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Elastic wave dispersion modelling within rotating functionally graded nanobeams in thermal environment

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Abstract. In the present research, wave propagation characteristics of a rotating FG nanobeam undergoing rotation is studied based on nonlocal strain gradient theory. Material properties of nanobeam are assumed to change gradually across the thickness of nanobeam according to Mori-Tanaka distribution model. The governing partial differential equations are derived for the rotating FG nanobeam by applying the Hamilton's principle in the framework of Euler-Bernoulli beam model. An analytical solution is applied to obtain wave frequencies, phase velocities and escape frequencies. It is observed that wave dispersion characteristics of rotating FG nanobeams are extremely influenced by angular velocity, wave number, nonlocal parameter, length scale parameter, temperature change and material graduation.

Keywords: functionally graded materials; nonlocal strain gradient theory; wave dispersion characteristics; rotating nanobeam

1. Introduction

Functionally graded materials (FGMs) are composed from a mixture of metal and ceramic and have a continuous material variation from one surface to another which is designed to reach the desirable and practical characteristics. Recently, many paper have been published concerning with analysis of FG nanostructures. Among them, Eltaher *et al*. (2012) explored free vibration behavior of nonlocal FG nanobeams using finite element method. Thermal loading influences on stability and vibrational behavior of nanoscale FGM beams is performed by Ebrahimi and Salari (2015c, d), Ebrahimi and Barati (2016a) employed nonlocal third order beam theory to vibration analysis of nanoscale FG beams. Ahouel *et al*. (2016) investigated size-dependent mechanical behavior of functionally graded trigonometric shear deformable nanobeams including neutral surface position concept. Vibration and buckling analysis of smart piezoelectrically actuated FG nanobeams subjected to magneto-electrical field is explored by Ebrahimi and Barati (2016b-d), In the case of rotating nanobeams, Ebrahimi and Shafiei (2016) examined the application of Eringen's nonlocal elasticity theory for vibration analysis of rotating FG nanobeams. Also, Ghadiri *et al*. (2016) displayed the surface effects on vibration behavior of a rotating FG nanobeam based on nonlocal elasticity theory.

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According to the nonlocal continuum theory, strain/stress state at any reference point is a function of corresponding states of other points of the continuum body. Narendar and Gopalakrishnan (2009) investigated small scale influences on wave propagation of multi-walled carbon nanotubes. Wang (2010) researched the wave propagation analysis of fluid-conveying single-walled carbon nanotubes applying strain gradient theory. Yang *et al*. (2011) investigated wave propagation of double-walled carbon nanotubes on the basis of size-dependent Timoshenko beam model. Wave propagation analysis of single-walled carbon nanotubes exposed to an axial magnetic field in the framework of nonlocal Euler–Bernoulli beam model studied by Narendar *et al*. (2012). Aydogdu (2014) performed longitudinal wave dispersion of carbon nanotubes. Also, Filiz and Aydogdu (2015) explored wave propagation analysis of functionally graded nanotubes conveying fluid embedded in elastic medium. A review on nonlocal elastic models for bending, buckling, vibrations, and wave propagation of nanoscale beams explored by Eltaher *et al*. (2016).

Nonlocal strain gradient theory accounts the stress for both nonlocal elastic stress field and the strain gradient stress field. It must be mentioned that the nonlocal strain gradient theory captures the true effect of the two length scale parameters on the physical and mechanical treatment of small scale structures. Also, nonlocal differential model is an approximate model. A closed form solution for a nonlocal strain gradient rod in tension is reported by Zhu and Li (2017), Li *et al*. (2016) investigated vibration analysis of nonlocal strain gradient FG nanobeams. Also, Şimşek (2016) examined nonlinear vibration behavior of FG nanobeams employing nonlocal strain gradient theory and a novel Hamiltonian approach. The effect of thickness on the mechanics of nanobeams is explored by Li *et al*. (2018) based on nonlocal strain gradient theory. In these works, both stiffness-softening and stiffness-hardening effects on mechanical behavior of FG nanobeams are reported.

Rotating nanostructures such as nanoscale molecular bearings, nano-gears, nano-turbines and multiple gear systems have gained great attention in research community (Srivastava 1997, Zhang *et al*. 2004).Thus, vibration and wave propagation analysis of such structures are very important for their accurate design. Pradhan and Murmu (2010) employed a nonlocal beam model to demonstrate the flapwise bending-vibration characteristics of a uniform rotating nanocantilever. Narendar and Gopalakrishnan (2011) reported the wave dispersion behavior of a rotating nanotube using the nonlocal elasticity theory. They mentioned that wave characteristics of rotating nantube is significantly affected by the angular velocity. Alizada and Sofiyev (2011) explored the modified Young's moduli of nanomaterials and works on the mechanical behavior of nano scale systems. Aranda-Ruiz *et al*. (2012) investigated free vibration of rotating nonuniform nanocantilevers according to the Eringen nonlocal elasticity theory. Ghadiri and Shafiei (2015) studied nonlinear bending vibration of a rotating nanobeam based on nonlocal Eringen's theory using differential quadrature method. Recently, Mohammadi *et al*. (2016) examined vibration analysis of a rotating viscoelastic nanobeam embedded in a visco-Pasternak elastic medium and in a nonlinear thermal environment.

