYS was studied by Kachlakev *et al*. (2001). They followed smeared cracking approach for FE modeling using Solid65 for concrete, Link8 for rebar, Solid46 for FRP composites and Solid45 for steel cushion at the location of supports and loading points. Fanning (2001) conducted FE analysis on 3000 mm RC beams and 9000 mm post-tensioned concrete beams with ANSYS V5.5 using smeared crack model to allow for concrete cracking with the option of modeling the reinforcement in a distributed or discrete manner. It was stated that, for RC beams internal reinforcement should be modelled discretely and for post-tensioned beams the post-tensioning tendons should be modelled discretely with any other additional reinforcement modelled in a distributed manner. Also, reported that, Young’s modulus and concrete tensile strength used in the numerical models can be calculated using the existing rules of thumb from the known compressive strength of concrete. Dahmani *et al*. (2010), studied the crack propagation in RC beams using ANSYS modelled with Solid65 element with smeared reinforcement approach, in which the concrete and the reinforcing were incorporated into elements with the same geometrical boundaries and the effects of reinforcing were averaged within the pertaining element. ANSYS Parametric Design Language (APDL) and batch mode approach was used by Vasudevan and Kothandaraman (2011a) for conducting analysis on multiple numbers of beam specimens and emphasized the advantages of APDL and batch mode approach for large size problems. Sallam *et al*. (2009) presented the results of peeling failure of FRP strengthened flexural beams using ANSYS and reported that the discrete crack approach was more accurate than smeared crack approach. Elavenil and Chandrasekar (2007) presented the results of the numerical models on the flexural behaviour of RC beams strengthened with ferro-cement. An elaborate parametric study on non-linear behaviour of RC beams was conducted using ANSYS and results were reported by Vasudevan and Kothandaraman (2011b) with regard to mesh density, material modeling, effect of excluding shear reinforcements in flexural behaviour, inclusion of steel cushion at the supports and loading points.

3. Beam specimens considered for the study

Two control beam specimens (RF-N-10, RF-H-10) and two sets (eight numbers) retrofitted beam specimens (ER-N-10-X, ER-H-10-X) using external bars with varying number of anchorages at the soffit were used for the non-linear finite element analysis. The intermediate anchorages of the external bars can be provided by welding pieces of straight rod of required length and numbers on the U-shaped bar (with only two end anchorages) to match with the planned hole locations of the beams. The external bar with welded additional anchorages can be inserted in to the holes filled with chemical adhesive. The results of the analysis were compared categorically and with available experimental results. The overall size of the specimen is 2000 mm x 250 mm x 200 mm with an effective span of 1800 mm. Two grades of concrete designated as N and H with targeted cube compressive strength of 30 MPa and 40 MPa were used for the study. Effective cover of 31.25 mm was used for the FE modeling. The beams were analysed for four-point bending with loading at a distance of 550 mm from either end of the support, so as to have a moment span of 700 mm. The details and other parameters used for the study are shown in Fig. 1 and Table 1.

Table 1 Details of beams

Sl. No.	Beam ID	Concrete cube compressive strength, $f_{ck,cube}$	Modulus of rupture, $f_{cr}=0.7(f_{ck})^{1/2}$ (MPa) [15]	Modulus of elasticity E_c=5000$(f_{ck})^{1/2}$ (MPa) (IS 456 2000)	Bonded (Internal) bars			External bars			Remark
					Area of steel (mm^2)	Yield strength (MPa)	Ultimate strength (MPa)	Area of steel (mm^2)	Yield strength (MPa)	Ultimate strength (MPa)	
1	RF-N-10	35.6	4.18	29833	157	556	590	-	-	-	Control
2	ER-N-10-10-A	38.2	4.33	30903	157	556	590	157	556	590	Retrofitted
3	ER-N-10-10-B	38.2	4.33	30903	157	556	590	157	556	590	
4	ER-N-10-10-C	38.2	4.33	30903	157	556	590	157	556	590	
5	ER-N-10-10-D	38.2	4.33	30903	157	556	590	157	556	590	
6	RF-H-10	45.2	4.71	33615	157	556	590	-	-	-	Control
7	ER-H-10-10-A	45.3	4.71	33653	157	556	590	157	556	590	Retrofitted
8	ER-H-10-10-B	45.3	4.71	33653	157	556	590	157	556	590	
9	ER-H-10-10-C	45.3	4.71	33653	157	556	590	157	556	590	
10	ER-H-10-10-D	45.3	4.71	33653	157	556	590	157	556	590	

Table 2 Materials properties for concrete and steel

Property		Property	
Yield strength of hanger bars	556 MPa	Shear transfer coefficient for open crack	0.3
Yield strength of stirrups	550 MPa	Shear transfer coefficient for closed crack	1.0
Tangent modulus for steel	20 MPa	Uni-axial crushing stress value	-1.0
Poisson’s ratio of concrete	0.2	Stiffness multiplier constant (T_c).	0.6

