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A refined nonlocal hyperbolic shear deformation beam model for bending and dynamic analysis of nanoscale beams

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Abstract. This paper proposes a new nonlocal higher-order hyperbolic shear deformation beam theory (HSBT) for the static bending and vibration of nanoscale-beams. Eringen's nonlocal elasticity theory is incorporated, in order to capture small size effects. In the present model, the transverse shear stresses account for a hyperbolic distribution and satisfy the free-traction boundary conditions on the upper and bottom surfaces of the nanobeams without using shear correction factor. Employing Hamilton's principle, the nonlocal equations of motion are derived. The governing equations are solved analytically for the edges of the beam are simply supported, and the obtained results are compared, as possible, with the available solutions found in the literature. Furthermore, the influences of nonlocal coefficient, slenderness ratio on the static bending and dynamic responses of the nanobeam are examined.

Keywords: nonlocal theory; nanosize beam; hyperbolic shear deformation theory; bending; vibration

1. Introduction

Recently, nanostructures such as nanobeams and nanoplates have gained extensive application in fabrication of nano-electro-mechanical systems with low mass and high sensitivity due to their precious chemical, mechanical and electrical properties (Li *et al*. 2003, Lavrik *et al*. 2004, Ekinci and Roukes 2005). These applications encourage new research area that focuses on investigating and predicting the behaviours of such nano structures. In these application fields, small-scale effects become prominent. A number of attempts have been conducted to analyze characteristic of small-scale structures including experiment and computer simulation. These approaches are often computationally expensive and difficult because the accurate manipulations of these behaviors are rather complex to find. Therefore, the mathematical and mechanical models which account for the small-scale effects are needed. On the other hand; it is shown that continuum mechanics is a reliable and appropriate approach to predict mechanical behavior of structures at various time and length scales. However, the classical continuum mechanics may lead up to incorrect results to accurately predict the mechanical behavior of nanostructures Ansari and Sahmani (2011), because the presence of small-scale effects at the nanoscale. Due to this gap, nonlocal continuum theories came into existence, in which additional material length scale parameters have been proposed to predict the accurate behavior of nanostructures. Among these theories, the nonlocal elasticity

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theory pioneered by Eringen (1972, 1983) has been widely used by the researchers. The nonlocal elasticity supposes that the stress state at a reference point is a function of the strain at all neighbor points of the body. Hence, this theory could take into consideration the effects of small scales.

Based on the nonlocal constitutive relation of Eringen, several studies are conducted by developing nonlocal beam models for prediction of mechanical behaviors of nanobeams. Peddieson *et al*. (2003) applied the nonlocal elasticity model to study Euler–Bernoulli micro and nanobeams. Reddy (2007) employed the nonlocal differential equations of Eringen to study the bending, buckling and vibration of nanobeams based on various beam theory. Based on nonlocal continuum mechanics, Wang and Liew (2007) investigated the static bending of micro- and nanoscale structures using Euler–Bernoulli and Timoshenko beam theory. Aydogdu (2009) established a general nonlocal beam theory for analysis bending, buckling and vibration of nanobeams using different beam theories. Thai (Thai 2012, Thai and Vo 2012) investigated the nonlocal behavior of the nanobeams using the nonlocal continuum theory of Eringen. Based on a nonlocal hyperbolic shear deformation theory, the bending, buckling and vibration of nanobeams have been studied by Benguediab *et al*. (2014) using nonlocal continuum mechanics. A comparison study of a variety of refined nonlocal shear deformation beam theories was made by Berrabah *et al*. (2013) to study the static bending, vibration and buckling of nanobeams using nonlocal elasticity theory.

Tounsi *et al*. (2013a) developed a novel nonlocal thickness-stretching sinusoidal shear deformation beam theory for the bending, and vibration of nanobeams based on the nonlocal theory. The thermal buckling behavior of nanobeams was studied by Tounsi *et al*. (2013b) using an efficient higher-order nonlocal beam theory. Recently, Zenkour and Sobhy (2015) proposed a new simplified three-unknown shear and normal deformation theory to study the thermal bending of nanobeams in a thermal environment. In this study the nonlocal constitutive equation of Eringen was used in the formulations. Behera and Chakraverty (2013) employed the boundary characteristic orthogonal polynomials in the Rayleigh-Ritz method to study vibration characteristics of nanobeams based on Euler-Bernoulli and Timoshenko beam theories.