Also in recent years the mechanical behavior of FG nanoplates is investigated based on various plate shear deformation plate theories (Ebrahimi and Barati 2016f-i, Ebrahimi *et al*. 2016d, Ebrahimi and Dabbagh 2016, Ebrahimi and Hosseini 2016a, b) while the analysis of nano-structure's mechanical behaviors is one of recent interesting research topics. (Ebrahimi and Barati 2016j-p, Ebrahimi and Barati 2017a). It can be seen that, most of the researches are devoted to buckling, static and vibration of FG nanobeams and just a few researchers are working in the field of wave propagation of FG small scale beams. Flexural wave propagation in size-dependent functionally graded beams based on nonlocal strain gradient theory is performed by Li *et al*.

(2015), In another work, Ebrahimi and Barati (2016e) explored flexural wave propagation analysis of embedded S-FGM nanobeams under longitudinal magnetic field. Narendar (2016) investigated Wave dispersion in functionally graded magneto-electro-elastic nonlocal rod. According to the literature, wave propagation analysis of temperature-dependent rotating FG nanobeams in thermal environment based on nonlocal strain gradient theory is a novel topic that has not been worked until now.

In this paper the wave propagation analysis of a spinning temperature-dependent functionally graded (FG) nanobeam is presented in thermal environment. The nonlocal strain gradient theory, in which the stress numerates for both nonlocal stress field and the strain gradient stress field is employed. Mori-Tanaka distribution model is considered to express the gradually variation of material properties across the thickness. The Hamilton's principle along with the Euler-Bernoulli beam theory is employed in order to derive the governing equations as a function of axial force due to centrifugal stiffening and displacements. The dispersion relations of rotating FG nanobeam are obtained by applying an analytical solution and solving an eigenvalue problem. It is concluded that the temperature change, wave number, angular velocity, gradient index, and nonlocality parameter have significant effects on the wave dispersion characteristics of rotating FG nanobeams and thus the results of this research can provide useful information for the next generation studies and accurate deigns of nanomachines.

2. Theory and formulation

2.1 Mori-Tanaka FGM nanobeam model

Material properties of an FG nanobeam with the length L, width b and the thickness h are assumed to vary according to Mori-Tanaka model about the spatial coordinate. Mori-Tanaka homogenization technique represents the local effective material properties of the FG nanobeam including effective local bulk modules K_e and shear modules μ_e in the form

$$\frac{K_e - K_m}{K_c - K_m} = \frac{V_c}{1 + V_m (K_c - K_m)/(K_m + 4\mu_m/3)} \tag{1}$$

$$\frac{\mu_e - \mu_m}{\mu_c - \mu_m} = \frac{V_c}{1 + V_m(\mu_c - \mu_m)/[(\mu_m + \mu_m(9K_m + 8\mu_m)/(6(K_m + 2\mu_m))]} \tag{2}$$

where, subscripts m and c denote metal and ceramic, respectively and their volume fractions are related to each other in the following form

$$V_c + V_m = 1 \tag{3}$$

In which

$$V_c = (\frac{z}{h} + \frac{1}{2})^p, \;\; V_m = 1 - (\frac{z}{h} + \frac{1}{2})^p \tag{4}$$

Here p represents the gradient index which explains gradual variation of material properties through the thickness of the nanobeam. Finally, the effective Young's modulus (E), poison ratio (v)

and mass density (ρ) can be represent by

$$E(z)=\frac{9K_e\mu_e}{3K_e+\mu_e} \tag{5}$$

$$v(z)=\frac{3K_e-2\mu_e}{6K_e+2\mu_e} \tag{6}$$

$$\rho(z)=\rho_c V_c+\rho_m V_m \tag{7}$$

And thermal expansion coefficient (α) and thermal conductivity (κ) may be expressed by

$$\frac{\alpha_e-\alpha_m}{\alpha_c-\alpha_m}=\frac{\frac{1}{K_e}-\frac{1}{K_m}}{\frac{1}{K_c}-\frac{1}{K_m}} \tag{8}$$

$$\frac{\kappa_e-\kappa_m}{\kappa_c-\kappa_m}=\frac{V_c}{1+V_m\frac{(\kappa_c-\kappa_m)}{3\kappa_m}} \tag{9}$$

Also, temperature-dependent coefficients of material phases can be expressed defined by the following relations (Ebrahimi *et al*. 2016b, 2017b)

$$P=P_0(P_{-1}T^{-1}+1+P_1T+P_2T^2+P_3T^3) \tag{10}$$

where, P_{-1}, P_0, P_1, P_2 and P_3 are the temperature-dependent constants which are tabulated in Table 1. The top and bottom surfaces of FG nanobeam are fully ceramic (Si_3N_4) and fully metal (SUS304), respectively.