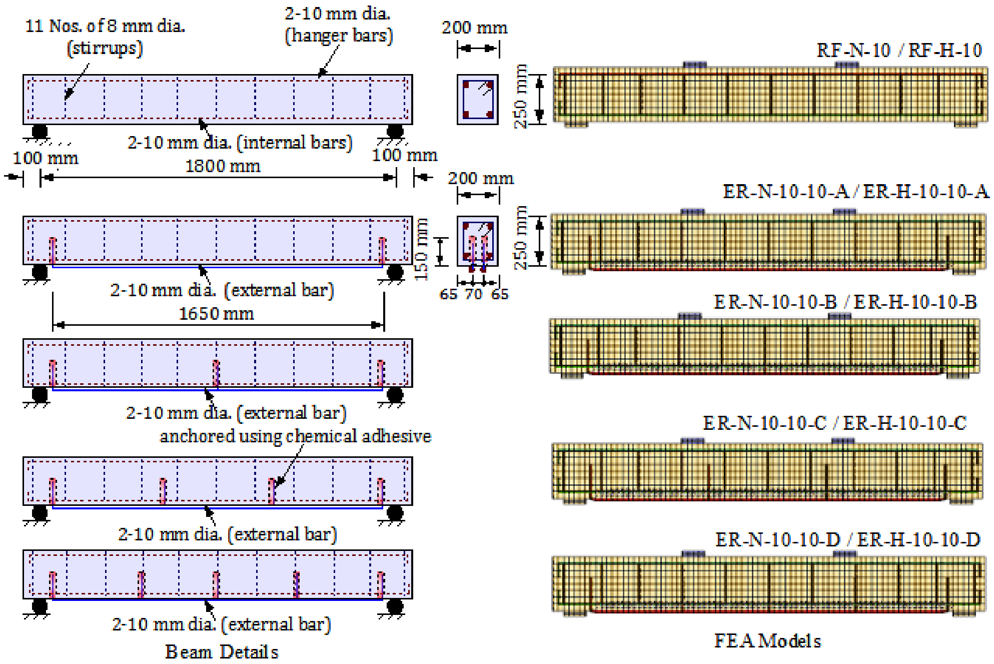

Fig. 1 Details of beam specimen and FEA Models

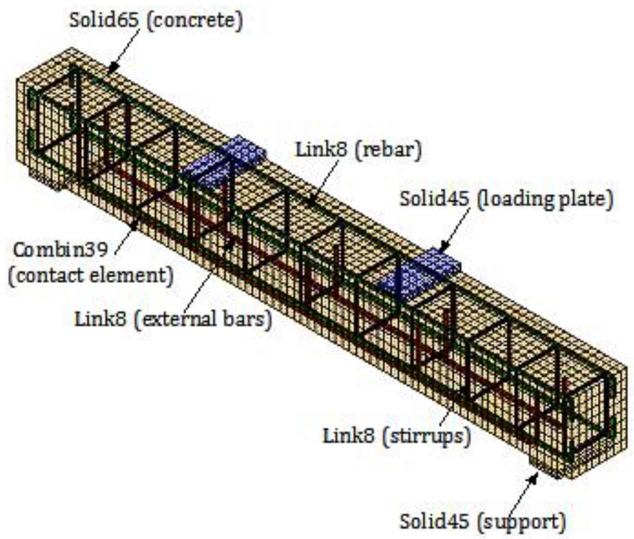

Fig. 2 Typical FEA model

4. Finite element modeling and analysis using ANSYS

RC beam specimens were modelled using eight noded SOLID65 element with three degrees of freedom at each node (translations in the nodal x, y, and z directions), capable of handling nonlinear behaviour, cracking in three orthogonal directions due to tension, crushing in compression and plastic deformation. The reinforcing bars were included in the finite element concrete model using two noded LINK8 spar element with three degrees of freedom at each node (translations in the nodal x, y and z directions), capable of handling plasticity, creep, swelling, stress stiffening and large deflection. The supports and loading points were modelled as steel cushion to avoid stress concentration problem using eight noded SOLID45 element with three degrees of freedom at each node (translations in the nodal x, y, and z directions), which handles plasticity, creep, swelling, stress stiffening, large deflection and strain. The contact between external bars and the soffit of the beam is modelled using COMBIN39, a unidirectional element with nonlinear generalized force-deflection capability. Material model for concrete used for the study was derived from IS 456 (2000). Other parameters used for the modeling is furnished in Tables 1 and 2. The steel reinforcement used for the FE models was assumed to be an elastic-perfectly plastic material, identical in tension and compression. The bi-linear elastic-plastic stress-strain for steel reinforcement to be used with LINK8 element was furnished in two sets of data. For the elastic range, modulus of elasticity of 200000 N/mm^2 and Poisson's ratio of 0.2 was used to setup a linear isotropic model. For bilinear isotropic hardening model of LINK8 element, the stress-strain curve of reinforcement follows the specified yield stress and continues along the second slope defined by the tangent modulus. It was experienced and suggested by earlier researchers (Dahmani *et al*. 2010, Fanning 2001, Kachlakev *et al*. 2001, Wolanski 2004) that a tangent modulus of 10 to 20 N/mm^2 is to be used to avoid loss of stability upon yielding and hence a value of 20 N/mm^2 was adopted in the present study. Parameters which are not stated in this report were taken as program default. The FE modeling was carried out in batch mode in sequence using, KEYPOINTS, LINES, LESIZE, VOLUME, VMESH and VSWEEP commands. The rebar elements were introduced in the nodes of the concrete elements using discrete reinforcement modeling which is most preferred for RC elements with well-defined reinforcement locations using E and EGEN commands. The support conditions were created using displacement (D) boundary conditions. The entire process of the non-linear finite element analysis such as geometrical modeling, material modeling, parameters for non-linear analysis, creation of load-steps, graphical post processing of results, generation of various graphs and images and output in the form of text file was generated using a single input file developed using the ANSYS Parametric Design Language (APDL) (ANSYS Commands Reference 2005). A typical FE model with discrete reinforcement model is shown in Fig.2. The external reinforced beams behave in a hybrid of flexural and tied arch action in addition to the frictional bonding by the soffit external bars. The external bars also follow the deflected shape of the beam due to loading and frictional bonding increases due to load increase. This is one of the additional advantages of providing external bars at the soffit when compared to the sides of the beam. The above behaviour was incorporated in the model by the use of COMBIN39 element between the external bar and the soffit of the beam. COMBIN39 is a unidirectional element with nonlinear generalized force-deflection capability. The element has longitudinal or torsional capability. The longitudinal option is a uniaxial tension-compression element with up to three degrees of freedom at each node such as translations in the nodal x, y, and z directions. Displacement along x and y directions were activated for the longitudinal and transverse COMBIN39 spring elements and all other options were set to default values. For incorporating the frictional bonding of longitudinal COMBIN39 elements, 5% of the bond strength of the fully bonded bar is assumed using IS 456 (2000) codal values. In order to obtain the correct fraction of bond strength achieved by frictional bonding between the external bars and the soffit, a large number of trial analyses were carried out on one of the beam

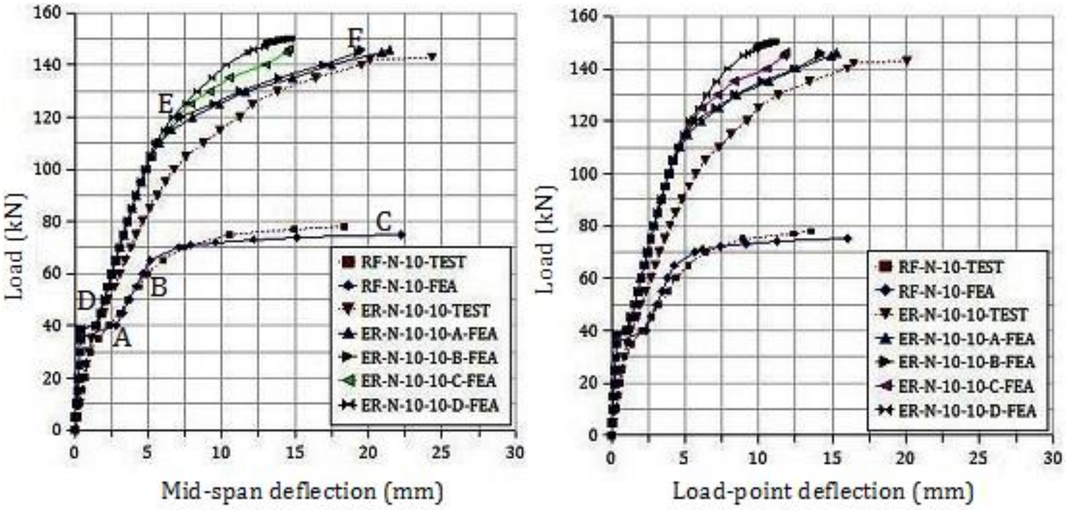

Fig. 3 Load versus deflection for beams in N-10 series

specimens with only two end anchorages (ER-N-10-12) by varying the bond percentage. The load versus mid-span behaviour was obtained for various bond percentages and was compared with experimental behaviour (Vasudevan and Kothandaraman 2014). It was found that at bond strength of nearly 5% of that of the fully bonded bar, the experimental load versus mid-span deflection curve is in close agreement with FEA load versus mid-span curve. Using the 5% of the frictional bonding, all the external beam specimens were analysed and categorically compared with experimental behaviour in each series of beams. Since the external bars restrained to displace independently along the transverse direction by the contact of concrete elements a full stiffness using the modulus of elasticity of concrete is used for transverse COMBIN39 elements.