It must be indicated that Euler-Bernoulli beam theory (EBT) is adequate for thin beams where the shear deformation effect is negligible. However, for moderately deep beams, it underpredicts deflection and overpredicts the free vibration owing to neglecting the shear deformation effect. Thus, the first order shear deformation theory (FSDT), also known Timoshenko beam theory (TBT) accounts for the shear deformation effect for short beams by assuming a constant shear strain through the height of the beam. However, a shear correction factor (SCF) is recommended to compensate the gap between the original stress case and the invariable stress one. To get an ideal prediction of response of deep beam and prevent the use of shear correction factor, several higher order shear deformation theories were developed based on the assumption of the higher-order variation of axial displacement through the height of the beam, notable among them are Levinson beam theory (1981), the sinusoidal shear deformation beam theory (SBT) suggested by Touratier (1991) (see also Thai and Vo (2012), Zenkour and Sobhy (2015) and Tounsi *et al*. (2013a, b)), hyperbolic shear deformation theory of Aissani *et al*. (2015), and the third-order shear deformation theory recommended by Reddy (2007) (RBT).

The present study is devoted to propose a refined nonlocal hyperbolic shear deformation beam theory for static bending, and vibration of nanoscale beams. The most attractive feature of this theory is that it accounts for a hyperbolic variation of the transverse shear strains through the thickness and satisfies the zero traction boundary conditions on the top and bottom surfaces of the nanobeam without employing a shear correction factors (SCFs). Nonlocal constitutive equations of

Eringen are used in the formulations. The governing equations of the nanobeams under uniform and point loads are solved analytically using Navier's type solution. The obtained results are compared with those available in the literature such as (Euler-Bernoulli beam theory (EBT), Timoshenko beam theory (TBT) and higher order beam theories Reddy's beam theory (RBT) , the model of Berrabah and Tounsi *et al*. (2013a)) to verify the exactness of the present solution. The impacts of nonlocal parameter and length to thickness ratio on the static and dynamic responses of the nanobeam are investigated in details.

2. General Mathematical model

We consider a nanosized beam with the length L and thickness h, and rectangular cross-section $b \times h$, with b being the width and h being the height (Fig. 1). A coordinate system x, y and z are positioned in the middle plane ($z = 0$) and its origin of the coordinate system is selected at the left end of the beam, whereas the x axis is considered along the length of the beam, the y axis in the width direction and the z axis is taken along the depth (height) direction.

2.1 The displacement field

The displacement field of the hyperbolic shear deformation beam theory (HSBDT) can be written as

$$u\left(x,z,t\right)=u_0\left(x,t\right)-z\frac{\partial w_0(x,t)}{\partial x}+f(z)\phi(x,t) \tag{1a}$$

$$w\left(x,z,t\right)=w_0\left(x,t\right) \tag{1b}$$

where u_0, w_0, ϕ are the unknown displacements of the midplane of the beam, $f(z)$ represents shape function representing the distribution of the transverse shear strains and stresses along the

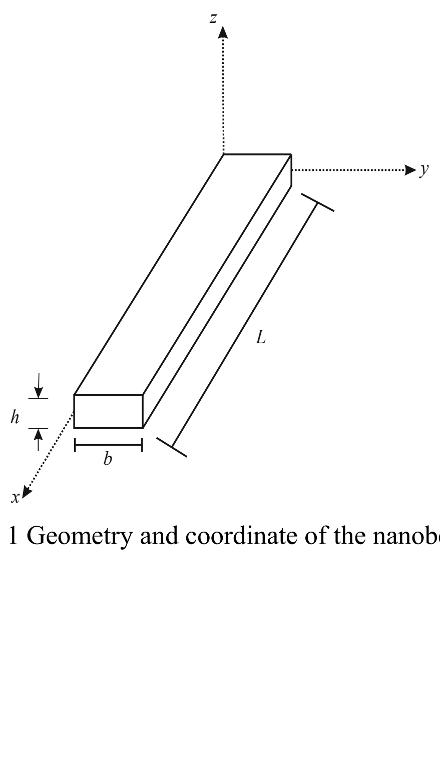

Fig. 1 Geometry and coordinate of the nanobeam

thickness. By supposing that $\phi_x = -\partial\varphi(x,t)/\partial x$ (Thai *et al*. 2014, Nguyen 2015), the displacement field of the present refined beam theory can be rewritten in a simpler form as

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w}{\partial x} - f(z)\frac{\partial \varphi}{\partial x} \tag{2a}$$

$$w(x,z,t) = w_0(x,t) \tag{2b}$$

It is worth noting that the displacement field of the recent refined nonlocal beam theories (Thai 2012, Berrabah *et al*. 2013, Tounsi *et al*. 2013a) is obtained by splitting the transverse displacement into bending and shear parts. Therefore, by making another supposition to existing ones, the displacement field and following equations of motion resulting in this study will be completely different with those given by (Thai 2012, Berrabah *et al*. 2013, Tounsi *et al*. 2013a).