Table 1 Temperature-dependent coefficients for Si_3N_4 and SUS304

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
Si_3N_4	E (Pa)	348.43e+9	0	-3.070e-4	2.160e-7	-8.946e-11
	α (K^{-1})	5.8723e-6	0	9.095e-4	0	0
	ρ (kg/m^3)	2370	0	0	0	0
	κ (W/mK)	13.723	0	-1.032e-3	5.466e-7	-7.876e-11
	v	0.24	0	0	0	0
SUS304	E (Pa)	201.04e+9	0	3.079e-4	-6.534e-7	0
	α (K^{-1})	12.330e-6	0	8.086e-4	0	0
	ρ (kg/m^3)	8166	0	0	0	0
	κ (W/mK)	15.379	0	-1.264e-3	2.092e-6	-7.223e-10
	v	0.3262	0	-2.002e-4	3.797e-7	0

In this study, the temperature varies nonlinearly through the thickness. Temperature distribution function can be obtained by dissolve the steady-state heat conduction equation with the boundary conditions on bottom and top surface of the nanobeam across the thickness

$$-\frac{d}{dz}\left(\kappa(z,T)\frac{dT}{dz}\right)=0 \tag{11}$$

Considering the boundary conditions as follows

$$T\left(\frac{h}{2}\right)=T_c,\quad T\left(-\frac{h}{2}\right)=T_m \tag{12}$$

By solving the above equations, we have

$$T=T_m+(T_c-T_m)\frac{\int_{-\frac{h}{2}}^{z}\frac{1}{\kappa(z,T)}dz}{\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{1}{\kappa(z,T)}dz} \tag{13}$$

where, $\Delta T = T_c - T_m$ in the temperature distribution.

2.2 Kinematic relations

In the framework of Euler-Bernoullibeam theory, the displacement field of nonlocal functionally graded nanobeam at any point given as

$$u_x(x,z)=u(x)-z\frac{\partial w}{\partial x} \tag{14}$$

$$u_z(x,z)=w(x) \tag{15}$$

where, u and w defines the components correspond to the longitudinal and bending displacement of a point on the beam's mid-surface, respectively. By considering some small deformations, non-zero strains of present beam model can be expressed as

$$\varepsilon_{xx}=\frac{\partial u}{\partial x}-z\frac{\partial^2 w}{\partial x^2} \tag{16}$$

Also, Hamilton's principle states as

$$\int_0^t \delta(U+V-K)dt=0 \tag{17}$$

Here U is strain energy, V is work done by external forces and K is kinetic energy. The virtual strain energy can be written as

$$\delta U = \int_v \sigma_{ij} \delta \varepsilon_{ij} \, dV = \int_v (\sigma_{xx} \delta \varepsilon_{xx}) \, dV \tag{18}$$

Substituting Eq. (16) into Eq. (18) yields

$$\delta U = \int_0^L (N \frac{d\delta u}{dx} - M \frac{d^2 \delta w}{dx^2}) dx \tag{19}$$

In which the new variables that used in above equation expressed as follows

$$N = \int_A \sigma_{xx} dA, \;\; M = \int_A z \sigma_{xx} dA \tag{20}$$

The first variation of the work done by external forces can be written in the following form

$$\delta V = \int_0^L ((N^T + N^R)(\frac{d(w_b)}{dx} \frac{d\delta(w_b)}{dx}))dx \tag{21}$$

where, N^R and N^T are applied force due to rotation and temperature respectively, which are defined by the following relations

$$N^R = b \int_x^L \int_{-h/2}^{h/2} (\rho(z) A \Omega^2 x) dx dz \tag{22}$$

$$N^T = \int_{-h/2}^{h/2} E(z,T) \, \alpha(z,T) \, (T - T_0) \, dz \tag{23}$$

where, Ω and T_0 denote the angular velocity and reference temperature, respectively. In this study, we suppose a uniform rotating nanobeam and maximum axial force is considered (Narendar and Gopalakrishnan 2011)

$$N^R_{\max} = b \int_0^L \int_{-h/2}^{h/2} (\rho(z) A \Omega^2 x) dx dz \tag{24}$$

The variation of kinetic energy can be defined as

$$\begin{aligned} \delta K = \int_0^L (I_0 [\frac{du}{dt} \frac{d\delta u}{dt} + (\frac{dw}{dt})(\frac{d\delta w}{dt})] - I_1 (\frac{du}{dt} \frac{d^2 \delta w}{dxdt} + \frac{d^2 w}{dxdt} \frac{d\delta u}{dt}) \\ + I_2 (\frac{d^2 w}{dxdt} \frac{d^2 \delta w}{dxdt}))dx \end{aligned} \tag{25}$$

where

$$(I_0, I_1, I_2) = \int_A \rho(z)(1, z, z^2) dA \tag{26}$$

Then, by inserting Eqs. (19)-(25) into Eq. (17) and setting the coefficients, the following Euler-Lagrange equations were obtained

$$\frac{\partial N}{\partial x}=I_0\frac{\partial^2 u}{\partial t^2}-I_1\frac{\partial^3 w}{\partial x\partial t^2} \tag{27}$$

$$\frac{\partial^2 M}{\partial x^2}+(N^R_{\max}+N^T)\frac{\partial^2 w}{\partial x^2}=I_0(\frac{\partial^2 w}{\partial t^2})+I_1\frac{\partial^3 u}{\partial x\partial t^2}-I_2\frac{\partial^4 w}{\partial x^2\partial t^2} \tag{28}$$