5. Results and discussion

5.1. Deflection behaviour

One of the important factors that affect the serviceability of RC beam is the deflection. The deflection is a function of load, length of span, second moment of area and modulus elasticity of material. The deflection at centre of mid-span and loading points were recorded at every 5 kN load increment and graphed. Load versus deflection behaviour for the beam specimens in both the series indicates three different zones as depicted in Figs. 3 and 4. The salient points of the load versus mid-span deflection curves are indicated in the graph for reference (points A, B, and C for RF-N-10 and RF-H-10) and retrofitted (D, E and F for ER-N-10-10-X and ER-H-10-10-X) beam specimens. As can be seen from the Figs. 3 and 4, the behaviour of the reference and the retrofitted beams are similar and closer to each other till the formation of the initial crack. However, the initial cracking load for the retrofitted beam is higher than the reference beam specimen. After the yielding of internal bars, the strength and the stiffness of the strengthened specimens were larger up to the ultimate stage when compared to the reference beam specimens. It is noted that the slope of the load-deflection curves of the retrofitted beam specimens after the yielding of internal bars

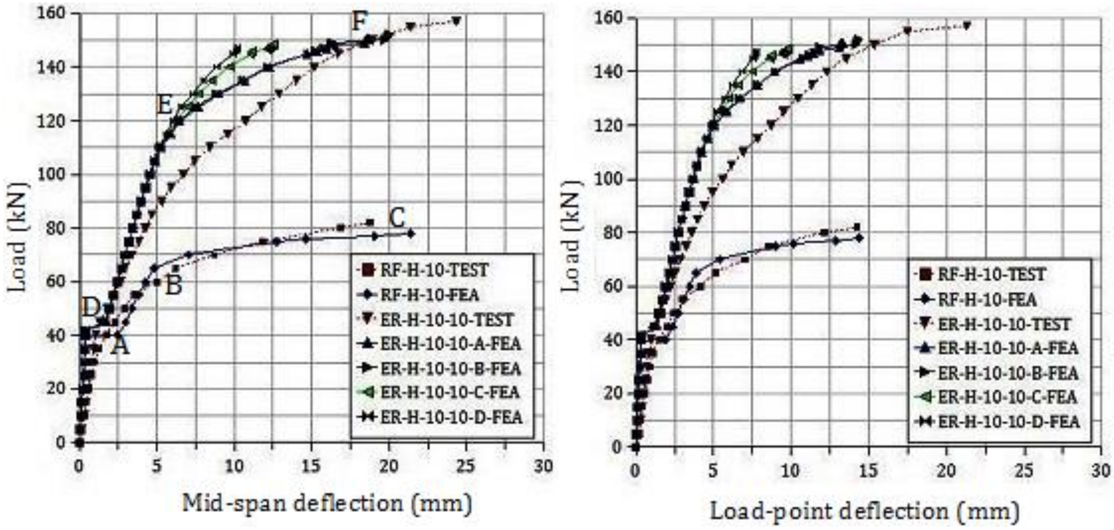

Fig. 4 Load versus deflection for beams in H-10 series

are steeper than the corresponding reference beam specimens, which indicates the increase in stiffness due to the addition of external bars at the soffit. Also for a particular load level, the deflections of the retrofitted beam specimens are less than the respective reference beam specimen in the series. At any load level, the deflections are reduced significantly thereby increasing the stiffness of the retrofitted beams. At ultimate load level of the reference beams, the retrofitted beams exhibit a deflection varying from 25% to 40% of the maximum-recorded deflection of the reference beam specimens. Comparing the load versus deflection curves of beams in N and H series for similar internal and external reinforcement ratios, the compressive strength appears to have some influence on the load versus deflection response of the beams with external bars. It is observed that the load versus deflection behaviour of reference and retrofitted beams follows similar trends, except the slight change in the performance after the internal steel yielding, which indicates that the unbonding of external bars did not significantly affect the load versus deflection of the beams. The similarity of the load-deflection behaviour of the reference and retrofitted beams further ensures the flexural and composite action of the external bars at the soffit. The plots also indicate that the additional external reinforcement enhances not only the moment carrying capacity of beams but also controls the deflections.

5.2 Effect of additional anchorages on the load-deflection behavior

The effect of additional anchorages in the external bars as seen from the load versus deflection behaviour clearly indicates that the performance of the beams with respect to varying number of anchorages does not show much variation till the yielding of internal bars. However, the deflection after the yielding of internal bars is reduced due to the provision of additional number of anchorages. At the ultimate stage, though the ultimate moment capacity remains practically the same, the maximum observed deflection reduces as the number of intermediate anchorages increases. Deflection behaviour also indicates that the provision of additional anchorages shows reduction in ductility performance of the retrofitted beam specimens.