The shape function $f(z)$ assigned according to the shearing stress distribution through the thickness of the nanobeam is chosen according to El Meiche *et al*. (2011) as

$$f(z) = \frac{(h/\pi)\sinh\left(\dfrac{\pi}{h}z\right) - z}{[\cos(\pi/2) - 1]} \tag{3}$$

The strains related with the displacements in Eq. (2) are written in following compact form

$$\varepsilon_x = \varepsilon_x^0 + z\kappa^b + f(z)\kappa^s \qquad \text{and} \qquad \gamma_{xz} = g(z)\gamma^0 \tag{4}$$

where

$$\begin{gathered} \varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \qquad \kappa^b = -\frac{\partial^2 w}{\partial x^2}, \qquad \kappa^s = -\frac{\partial^2 \varphi}{\partial x^2} \\ \gamma^0 = \frac{\partial \varphi}{\partial x}, \qquad g(z) = 1 - f'(z), \qquad \text{and} \qquad f'(z) = \frac{df(z)}{dz} \end{gathered} \tag{5}$$

2.2 The nonlocal model and constitutive relations

In the case of nonlocal elasticity theory, it is supposed that the stress at every point in the body is a function of strains at all points in the continuum. The nonlocal constitutive law given by Eringen (1972, 1983) is given as

$$\sigma_x - \mu\frac{d^2\sigma_x}{dx^2} = E\varepsilon_x \tag{6a}$$

$$\tau_{xz} - \mu\frac{d^2\tau_{xz}}{dx^2} = G\gamma_{xz} \tag{6b}$$

where E and G are the Young modulus and shear modulus of the nanobeam, correspondingly; $\mu = (e_0 a)^2$ is the nonlocal parameter, e_0 is a constant proper to each material and $\boldsymbol{a}$ is an interior characteristic length. The nonlocal parameter depends on the boundary conditions, chirality, mode shapes, number of walls, and type of motion (Arash and Wang 2012).

2.3 Equations of motion in terms of displacements

Hamilton's rule is applied here to obtain the equations of motion. The concept can be expressed in analytical form as (Reddy 2002, Bourada *et al*. 2015, Kheroubi *et al*. 2016)

$$\delta\int_0^T (U+V-K)dt = 0 \tag{7}$$

where δU is the variation of the strain energy; δV represents the potential energy; and the variation of the kinetic energy is given by δK. The variation of the strain energy of the beam can expressed by the following form

$$\delta U = \int_0^L\int_A (\sigma_x\delta\varepsilon_x + \tau_{xz}\delta\gamma_{xz})dAdx = \int_0^L\left(N\frac{d\delta u_0}{dx} - M\frac{d^2\delta w_0}{dx^2} - P\frac{d^2\delta\phi}{dx^2} + Q\frac{d\delta\phi}{dx}\right)dx \tag{8}$$

where M, P and Q represent the stress resultants and they are expressed as

$$(M,P) = \int_A (z,f)\sigma_x dA \qquad \text{and} \qquad Q = \int_A g\,\tau_{xz}dA \tag{9}$$

The variation of the potential energy caused by the practical loads can be given as

$$\delta V = -\int_0^L q\,\delta w\,dx \tag{10}$$

where q is the external transverse load.

The variation of the kinetic energy can be derived as follows

$$\begin{aligned}\delta K &= \int_0^L\int_A \rho\left[\dot{u}\delta\dot{u} + \dot{w}\delta\dot{w}\right]dAdx \\ &= \int_0^L\left\{I_0\left[\dot{w}_0\delta\dot{w}_0\right] + I_2\left(\frac{d\dot{w}_0}{dx}\frac{d\delta\dot{w}_0}{dx}\right) + K_2\left(\frac{d\dot{\phi}}{dx}\frac{d\delta\dot{\phi}}{dx}\right) + J_2\left(\frac{d\dot{w}_0}{dx}\frac{d\delta\dot{\phi}}{dx} + \frac{d\dot{\phi}}{dx}\frac{d\delta\dot{w}_0}{dx}\right)\right\}dx\end{aligned} \tag{11}$$