2.3 The nonlocal FG nanobeam strain gradient model

Nonlocal strain gradient elasticity theory, (Li *et al*. 2015) enumerates the stress for both nonlocal elastic stress and strain gradient stress fields. Hence, the stress can be defined as follows

$$\sigma_{ij}=\sigma_{ij}^{(0)}-\frac{d\sigma_{ij}^{(1)}}{dx} \tag{29}$$

where the stresses $\sigma_{xx}^{(0)}$ and $\sigma_{xx}^{(1)}$ are correspond to strain ε_{xx} and strain gradient $\varepsilon_{xx,x}$, respectively and are defined as follow

$$\sigma_{ij}^{(0)}=\int_0^L C_{ijkl}\alpha_0(x,x',e_0a)\varepsilon'_{kl}(x')dx' \tag{30}$$

$$\sigma_{ij}^{(1)}=l^2\int_0^L C_{ijkl}\alpha_1(x,x',e_1a)\varepsilon'_{kl,x}(x')dx' \tag{31}$$

in which C_{ijkl} are the elastic constants and e_0a and e_1a enurmerate the effect of nonlocal stress field and l is the length scale parameter of material and represents the influence of higher order strain gradient stress field. When the nonlocal functions α_0 (x, x', e_0a) and α_1 (x, x', e_1a) satisfy the developed conditions by Eringen (1983), the constitutive relation for a functionally graded nanobeam can be stated as

$$[1-(e_1a)^2\nabla^2][1-(e_0a)^2\nabla^2]\sigma_{ij}=C_{ijkl}[1-(e_1a)^2\nabla^2]\varepsilon_{kl}-C_{ijkl}\,l^2[1-(e_0a)^2\nabla^2]\nabla^2\varepsilon_{kl} \tag{32}$$

In which ∇^2 denotes the Laplacian operator. Assuming $e_1 = e_0 = e$ and discarding terms of order $O(\nabla^2)$, the general constitutive relation in Eq. (32) can be rewritten as (Li *et al*. 2015)

$$[1-(ea)^2\nabla^2]\sigma_{ij}=C_{ijkl}[1-l^2\nabla^2]\varepsilon_{kl} \tag{33}$$

Thus, the constitutive relations for a nonlocal Euler-Bernoulli FG nanobeam can be stated as

$$\sigma_{xx}-\mu^2\frac{\partial^2\sigma_{xx}}{\partial x^2}=E(z)(\varepsilon_{xx}-\lambda^2\frac{\partial^2\varepsilon_{xx}}{\partial x^2}) \tag{34}$$

where, $\mu = ea$ and $\lambda = l$. By integrating Eq. (34) over the cross-section area of nanobeam provides the following nonlocal relations for FGM beam model as

$$N - \mu^2 \frac{\partial^2 N}{\partial x^2} = (1 - \lambda^2 \frac{\partial^2}{\partial x^2})(A \frac{\partial u}{\partial x} - B \frac{\partial^2 w}{\partial x^2}) \tag{35}$$

$$M - \mu^2 \frac{\partial^2 M}{\partial x^2} = (1 - \lambda^2 \frac{\partial^2}{\partial x^2})(B \frac{\partial u}{\partial x} - D \frac{\partial^2 w}{\partial x^2}) \tag{36}$$

where the cross-sectional rigidities are defined as the following forms

$$(A, B, D) = \int_A E(z)\,(1, z, z^2)\,dA \tag{37}$$

The governing equations of Euler-Bernoulli FGM nanobeams in terms of displacements are obtained by inserting for N, M from Eqs. (35) and (36), respectively, into Eqs. (27) and (28) as follows

$$\begin{aligned} &A(1 - \lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^2 u}{\partial x^2}) - B(1 - \lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^3 w}{\partial x^3}) - I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^3 w}{\partial x \partial t^2} \\ &+ \mu^2 \,(I_0 \frac{\partial^4 u}{\partial x^2 \partial t^2} - I_1 \frac{\partial^5 w}{\partial x^3 \partial t^2}) = 0 \end{aligned} \tag{38}$$

$$\begin{aligned} &B(1 - \lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^3 u}{\partial x^3}) - D(1 - \lambda^2 \frac{\partial^2}{\partial x^2})(\frac{\partial^4 w}{\partial x^4}) - (N^T + N^R_{\max}) \frac{\partial^2 (w)}{\partial x^2} - I_0 (\frac{\partial^2 w}{\partial t^2}) - I_1 \frac{\partial^3 u}{\partial x \partial t^2} \\ &+ I_2 \frac{\partial^4 w}{\partial x^2 \partial t^2} + \mu^2 \left((N^T + N^R_{\max}) \frac{\partial^4 (w)}{\partial x^4} + I_0 (\frac{\partial^4 w}{\partial x^2 \partial t^2}) + I_1 \frac{\partial^5 u}{\partial x^3 \partial t^2} - I_2 \frac{\partial^6 w}{\partial x^4 \partial t^2} \right) = 0 \end{aligned} \tag{39}$$