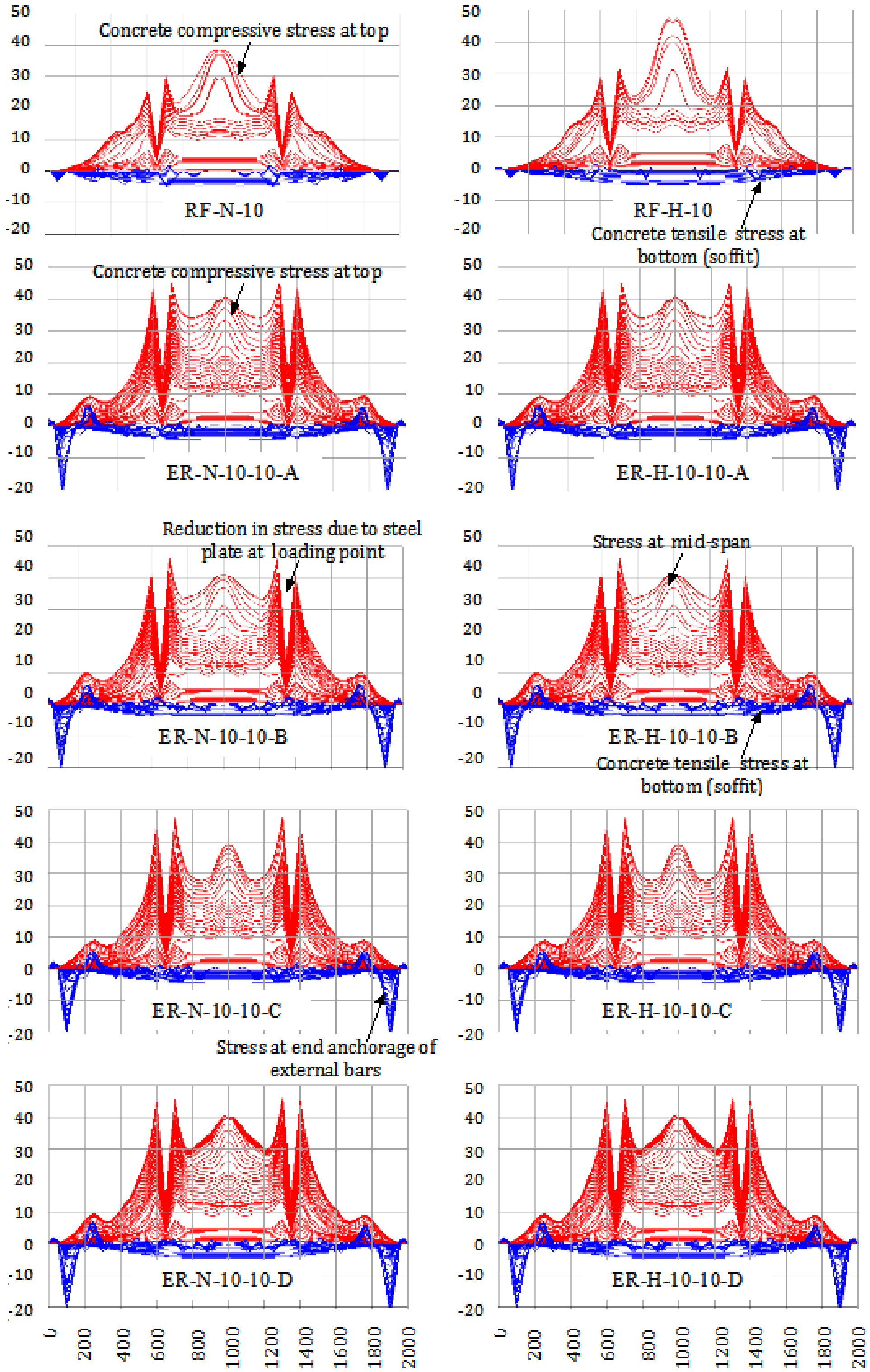

Fig. 5 Stress distribution in concrete top (compression) and bottom (tension) surfaces

5.3. Stress variation along the length of the beam at various loading stages

Graphical display of concrete surface stress variation along the length of the beam at top (compression) and bottom (tension) is depicted in Fig. 5. For the reference beam specimens (RF-N-10, RF-H-10), the compressive stress variation along the span indicates four-point flexure behaviour. It is to be noted that, at the loading point location, the concrete compressive stress is drastically reduced due to the provision of steel cushion plates. At the ultimate stage, after the yielding of internal bars, the mid-span section of the beam experiences higher stress values close to the compressive strength of concrete. At the soffit of the beam, the tensile stress variation in concrete along the length of the span indicates pure flexure behaviour. For the retrofitted beam specimens (ER-N-10-10-A, ER-N-10-10-B, ER-N-10-10-C, ER-N-10-10-D, ER-H-10-10-A, ER-H-10-10-B, ER-H-10-10-C, ER-H-10-10-D), the concrete stress variation along the length of the beam is uniform in the constant moment region, which can be considered as one of the improvement attained out of the external reinforcing bars. As in the case of reference beam specimens, at the loading point locations local reduction in stress is observed due to the provision of steel plates. At the end anchorage locations of the external bars, local increase in concrete tensile stress is observed, which leads to cracking of concrete at the anchorage locations. It is important to note that the provision of additional intermediate anchorages does not show any improvement or changes in the concrete stress behaviour.