where dot-superscript sign defines the differentiation with sense to the time variable t; ρ is the mass density; and (I_0, I_2, J_2, K_2) are the mass inertias expressed as

$$(I_0, I_2, J_2, K_2) = \int_A (1, z^2, z, f, f^2)\rho dA \tag{12}$$

The explicit equations of motion of the new proposed nonlocal beam theory are obtained by substituting the expressions for δU, δV, and δK from Eqs. (8), (10) and (11) into Eq. (7), integrating by parts, and collecting the coefficients of δw, and $\delta\varphi$, and which are given as follows

$$\delta w: \frac{d^2 M}{dx^2} + q = I_0 \ddot{w} - I_2 \frac{d^2 \ddot{w}}{dx^2} - J_2 \frac{d^2 \ddot{\varphi}}{dx^2}, \tag{13a}$$

$$\delta\varphi: \frac{d^2 P}{dx^2} + \frac{dQ}{dx} = -J_2 \frac{d^2 \ddot{w}}{dx^2} - K_2 \frac{d^2 \ddot{\varphi}}{dx^2}, \tag{13b}$$

However, the Euler–Bernoulli beam theory can be obtained from the equilibrium equations in Eq. (13), by neglecting the shear deformation effect ($\varphi = 0$).

The stress resultants are obtained by substituting Eq. (4) into Eq. (6) and the subsequent results into Eq. (9), and are given as follows

$$M - \mu \frac{d^2 M}{dx^2} = -D \frac{d^2 w}{dx^2} - D_s \frac{d^2 \varphi}{dx^2} \tag{14a}$$

$$P - \mu \frac{d^2 P}{dx^2} = -D_s \frac{d^2 w}{dx^2} - H_s \frac{d^2 \varphi}{dx^2} \tag{14b}$$

$$Q - \mu \frac{d^2 Q}{dx^2} = A_s \frac{d\varphi}{dx} \tag{14c}$$

where

$$(D, D_s, H_s) = \int_A (z^2, zf, f^2) E dA, \qquad A_s = \int_A g^2 G dA \tag{15}$$

The nonlocal equations of motion of the present refined nano beam model can be written in terms of displacements (w_0, φ) by substituting Eq. (14) into Eq. (13) as

$$-D\frac{d^4 w_0}{dx^4} - D_s \frac{d^4 \phi}{dx^4} + q - \mu \frac{d^2 q}{dx^2} = I_0 \left(\ddot{w}_0 - \mu \frac{d^2 \ddot{w}_0}{dx^2} \right) - I_2 \left(\frac{d^2 \ddot{w}_0}{dx^2} - \mu \frac{d^4 \ddot{w}_0}{dx^4} \right) - J_2 \left(\frac{d^2 \ddot{\phi}}{dx^2} - \mu \frac{d^4 \ddot{\phi}}{dx^4} \right) \tag{16a}$$

$$-D_s \frac{d^4 w_0}{dx^4} - H_s \frac{d^4 \phi}{dx^4} + A_s \frac{d^2 \phi}{dx^2} = I_0 \left(\ddot{w}_0 - \mu \frac{d^2 \ddot{w}_0}{dx^2} \right) - J_2 \left(\frac{d^2 \ddot{w}_0}{dx^2} - \mu \frac{d^4 \ddot{w}_0}{dx^4} \right) - K_2 \left(\frac{d^2 \ddot{\phi}}{dx^2} - \mu \frac{d^4 \ddot{\phi}}{dx^4} \right) \tag{16b}$$

By setting the scale parameter μ equal to zero the equations of motion of local beam theory can be derived from Eq. (16).

3. Exact solutions for simply supported nanobeams

In this part, analytical solutions are provided for simply supported nanosized beams in the case of bending and free vibration.