3. Solution procedure

The solution of governing equations of nonlocal FGM nanobeam can be presented by

$$u(x,t) = U_n \exp[i(\beta x - \omega t)] \tag{40}$$

$$w\,(x,t) = W_n \exp[i(\beta x - \omega t)] \tag{41}$$

where (U_n , W_n) are the wave amplitudes. By inserting Eqs. (40) and (41) into Eqs. (38) and (39) respectively, we have

$$\left\{ \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} - \omega^2 \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \right\} \begin{Bmatrix} U_n \\ W_n \end{Bmatrix} = 0 \tag{42}$$

where

$k_{1,1} = -A\beta^2 - \lambda^2 A\beta^4$, $k_{1,2} = -iB\beta^3 - \lambda^2 B\beta^5$,

$k_{2,1} = +iB\beta^3 - i\lambda^2 B\beta^3$

$k_{2,2} = (N^T + N^R_{\max})\beta^2(1 + \mu^2\beta^2) - D\beta^4 - \lambda^2 D\beta^6$

$$m_{1,1} = I_0(1+\mu^2\beta^2),\quad m_{1,2} = +iI_1\beta(1+\mu^2\beta^2)\ ,$$
$$m_{2,1} = -iI_1\alpha(1+\mu^2\beta^2)$$
$$m_{2,2} = I_0(1+\mu^2\beta^2)+I_2\beta^2+\mu^2 I_2\beta^4$$

By setting the determinant of above matrix to zero, the circular frequency ω can be obtained. Also, the phase velocity of waves can be calculated by the following relation

$$c_p = \frac{\omega}{\beta} \tag{43}$$

which displays the dispersion relation of phase velocity c_p and wave number β for the FGM nanobeam. Also, the escape frequencies of the FG nanobeam can be obtained by setting $\beta \to \infty$. It is worth mentioning that after the escape frequency, the flexural waves will not propagate anymore.

4. Numerical results and discussions

This section is devoted to investigate the propagation behavior of temperature-dependent functionally graded nanobeam undergoing rotation in thermal environment. The nanobeam is modeled based on Euler-Bernoulli beam theory. An FG nanobeam with width b = 1 nm and length L = 10 nm is considered according to Fig. 1. The material properties of such FG nanobeam is presented in Table 1. The frequencies are verified with those of Eltaher *et al.* (2012) for various nonlocal parameters and a good agreement is observed as presented in Table 2. Variation of the

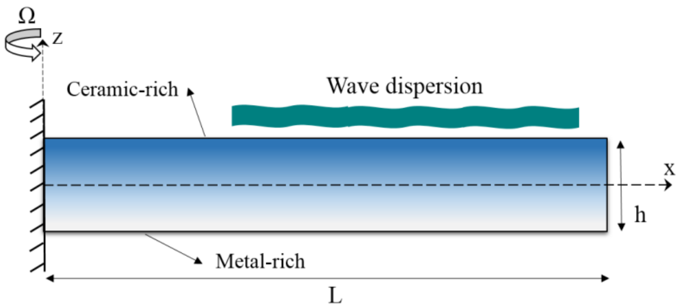
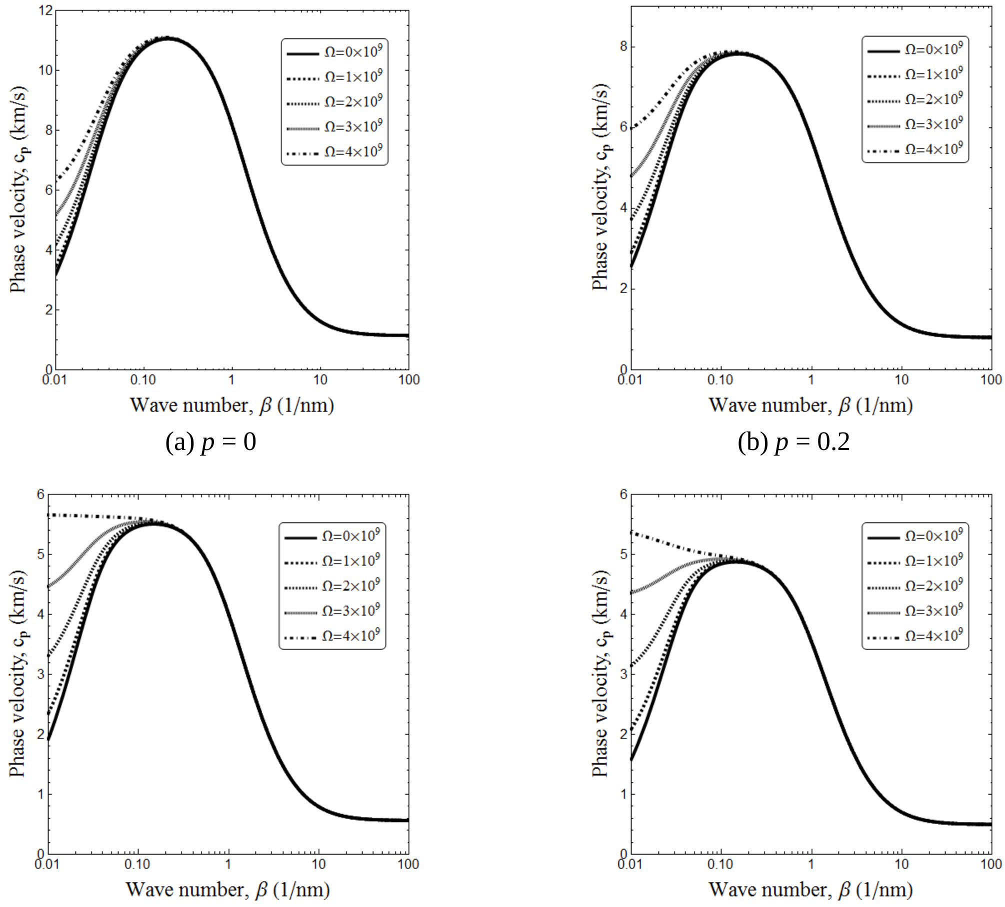