5.4. Steel bar stress variation along the length of the beam at various loading stages

Stress in internal tension bar, compression bar and in the external bar at the soffit was extracted from the FE analysis and are presented in Fig. 6. For the reference beam specimens (RF-N-10, RF-H-10), stress variation in the internal tension bar shows four-point bending behaviour. For specimens RF-N-10 and RF-H-10, the stress in internal bars varies along the length of the beam with zero at the supports to maximum at the mid-span section of the beam with variation in bending moment. For retrofitted beam specimens with only two anchorages (ER-N-10-10-A, ER-H-10-10-A), the stress variation in the internal bars shows a little variation when compared to the reference beam specimens. For retrofitted beams with two end anchorages and additional middle anchorage (ER-N-10-10-B, ER-H-10-10-B), almost similar behaviour is observed, which shows no additional benefit out of the additional middle anchorage. For retrofitted beam specimens with end anchorages and two additional internal anchorages (ER-N-10-10-C, ER-H-10-10-C), sudden variation in stress distribution in the three segments of external bars between the anchorages is observed. However, the stress distribution in the internal tension and compression bars are similar. Retrofitted beam specimens with two end anchorages and three additional interior anchorages (ER-N-10-10-D, ER-H-10-10-D), shows stress distribution similar to beam specimens with two additional interior anchorages (ER-N-10-10-C, ER-H-10-10-C). It is observed that, the beam specimens with additional anchorage at the mid-span section of the beam (ER-N-10-10-B, ER-H-10-10-B, ER-N-10-10-D, ER-H-10-10-D) does not show any change in the behaviour. Hence, it may be concluded that the provision of additional anchorages at the mid-span section of the beam is not benefitted to the external bar at the soffit. One of the general observations is that the stress distribution in the internal tension and compression bar remains same for all the retrofitted beam specimens. Finally, it can be concluded that the provision of additional interior anchorages does not show any improvement in the strength performance of the beam with external bars at the soffit. Also, reduction in ductility is observed due to the loss of tied