However, in this study the boundary conditions of (S-S) simply supported nanobeams are defined as follows

$$w = \varphi = M = P = 0 \text{ at } x = 0, L$$

To satisfy the boundary conditions and governing equations of motion, the displacements fields are adopted to be of the form

$$\begin{Bmatrix} w \\ \phi \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} W_n \sin(\alpha x) e^{i\omega t} \\ \varphi_n \sin(\alpha x) e^{i\omega t} \end{Bmatrix}, \tag{17}$$

where W_n, and Φ_n are arbitrary parameters to be determined, ω is the eigenfrequency associated with nth eigenmode, and $\alpha = n\pi/L$. The transverse applied load q is also expressed in the Fourier series as

$$q(x) = \sum_{n=1}^{\infty} Q_n \sin(\alpha x), \qquad Q_n = \frac{2}{L}\int_0^L q(x)\sin(\alpha x)dx \tag{18}$$

The Fourier coefficients $\boldsymbol{Q_n}$ related with some typical loads are given as follows

$$Q_n = q_0, \quad n = 1 \qquad \text{for sinusoidal load} \tag{19a}$$

$$Q_n = \frac{4q_0}{n\pi}, \quad n = 1, 3, 5.... \qquad \text{for uniform load} \tag{19b}$$

$$Q_n = \frac{2q_0}{L}\sin\frac{n\pi}{2}, \quad n = 1, 2, 3.... \qquad \text{for point load } Q_0 \text{ at the midspan} \tag{19c}$$

By replacing the expansions of w, φ and q from Eqs. (17) and (18) into Eq. (16), one can obtain the closed form solutions from the following equations

$$\left(\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} - \lambda\omega^2 \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} \right) \begin{Bmatrix} W_n \\ \phi_n \end{Bmatrix} = \begin{Bmatrix} \lambda Q_n \\ 0 \end{Bmatrix} \tag{20}$$

where

$$\begin{gathered} P_{11} = D\alpha^4, \quad P_{12} = D_s\alpha^4, \quad P_{22} = H_s\alpha^4 + A_s\alpha^2, \quad \lambda = 1 + \mu\alpha^2 \\ m_{11} = I_0 + I_2\alpha^2, \quad m_{12} = I_0 + J_2\alpha^2, \quad m_{22} = I_0 + K_2\alpha^2 \end{gathered} \tag{21}$$

3.1 Static bending

The static deflection is derived from Eq. (20) by making all time derivatives equal to zero

$$w(x) = \sum_{n=1}^{\infty} \frac{(P_{11} + P_{22} - 2P_{12})}{(P_{11}P_{22} - P_{12}^2)} \lambda Q_n \sin\alpha x \tag{22}$$

3.2 Free vibration

The natural frequency can be obtained from the equation shown below by setting ***q*** in Eq. (20) equal to zero, so we have

$$\left(m_{11}m_{22}-m_{12}^2\right)\lambda^2\omega^4+\left(2P_{12}m_{12}-P_{11}m_{22}-P_{22}m_{11}\right)\lambda\omega^2+\left(P_{11}P_{22}-P_{12}^2\right)=0 \tag{23}$$

4. Numerical results and discussions

Through this section, numerical results are given for analytical solutions provided in the preceding sections. For all computations, the Poisson's ratio ν is taken as 0.3. However, for calculation carried out using TBT, the shear correction factor is taken as 5/6. The length of nanobeam L is supposed to be 10 nm. Calculations are executed considering the non-dimensional form of deflection and natural frequencies as follows

$$\bar{w}=100w\frac{EI}{q_0L^4} \quad \text{for uniform load;}$$

$$\bar{\omega}=\omega L^2\sqrt{\frac{I_0}{EI}} \quad \text{frequency parameter.}$$

Table 1 displays the nondimensional maximum deflections $\bar{w}$ of a simply supported nano beam using different beam theories for different values of the length-to-thickness ratio L/h and nonlocal coefficient $\boldsymbol{\mu}$. The calculated values are obtained using 100 terms in series in Eqs. (17) and (18). Timoshenko beam theory (TBT), Reddy's beam theory (RBT), hyperbolic beam theory (HBT) of Berrabah *et al*. (2013), and Sinusoidal (ε_z = 0) of Tounsi *et al*. (2013a) and the theory developed by Tounsi, Benguediab *et al*. (2013a). It can be observed that the results of the proposed nonlocal hyperbolic shear deformation theory are in excellent agreement with those predicted by TBT and RBT and the other higher order shear deformation beam model for all values of thickness ratio L/h and scale parameter μ. An increase in the values the nonlocal scale parameter yields to an increment in the deflection at a specified slenderness ratio, and that for all theories showed in table 1, because the nonlocal parameter tends to soften the nanobeam. EBT underestimates deflections for minor thickness ratios which corresponds to moderately thick beam (L/h = 10), where the discrepancy between EBT and other theories is very noticeable.

The difference between EBT and shear deformation theories is negligible for thin nanobeams and considerable for thick nanobeams. This is due to the fact that the EBT is unable to take into account the effects of transverse shear deformation.