Fig. 1 Configuration of rotating FG nanobeam

Table 2 Comparison ofthe frequency for power-law FG nanobeams

μ	p = 0.1		p = 0.5		p = 1	
	Eltaher *et al.* (2012)	Present	Eltaher *et al.* (2012)	Present	Eltaher *et al.* (2012)	Present
0	9.2129	9.1887	7.8061	7.7377	7.0904	6.9885
1	8.7879	8.7663	7.4458	7.3820	6.7631	6.6672
2	8.4166	8.3972	7.1312	7.0712	6.4774	6.3865
3	8.0887	8.0712	6.8533	6.7966	6.2251	6.1386



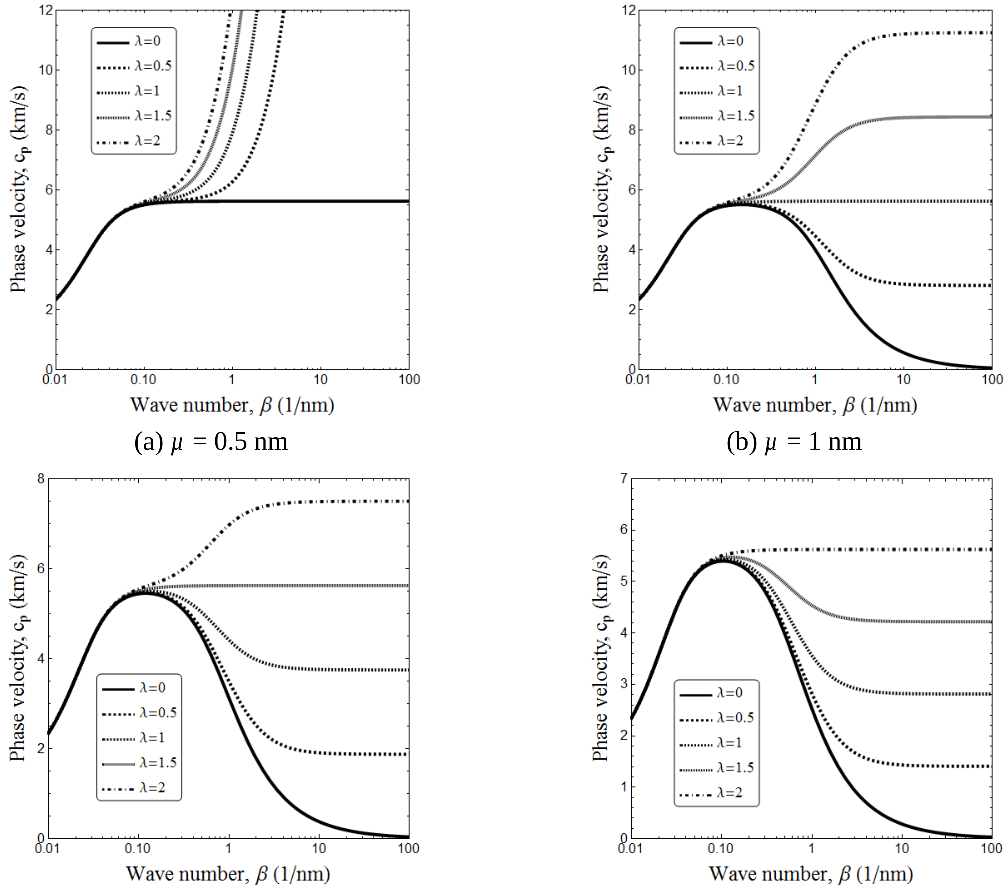
(a) p = 0 (b) p = 0.2

(c) p = 1 (d) p = 5

Fig. 2 Variation of phase velocity of rotating FG nanobeam versus wave number for various angular velocities and gradient indices (μ = 1 nm, λ = 0.1 nm, ΔT = 200)

phase velocity (c_p) of rotating FG nanobeam versus wave number (β) for various angular velocities (Ω) and gradient indices (p) at a constant value of nonlocality parameter (μ = 1 nm) length scale parameter (λ = 0.1 nm) and temperature ΔT = 200 is plotted in Fig. 2. It is clear that, with the increase in wave number, the phase velocity increases but for β > 0.1 the phase velocity will decrease and in $\beta \geq 10$ tends to a constant value and don't change anymore. Also, at a constant value of wave number with the increase in angular velocity, phase velocity will increase too. However, at $\beta \leq 0.1$ diagrams of different angular velocities are more distinguished. So, angular velocity of rotating FG nanobeams has no considerable effect on phase velocities at higher values of wave number. In addition, phase velocity will decrease with the increase in gradient index. This is due to higher portion of metal phase by increase of gradient index.