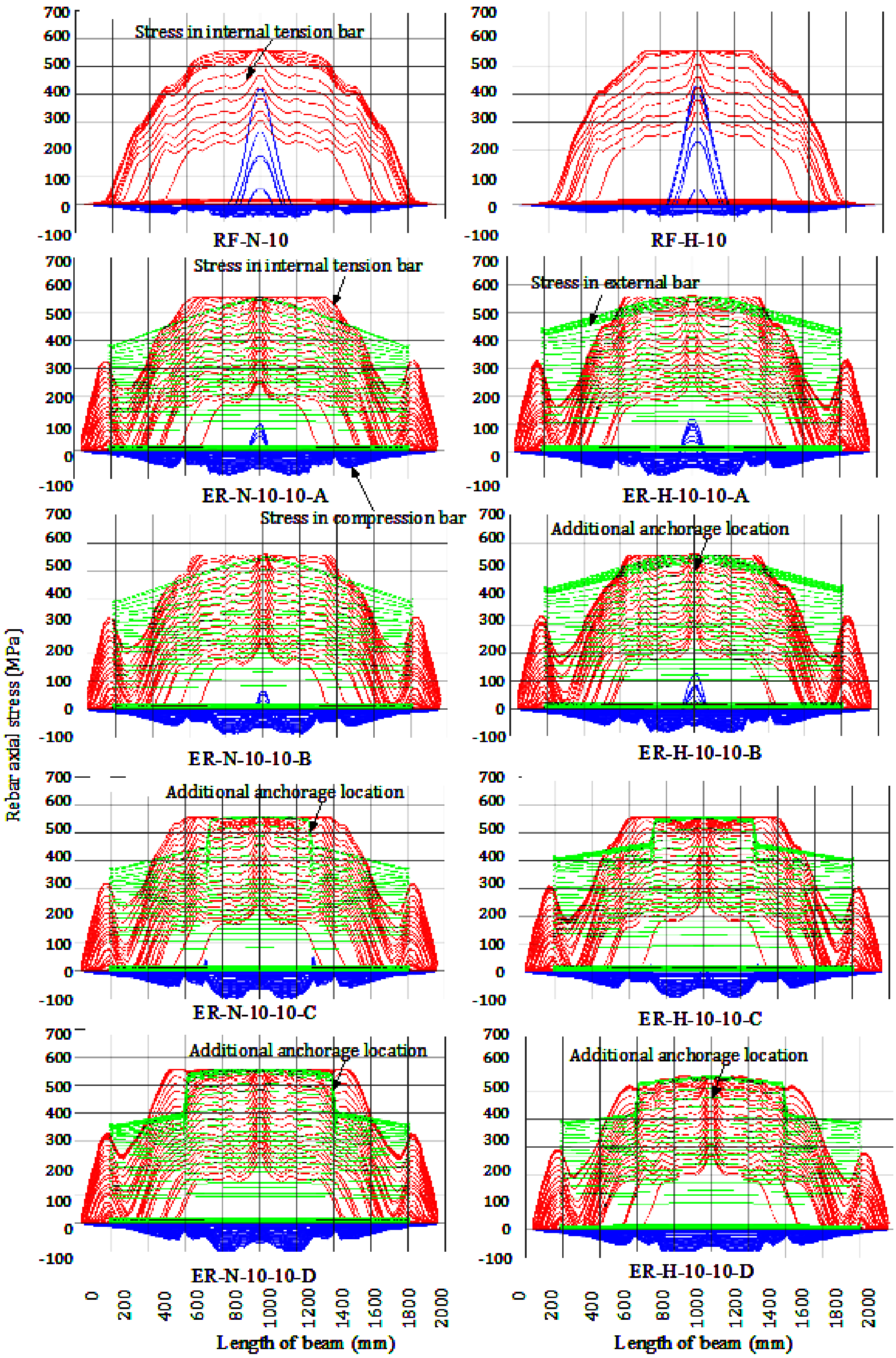

Fig. 6 Axial stress in internal, external and compression bars

Table 3 Comparison moment at critical stages and deflection ductility indices

Beam ID	Moment (kNm) at			Deflection ductility index
	Initial crack	Yielding of internal steel	Ultimate stage	δ_u / δ_y
RF-N-10-TEST	8.0	17.9	21.5	3.07
RF-N-10-FEA	9.7	17.9	20.6	5.25
ER-N-10-10-TEST	10.7	27.5	39.3	3.6
ER-N-10-10-A-FEA	10.6	31.6	40.2	4.08
ER-N-10-10-B-FEA	10.6	33.0	40.2	3.44
ER-N-10-10-C-FEA	10.6	35.8	40.2	3.3
ER-N-10-10-D-FEA	10.6	38.5	41.3	1.43
RF-H-10-TEST	9.6	17.9	22.6	3.03
RF-H-10-FEA	10.9	17.9	21.4	4.46
ER-H-10-10-TEST	11.6	28.9	43.2	3.27
ER-H-10-10-A-FEA	11.5	33.0	41.5	2.89
ER-H-10-10-B-FEA	11.4	33.0	41.8	3.1
ER-H-10-10-C-FEA	11.5	35.8	40.8	1.64
ER-H-10-10-D-FEA	11.4	38.5	40.5	1.15

arch action due to the provision of additional anchorages at the intermediate locations.

5.5. Behaviour at initial cracking, internal steel yielding and ultimate stage

Loads at initial crack formation were observed and corresponding bending moment values were calculated for all the beam specimens and are compared as depicted in Table 3. It is to be noted that for the conventional beams (without external bars) the load at first crack essentially depends upon the strength of concrete. On the contrary, the provision of external bars has contributed significantly to enhance the load at initial crack formation. However, the provision of additional intermediate anchorages in the external bars shows no improvement in initial cracking behaviour. The load at internal steel yielding stage is noted from the load versus mid-span deflection curves, depicted in Figs. 3 and 4 corresponding to second change in the slope of the curves (point - B for reference and E for retrofitted beam specimens). The corresponding moment values are calculated and reported in Table 3. It is to be noted that for the reference beam specimens, well defined yield point is observed when compared to retrofitted beam specimens. For the retrofitted beam specimens, even after the yielding of internal bars the strain in the external bar is within the yield limit up to the ultimate failure stage. This is one of the special advantages of the proposed technique by which the retrofitted beam members are capable of recovering from the maximum deflection. Due to the provision of additional anchorages in the external bars, marginal increase in moment at internal steel yielding (maximum of 16%) was noted. The ultimate moment values calculated from the maximum observed loads are presented in Table 3. Graphical comparisons of the critical values are shown in Figs. 7 and 8. From the tabulated values, it is noted that the provision of additional anchorages slightly reduces ultimate moment capacity. The reduction in ultimate moment capacity may be due to the formation of concentrated cracks at the additional