Table 2 and 3 exhibit the nondimensional frequency $\bar{\omega}$ of a simply supported (S-S) nanobeam for different values of thickness ratio L/h and scale parameter. It can be seen that the obtained results agree very well with those predicted by TBT and RBT and those of Berrabah, and of Tounsi *et al*. (2013a) (ε_z = 0), for various values of thickness ratio L/h and nonlocal parameter μ. Furthermore, the effect of transverse shear deformation on the fundamental frequency is crucial as the value of L/h becomes low. The first three fundamental frequencies frequencies are presented in Table 3 for various values of the nonlocal parameter μ, thickness ratio L/h and for different theories. It can be concluded from the table that by decreasing slenderness ratio, the frequencies of the nanobeam increase. It is also observed that as the nonlocal parameter rises the fundamental

Table 1 Non-dimensional deflection $\overline{w}$ under uniform load for simply supported nanobeams

L/*h*	μ (nm^2)	EBT	TBT	RBT	Berrabah *et al*. (2013)	Tounsi *et al*. (2013a)	Present
5	0	1.3021	1.4321	1.4320	1.4320	1.4317	1.4318
	1	1.4271	1.5674	1.5673	1.5674	1.5617	1.5672
	2	1.5521	1.7028	1.7027	1.7028	1.7025	1.7025
	3	1.6771	1.8381	1.8381	1.8382	1.8379	1.8379
	4	1.8021	1.9734	1.9735	1.9736	1.9733	1.9733
10	0	1.3021	1.3346	1.3346	1.3346	1.3345	1.3345
	1	1.4271	1.4622	1.4622	1.4622	1.4621	1.4621
	2	1.5521	1.5898	1.5898	1.5898	1.5897	1.5897
	3	1.6771	1.7173	1.7174	1.7174	1.7173	1.7172
	4	1.8021	1.8489	1.8450	1.8450	1.8449	1.8448
20	0	1.3021	1.3102	1.3102	1.3102	1.3102	1.3102
	1	1.4271	1.4359	1.4359	1.4359	1.4358	1.4358
	2	1.5521	1.5615	1.5615	1.5615	1.5615	1.5614
	3	1.6771	1.6871	1.6872	1.6872	1.6871	1.6871
	4	1.8021	1.8128	1.8128	1.8128	1.8128	1.8127
100	0	1.3021	1.3024	1.3024	1.3024	1.3024	1.3024
	1	1.4271	1.4274	1.4274	1.4274	1.4274	1.4274
	2	1.5521	1.5525	1.5525	1.5525	1.5525	1.5524
	3	1.6771	1.6775	1.6775	1.6775	1.6775	1.6774
	4	1.8021	1.8025	1.8025	1.8025	1.8025	1.8025

frequencies decrease. Moreover, it is seen that the dissimilarity between EBT and the other shear deformation theories (i.e., TBT, RBT, HBT, SBT and present theory) is negligible for slender nanosized beams and more significant for higher values of the vibrational mode number n.

It can be also observed from Tables 1-3 that the local theory underestimates the static deflections and overestimates the natural frequencies of the nanobeams compared to the nonlocal theory, and the difference between local and nonlocal theories is most important for large value of the scale factor. This is because that the local theory fails to capture the small scale effect of the nanobeams. Also, the inclusion of the shear deformation and nonlocal impacts grows the deflections and decreases the natural frequencies in particular at elevated modes (as shown in Table 3).

Fig. 2 displays the variation of bending and vibration responses of nanobeam versus slenderness ratio (L/h) and μ = 1 nm. In this case, the deflection / frequency ratios (Fig. 2) are obtained as the ratios of $(\overline{w}, \overline{\omega})$ calculated by the present model to the correspondences obtained by EBT where the transverse shear deformation effect is omitted. It can be deduced that, the inclusion of shear deformation effect makes the beam more flexible, and hence, leads to an increase of deflections, and a reduction of the natural frequencies, and this effect can be clearly observed when there are elevated vibration modes (as shown in Fig. 3). This is attributed to the shear deformation effect, which leads in a reduction of the beam stiffness.