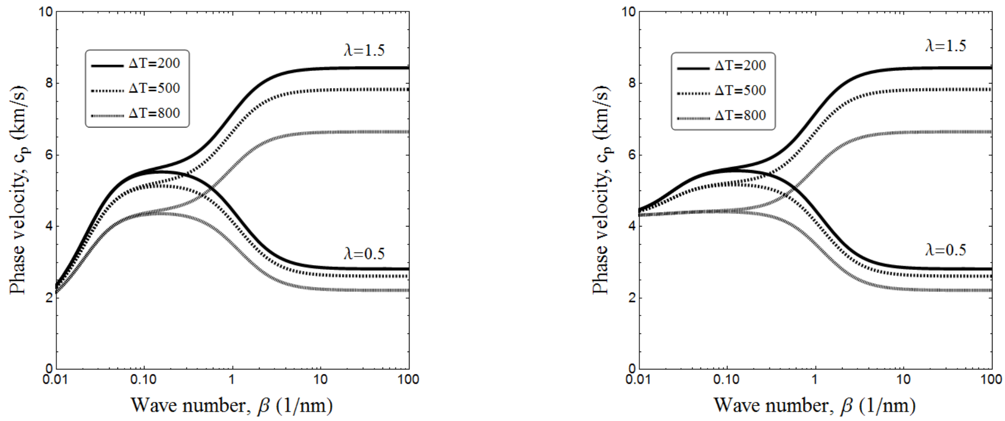
Fig. 4 shows the variation of phase velocity (c_p) of rotating FG nanobeam versus wave number (β) for various length scale parameters (λ) and temperature changes (ΔT) at constant values of nonlocality parameter (μ = 1 nm) and gradient index (p = 1), It is observable that, in $\beta \leq 0.1$ with the increase in wave number, phase velocity increases, but for $\beta \geq 0.1$ diagram of various length



(a) μ = 0.5 nm (b) μ = 1 nm

(c) μ = 1.5 nm (d) μ = 2nm

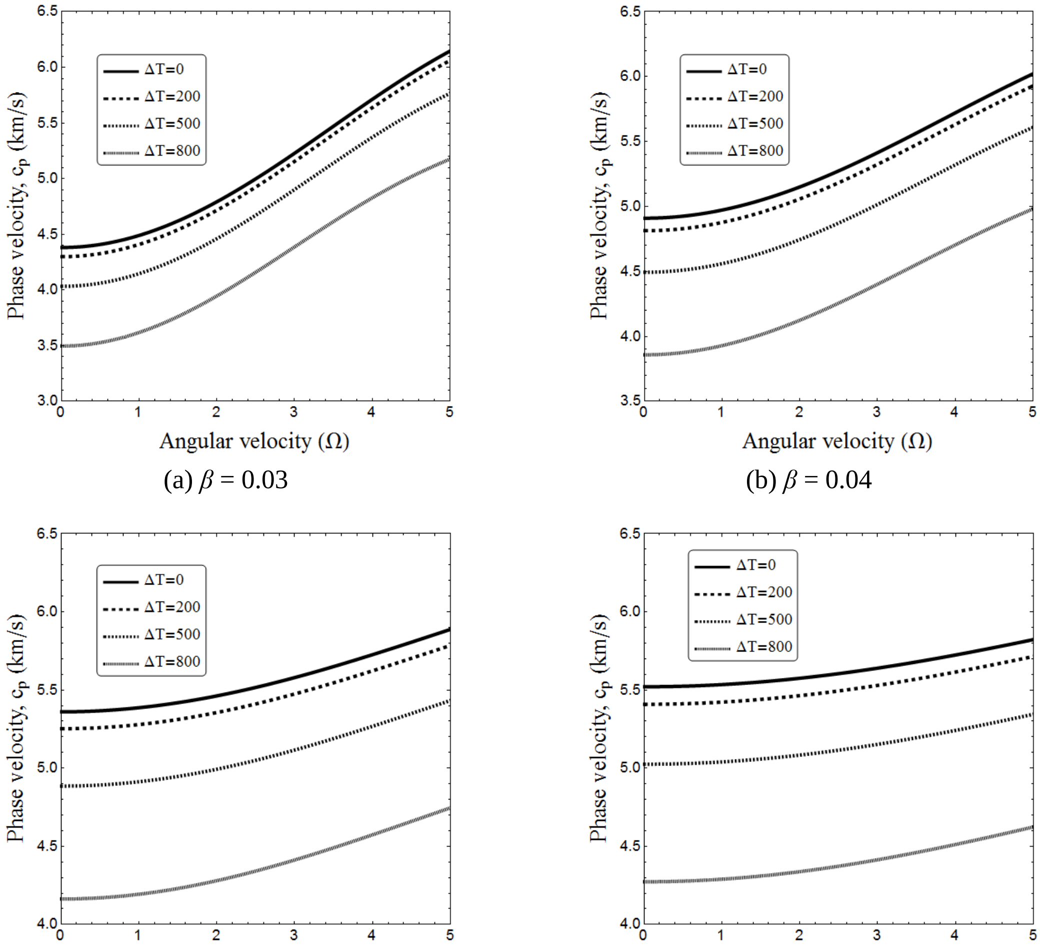
Fig. 3 Variation of phase velocity of rotating FG nanobeam for various length scale and nonlocal parameters (Ω = 1, ΔT = 200, p = 1)



(a) Ω = 1 (b) Ω = 3

Fig. 4 Variation of phase velocity of rotating FG nanobeam versus wave number for various length scale parameters and temperature changes (μ = 1 nm, p = 1) scale parameters are distinguished. Also, phase velocity does not change with the increase in wave number in $\beta \geq 10$ for every value of temperature change. In addition, at a constant value of wave number increasing in temperature leads to lower phase velocities, especially at higher wave numbers.