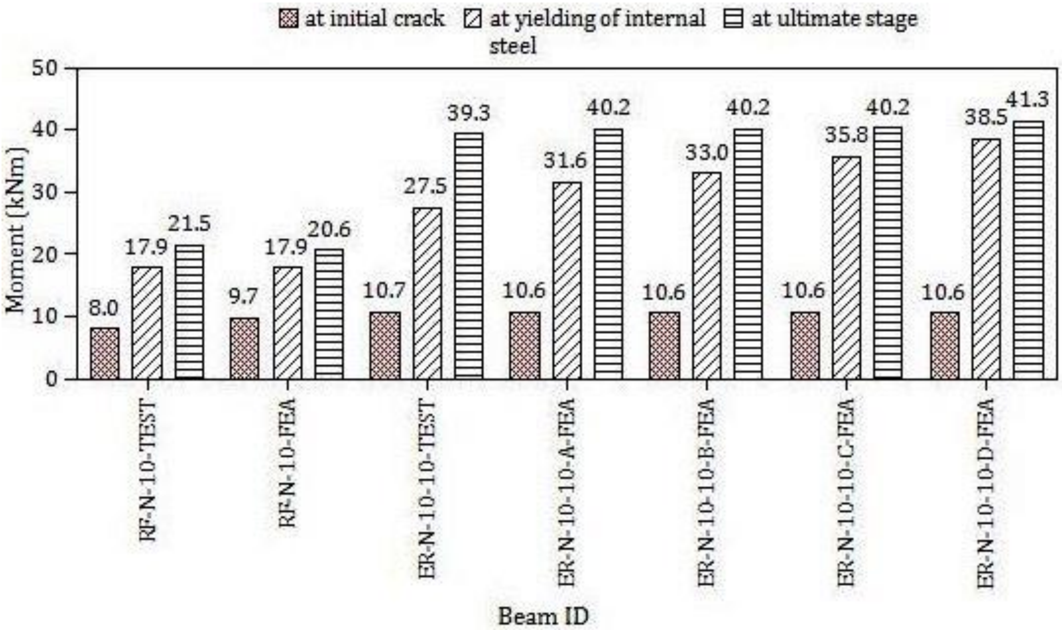

Fig. 7 Comparison of moment at critical stages– N-10 series

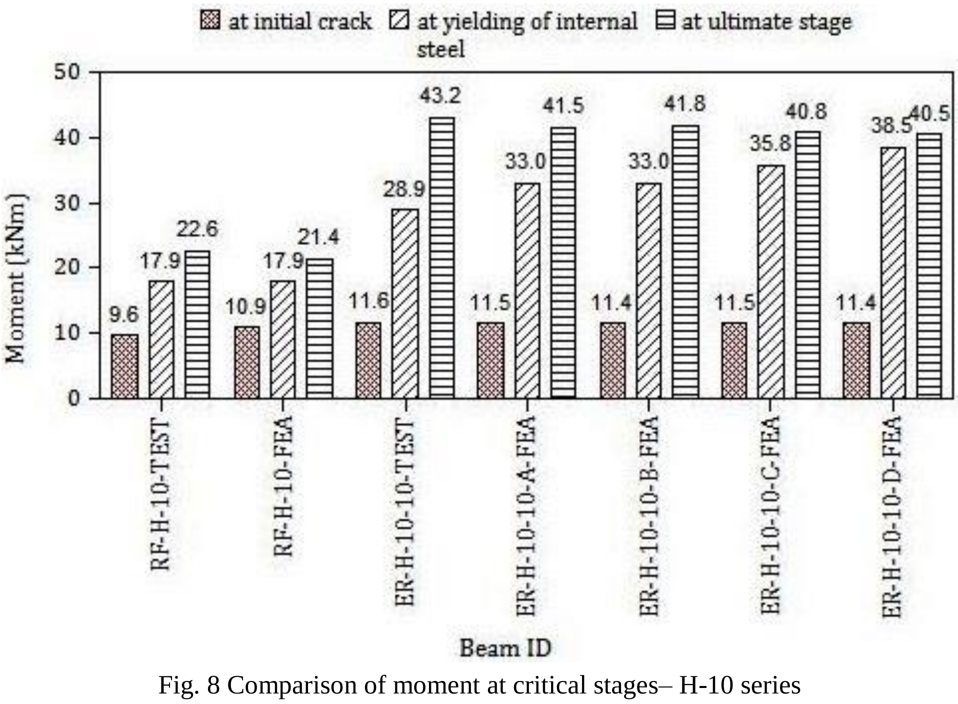

Fig. 8 Comparison of moment at critical stages– H-10 series

anchorage locations and loss of tied arch action due to the provision of additional anchorages. In general, the provision additional intermediate anchorages have not shown any remarkable improvement in performance.

6. Conclusions

Based on the above Non-Linear Finite Element Analysis (NLFEA) of two numbers of reference beam specimens and eight numbers of RC beam specimens retrofitted with external bars at the soffit with varying numbers of additional intermediate anchorages the following conclusions are made.

- Retrofitted beam specimens using external bar at soffit with two end anchorages has shown remarkable improvement in behaviour as reported in the earlier study (Kothandaraman and Vasudevan 2010, Vasudevan and Kothandaraman 2014).
- Deflection behaviour indicates that the provision of additional anchorages shows reduction in ductility performance of the retrofitted beam specimens.
- Provision of additional anchorages marginally reduces ultimate moment capacity.
- In general the additional intermediate anchorages has not shown any improvement in behaviour due to loss of tied-arch action of external bars at soffit and hence not recommended.

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