Table 2 Non-dimensional fundamental frequency $\overline{\omega}$ for simply supported nanobeams

L/h	μ (nm^2)	EBT	TBT	RBT	Berrabah *et al*. (2013)	Tounsi *et al*. (2013a)	Present
5	0	9.7112	9.2740	9.2745	9.2745	9.2752	9.2748
	1	9.2647	8.8477	8.8482	8.8482	8.8488	8.4884
	2	8.8747	8.4752	8.4757	8.4757	8.4763	8.4759
	3	8.5301	8.1461	8.1466	81466	8.1472	8.1468
	4	8.2228	7.8526	7.8530	7.8530	7.8536	7.8533
10	0	9.8293	9.7075	9.7075	9.7075	9.7077	9.7076
	1	9.3774	9.2612	9.2612	9.2612	9.2614	9.2613
	2	8.9826	8.8713	8.8714	8.8713	8.8715	8.8714
	3	8.6338	8.5269	8.5269	8.5269	8.5271	8.5269
	4	8.3228	8.2196	8.2197	8.2196	8.2198	8.2197
20	0	9.8595	9.8281	9.8281	9.8282	9.8282	9.8281
	1	9.4062	9.3763	9.3763	9.3764	9.3764	9.3763
	2	9.0102	8.9816	8.9816	8.9816	8.9816	8.9816
	3	8.6604	8.6328	8.6328	8.6329	8.6329	8.6328
	4	8.3483	8.3218	8.3218	8.3218	8.3218	8.3218
100	0	9.8692	9.8679	9.8679	9.8722	9.8679	9.8679
	1	9.4155	9.4143	9.4143	9.4184	9.4143	9.4142
	2	9.0191	9.0180	9.0180	9.0219	9.0180	9.0179
	3	8.6689	8.6678	8.6678	8.6716	8.6678	8.6678
	4	8.3566	8.3555	8.3555	8.3592	8.3555	8.3555

Table 3 Different first three non-dimensional frequencies $\overline{\omega}$ of simply supported nanobeam (L/h = 5)

L/h	μ (nm^2)	EBT	TBT	RBT	Berrabah *et al*. (2013)	Tounsi *et al*. (2013a)	Present
1	0	9.7112	9.2740	9.2745	9.2746	9.2752	9.2748
	1	9.2647	8.8477	8.8482	8.8483	8.8488	8.4884
	2	8.8747	8.4752	8.4757	8.4758	8.4763	8.4759
	3	8.5301	8.1461	8.1466	81467	8.1472	8.1468
	4	8.2228	7.8526	7.8530	7.8531	7.8536	7.8533
2	0	37.1120	32.1665	32.1847	32.1851	32.1948	32.1854
	1	31.4239	27.2364	27.2519	27.2522	27.2604	27.2525
	2	27.7422	24.0453	24.0589	24.0592	24.0664	24.0594
	3	25.1104	21.7642	21.7765	21.7768	21.7833	21.7770
	4	23.1088	20.0293	20.0407	20.0409	20.0470	20.0411
3	0	78.0234	61.4581	61.5746	61.5718	61.6192	61.5666
	1	56.7798	44.7247	44.8095	44.8075	44.8420	44.8037
	2	46.8246	36.8831	36.9531	36.9514	36.9798	36.9483
	3	40.7568	32.1036	32.1645	32.1631	32.1878	32.1603
	4	9.7112	9.2740	28.8569	28.8556	28.8778	28.8532

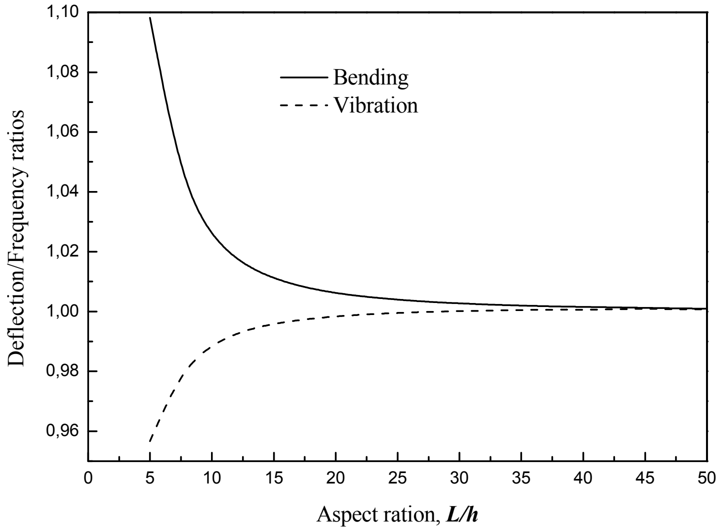

Fig. 2 Effect of transverse shear deformation on the static deflection, and fundamental frequency ratios for (S-S) nanobeam with $\mu = 1$ nm^2