Variation of phase velocity (c_p) of rotating FG nanobeam versus angular velocity (Ω) for various temperature changes (ΔT) and wave numbers at μ = 1 nm, λ = 0.5 nm and p = 1 is plotted in Fig. 5. It can be seen that, with the increase in angular velocity, phase velocity increases for every value of temperature change. But, this increment in phase velocity is significantly influenced by the value of wave number. In fact, increase of phase velocity with the rise of angular velocity is more prominent at lower wave numbers. Also, at a constant value of angular velocity, increasing in temperature causes the decrease in phase velocities, since stiffness of nanobeams degrades with increase of temperature.



(a) β = 0.03 (b) β = 0.04

(c) β = 0.06 (d) β = 0.08

Fig. 5 Variation of phase velocity of rotating FG nanobeam versus angular velocity for various temperature changes (μ = 1 nm, λ = 0.5 nm, p = 1)

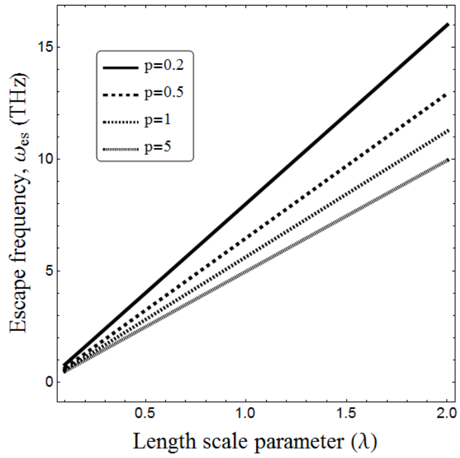


Fig. 6 Variation of escape frequency of rotating FG nanobeam versus length scale parameter for various gradient indices (μ = 1 nm, ΔT = 200)

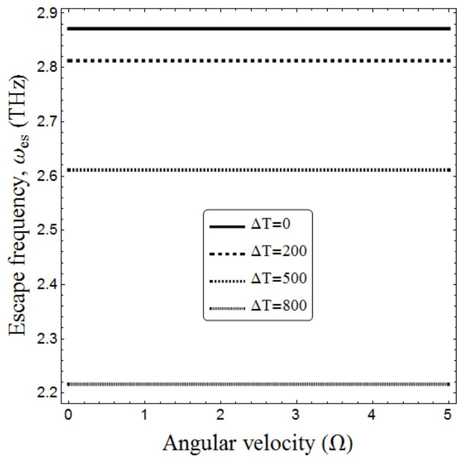


Fig. 7 Variation of escape frequency of rotating FG nanobeam versus angular velocity for various temperature changes (μ = 1 nm, λ = 0.5 nm, p = 1)

In Fig. 6 variation of escape frequency (ω_{es}) of rotating FG nanobeam versus length scale parameter (λ) for various gradient indices (P) is plotted at constant values of nonlocality parameter (μ = 1 nm) and temperature (ΔT = 200), It is clear from the figure that, increase in length scale parameter, causes the increase in escape frequency. Also, it is observable that, with the increase in gradient index, the slope of diagram of various gradient indices decreases.

Fig. 7 shows the variation of escape frequency (ω_{es}) of rotating FG nanobeam versus angular velocity (Ω) for various temperature changes (ΔT) with the constant values of nonlocality parameter (μ = 1 nm), length scale parameter (λ = 0.5 nm) and gradient index (p = 1). It is observable that, with the increase in angular velocity, escape frequency remains constant. Because, escape frequencies are obtained by setting wave number to infinity. Although, increase in temperature causes decrease of escape frequency, regardless of the value of angular velocity.

5. Conclusions

In this paper, wave dispersion characteristics of a rotating functionally graded (FG) nanobeam are explored based on Euler-Bernoulli beam theory. Material properties of rotating nanobeam are supposed to be graded according to Mori-Tanaka distribution function. Finally, through some parametric study, the effect of different parameters such as angular velocity, gradient index, nonlocality parameter, temperature rise and wave number on wave dispersion behavior of rotating FG nanobeam are studied. It is found that increasing in the angular velocity causes the increase in phase velocity. However, effect of angular velocity on wave frequency and phase velocity is significant at lower wave numbers. Also, the increasing in nonlocality parameter causes decrease in wave frequency and phase velocity at a constant angular velocity. Length scale parameter introduces a stiffness-hardening effect on the nanobeam structure and increases the phase velocities and escape frequencies. However, the escape frequency is not influenced by the change in angular velocity. Also, phase velocities and escape frequencies of rotating FG nanobeam will decrease with increase of temperature, especially at higher wave numbers.

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