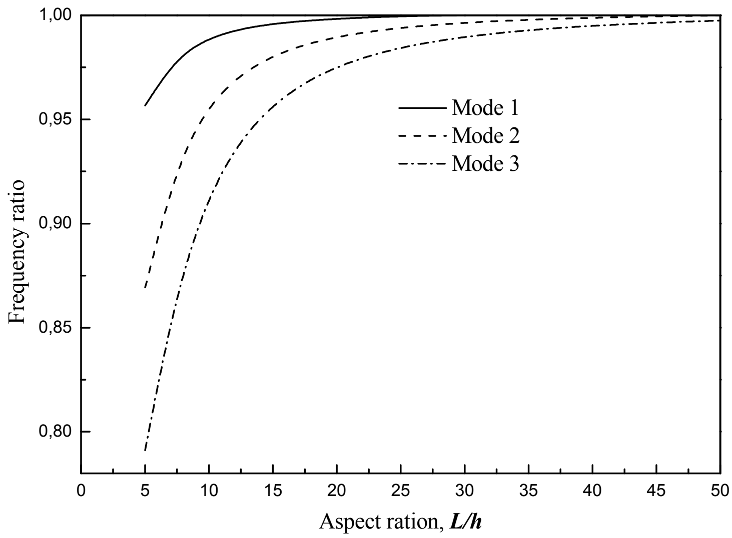

Fig. 3 Impact of the aspect ratio on higher frequency ratios for (S-S) nanobeam with $\mu = 1$ nm^2

Fig. 4 explores the effect of the small scale parameter (e_0a) on the bending and vibration responses of nanobeam using the present new nonlocal higher order shear deformation theory. The deflection and frequency ratios are defined as mentioned in previous section above. It can be seen that the deflection ratio is bigger than unity, whereas the frequency ratios are less important than unity. This is caused by the local theory, which underestimates the deflections and overestimates the natural frequencies of the nanosize beams compared to the nonlocal one. Also it is attributed to the local elasticity theory, which fails to determine the small scale effect of the nanobeams. Moreover, it is seen that the variation among the classical local and nonlocal theories is especially

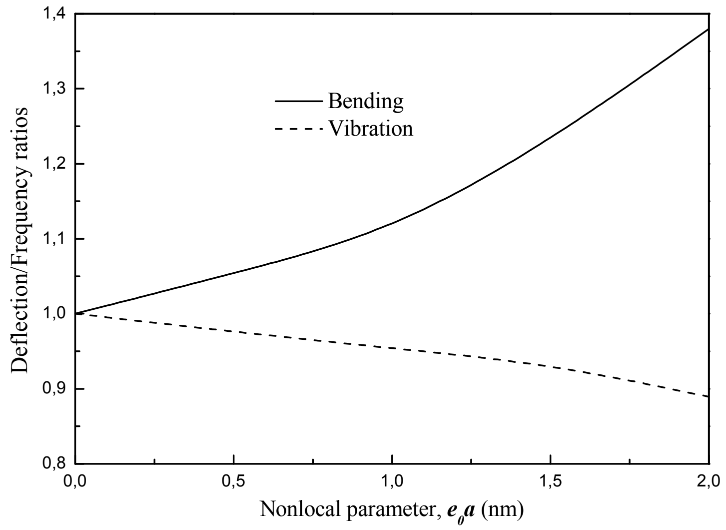

Fig. 4 Impact of small scale on the deflection, and fundamental frequency ratios for (S-S) a simply supported nanobeam with L/h = 10

important for the elevated modes (see Fig. 4).

5. Conclusions

A refined nonlocal higher-order hyperbolic shear deformation nanobeam theory is presented in this paper for the study bending, and free vibration of nanobeams. The present model accounts for hyperbolic variation of transverse shear stress, and takes into account the traction-free boundary conditions on the upper and base surfaces of the nanosized beam and no need to use a shear correction factor (SFCs). On the basis of the nonlocal differential constitutive relation of Eringen, the governing nonlocal differential equations of the proposed theory are derived by implementing Hamilton's principle. Navier type solution for static deflection and natural frequency are provided for a simply supported nanobeam, and the accuracy of the obtained results are in well agreement with those predicted by the RBT and TBT. Numerical results show that the nonlocal effects play an important role on the bending and vibration responses of the nanobeam. Therefore, the nonlocality effects should be considered in the investigation of mechanical behaviors of nanobeams..

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